Mechanisms of enhancement of many-body delocalization

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Collaboration:

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arXiv:1611.02681
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QCTMBS17, MPIPKS, Dresden

Outline

1. Enhancement of many-body delocalization by spectral diffusion

2. Many-body delocalization in continuum

The Problem

$D \leq 2:$ **ANDERSON LOCALIZATION**

$+$ **ELECTRON-ELECTRON INTERACTION**

$+$ **vanishing coupling to the external world (phonons, etc.)**

temperature $T \neq 0:$ **conductivity** $\sigma(T) \; = ?$

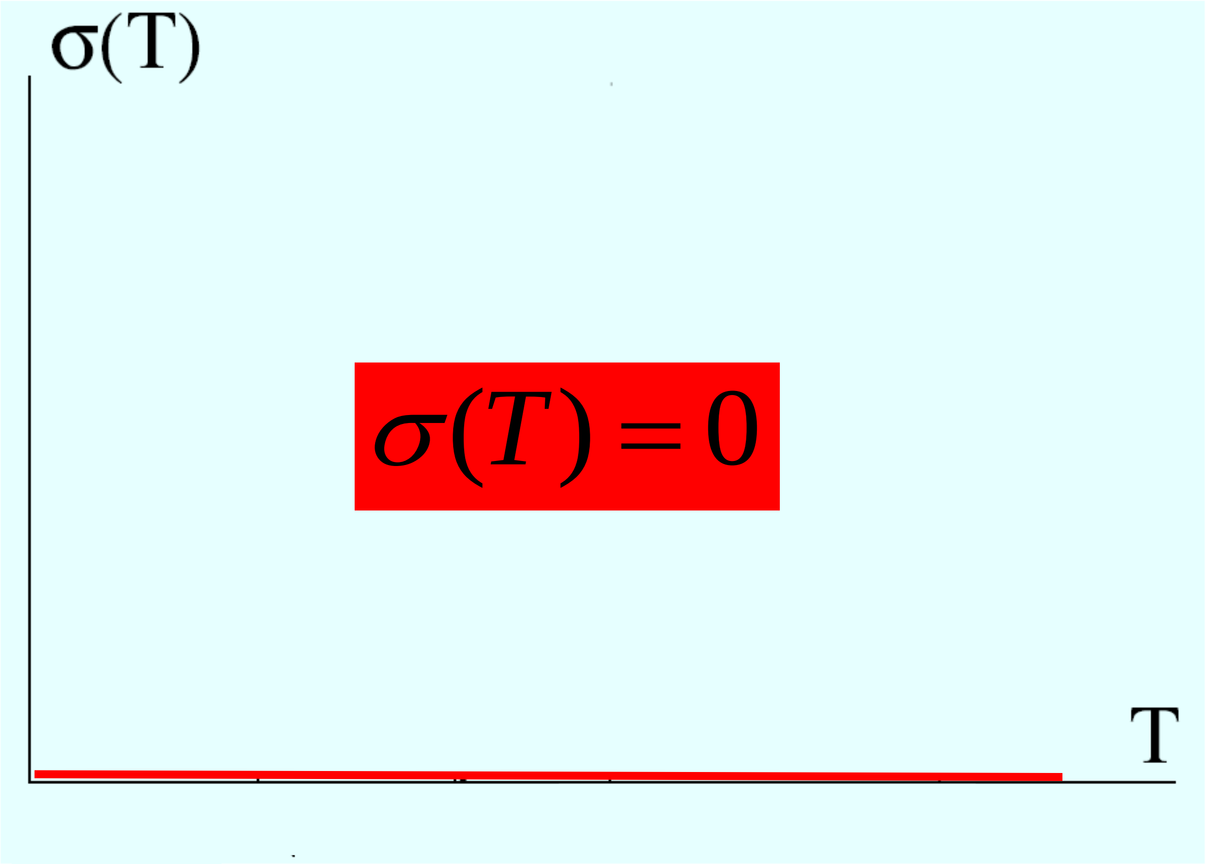
Why interesting ? **No interaction** $\Longrightarrow \sigma(T) \equiv 0$ **for any** T

T: distribution function over localized states

$\sigma(T)$ is (possibly) nonzero due to e-e interaction only !

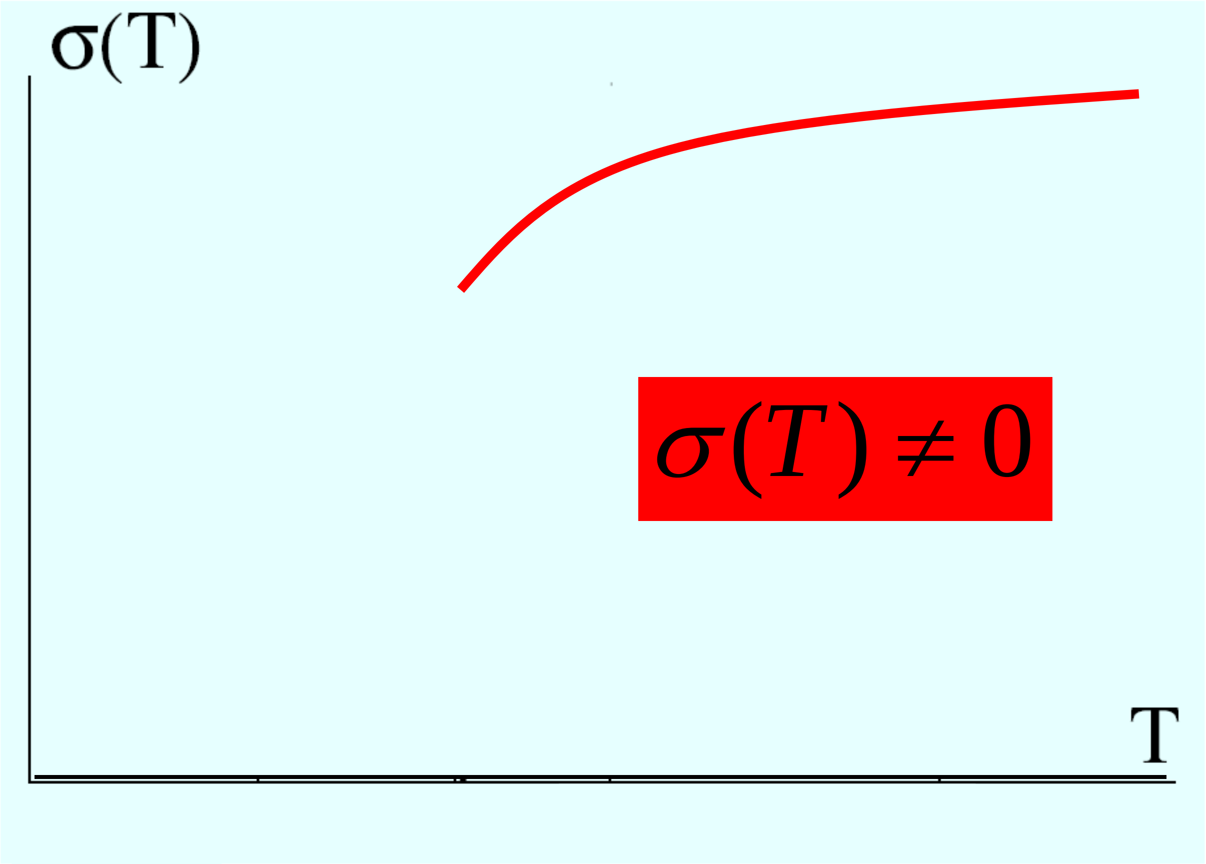
slide from 2004

T-dependence of conductivity (1958, 1980)



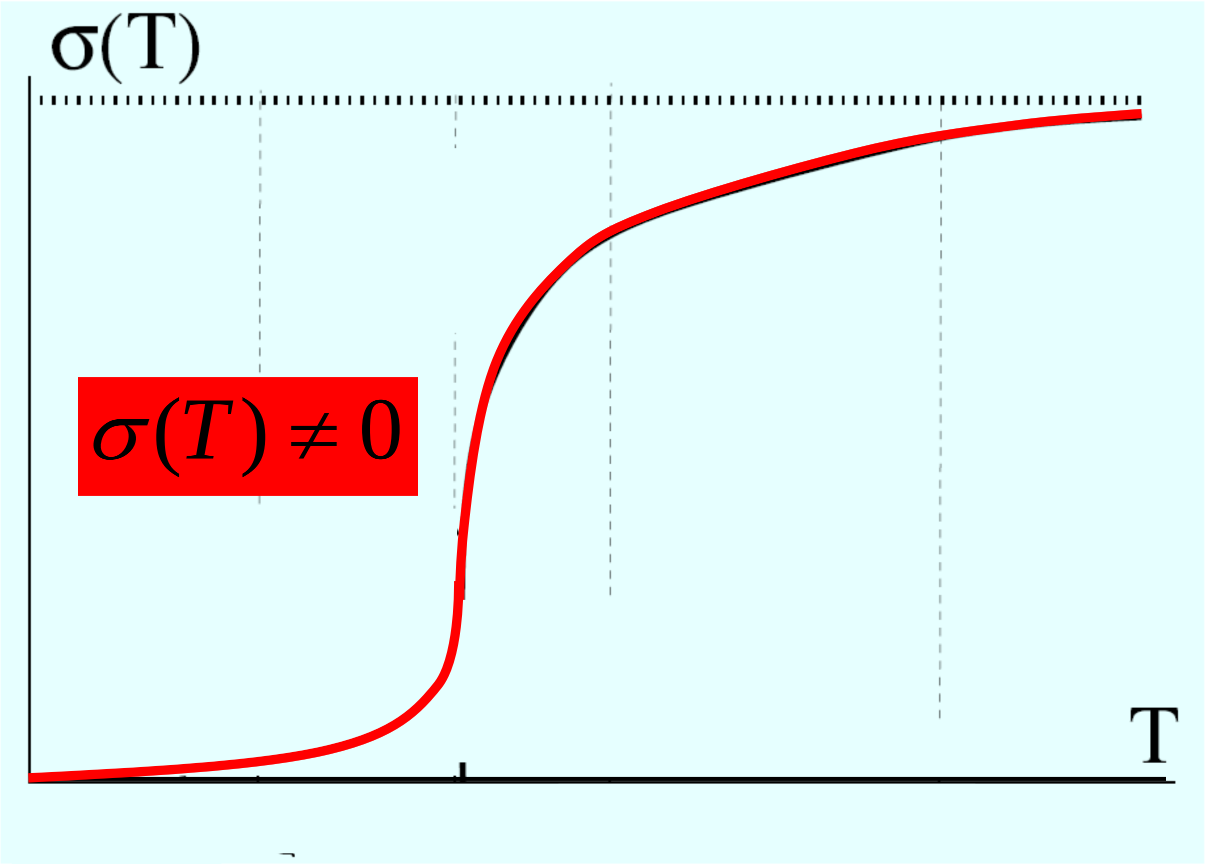
No interaction: Anderson ’58; short-range interactions: Fleishman & Anderson ‘80

T-dependence of conductivity (1982)



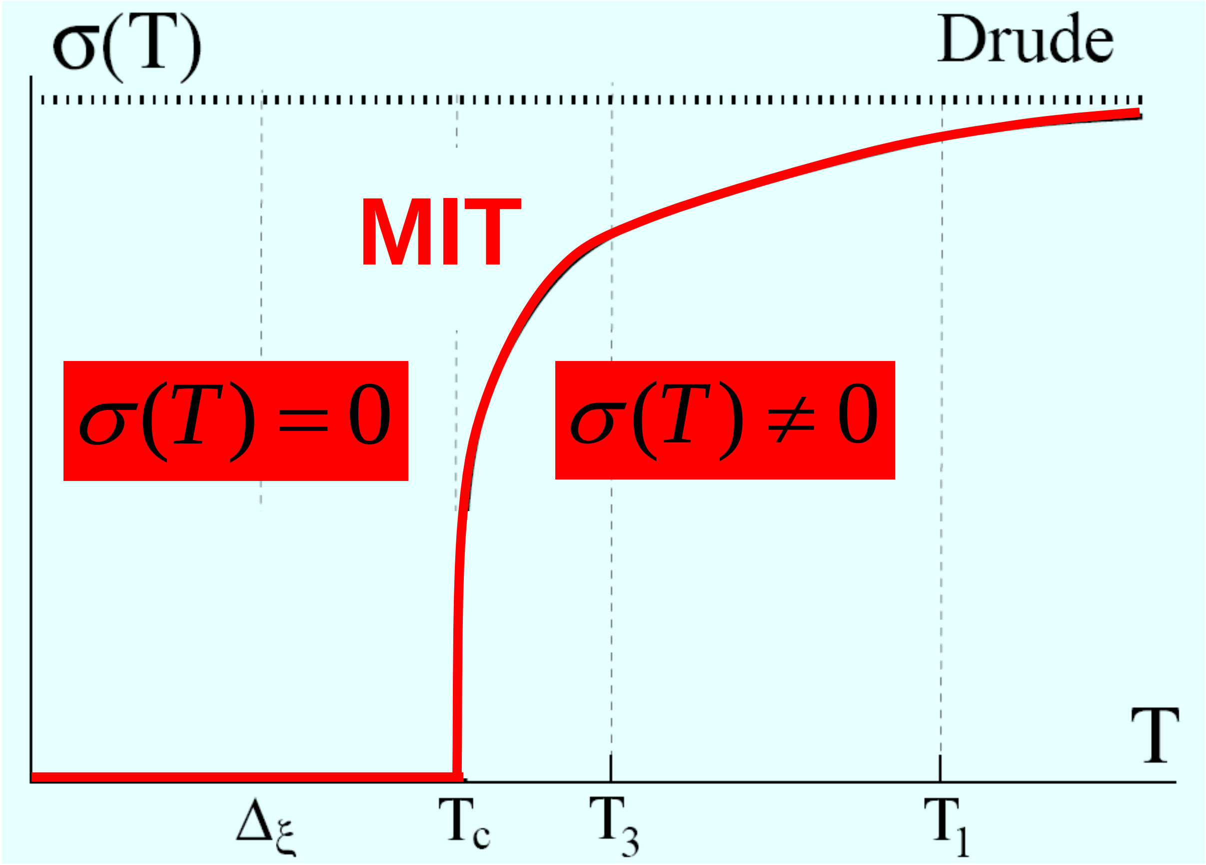
Dephasing of weak localization. Altshuler, Aronov, Khmelnitskii '82

T-dependence of conductivity (2004)



IVG, Mirlin, Polyakov, arXiv:cond-mat/0407305v1
Localization in Fock space (Altshuler, Gefen, Kamenev, Levitov ’97), but error in counting terms (“n! problem”) – “too classical”, as pointed out by BAA

T-dependence of conductivity (2005)



GMP‘05 (~GMP‘04 with corrected factorial)
Basko, Aleiner, Altshuler ‘05

$$T_c \sim \frac{\Delta_\xi}{\alpha \ln(1/\alpha)}$$

What might be missing here?

- **Spectral diffusion**
- **Divergence of single-particle localization length**
- **Rare events**
- **Long-range Coulomb interaction**

Spectral diffusion

J.R. Klauder, P.W. Anderson (1962); J. L. Black, B. I. Halperin (1977);
Yu.M. Gal'perin, V.L. Gurevich, D.A. Parshin (1983,1988);
A.L. Burin, K.N. Kontor, L.A. Maksimov (1990); A.L. Burin, Yu. Kagan (1995)
(spin resonance, relaxation in glasses)

transitions in interacting many-body systems affect resonant conditions for other transitions – “true many-body” relaxation mechanism

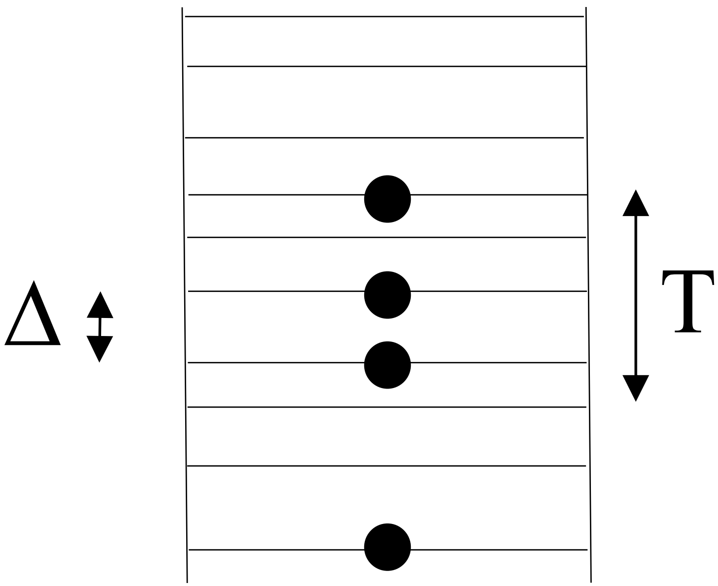
delocalization for power-law interactions:
A.L. Burin (2015); D.B. Gutman *et al.* (2016)

This talk: parametric enhancement of many-body delocalization by spectral diffusion for short-range interactions

“Anderson vs. Anderson” (Fleishman & Anderson ’80 vs. Klauder & Anderson ‘62)

Relaxation in a quantum dot

B.L. Altshuler, Y. Gefen, A. Kamenev, L.S. Levitov (1997)



diffusive quantum dot, conductance: $g \gg 1$

mean single-particle level spacing: Δ

Thouless energy: $E_T = g\,\Delta$

random interaction, r.m.s. $V \sim \Delta / g$

(within energy band of width E_T)

typical many-body states with energy E :

$N = \sqrt{E / \Delta}$ excited particles

"effective temperature" $\sqrt{E\Delta} = T$

Localization in Fock space

B.L. Altshuler, Y. Gefen, A. Kamenev, L.S. Levitov (1997)

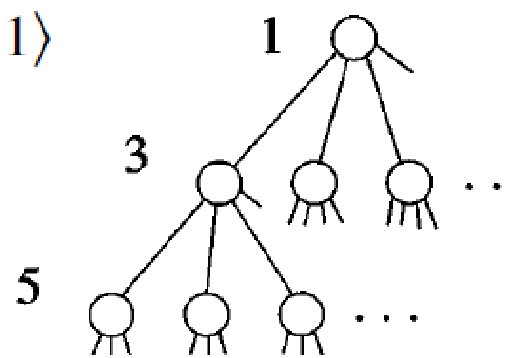
$$\mathcal{H}_0 + \mathcal{H}_1 = \sum_{\alpha} \epsilon_\alpha c_\alpha^\dagger c_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\gamma\delta}^{\alpha\beta} c_\gamma^\dagger c_\delta^\dagger c_\beta c_\alpha$$

Mapping onto localization problem for hopping over graph in Fock space:

$$\Upsilon^{\alpha} = c_\alpha^\dagger |N-1\rangle$$

$$\Upsilon_{\gamma}^{\alpha\beta} = c_\alpha^\dagger c_\beta^\dagger c_\gamma |N-1\rangle$$

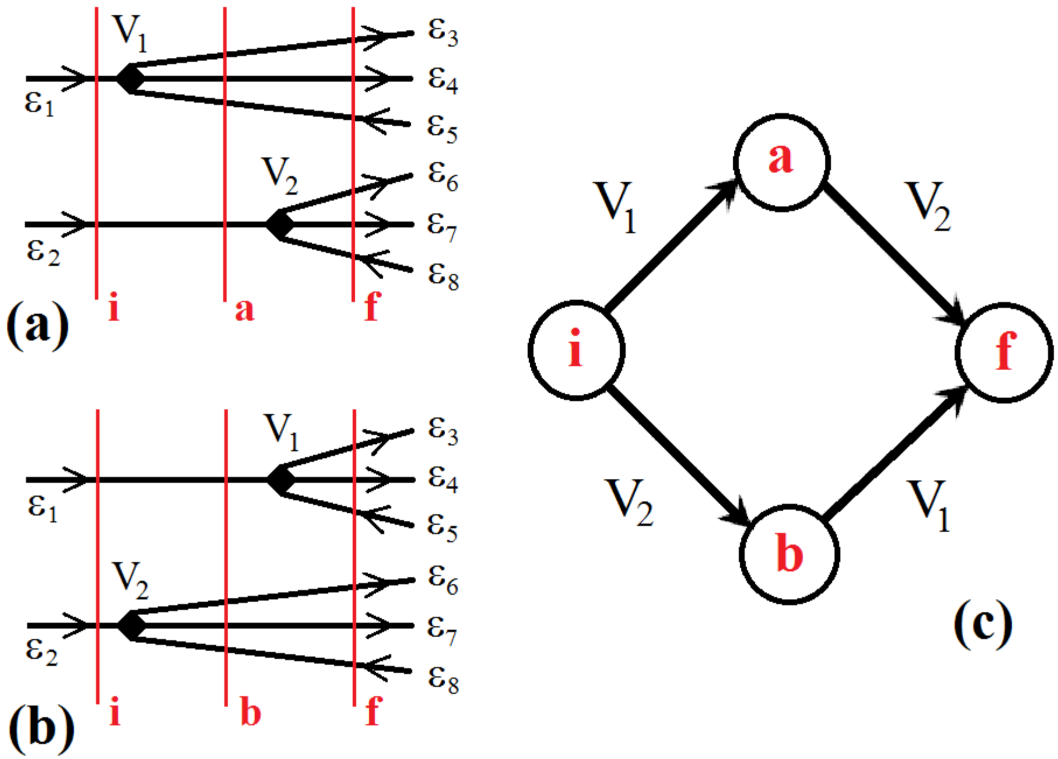
$$\Upsilon_{\lambda\mu}^{\alpha\beta\gamma} = c_\alpha^\dagger c_\beta^\dagger c_\gamma^\dagger c_\lambda c_\mu |N-1\rangle$$



Interactions → hybridization of Fock-space basis states (Slater determinants)
→ noninteracting single-particle problem on Fock-space graph
(node = basis state, hopping = interaction)

Anderson transition on graph ➔ Localization-delocalization in Fock space

Many-body interference



Silvestrov (1997);
Basko, Aleiner, Altshuler (2005);
Ros, Mueller, Scardicchio (2014);
IVG, Mirlin, Polyakov (2016)

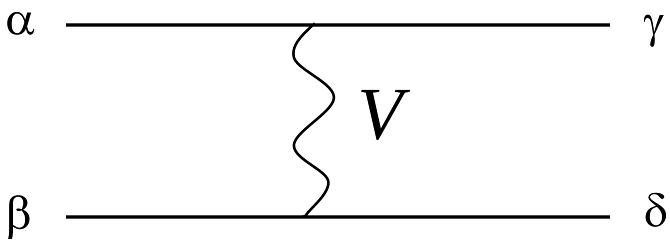
No spectral diffusion:
Loops in Fock space
→ destructive interference

Decay of n-particle state: elementary interactions → n! ways to be time-ordered
→ n! different paths in Fock space

but not statistically independent: correlated amplitudes

$$\boxed{\sum_{\text{permutations}} \frac{1}{\mathcal{E}_1(\mathcal{E}_1+\mathcal{E}_2)\ldots(\mathcal{E}_1+\mathcal{E}_2+\ldots+\mathcal{E}_n)} = \frac{1}{\mathcal{E}_1\mathcal{E}_2\ldots\mathcal{E}_n}}$$

Counting resonances



elementary resonance:

$$|\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta| \lesssim V$$

level spacing of all basis states with energy E to which a given basis state is connected:

$$\Delta^{(N)} \sim \frac{\Delta}{N^3} \sim \Delta \left(\frac{\Delta}{E}\right)^{3/2}$$

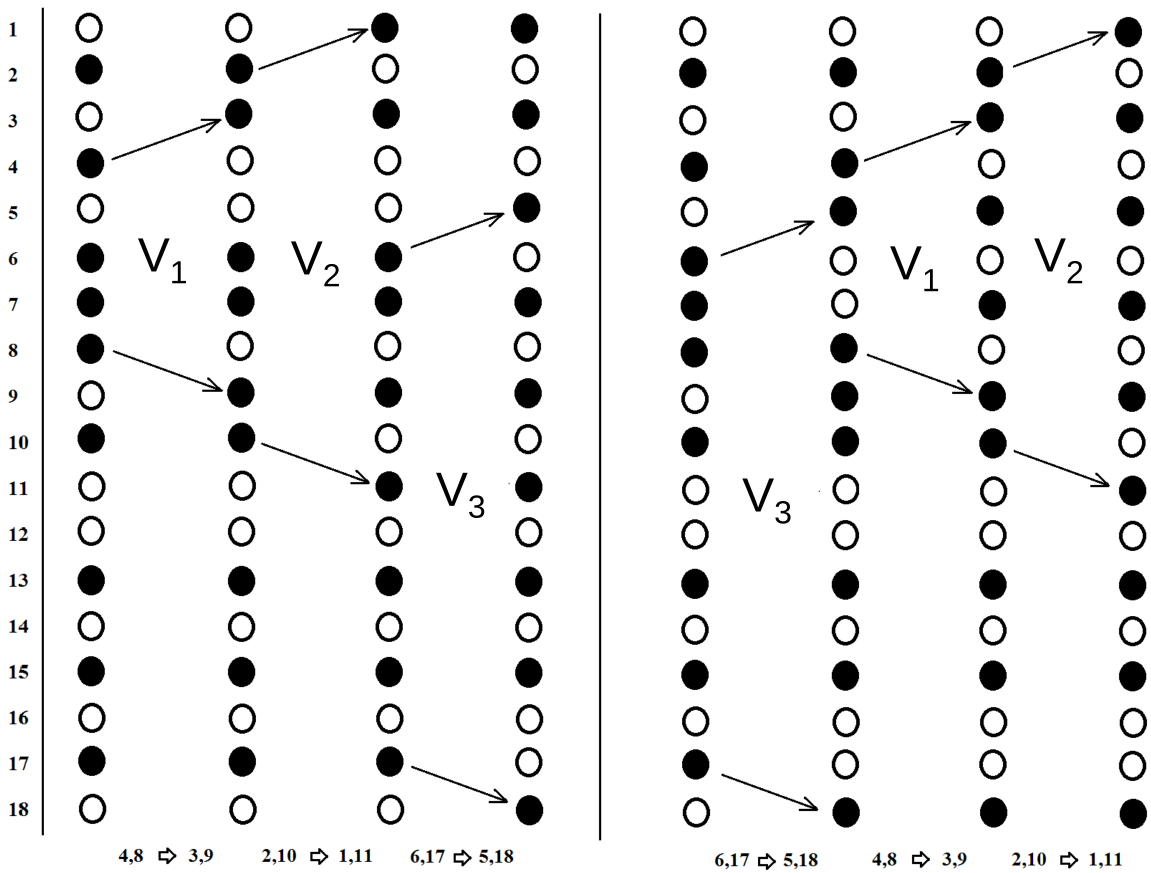
number of resonances for N-particle state:

$$p \sim \frac{V}{\Delta^{(N)}} \sim \frac{N^3}{g}$$

no spectral diffusion (no energy shifts) ➔ conservative estimate:

$$p = N \quad \Longrightarrow \quad E \sim g\Delta$$

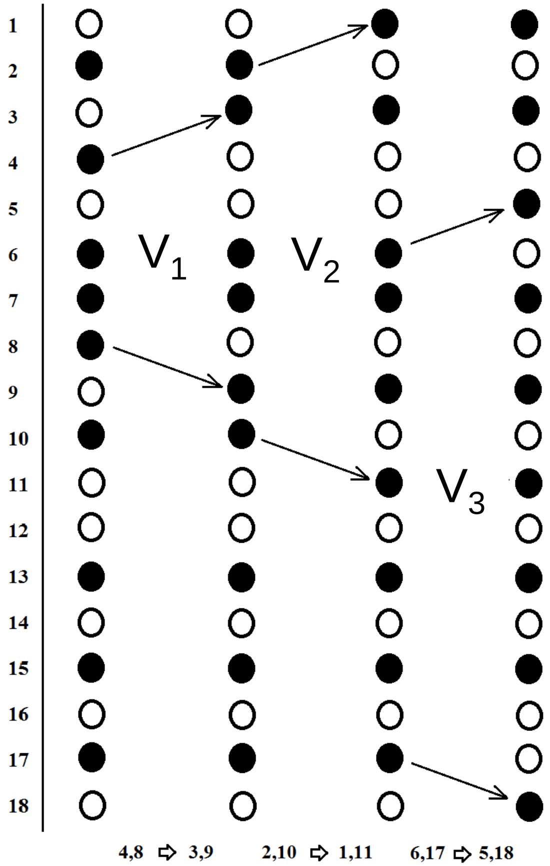
Quantum dot: Counting resonances

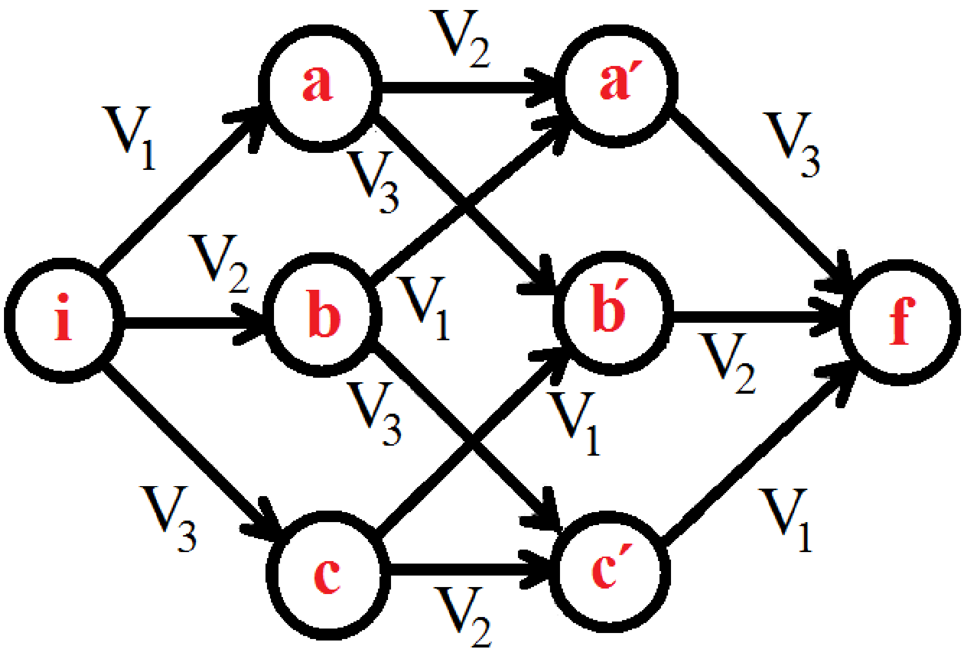


+ other permutations

$$N = 6, \quad p = 3$$

Quantum dot: Counting resonances





$$N = 6, \quad p = 3$$

Quantum dot with spectral diffusion

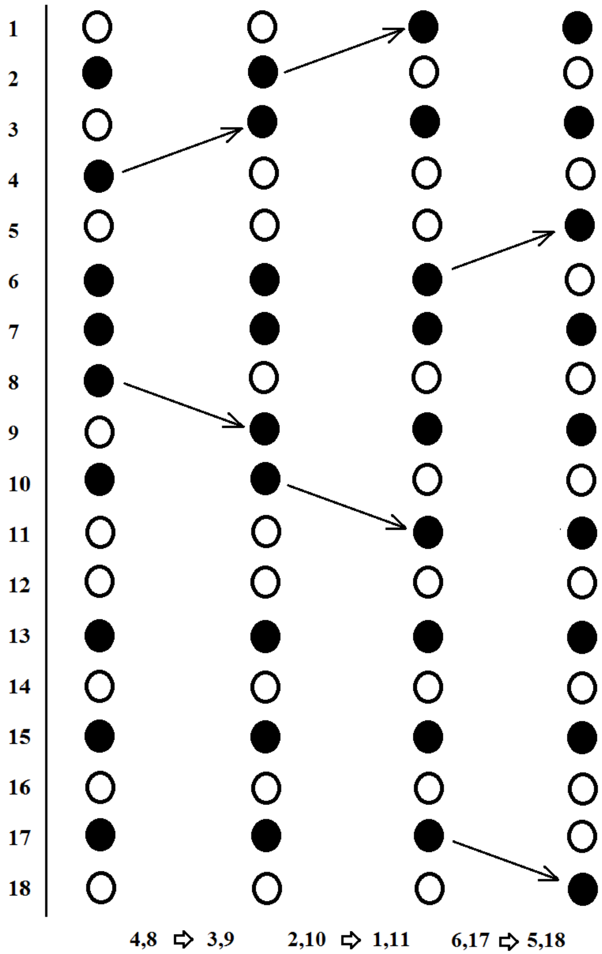
$$H = \sum_\alpha \epsilon_\alpha c^\dagger_\alpha c_\alpha + \frac{1}{2} \sum V_{\alpha\beta\gamma\delta} c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta$$

diagonal matrix elements of interaction do not connect sites on Fock graph

$$V_{\alpha\beta\beta\alpha} = -V_{\alpha\beta\alpha\beta}$$

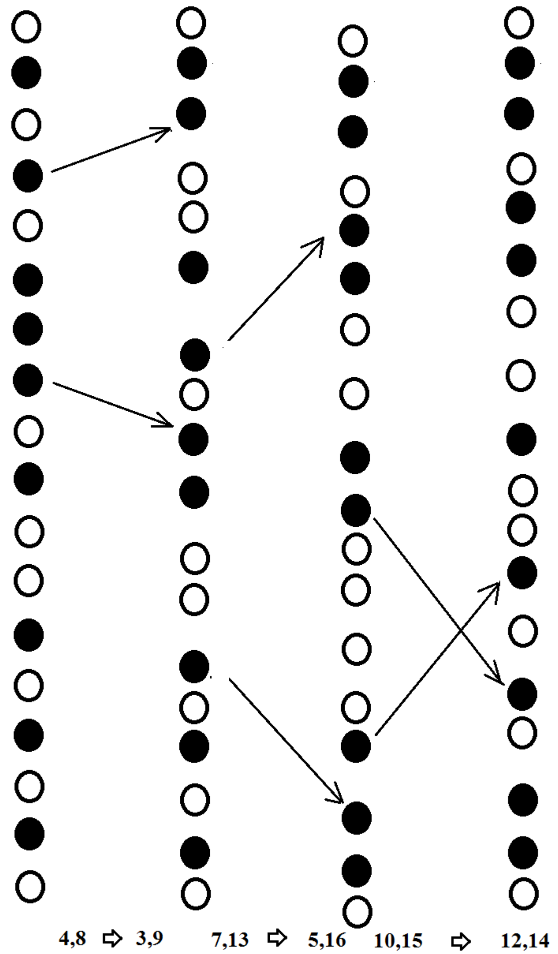
Energies of sites on Fock graph (basis states) include interactions between all single-particle states forming given many-body state

Without and with spectral diffusion



without spectral diffusion

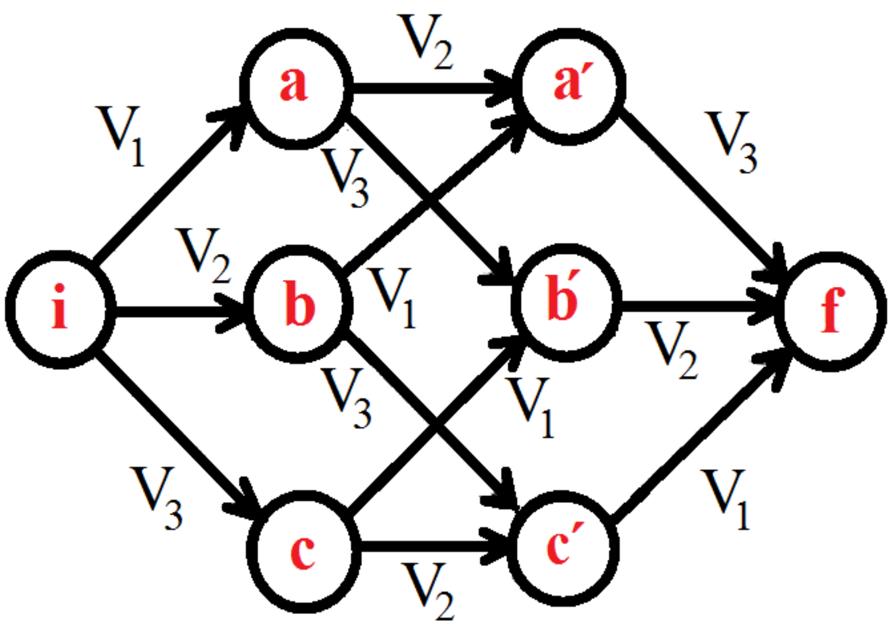
$$E \sim g\Delta$$



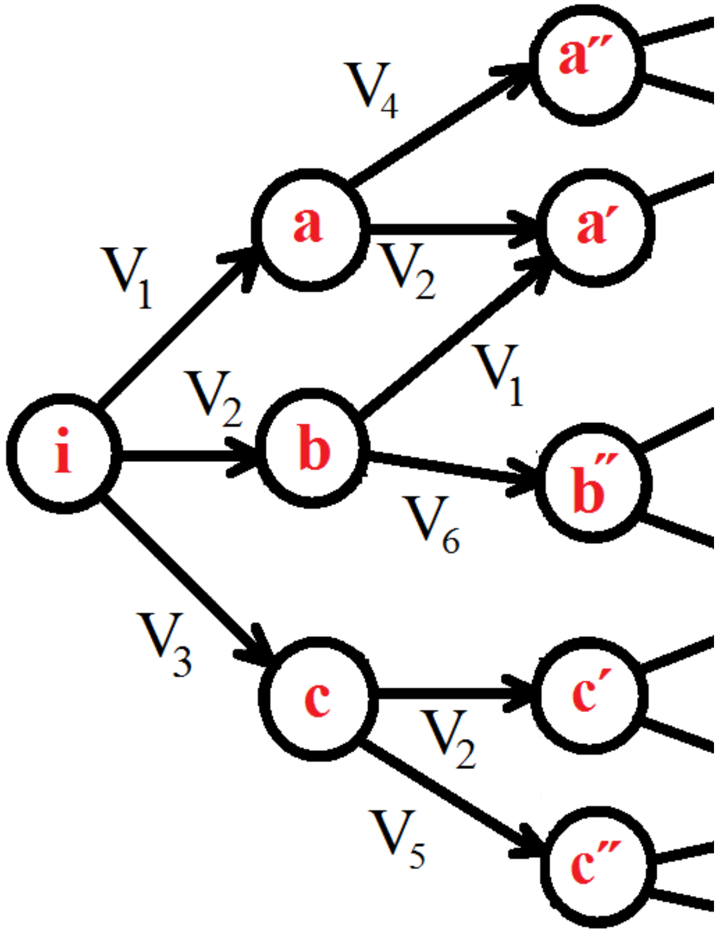
with spectral diffusion

$$E \sim g^{4/5}\Delta$$

Quantum dot: Structure of resonant graph

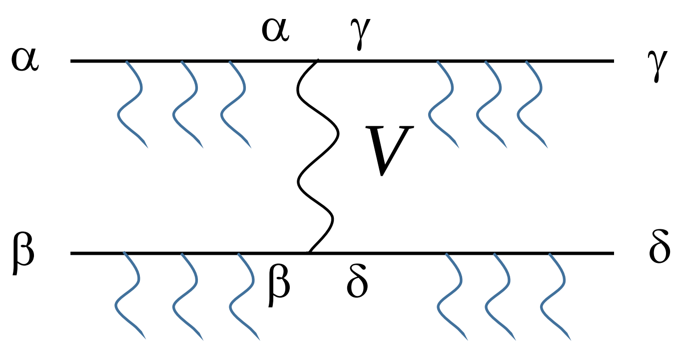


without spectral diffusion:
finite cluster of resonant nodes



with spectral diffusion:
very similar to Bethe lattice

Reinitialization of resonances



elementary resonance:

$$|\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta| \lesssim V$$

number of resonances for N-particle state: $$p \sim \frac{V}{\Delta^{(N)}} \sim \frac{N^3}{g}$$

spectral diffusion (diagonal interaction): each step (two electrons change states)

➔ all other single-particle energies are shifted by random amount $V \sim \Delta / g$

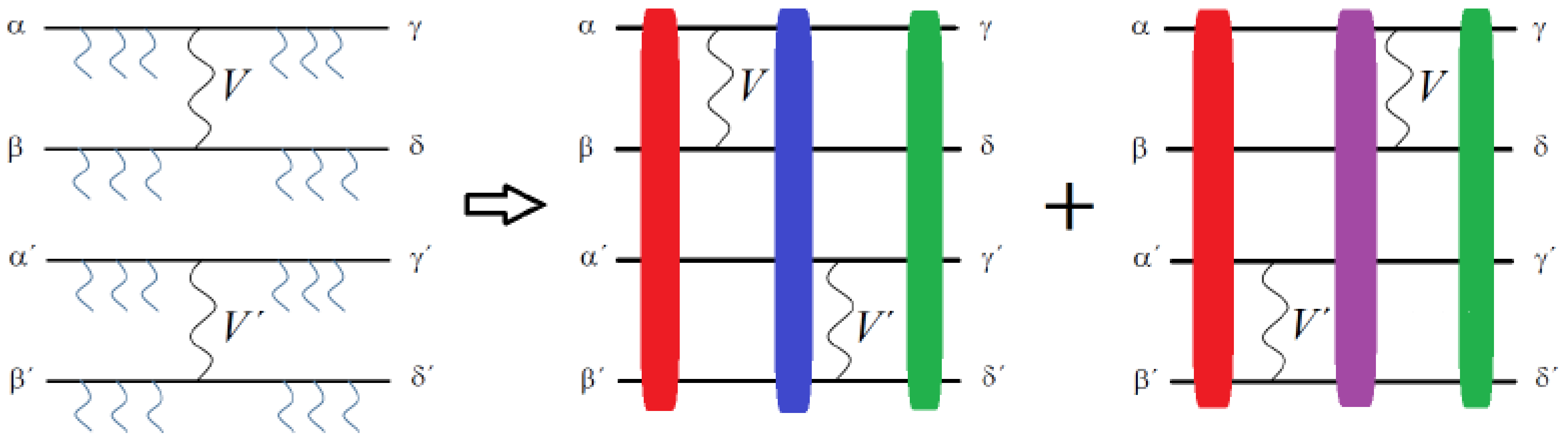
➔ ~ $p/2$ original resonances are replaced by new resonances ("reinitialization" of resonances at each step)

after m steps, pairs with original mismatch $|\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta| \sim m^{1/2} V$ become resonant; hybridization proceeds until

$$m\Delta^{(N)} \sim m^{1/2} V \quad \Rightarrow \quad m \sim p^2$$

$$m = N \quad \Rightarrow \quad \boxed{N \sim g^{2/5}, \qquad E \sim g^{4/5}\Delta}$$

Quantum dot: Higher-order processes



Generalization to *k* nonresonant pairs:

$$V^{(k)} = V_k \sum_{\{t\}_k} \prod_{i=1}^{k-1} \frac{V_i}{\sum_{j=1}^{i} \tilde{\epsilon}_{t_j} + \sum_{j'<j\leq i} \tilde{U}_{t_j t_{j'}}}$$

Quantum dot: Higher-order processes

with diagonal matrix elements (spectral diffusion):

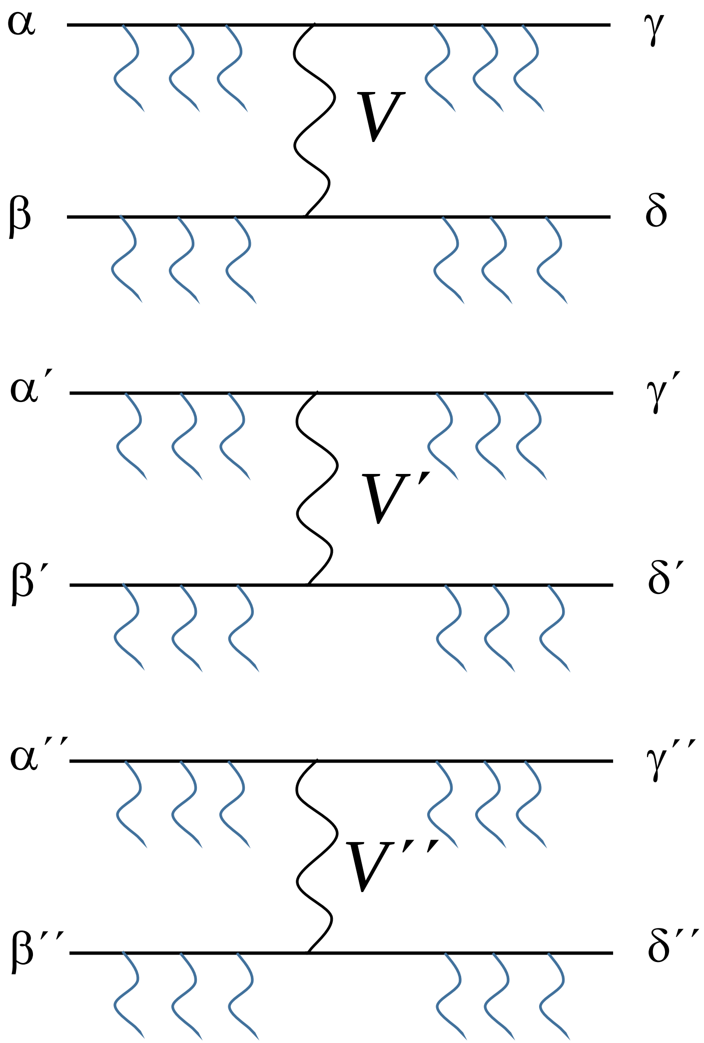
$$\eta_3 = \frac{V^{(3)}}{\epsilon^{(3)}} = \frac{V_1 V_2 V_3}{\tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{U}_{12} + \tilde{U}_{13} + \tilde{U}_{23}} \left[\frac{1}{\tilde{\epsilon}_1(\tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{U}_{12})} + \frac{1}{\tilde{\epsilon}_1(\tilde{\epsilon}_1 + \tilde{\epsilon}_3 + \tilde{U}_{13})} \right.$$
$$\left. + \frac{1}{\tilde{\epsilon}_2(\tilde{\epsilon}_1 + \tilde{\epsilon}_2 + \tilde{U}_{12})} + \frac{1}{\tilde{\epsilon}_2(\tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{U}_{23})} + \frac{1}{\tilde{\epsilon}_3(\tilde{\epsilon}_1 + \tilde{\epsilon}_3 + \tilde{U}_{13})} + \frac{1}{\tilde{\epsilon}_3(\tilde{\epsilon}_2 + \tilde{\epsilon}_3 + \tilde{U}_{23})} \right]$$

without diagonal matrix elements (no spectral diffusion):

$$\eta_3 = \frac{V_1 V_2 V_3}{\epsilon_1 + \epsilon_2 + \epsilon_3} \left[\frac{1}{\epsilon_1(\epsilon_1 + \epsilon_2)} + \frac{1}{\epsilon_1(\epsilon_1 + \epsilon_3)} + \frac{1}{\epsilon_2(\epsilon_1 + \epsilon_2)} \right.$$
$$\left. + \frac{1}{\epsilon_2(\epsilon_2 + \epsilon_3)} + \frac{1}{\epsilon_3(\epsilon_1 + \epsilon_3)} + \frac{1}{\epsilon_3(\epsilon_2 + \epsilon_3)} \right] = \frac{V_1 V_2 V_3}{\epsilon_1 \epsilon_2 \epsilon_3}$$

Quantum dot: Higher-order processes

Generalization to k nonresonant pairs:



$$N_c^{(k)} \sim g^{2k^2/(6k^2-1)}$$

Many-body delocalization threshold at

$$N \sim g^{1/3}, \text{ or, equivalently, } E \sim g^{2/3}\Delta$$

Single resonance (p~1) is sufficient

("Aberg criterion")

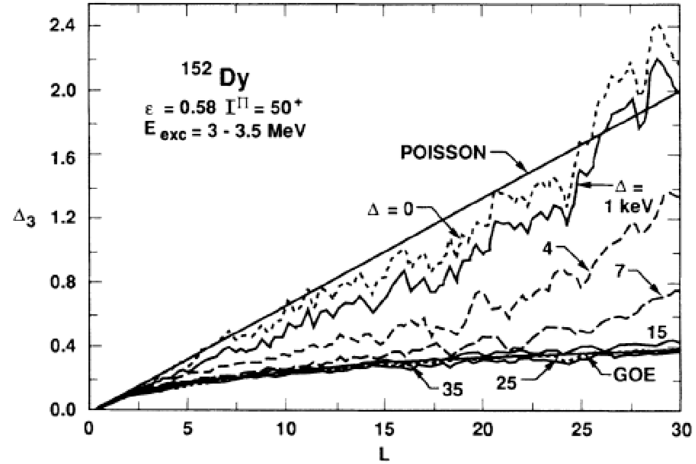
Onset of quantum chaos in nuclei

Aberg '90, '92

Two-body random interaction model,

highly excited states;

level statistics: crossover from Poisson to Wigner-Dyson



Criterion conjectured on the basis of numerics:

By comparing these results to the average level distances shown in fig. 6 we conclude that chaos seems to set in when the average size of the two-body matrix element is

$$\Delta \approx \left(\tfrac{1}{2} - \tfrac{1}{3}\right) \bar{d}_{2p2h} \,. \tag{22}$$

Δ – interaction matrix element,

d_{2p2h} – level spacing of Fock-space basis states directly connected to the given one by interaction

Extended systems

Inclusion of diagonal matrix elements ➔ spectral diffusion
➔ enhancement of delocalization

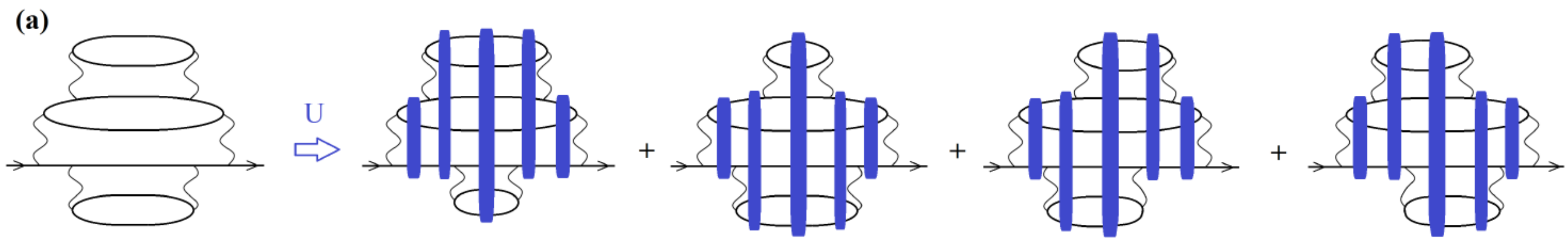
$$T_c = \frac{\Delta_\xi}{\alpha \ln \alpha^{-1}} \quad \longrightarrow \quad T_c = \frac{\Delta_\xi}{\alpha^{1/2} \ln^{\mu} \alpha^{-1}} \,, \quad 0 \leq \mu \leq 1/2$$

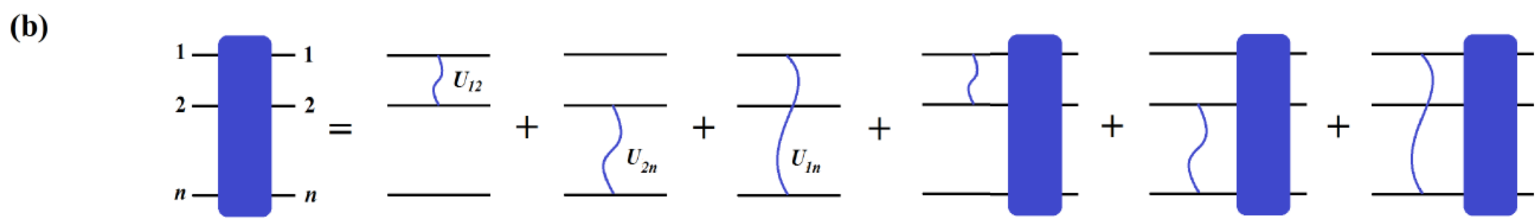
without spectral diffusion (2005) | with spectral diffusion

Higher-order processes important at

$$T_c < T \ll \Delta_\xi / \alpha^{2/3}$$

Diagrams and factorials





Quantum dot:

$$\eta_k \sim \left(\frac{N^2}{g}\right)^k k! \sim \left(\frac{N^2 k}{g}\right)^k \overset{k=N}{\rightarrow} \left(\frac{N^3}{g}\right)^N$$

Extended system (M “quantum dots”): $k \; = \; MN$ $\qquad (N!)^M \sim N^{NM} = N^k$

$$\eta_k \sim \left(\frac{V}{\Delta_3(T)}\right)^k (N!)^M \sim \left(\frac{\alpha T N}{\Delta_\xi}\right)^k \sim \left(\alpha \frac{T^2}{\Delta_\xi^2}\right)^k$$

Conclusions I

1. Spectral diffusion (diagonal matrix elements)

   ➔ enhancement of many-body delocalization

3. Relaxation in a quantum dot revisited:

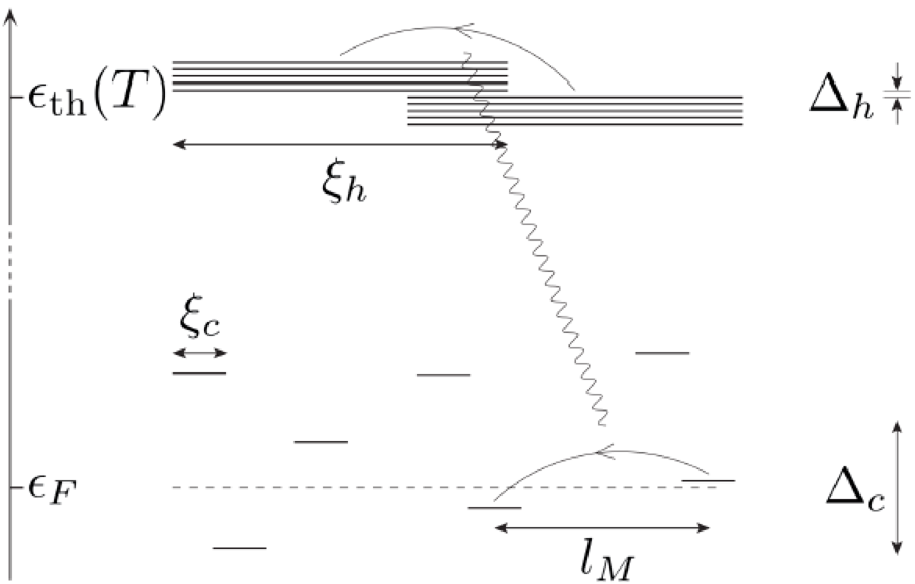
   Localization threshold at $E_c \sim g^{2/3}\Delta$

4. Extended systems with localized single-particle states:

   Many-body delocalization down to

$$T_c \sim \Delta_\xi / \alpha^{1/2}$$

Absence of MBL in continuum



IVG, Mirlin, Müller, Polyakov,
Ann. der Phys. 2017, 1600365

Localization length $\xi(\epsilon)$ increases with energy ϵ

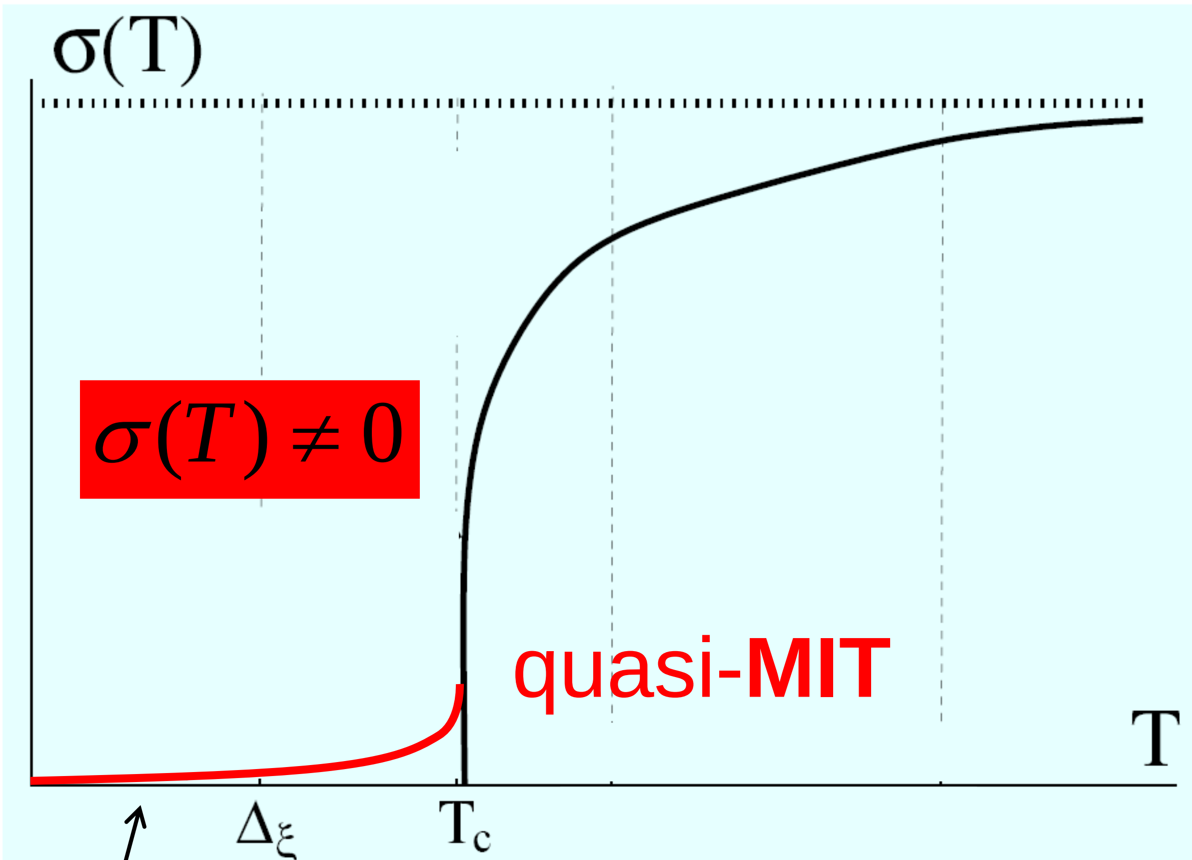
$\longrightarrow$ **delocalization of high-energy states**

The system remains conducting down to arbitrarily small $T \neq 0$.

In particular, low-T conductivity in 2D:

$$\ln \sigma(T) \simeq -\frac{1}{2\pi T \tau} \ln \left[\frac{1}{\alpha} \left(\frac{\Delta_c}{T} \right)^3 \right]$$

T-dependence of conductivity (continuum)



Superactivation

$$\ln \sigma(T) \sim -\frac{1}{T\tau} \ln\left(\frac{E_0}{T}\right)$$

Conclusions II

In **continuum** [unbounded localization length $\xi(E)$],
an ***effective mobility edge*** E_m emerges
due to hot-cold hybridization;
$E_m(T)$ diverges (rather slowly) as $T \to 0$.

Thus there is **no genuine MBL** in continuum:
the conductivity is zero only for $T = 0$.

Superactivated transport as $T \to 0$