Dresden 2017

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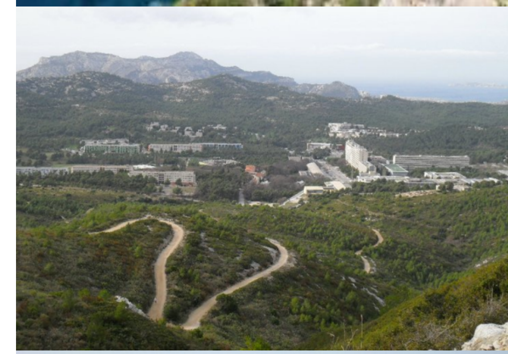
Hanbury-Brown and Twiss noise Correlations in the topological superconductor beam splitter

arXiv:1611.03776
Phys. Rev. B 95, 054514 (2017)






Collaborators:

- **T. Jonckheere**, **J. Rech**, D. Chevallier

(Superconducting beam splitter, with(out) interactions)

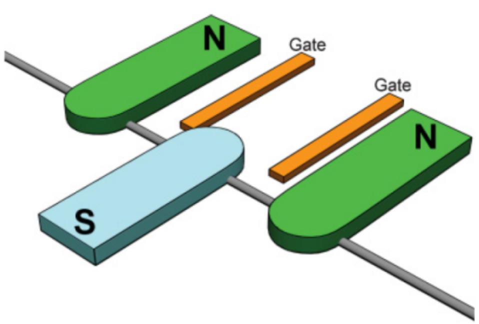
- D. Feinberg, R. Melin, D. Douçot, **T.J., J.R.**

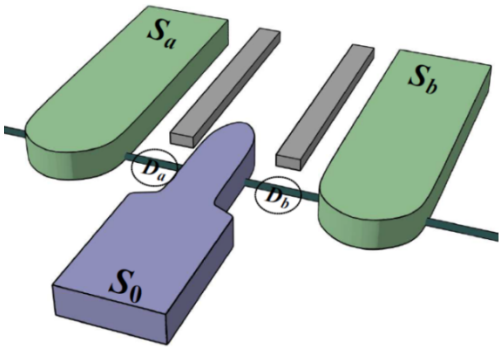
(3 superconductors , quartets, equilibrium…)

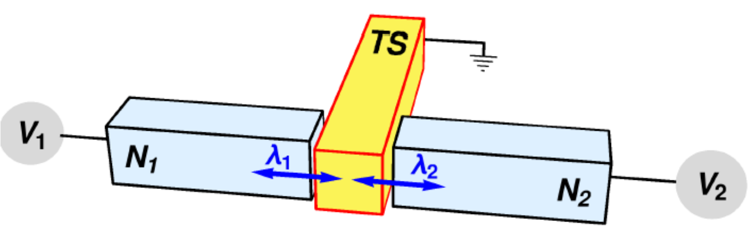
- **A. Zazunov**, **R. Egger, T.J., J.R.** (topological superconductors)

OUTLINE:

- Crossed Andreev Reflection in NSN forks
- Multipair DC Josephson resonances in SSS forks
- Topological Superconductor beam splitter (TS fork)

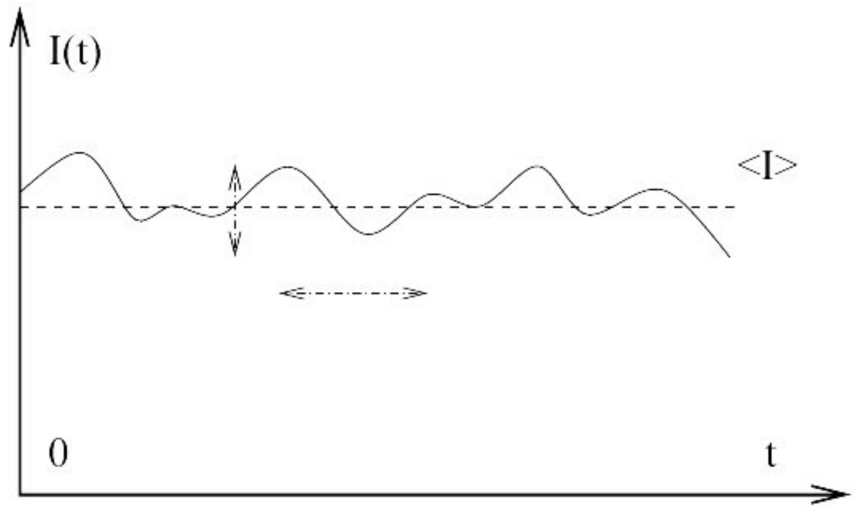






Measurable quantities: current and noise

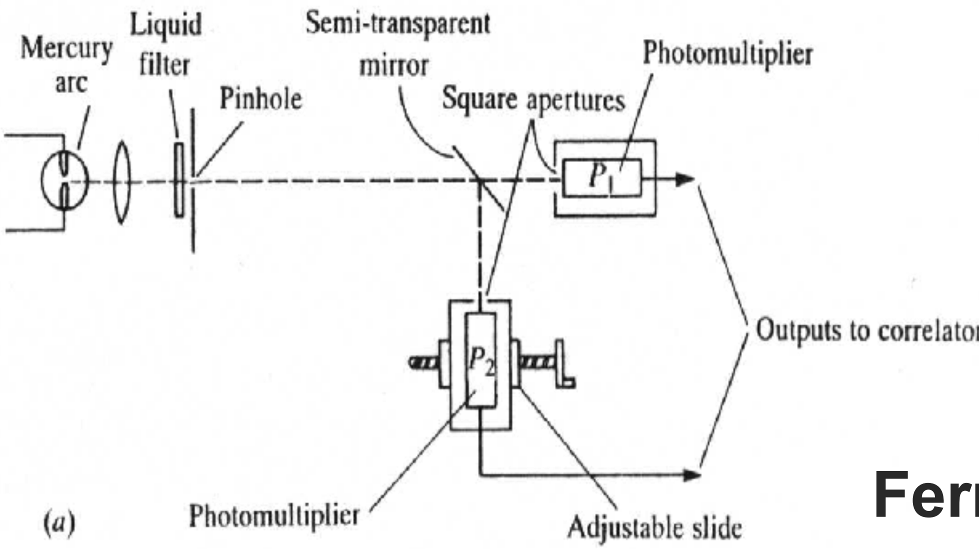
« the noise is the signal »



R. Landauer

$$S_{ij}(\omega) = \lim_{T \to +\infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int_{-\infty}^{+\infty} dt' e^{i\omega t'} (\langle I_i(t) I_j(t+t') \rangle - \langle I_i \rangle \langle I_j \rangle).$$

Hanbury Brown and Twiss experiment



Bunching effect: positive correlations

for thermal photons

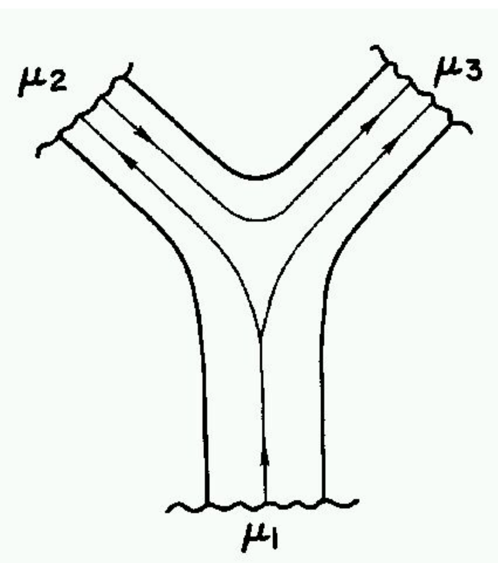
Fermions in nanophysics:

Negative correlations

T.M.+R. Landauer

M. Buttiker,

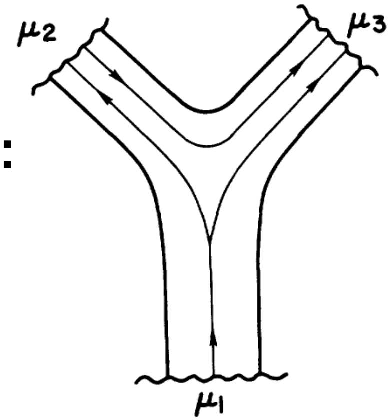
Phys Rev B 's 92)



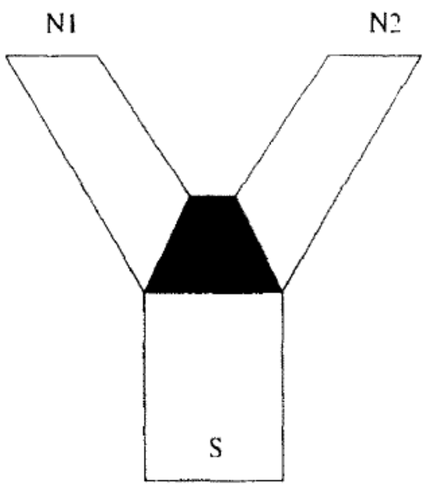
Experiments: Schonenberger 99,

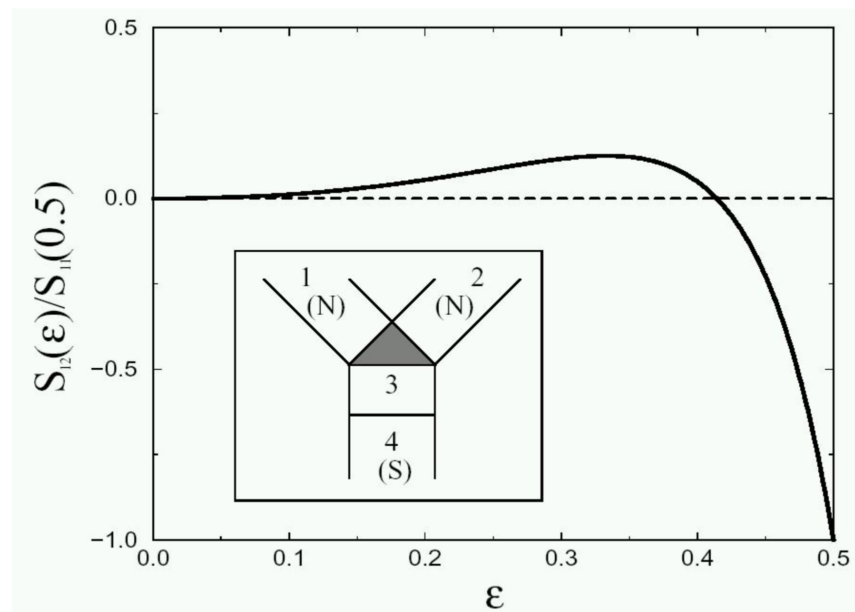
Yamamoto 99 (Science)

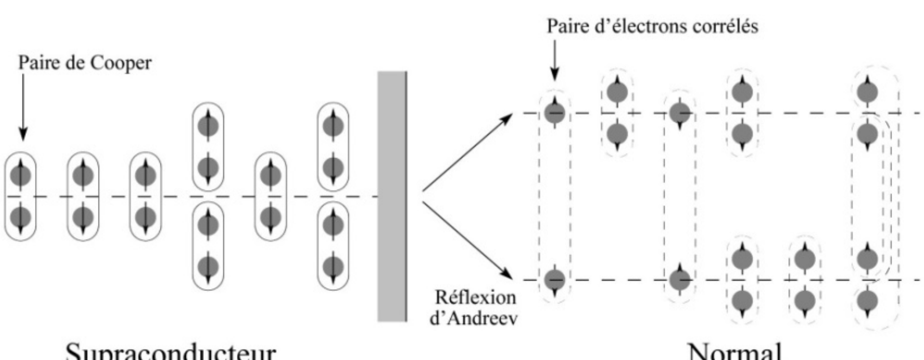
Normal metal forks: noise crossed correlations < 0 (partition of a filled Fermi sea)



Superconducting source connected to normal leads:
Noise crossed correlations > 0 or < 0
Martin, Phys Lett. A 1996, Anantram Data PRB96,
Torrès Martin EPJB 1999







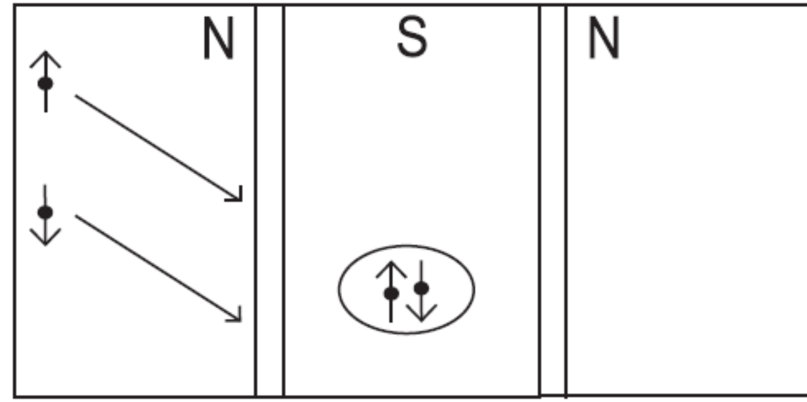
Crossed Andreev reflection

3 competing Processes:

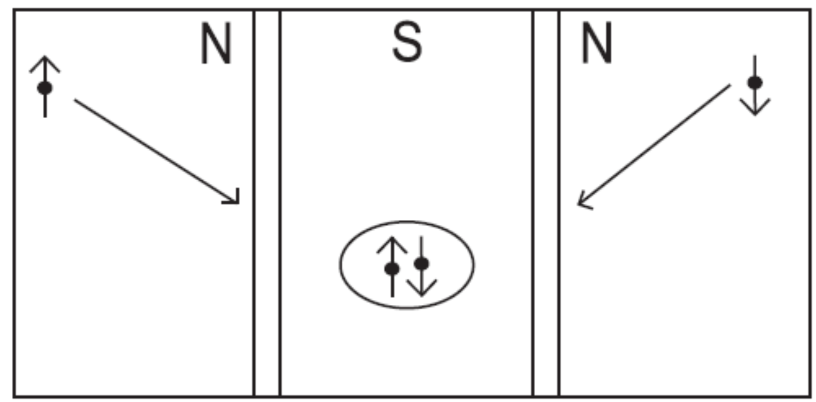
Direct Andreev reflection DAR

Crossed Andreev Reflection CAR (favored for equal voltages at output)

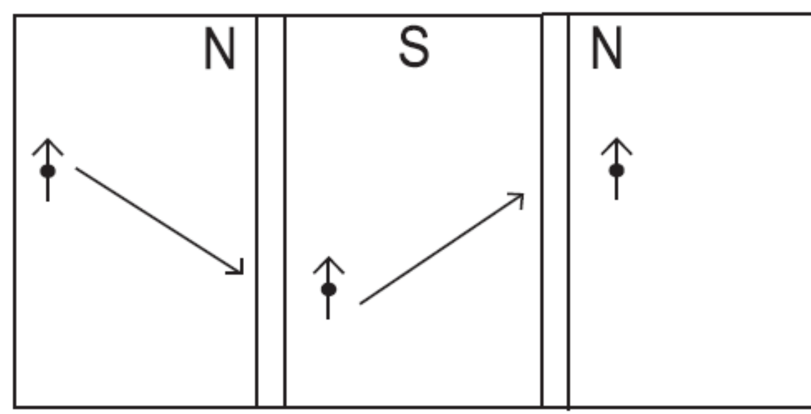
Electron cotunelling (EC) through S (favored for unequal voltages at output)



(a) Direct Andreev Reflection



(b) Crossed Andreev Reflection

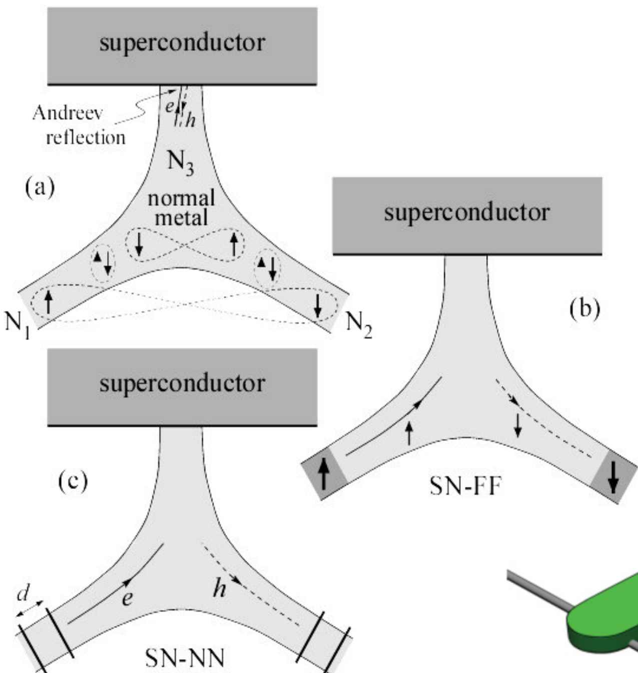


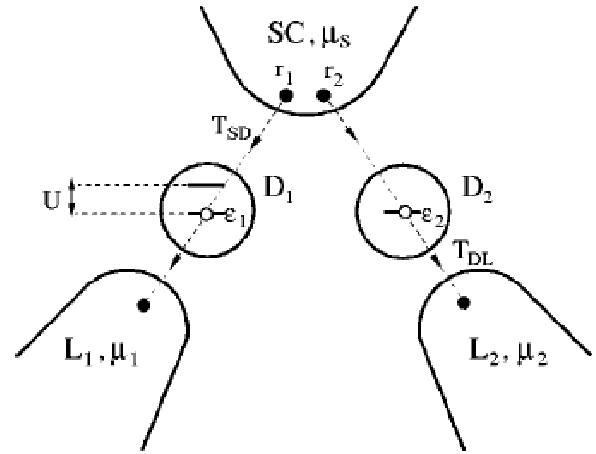
(c) Elastic Cotunneling

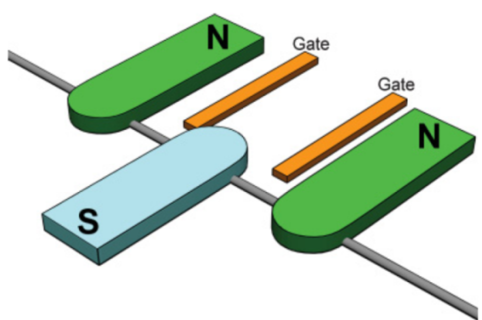
Superconductor: a source of entangled electrons

Lesovik Martin Blatter EPJB 2001 Recher, Sukorukov, Loss PRB 2001

Chtchelkatchev et al. PRB 2002 (Bell test)







T matrix calculation of the current: shows singlet state on dots

Positive noise cross correlations for energy filters or spin filters Chevallier... PRB 2011, Rech…PRB 2012

Also Börlin, Belzig, Bruder PRL 02 **FCS**
Samuelsson Buttiker Chaotic PRL 02

Experimental evidence for CAR

Karlsruhe

VOLUME 93, NUMBER 19 PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2004

Evidence for Crossed Andreev Reflection in Superconductor-Ferromagnet Hybrid Structures

D. Beckmann* and H. B. Weber
Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany

H. v. Löhneysen
Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany,
ıes Institut, Universität Karlsruhe, D-76128 Karlsruhe, Germany
ceived 16 April 2004; published 4 November 2004)

PRL 95, 027002 (2005) PHYSICAL REVIEW LETTERS week ending 8 JULY 2005

Experimental Observation of Bias-Dependent Nonlocal Andreev Reflection

S. Russo, M. Kroug, T. M. Klapwijk, and A. F. Morpurgo
Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
(Received 21 January 2005; published 8 July 2005)

Delft

PRL 97, 237003 (2006) PHYSICAL REVIEW LETTERS week ending 8 DECEMBER 2006

Nonlocal Correlations in Normal-Metal Superconducting Systems

P. Cadden-Zimansky and V. Chandrasekhar
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208, USA
(Received 18 May 2006; published 6 December 2006)

…Evanston

nature Vol 461|15 October 2009|doi:10.1038/nature

LETTERS

Basel

Cooper pair splitter realized in a two-quantum-dot Y-junction

L. Hofstetter[1]*, S. Csonka[1,2]*, J. Nygård[3] & C. Schönenberger[1]

PRL 104, 026801 (2010) PHYSICAL REVIEW LETTERS week ending 15 JANUARY 2010

Carbon Nanotubes as Cooper-Pair Beam Splitters

Paris/Regensburg

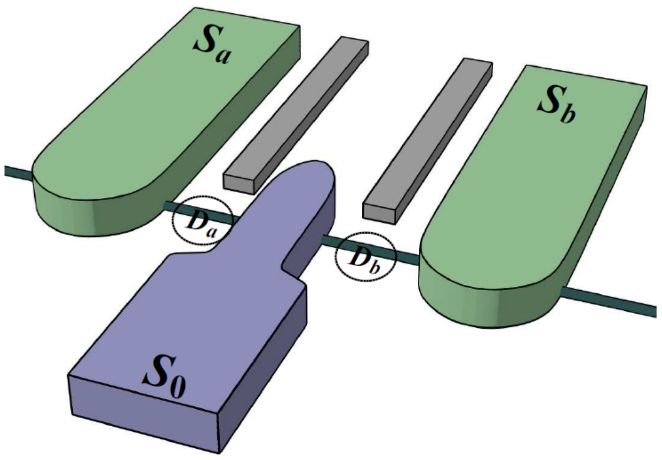
L. G. Herrmann,[1,2,5] F. Portier,[3] P. Roche,[3] A. Levy Yeyati,[4] T. Kontos,[1,2,*] and C. Strunk[5]

Evidence of entangled electrons born from Cooper pairs splitting via current and noise correlations

Rehovot

Anindya Das, Yuval Ronen, Moty Heiblum,*
Diana Mahalu, Andrey V. Kretinin and Hadas Shtrikman

Jonckheere et al. PRB 2013: superconductor bi-junction off equilibrium

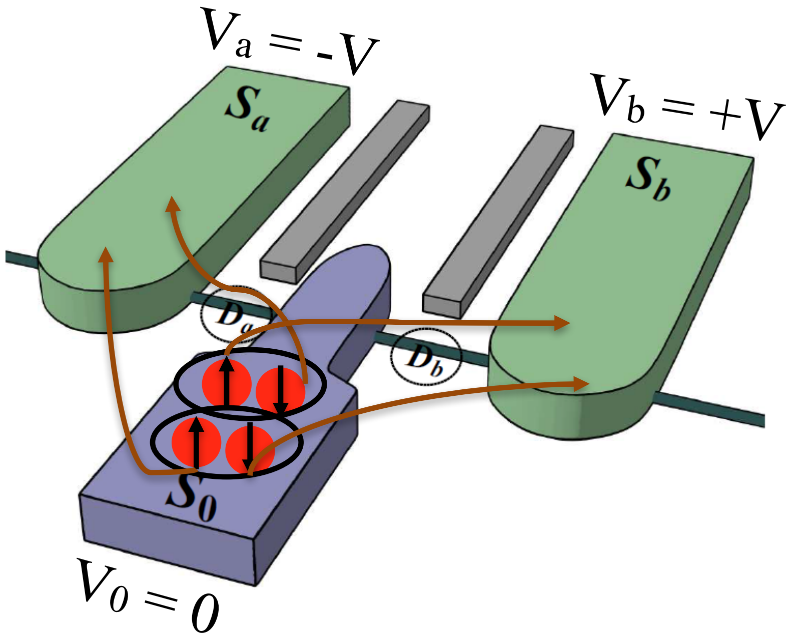


« Multiple Cooper Pair Resonnances »

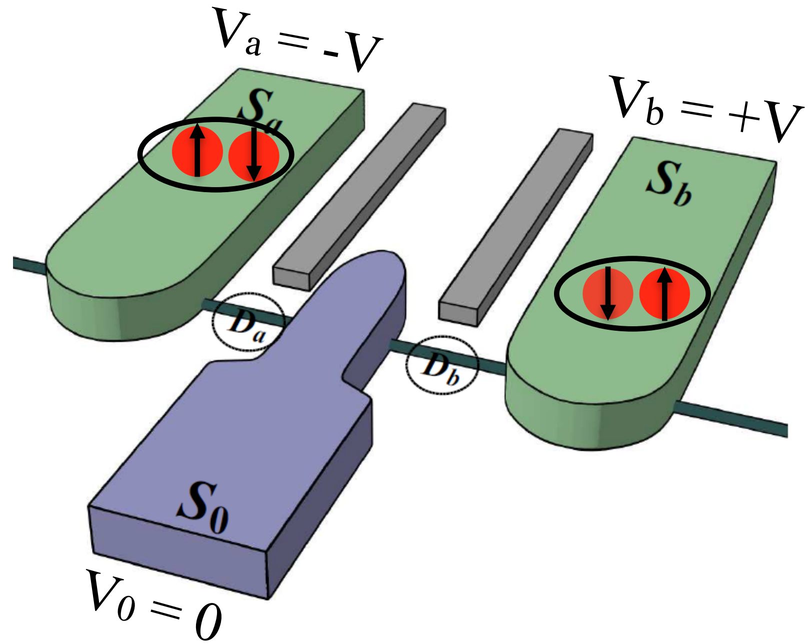
3 Superconductors separated by quantum dots
Dots (generated by nanowires) between pairs of superconductors (S)
Phases applied on each S

→ DC Josephson signal dependant on linear combinations of the 2 phase differences

The « Quartet » process



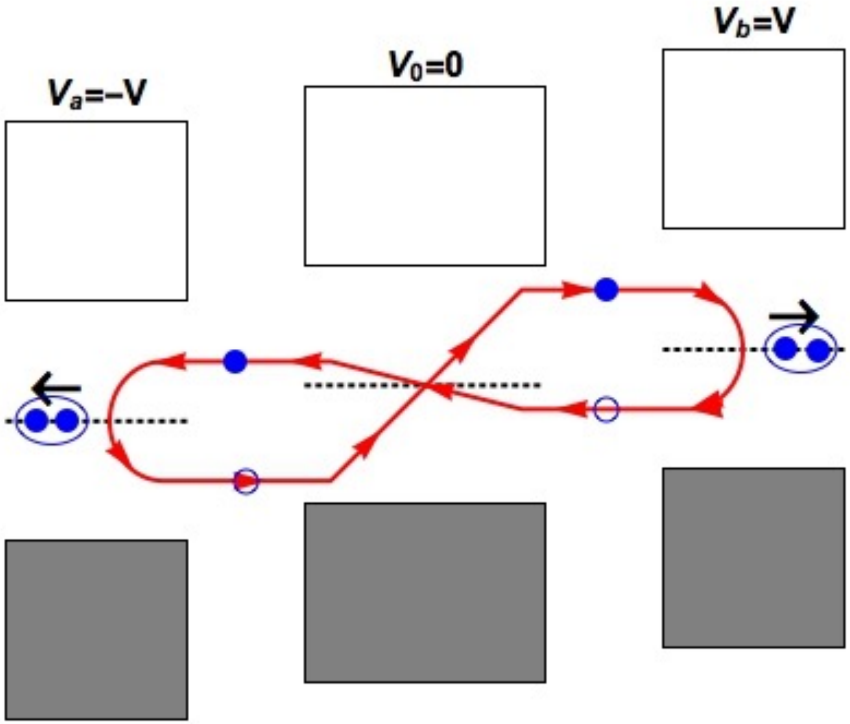
Initial State :
2 Cooper pairs at V=0



Final State :
1 pair at +V, 1 pair at -V

Energy conserving process
Transfer of 2 Cooper pairs
« quartet » of electrons

Energy Diagram of the Quartet process



Combination of 2 crossed Andreev reflections in the central superconductors, and 2 standard Andreev reflections

Main message

At commensurate voltages $nV_1 + mV_2 = 0$ → synchronization of 2 AC Josephson effects → **DC Josephson resonances**

Pi shift for « quartet » resonances $n=m=1$ at low bias $V_1=-V_2$

$$I_Q = I_{Q0} \sin(\varphi_a + \varphi_b - 2\varphi_0)$$

Tunability (enhancement of DC resonances) when gates tune the position of dot levels

Other processes, such as DC quasiparticle-pair interference effects, also contribute → phase dependent MAR

Ingredients

(Nambu spinor notation)

$$\hat{\mathcal{H}}_T(t) = \sum_{jk\alpha} \Psi^\dagger_{jk}\, t_{j\alpha} e^{i\sigma_z \varphi_j/2}\, \mathbf{d}_\alpha + \mathrm{h.c.}$$

$$\varphi_j(t) = \varphi_j^{(0)} + 2eV_j t/\hbar$$

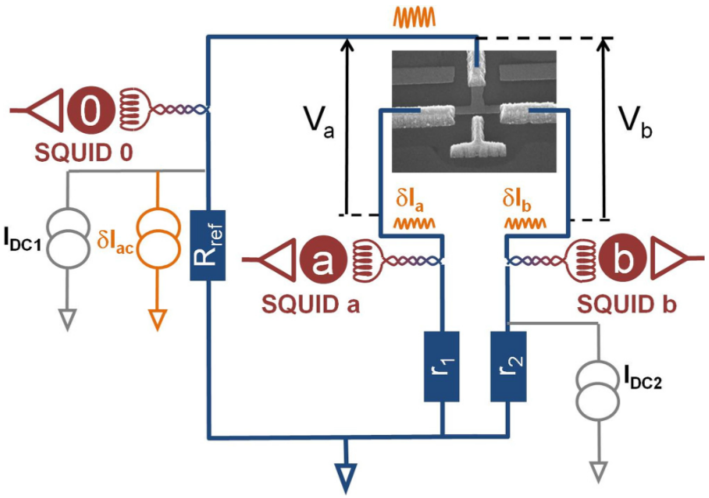
Current

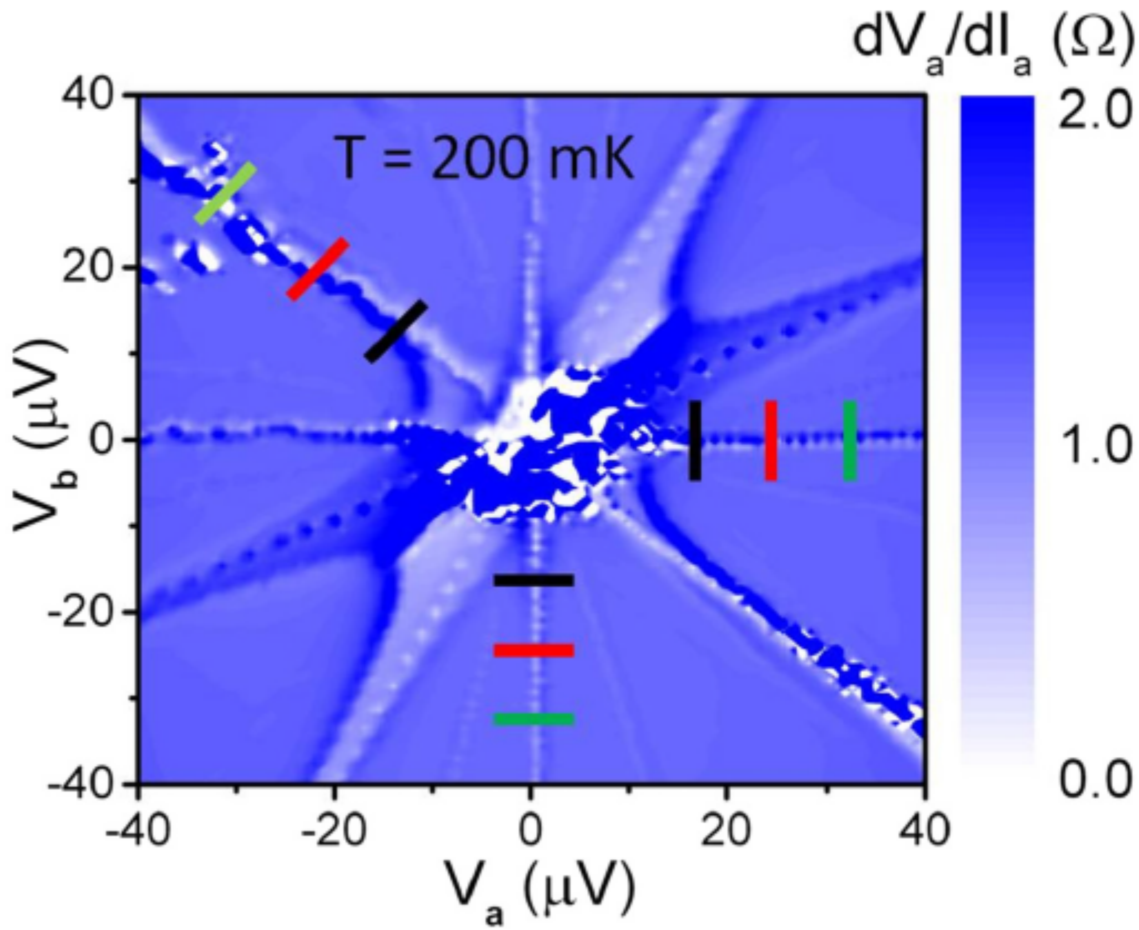
$$\langle I_j(t)\rangle = -2\mathrm{Re}\left\{\mathrm{tr}\left[\sigma_z \left(\hat{\Sigma}_j \circ \hat{G}\right)^{+-}(t,t)\right]\right\}$$

« Meir Wingreen » formula with dot Greens function

Possible first evidence of multiple pair resonances

Subgap structure in the conductance of a three-terminal Josephson junction
A.H. Pfeffer, J. E. Duvauchelle,H. Courtois, R. M´elin, D. Feinberg, F. Lefloch
PRB **90**, 075401 (2014)



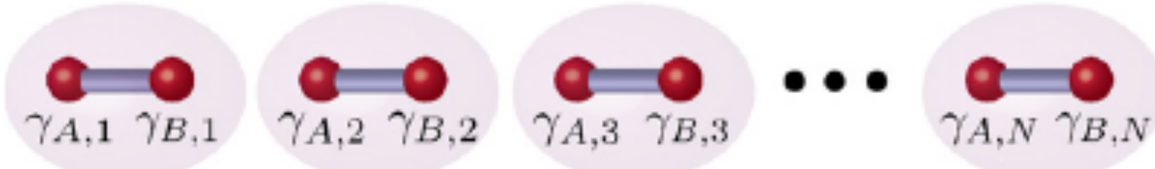


(end of intro)

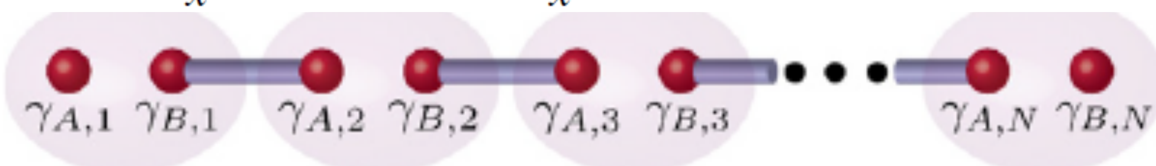
Hanbury-Brown and Twiss noise Correlations in the topological superconductor beam splitter

The hunt for Majorana Fermions in condensed matter

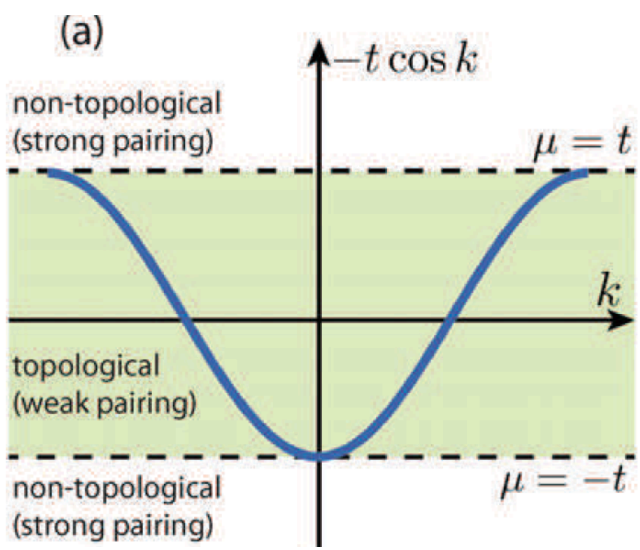
Topological superconductor: Kitaev model (p wave+hopping)



$$H = -\mu \sum_x c_x^\dagger c_x - \frac{1}{2} \sum_x (t c_x^\dagger c_{x+1} + \Delta e^{i\phi} c_x c_{x+1} + \text{H.c.})$$



t=Δ μ=0



$$\gamma_1 \equiv \gamma_{A,1} \qquad \gamma_2 \equiv \gamma_{B,N}$$

$$f = \frac{1}{2}(\gamma_1 + i\gamma_2)$$

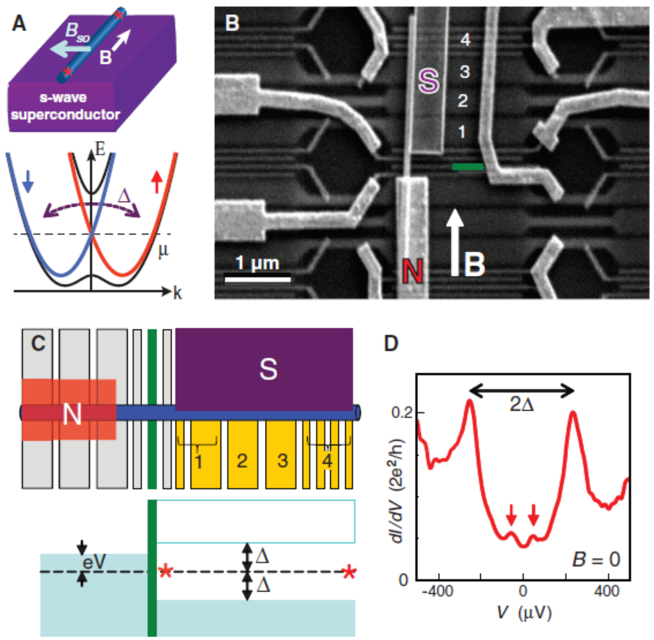
delocalized fermion

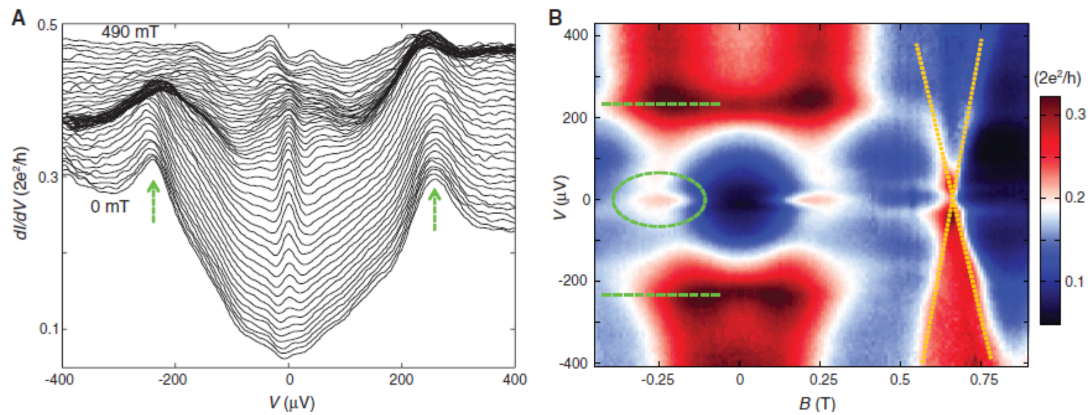
Alicea et al. Nature Physics 2011

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik *et al.*

Science **336**, 1003 (2012);





Zero bias anomaly is a potential signature of Majoranas

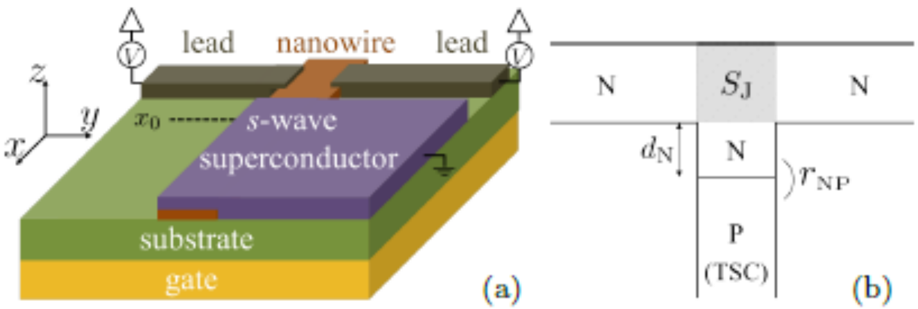
Is it the smoking gun ?

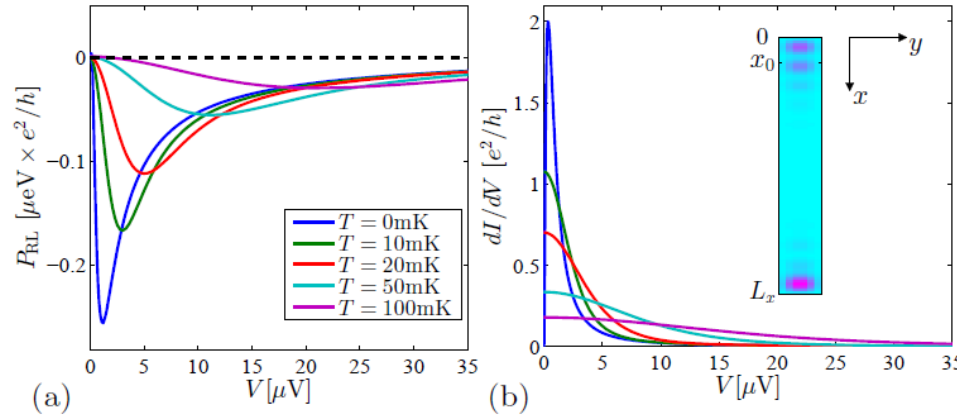
Several more experiments…

More proposals are needed

Previous works on the Majorana beam splitter:

Haim et al. 2015 (Y. Oreg/F. von Oppen)
Below gap, Landauer-Buttiker scattering theory





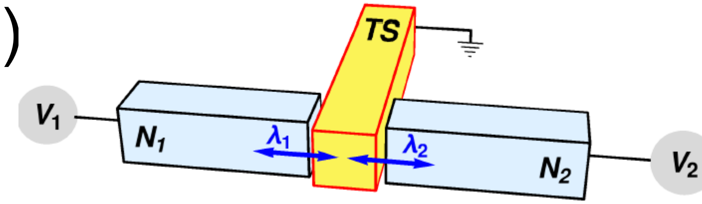
Predicts negative noise crossed correlations when Normal leads are at the same voltage

Also: Valentini et al. (2016)

Remark: this behavior is identical to the properties of an all normal metal beam splitter (Martin Landauer 92)

Goal here: (continuum version of Kitaev)
(Levy-Yeyati, Zazunov, Egger PRB16)

- Treat below and above gap with microscopic Keldysh Green's function
- Arbitrary +/- voltages



$$H = H_{TS} + H_N + H_t$$ (TS + Normal + tunneling)

$$H_{TS} = \int_0^\infty dx\, \Psi^\dagger_{TS}(x)\left(-iv_F\partial_x\sigma_z + \Delta\sigma_y\right)\Psi_{TS}(x)$$

$$\Psi_{TS}(x) = (c_r, c_l^\dagger)^T$$

$$H_t = \frac{1}{2}\sum_{j,j'}\Psi^\dagger_j W_{jj'}\Psi_{j'}$$

Boundary Green's function

$$\check{g}_{TS}(t-t') = -i\left\langle \mathcal{T}_C \Psi(t)\Psi^\dagger(t')\right\rangle$$

$$\Psi = (c, c^\dagger)^T \qquad c = [c_l + c_r](x=0)$$

Current and noise in terms of Keldysh Green's function (Dyson solved to all orders in tunneling)

« Nozières» formula (1971)

$$I_j = \frac{1}{2}\frac{e}{\hbar}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{j'\neq j}\mathrm{tr}_N\left[\sigma_z W_{jj'}G^K_{j'j}(\omega)\right]$$

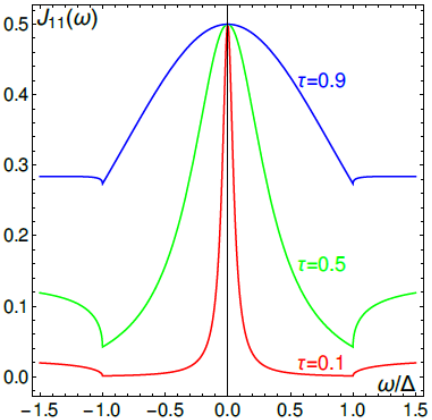
$$S_{jj'} = \int_{-\infty}^{\infty} d\tau \left\langle \delta\hat{I}_j(\tau)\delta\hat{I}_{j'}(0)\right\rangle$$

$$S_{jj'} = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{j_1\neq j}\sum_{j_2\neq j'}\mathrm{tr}_N\Big\{\lambda_{jj_1}\Big[G^{-+}_{j_1j_2}(\omega)\lambda_{j_2j'}G^{+-}_{j'j}(\omega)$$

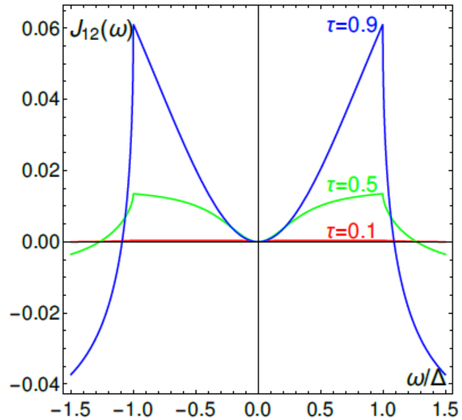
$$-G^{-+}_{j_1j'}(\omega)\lambda_{j'j_2}G^{+-}_{j_2j}(\omega)\Big]\Big\},$$

From Wick's theorem

Current and differential conductance



Local



Non-local

$$\tau = 4\Lambda^2/(1+\Lambda^2)^2$$

transparency

$$\Lambda = \sqrt{\lambda_1^2 + \lambda_2^2}$$

Coupling to TS

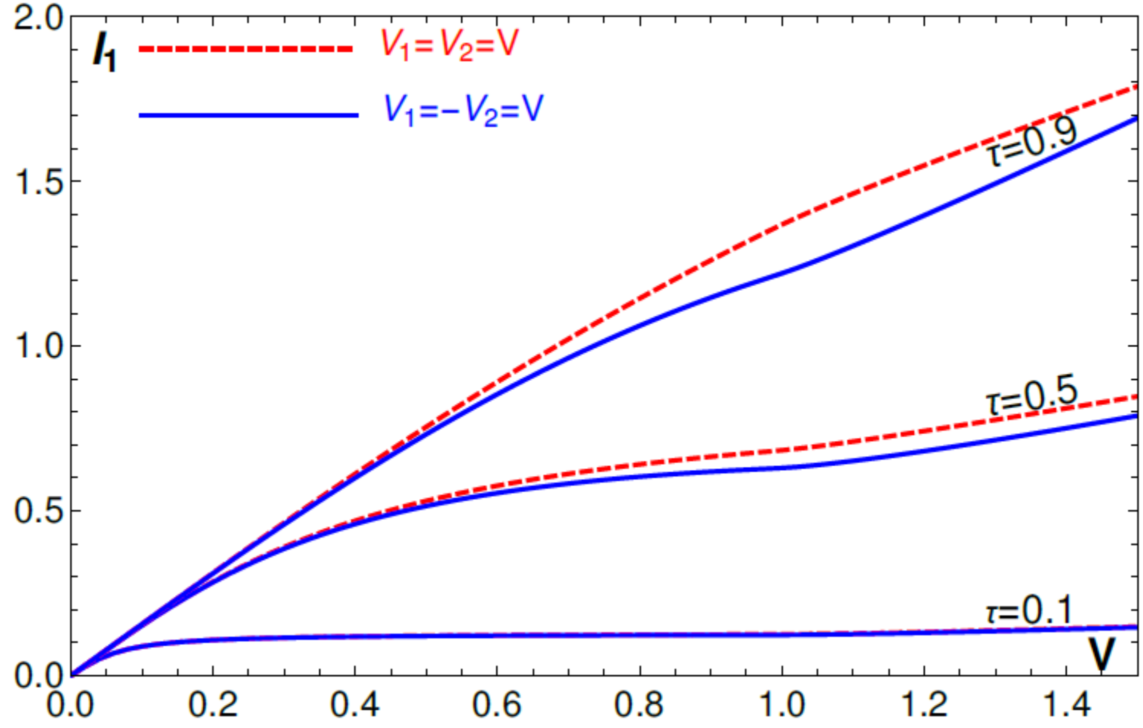
$$I_j = \frac{e}{h}\int_{-\infty}^{\infty} d\omega \sum_{k=1,2}\sum_{s=\pm} s\, n_F(\omega - s\, eV_k) J_{jk}(\omega)$$

Landauer formula

$$J_{11}(\omega) = -J_{12}(\omega) + \frac{4\lambda_1^2\Lambda^2}{(1-\Lambda^4)^2\frac{\omega^2}{\Delta^2} + 4\Lambda^4}$$

$$J_{12}(\omega) = \frac{2\lambda_1^2\lambda_2^2\left(1-\Lambda^4\right)\frac{\omega^2}{\Delta^2}}{(1-\Lambda^4)^2\frac{\omega^2}{\Delta^2} + 4\Lambda^4},$$

Non-local differential conductance (measurable experimentally)



For equal or opposite voltages, the current in 1 depends weakly on the symmetric/antisymmetric voltage configuration.

Differences occur at high transparency

Hanbury-Brown and Twiss noise crossed correlations

•equal voltages

$$S_{12}(V_1=V_2=V)=-\frac{2e^2}{h}\frac{\Gamma^2}{4}\frac{|eV|}{(eV)^2+\Gamma^2}$$

$$\Gamma=2\Delta\Lambda^2/(1-\Lambda^4)$$

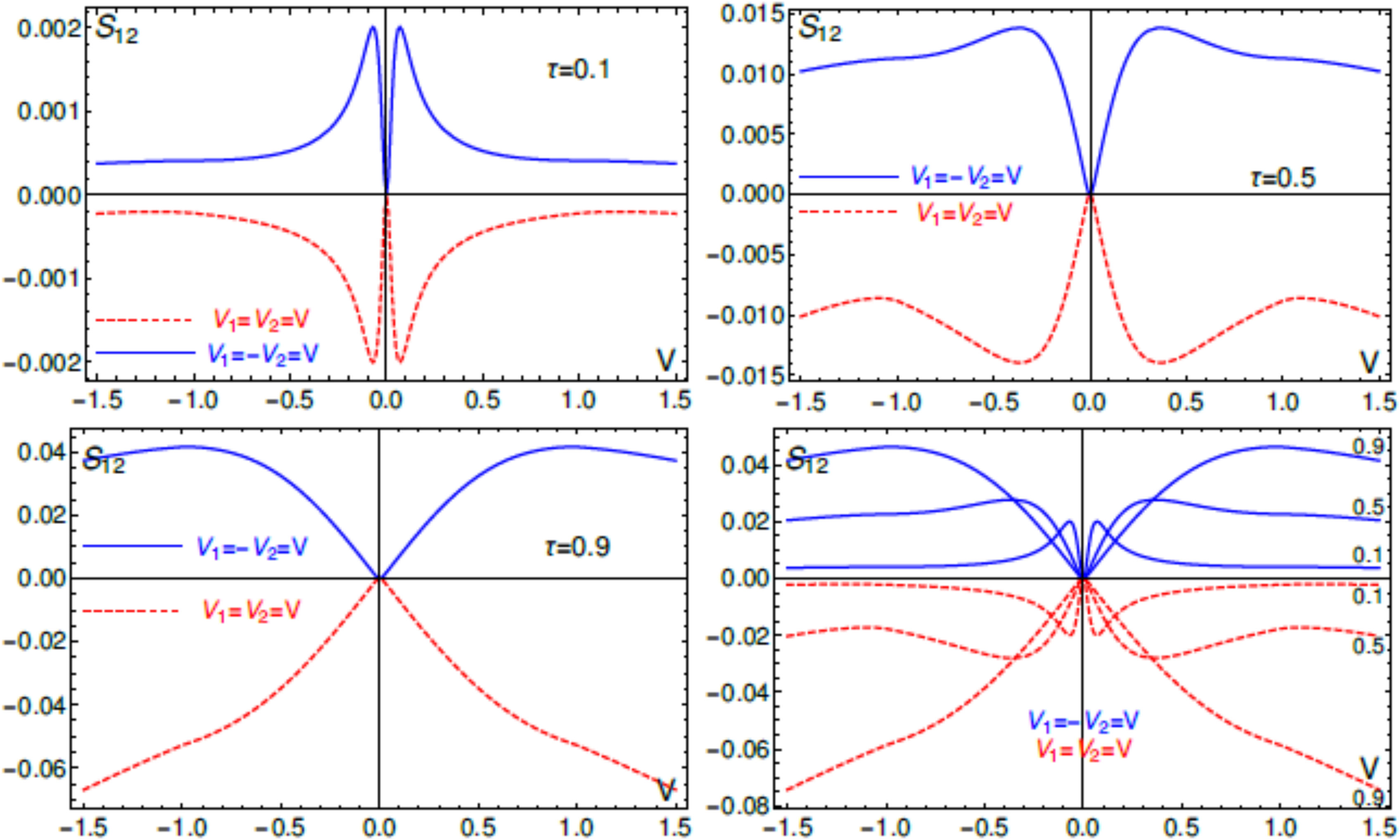
negative noise correlations like all fermionic system

•opposite voltages
positive noise
crossed correlations

$$S_{12}(V_1=-V_2=V)=\frac{2e^2}{h}\frac{\Gamma^2}{4}\Big[\frac{|eV|}{(eV)^2+\Gamma^2}+\frac{2\Gamma^2+(eV)^2}{(eV)^2+\Gamma^2}\frac{|eV|}{\Delta^2}-\frac{2\Gamma}{\Delta^2}\tan^{-1}\left(\frac{|eV|}{\Gamma}\right)\Big]$$

•General relation bewteen auto and crossed correlations
(Martin Landauer PRB 1992)

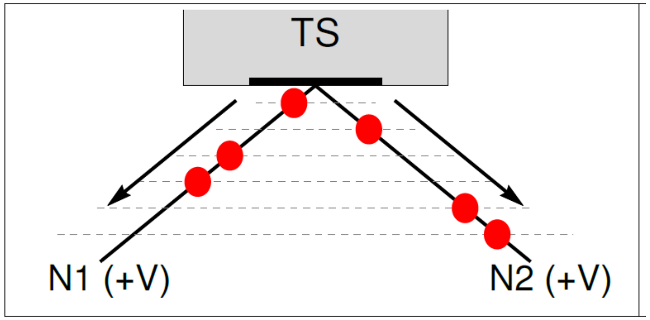
$$S_{00}=S_{11}+S_{22}+2\,S_{12}$$



Noise crossed correlations for equal and opposite voltages on the normal leads as a function of transparency

Physical interpretation: equal voltages

(low voltage behavior)



$$S_{00} = 2\Gamma \left[\tan^{-1}\left(\frac{|eV|}{\Gamma}\right) - \frac{|eV|/\Gamma}{1+(eV/\Gamma)^2} \right] \simeq 0$$

$$S_{11} = \Gamma \tan^{-1}\left(\frac{|eV|}{\Gamma}\right) - \frac{1}{2}\frac{|V|}{1+(eV/\Gamma)^2} \simeq \frac{|eV|}{2}$$

$$S_{12} = -\frac{1}{2}\frac{|eV|}{1+(eV/\Gamma)^2} \simeq -\frac{|eV|}{2}$$

$$\Gamma = 2\Delta\Lambda^2/(1-\Lambda^4)$$

Injection current is noiseless due to a zero bias resonance (ideal transmission)

$$I_0 = 2(e^2/h)V$$

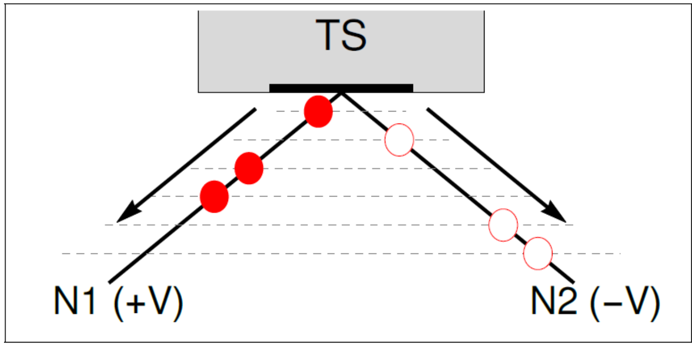
The injection current
Is partitioned in 1 an 2
The normal leads noise is

$$S_{jj} \equiv eI_j(1-T) = \frac{e^2}{h}\frac{|eV|}{2}$$

$$S_{12} = -S_{11}$$

Crossed correlations
(all fermionic behavior) → negative crossed correlations

Interpretation: opposite voltages



$$S_{00} = 2\Gamma \tan^{-1}\left(\frac{|eV|}{\Gamma}\right) \simeq 2|eV|$$

$$S_{11} = \Gamma \tan^{-1}\left(\frac{|eV|}{\Gamma}\right) - \frac{|eV|/2}{1+(eV/\Gamma)^2} - f(V,\Gamma) \simeq \frac{|eV|}{2}$$

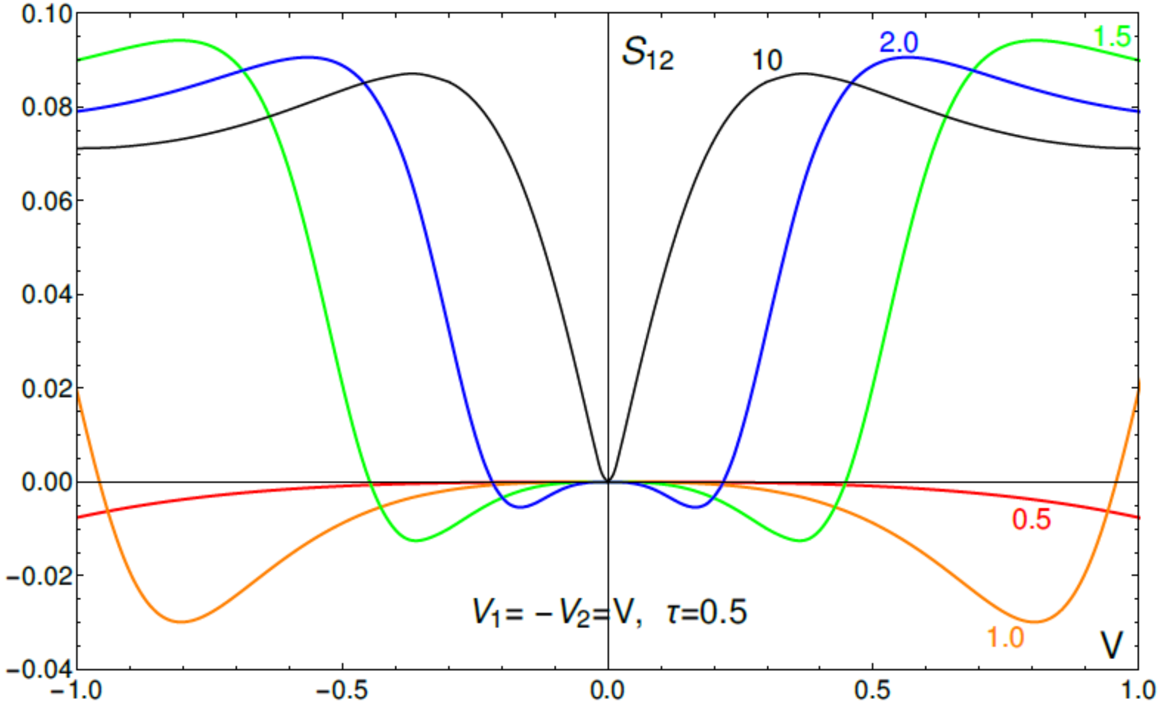
$$S_{12} = \frac{1}{2}\frac{|eV|}{1+(eV/\Gamma)^2} + f(V,\Gamma) \simeq \frac{|eV|}{2}$$

Same ingredients, as coupling to Majorana is e-h symmetric
1 collects e, 2 collects h
→ TS particle current is noiseless, TS charge current is noise-full
e-h partitioning leads to $I_1 = -I_2 = (e^2/h)V$

Autocorelation noise $S_{11} = S_{22} = (e^3/h)|V|/2$
Crossed correlation noise are the same (positive, as carriers bear opposite charge)

TS lead noise is thus $S_{00} = 2(e^3/h)|V|$

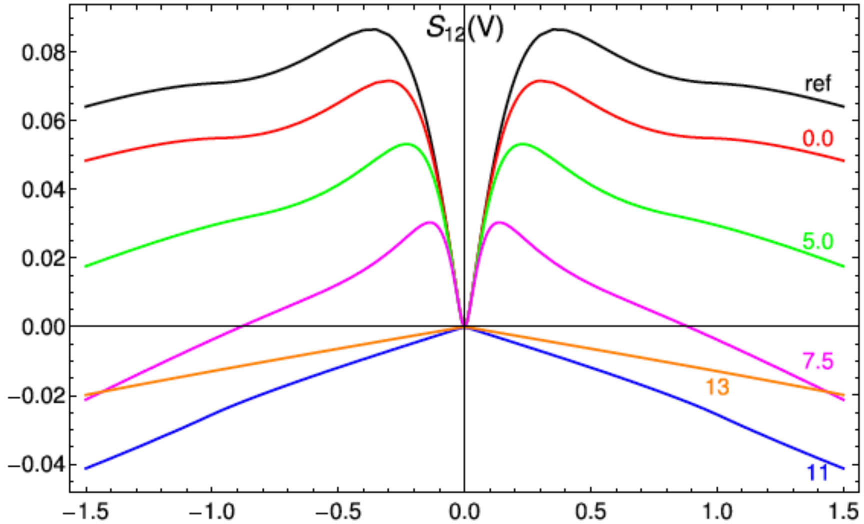
Extension 1: Finite length TS wire, opposite voltages
(the two Majorana's « communicate » for small wire length L)



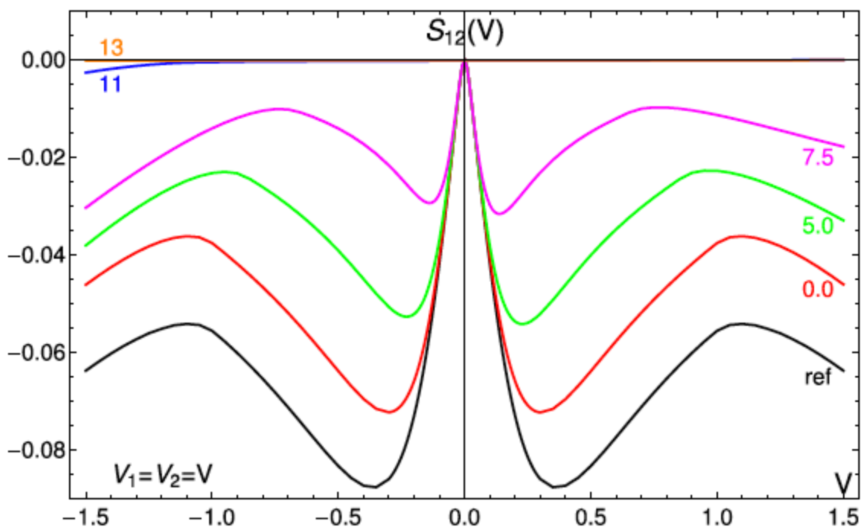
L in units of ξ
(TS coh. length)

Reversal of sign of noise crossed correlations for small TS wire length.

Varying the intrinsic chemical potential of the TS wire (allows to drive the TS to a topologically trivial phase)



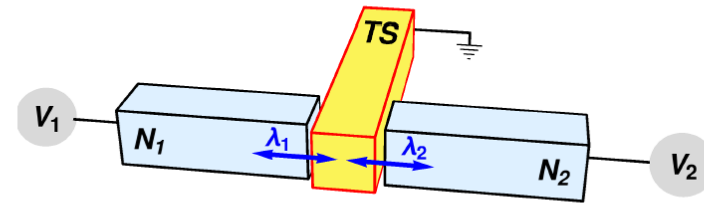
reversal of the sign of crossed correlation for opposite voltages



equal voltages

asymmetry develops when topologically trivial phase is reached

CONCLUSIONS:



•Keldysh Green's function approach to hybrid N-TS-N systems, treat: below/above gap, finite TS, doping of TS
•Non local differential conductance
•Crossed correlations < 0 (fermionic) at equal voltages
•Crossed correlations > 0 at opposite voltages: Majorana converts electrons into holes.
•Reversal of noise crossed correlations (opposite V) when 2 Majoranas overlap.
•Transition of noise crossed correlations when driving to topologically trivial phase.
•Extensions ? Interactions (dots…) from Keldysh Green's functions

NSN Beam splitter: PRB 83, 125421 (2011); PRB 85, 035419 (2012)
Quartets: PRB 87, 214501 (2013); PRB 90, 075419 (2014)
TS Beam splitter : arXiv:1611.03776, Phys Rev B 95, 054514 (2017)

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Boundary Green's functions

Semi infinite TS

$$g_{TS}^{R/A}(\omega) = \frac{\sqrt{\Delta^2 - (\omega \pm i0^+)^2}\,\sigma_0 + \Delta\sigma_x}{\omega \pm i0^+}$$

$$g_{TS}^{K}(\omega) = (1 - 2n_F(\omega))\left[g_{TS}^{R}(\omega) - g_{TS}^{A}(\omega)\right]$$

Dyson

$$\begin{aligned} G^K(\omega) =& G^R(\omega)F(\omega) - F(\omega)G^A(\omega) \\ &+ G^R(\omega)\left[F(\omega)W - WF(\omega)\right]G^A(\omega) \end{aligned}$$

$$F_{jk}(\omega) = \delta_{jk}\left[1 - 2n_F(\omega - \mu_j\sigma_z)\right]$$

Finite size TS Green's function

$$g^{R/A}(\omega) = \omega \tanh(\zeta_\omega L)\frac{\zeta_\omega \sigma_0 - \tanh(\zeta_\omega L)\Delta\sigma_x}{(\omega \pm i0^+)^2 - \epsilon_\omega^2}$$