Cold atoms in quasi-1D traps: beyond the zero-range approximation

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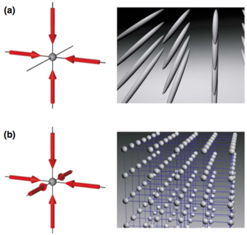
Universität Stuttgart

Why ultracold atoms?

- large number of bosons/fermions close to zero temperature
- easy to manipulate with optical fields
- optical lattices, reduced dimensional systems
- control of the type and strength of interactions
- precise measurements
- great toolkit for quantum simulations



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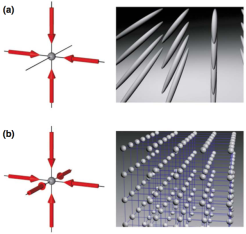

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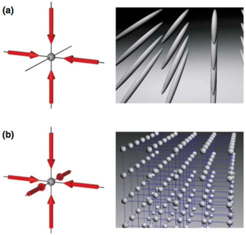

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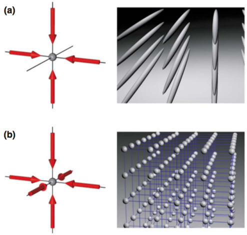

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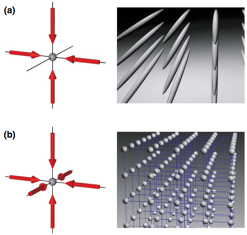

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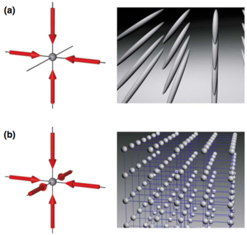

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Realization of Lieb-Liniger model

- low density, quasi-1D gas with short-range interactions
- $V(x) \approx g_{1D}\delta(x)$
- control over g to probe different regimes achieved using Feshbach resonances
- realization of Tonks-Girardeau gas: Kinoshita, Wenger, and Weiss, Science 2004; Paredes et al, Nature 2004;
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Outline

- Motivation
- Cold atomic collisions
- Scattering in a waveguide
- Generalized Lieb-Liniger model
- Conclusions

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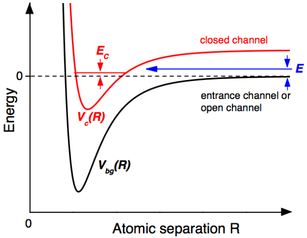
Interactions & Feshbach resonances

- neutral, closed-shell atoms - van der Waals interactions $V(r) \overset{r\to\infty}{\longrightarrow} -\frac{C_6}{r^6}$
- length $R_6 = (2\mu C_6/\hbar^2)^{1/4}$ or $\bar{a} \approx 0.477 R_6$; $E_6 = \hbar^2/2\mu R_6^2$
- collision energy $E \sim$nK, very dilute gas, s-wave scattering is enough (bosons)
- scattering length $a_{3D} = \lim_{k\to 0}\left(-\frac{\tan\delta(k)}{k}\right)$ can be tuned using Feshbach resonances (see Chin et al, RMP 2010)



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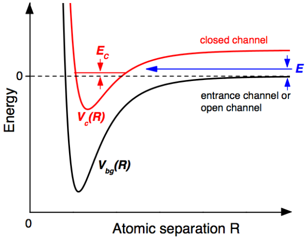

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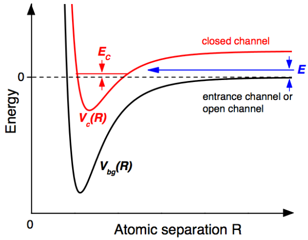

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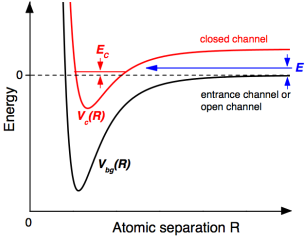

Interactions & Feshbach resonances

- scattering length $a_{3D} = a_{\mathrm{bg}} \left(1 - \frac{\Delta}{B - B_0}\right)$
- universal weakly bound state $E_b \propto 1/a^2$

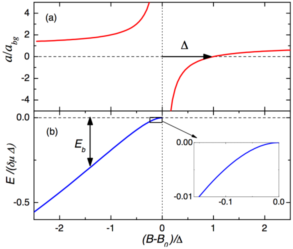

Figure: Scattering length and bound state energy near a Feshbach resonance (Chin et al, RMP 2010).

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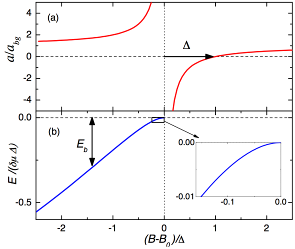

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Introducing the pseudopotential

- replace the interaction by pseudopotential which reproduces scattering properties

$$V_{\mathrm{eff}} = \frac{2\pi\hbar^2 a_{3D}}{\mu}\delta(\mathbf{r})\frac{\partial}{\partial r}r$$

- energy dependence can be included using effective range

$$k\cot\delta_{3D}(k) = -\frac{1}{a_{3D}(k)} = -\frac{1}{a_{3D}} + \frac{1}{2}r_{3D}k^2 + \dots$$

- energy-dependent pseudopotential for trapped particles $V(\mathbf{r}) = -\frac{2\pi\hbar^2}{\mu}\frac{\tan\delta_{3D}(k)}{k}\delta(\mathbf{r})\frac{\partial}{\partial r}r$

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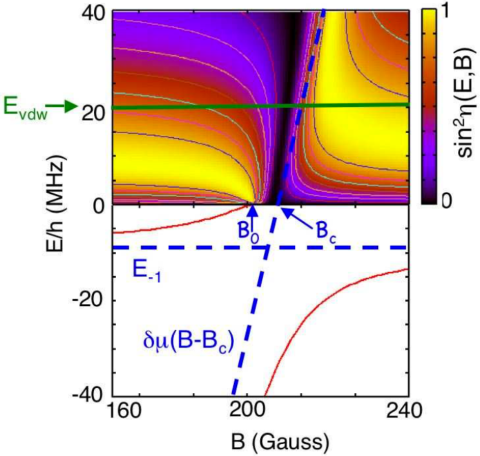
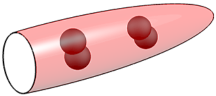


Figure: Energy dependence of the phase shift (Chin et al, RMP 2010)

Energy and length scales

- transverse harmonic confinement $U_{\mathrm{tr}} = \frac{1}{2}\mu\omega^2\rho^2$
- new length scale $d = \sqrt{\frac{\hbar}{\mu\omega}}$, energy scale $\hbar\omega$
- typically $d \gg R_6$



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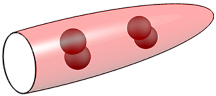

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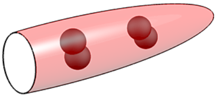

Solving energy-dependent problem

- start with Schrödinger equation

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(\mathbf{r}) + \frac{1}{2}\mu\omega^2\rho^2\right)\psi = E\psi$$

- asymptotic boundary conditions

$$\psi \overset{r\to\infty}{\longrightarrow} \psi_{nm}(\rho)e^{ipz} + \sum_{n'm'} f^{(+)}_{nm,n'm'}(p)\psi_{n'm'}e^{ip'|z|}$$

- odd part vanishes (bosons)
- restrict to lowest transverse mode
- extract the scattering amplitude $f^{(+)}$

Olshanii, PRL 1999; Naidon et al, NJP 2007; Idziaszek et al, NJP 2015

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- one-dimensional phase shift δ_{1D}

$$f^{(+)}(p) = -\frac{1}{1 + i \cot \delta_{1D}(p)}$$

- 1D even scattering length

$$a_{1D}^{(+)}(p) = \frac{1}{p \tan \delta_{1D}(p)}$$

- effective interaction strength in 1D $V_{\text{eff}}(x) = g_{1D}\delta(x)$

$$g_{1D}(p) = -\frac{\hbar^2}{\mu a_{1D}(p)}$$

(note that $g_{3D} = \frac{2\pi\hbar^2 a_{3D}}{\mu}$)

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Energy-dependent interaction strength

- general result for $a_{1D}(p)$ ($\mathcal{C} = -\zeta(1/2)$) (Olshanii, Naidon)

$$a_{1D}(p) = -\frac{d^2}{2a_{3D}(k)}\left(1 - \mathcal{C}\frac{a_{3D}(k)}{d}\right)$$

- incorporate energy dependence

$$a_{3D}(k) \approx \frac{a_{3D}}{1 - k^2 r_{3D} a_{3D}/2}$$

- include zero-point energy of the transverse oscillator! $k^2 = p^2 + 2/d^2$
- low energy expansion $g_{1D}(p) = g_{1D}(1 + g' p^2) + \dots$
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Numerical verification

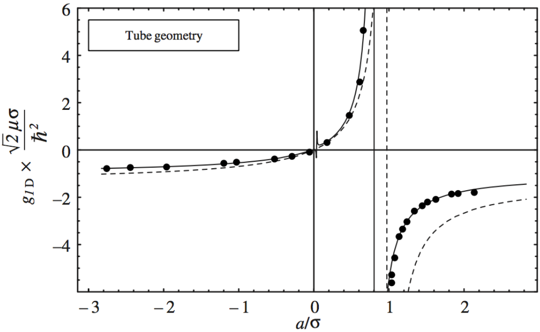
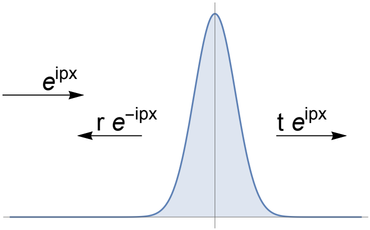


Figure: Naidon, NJP 2007; Bergeman, PRL 2003, Lennard-Jones potential

theory remains valid for $d \gtrsim \bar{a}$, independently of a_{3D}

Transmission coefficient

convenient analysis in terms of transmission coefficient



$$T(p) = |1+f|^2 = \frac{1}{1+\tan^2 \delta_{1D}(p)}$$

Role of effective range

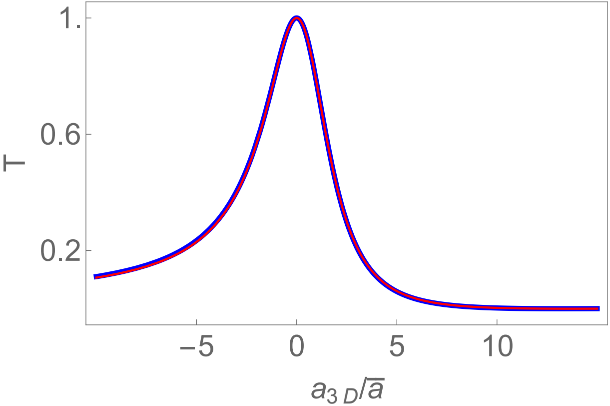


Figure: Wide trap $d = 20\bar{a}$, wide resonance - typical conditions in Innsbruck experiment

Wide vs. narrow resonances

- role of closed-channel contribution close to the Feshbach resonance
- “pole strength“ $s_{\mathrm{res}} = \frac{a_{\mathrm{bg}}}{\bar{a}} \frac{\delta\mu\Delta}{\bar{E}}$
- large s_{res} - open channel-dominated (“broad“)
- $s_{\mathrm{res}} \ll 1$ “narrow“ resonance
- effective range at the broad resonance - single-channel formula

$$r_{3D} = \frac{\Gamma(1/4)^2 \bar{a}}{6\pi^2} \left(1 - \frac{2\bar{a}}{a_{3D}} + \frac{2\bar{a}^2}{a_{3D}^2}\right)$$

- narrow resonances - nonuniversal behavior

$$r_{3D} \approx \frac{v + r_0 (a_{3D} - a_{ex})^2}{a_{3D}^2}$$

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Role of effective range

Cs, $\sim$ 47G resonance with very small s_{res}

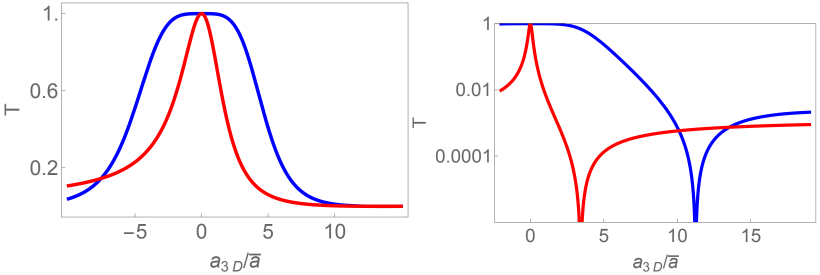

Figure: left: $d = 20\bar{a}$, right: $d = 5\bar{a}$

Theory without effective range corrections fails!

Role of effective range II

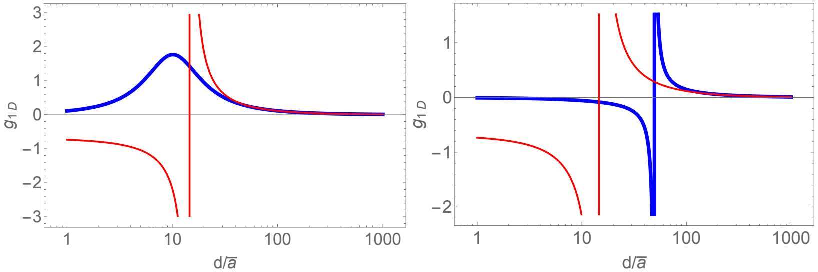

Figure: g_{1D} with (blue) and without corrections for two different narrow resonances at $a_{3D} = 10\bar{a}$; note that red curve remains the same!

Introducing the GLL

- length scale $\ell = \sqrt{2|g'|}$ associated with the correction
- $V(x)\psi(x) = g_{1D}\delta(x)\left(1 - g'\partial_x^2\right)\psi(x)$
- discretize the derivative
- resulting effective model
  $V(x) = c_0\delta(x) + c_\ell(\delta(x-\ell) + \delta(x+\ell))$
  $c_0 = 2g_{1D}$, $c_\ell = -g_{1D}/2$ or $c_0 = 0$, $c_\ell = g_{1D}/2$ depending on the sign of g'
- mapping on the Lieb-Liniger model for dilute system

$$c_{\mathrm{eff}} = c_0 + 2c_\ell + \frac{\frac{mc_\ell \ell}{\hbar^2}\left(2c_0 + 2c_\ell + \frac{mc_0c_\ell\ell}{\hbar^2} + \frac{mc_0^2\ell}{2\hbar^2}\right)}{1 - \frac{m^2c_0c_\ell\ell^2}{2\hbar^4} - \frac{m_c\ell l}{\hbar^2}}$$

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validity of GLL

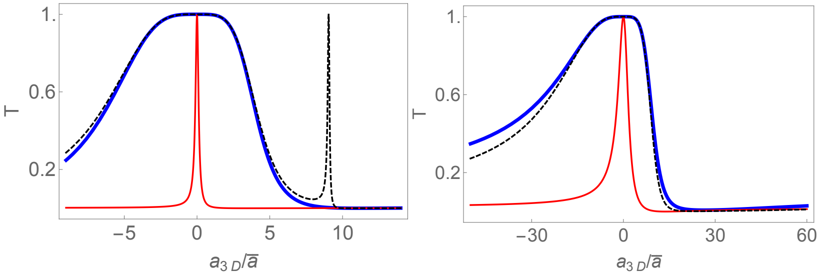

Figure: Transmission for two narrow resonances; GLL denoted by black dashed line

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- can be described in terms of universal quantities
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