

Single-particle interference and two-particle collisions

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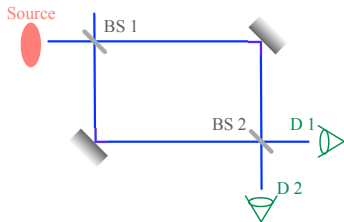
**NANOSCIENCE AND
NANOTECHNOLOGY**
A CHALMERS
AREA OF ADVANCE

MPI Dresden, 16 February 2017

- Introduction
 - Electronic interferometers in the QH regime
 - Single-electron sources
 - Tunable two-particle effects
- Single-electron coherence from Mach-Zehnder interferometry
- Single-particle interference versus two-particle collision
 - Two-particle effects in the charge current
 - Confirmation of observed effects from the noise
- Counter-intuitive results from the energy current

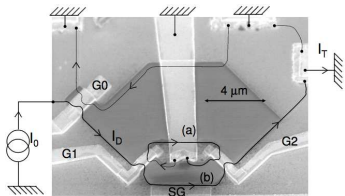
Introduction

Electronic interferometer in the quantum Hall regime

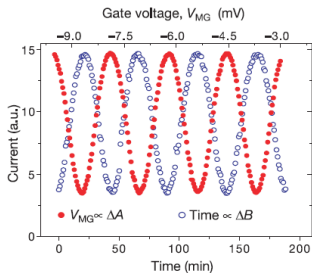


⇒ Interference pattern as a coherence effect.

Electronic realization in a Quantum Hall setup

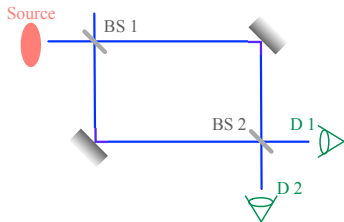


Y. Ji, *et al.*, Nature **422**, 415 (2003); P. Roulleau, *et al.*, Phys. Rev. B **76**, 161309(R) (2007).



Visibility as a measure of coherence!

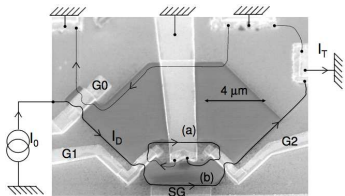
Electronic interferometer in the quantum Hall regime



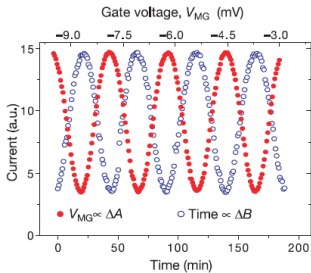
- Particles emitted into quantum Hall edges (waveguides)
- Particles are scattered at quantum point contacts (beam splitters)

⇒ Interference pattern as a coherence effect.

Electronic realization in a Quantum Hall setup

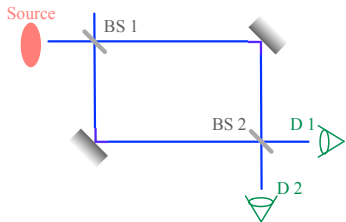


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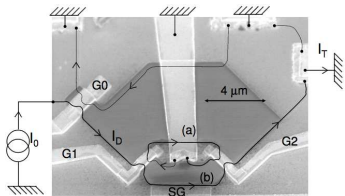
Electronic interferometer in the quantum Hall regime



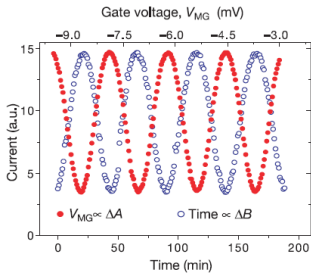
- Particles emitted into quantum Hall edges (waveguides)
- Particles are scattered at quantum point contacts (beam splitters)
- Coherent single-particle sources \Rightarrow ???
 \Rightarrow "Quantum optics with electrons."

\Rightarrow Interference pattern as a coherence effect.

Electronic realization in a Quantum Hall setup



Y. Ji, *et al.*, Nature **422**, 415 (2003); P. Roulleau, *et al.*, Phys. Rev. B **76**, 161309(R) (2007).



Visibility as a measure of coherence!

Single-electron sources

Aim: Inject single, well-separated electrons into an electronic circuit.

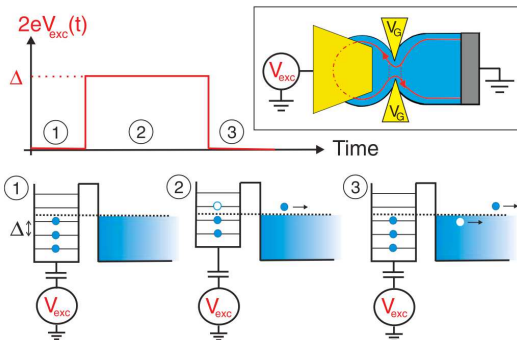
- ▶ (Superconducting) single-electron turnstiles

see, e.g., J. Pekola, *et al.*, Nat. Phys. 4, 120 (2008).

- ▶ Single-electron pumps

see, e.g., H. Pothier, *et al.*, Europhys. Letts. 17, 249 (1992); B. Kaestner, *et al.*, Phys. Rev. B 77, 153301 (2008); B. Roche, *et al.*, Nat. Comm. 4, 1581 (2013)

- ▶ Gate-tuned mesoscopic capacitor



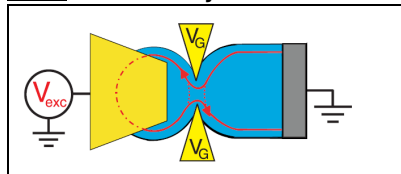
G. Fève, *et al.*, Science 316, 1169 (2007).

- ▶ Time-dependent bias-pulses (Levitons)

J. Dubois, *et al.*, Nature 502, 659 (2013).

Single-electron sources

Here: **adiabatically driven mesoscopic capacitor** (\sim Levitons)



Adiabaticity condition:

$$\Omega \ll T/\tau$$

with driving frequency Ω , QPC transmission T , time of flight τ .

Relate incoming and outgoing states to this (and other!) setups by a time-dependent/Floquet scattering matrix:

$$S(t, t') \Leftrightarrow S(t, E) \Leftrightarrow S(E + m\hbar\Omega, E)$$

Adiabatic limit **for the source**: \Rightarrow Frozen scattering matrix:

$$S_{SES}(t) = \frac{\pm(t - t^{e/h}) - i\sigma}{\pm(t - t^{e/h}) + i\sigma}$$

Here: Neglect Coulomb interaction (strong screening due to top gates)

otherwise, see e.g. C. Wahl, J. Rech, T. Jonckheere, and T. Martin, Phys. Rev. Lett. **112**, 046802 (2014) [interaction between edge states]; J. Schulenburg, R. B. Saptsov, F. Haupt, J. Splettstoesser, and M. R. Wegewijs, Phys. Rev. B **93**, 081411(R) (2016) [Onsite Coulomb interaction].

Single-electron sources

Scattering matrix adds a **phase** to the wave-function

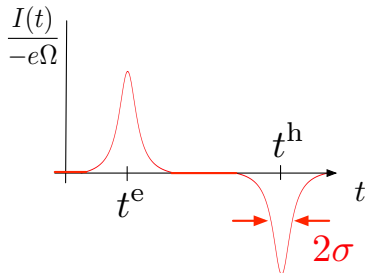
$$\Psi_E(t, x) = S_{\text{SES}}(t - x/v_D, E) e^{i(k_E x + \omega t)}$$

which has a nontrivial effect on the current:

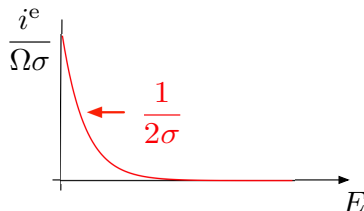
$$I_{\text{SES}}(t, x) = \frac{-ie}{2\pi} S_{\text{SES}}(t - x/v_D, \mu) \frac{\partial S_{\text{SES}}^*(t - x/v_D, \mu)}{\partial t}$$

Pulses of duration $2\sigma \sim$ Emission/absorption of particles!

Lorentzian time-dependence



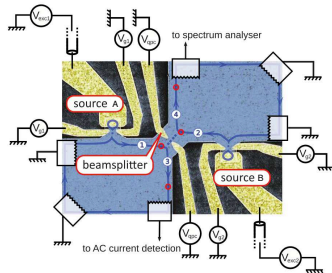
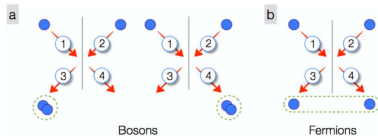
Exponential energy-distribution



Up to here: Single-particle properties.

Tunable two-particle (exchange) effects

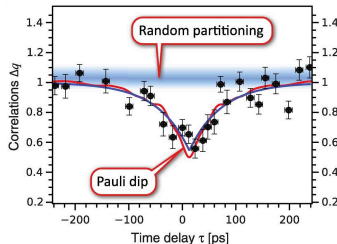
Hong-Ou-Mandel effect for electrons (antibunching)



Charge current noise (correlations):

$$\mathcal{P} = \frac{e^2}{T} T(1 - T) \left(1 - \frac{4\sigma_A\sigma_B}{(I_A^e - I_B^e)^2 + (\sigma_A + \sigma_B)^2} \right)$$

S. Ol'khovskaya, J. Splettstoesser, M. Moskalets, and M. Büttiker, *Phys. Rev. Lett.* **101**, 166802 (2008).



Figures from Experiment: E. Bocquillon, V. Freulon, J. M. Berroir, P. Degiovanni, B. Plaças, A. Cavanna, Y. Jin, and G. Fève, *Science* **339**, 1054 (2013).

Single-electron coherence from Mach-Zehnder interferometry

Single-electron coherence from Mach-Zehnder interferometry

Spectral analysis of emitted signal - some alternative approaches:

- ▶ **Full quantum state tomography from noise**

C. Grenier, R. Hervé, E. Bocquillon, F. D. Parmentier, B. Plaçais, J.-M. Berroir, G. Fève, P. Degiovanni, *New J. Phys.* **13**, 093007 (2011).

- ▶ **Energy current, power fluctuations and mixed noise**

F. Battista, F. Haupt, and J. Splettstoesser, *Phys. Rev. B* **90**, 085418 (2014);

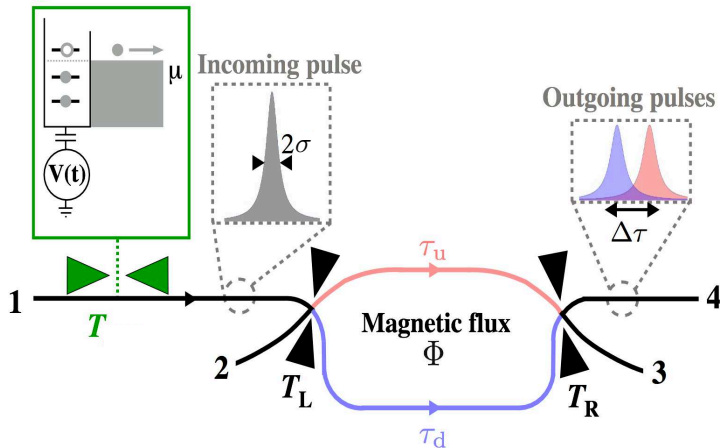
F. Battista, F. Haupt, and J. Splettstoesser, *J. Phys.: Conf. Ser.* **568**, 052008 (2014).

N. Dashti, M. Misiorny, P. Samuelsson, and J. Splettstoesser, in preparation;

N. Dashti, P. Samuelsson, M. Misiorny, and J. Splettstoesser, in preparation.

Single-electron coherence from Mach-Zehnder interferometry

Single electron source



Single-electron coherence from Mach-Zehnder interferometry

$$I_4(t, \Phi) = I_4^{(\text{cl})}(t) + I_4^{(\text{int})}(t, \Phi)$$

$$I_4^{(\text{int})}(t, \Phi) \propto \text{Re} \left\{ e^{-i\phi} C_{\text{SES}}(t, \Delta\tau) \right\}$$

Single-particle coherence:

$$C_{\text{SES}}(t^-, \Delta\tau) = \frac{1}{1 - i\Delta\tau/\sigma}$$

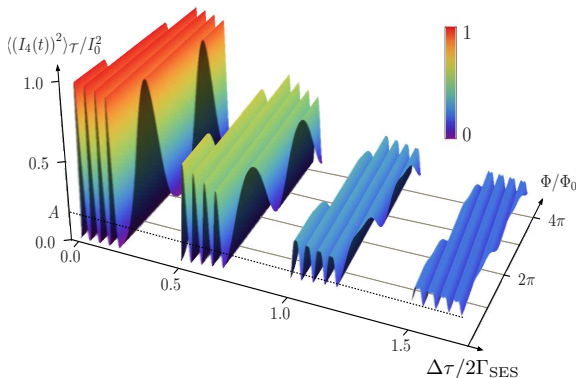
Extract

Single-particle coherence length

$$\Lambda = v_D \sigma$$

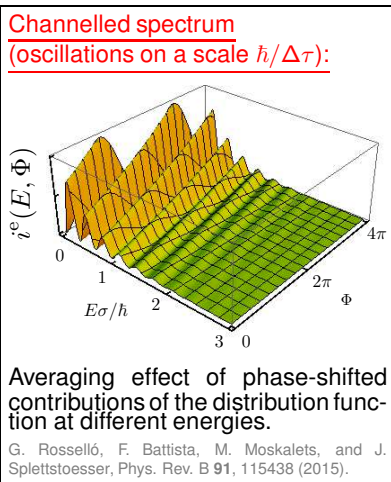
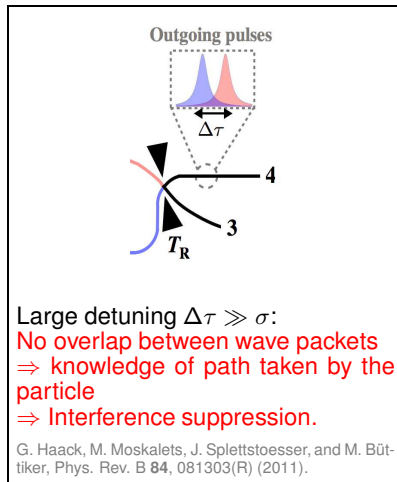
from the visibility of a time-averaged measurement.

G. Haack, M. Moskalets, J. Splettstoesser, and M. Büttiker, Phys. Rev. B **84**, 081303(R) (2011).



Single-electron coherence from Mach-Zehnder interferometry

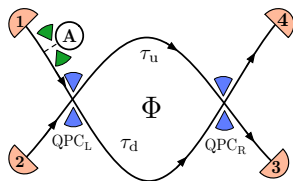
Two interpretations:



\Rightarrow Particle interpretation of a superposition of plane waves (wave packet)

[Strong energy-dependence of MZI-transmission leads to different currents of electrons and holes!]

Single-electron coherence from Mach-Zehnder interferometry



$$\mathcal{P} = \frac{1}{2} \int_0^T \frac{dt'}{\mathcal{T}} \int_{-\infty}^{\infty} d(t-t') [\langle \hat{I}_3(t) \hat{I}_4(t') + \hat{I}_3(t') \hat{I}_4(t) \rangle - 2 \langle \hat{I}_3(t) \rangle \langle \hat{I}_4(t') \rangle]$$

Noise of a **single-particle** process (partition noise $\propto T(1-T)$):

$$\mathcal{P} = -\frac{e^2}{\mathcal{T}} T_{41}^{\text{eff}} T_{31}^{\text{eff}}$$

Interpretation of the classical part:

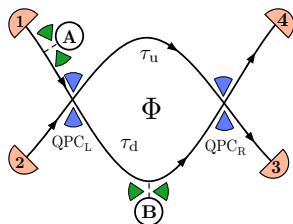
$$\propto T_R R_R + T_L R_L (T_R - R_R)^2$$

Noise of QPC R

Noise of QPC L (in presence of QPC R)

Single-particle interference versus two-particle collision

Single-particle interference versus two-particle collision



Two electrons collide at QPC R:

$$\Delta t_u^{ee} = 0$$

Two electrons collide at source B:

$$\Delta t_d^{ee} = 0$$

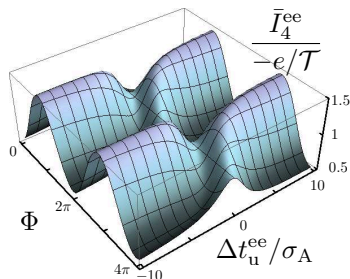
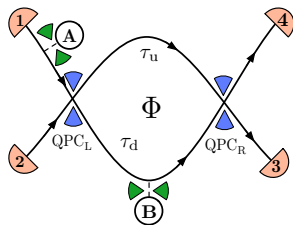
Time-averaged charge current

$$\frac{\bar{I}_4^{ee}}{-e/\mathcal{T}} = R_L R_R + T_L T_R + T_R \\ - 2\gamma \text{Re} \left\{ e^{-i\phi} \frac{-2i\sigma_A}{\Delta\tau - 2i\sigma_A} \left(1 - \frac{-2i\sigma_B}{\Delta t_u^{ee} - i(\sigma_A + \sigma_B)} \right) \right\}$$

Two-particle effect visible in a single-particle quantity!

(Similar for h-h collision at QPC R and for e-h collision (absorption) at source B)

Single-particle interference versus two-particle collision



Two interpretations:

- 1.) Collision of electrons at QPC R suppresses fluctuations due to the Hong-Ou-Mandel effect!

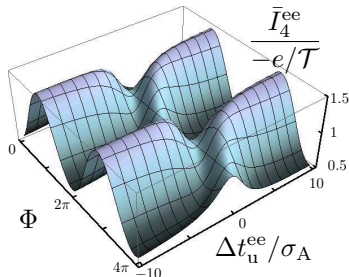
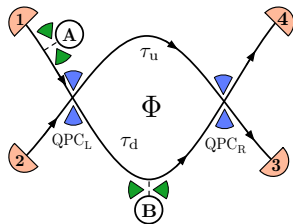
Noise at $\Delta t_u^{ee} = 0$:

$$2T_L \cdot T_R R_R + T_L R_L (T_R - R_R)^2$$

Noise of two particles at QPC R Noise of QPC L (in presence of QPC R)

⇒ Which-path detection suppressing interference.

Single-particle interference versus two-particle collision

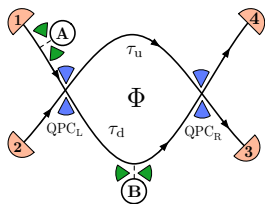


Two interpretations:

- 1.) Collision of electrons at QPC R suppresses fluctuations due to the Hong-Ou-Mandel effect!
 \Rightarrow Which-path detection suppressing interference.
- 2.) Time-deference Δt_d^{ee} introduces new energy-scale $\hbar/\Delta t_d^{ee}$ on which the spectral current oscillates.
 \Rightarrow Averaging effect from the channelled spectrum of the **combined** time-dependent scatterer (MZI + Source B).

Counter-intuitive results from the energy current

Counter-intuitive results from the energy current



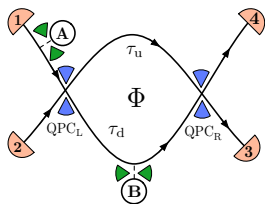
Energy current J

carried by electrons (and holes)
in the vicinity of e-e collision

Independent transmission/reflection of energy injected from the sources.

$$\begin{aligned} \bar{j}^{ee} = & \frac{\hbar}{2\sigma_A \mathcal{T}} (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B \mathcal{T}} T_R \\ & + T_L T_R \left(\frac{\hbar}{2\sigma_A \mathcal{T}} + \frac{\hbar}{2\sigma_B \mathcal{T}} \right) \frac{4\sigma_A \sigma_B}{\Delta t_d^{ee2} + (\sigma_A + \sigma_B)^2} \\ & - 2\gamma \frac{\hbar}{2\sigma_A \mathcal{T}} \operatorname{Re} \left\{ e^{-i\phi} \left(\frac{-2i\sigma_A}{\Delta\tau - 2i\sigma_A} \right)^2 \left(1 - \frac{-2i\sigma_B}{\Delta t_U^{ee} - i(\sigma_A + \sigma_B)} \right) \right\} \\ & + 2\gamma \frac{\hbar}{2\sigma_B \mathcal{T}} \operatorname{Re} \left\{ e^{-i\phi} \frac{-2i\sigma_A}{\Delta\tau - 2i\sigma_A} \left(\frac{-2i\sigma_B}{\Delta t_U^{ee} - i(\sigma_A + \sigma_B)} \right)^2 \right\} \end{aligned}$$

Counter-intuitive results from the energy current



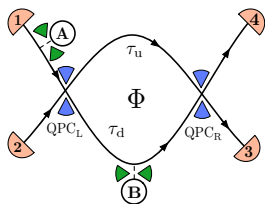
Energy current J

carried by electrons (and holes)
in the vicinity of e-e collision

Increased energy current when particle B is injected on top of particle A.

$$\begin{aligned} \mathcal{J}^{ee} &= \frac{\hbar}{2\sigma_A \mathcal{T}} (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B \mathcal{T}} T_R \\ &+ T_L T_R \left(\frac{\hbar}{2\sigma_A \mathcal{T}} + \frac{\hbar}{2\sigma_B \mathcal{T}} \right) \frac{4\sigma_A \sigma_B}{\Delta t_d^{ee^2} + (\sigma_A + \sigma_B)^2} \\ &- 2\gamma \frac{\hbar}{2\sigma_A \mathcal{T}} \operatorname{Re} \left\{ e^{-i\phi} \left(\frac{-2i\sigma_A}{\Delta \mathcal{T} - 2i\sigma_A} \right)^2 \left(1 - \frac{-2i\sigma_B}{\Delta t_u^{ee} - i(\sigma_A + \sigma_B)} \right) \right\} \\ &+ 2\gamma \frac{\hbar}{2\sigma_B \mathcal{T}} \operatorname{Re} \left\{ e^{-i\phi} \frac{-2i\sigma_A}{\Delta \mathcal{T} - 2i\sigma_A} \left(\frac{-2i\sigma_B}{\Delta t_u^{ee} - i(\sigma_A + \sigma_B)} \right)^2 \right\} \end{aligned}$$

Counter-intuitive results from the energy current



Energy current J

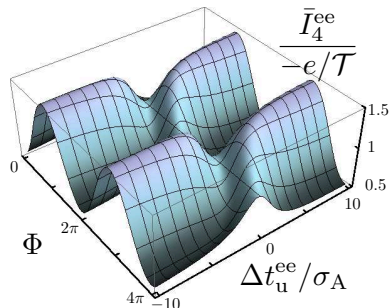
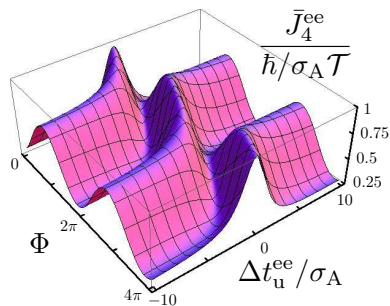
carried by electrons (and holes)
in the vicinity of e-e collision

**Impact on interference part - but no suppression!! -
when electrons collide at QPC R.**

$$\begin{aligned} \bar{j}^{ee} &= \frac{\hbar}{2\sigma_A \mathcal{T}} (R_L R_R + T_L T_R) + \frac{\hbar}{2\sigma_B \mathcal{T}} T_R \\ &+ T_L T_R \left(\frac{\hbar}{2\sigma_A \mathcal{T}} + \frac{\hbar}{2\sigma_B \mathcal{T}} \right) \frac{4\sigma_A \sigma_B}{\Delta t_d^{ee2} + (\sigma_A + \sigma_B)^2} \\ &- 2\gamma \frac{\hbar}{2\sigma_A \mathcal{T}} \operatorname{Re} \left\{ e^{-i\phi} \left(\frac{-2i\sigma_A}{\Delta\tau - 2i\sigma_A} \right)^2 \left(1 - \frac{-2i\sigma_B}{\Delta t_u^{ee} - i(\sigma_A + \sigma_B)} \right) \right\} \\ &+ 2\gamma \frac{\hbar}{2\sigma_B \mathcal{T}} \operatorname{Re} \left\{ e^{-i\phi} \frac{-2i\sigma_A}{\Delta\tau - 2i\sigma_A} \left(\frac{-2i\sigma_B}{\Delta t_u^{ee} - i(\sigma_A + \sigma_B)} \right)^2 \right\} \end{aligned}$$

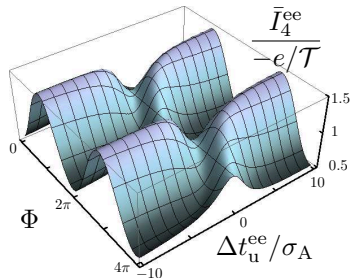
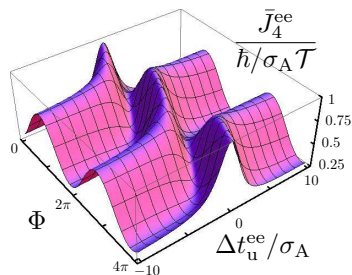
Counter-intuitive results from the energy current

Impact on interference part - **but no suppression!!** -
when electrons collide at QPC R.



Separation of charge/energy current due to the electronic HOM effect!

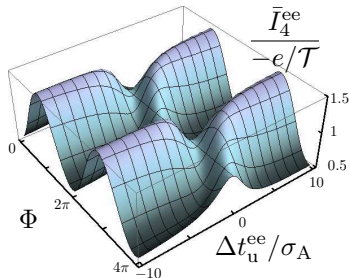
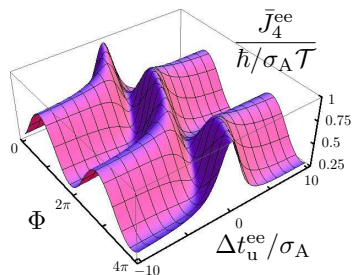
Counter-intuitive results from the energy current



For electron-electron collisions ($\Delta t_u^{ee} \approx 0$):

- Interference in energy current - **no** interference in charge current.
No consistent particle picture can be employed!
(separation, sometimes debated as "quantum cheshire cat".)

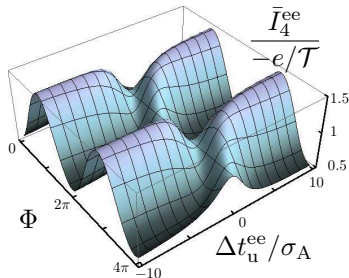
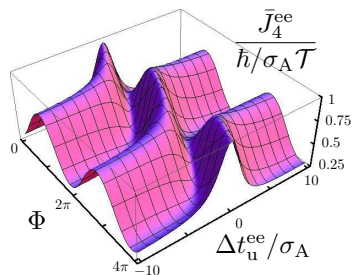
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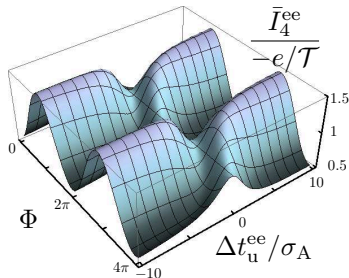
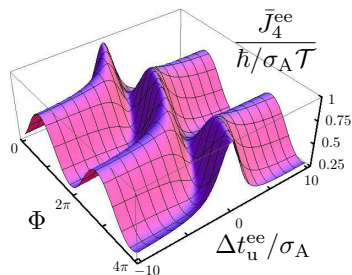
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- No which-path information from increased energy current at $\Delta t_d^{ee} \approx 0$.
- Spectral current oscillates on energy scales $\hbar/\Delta\tau$ and $\hbar/\Delta t_d^{ee}$
 \Rightarrow exactly combines to $\Delta t_u^{ee} = \Delta\tau + \Delta t_d^{ee}$ in interference part.

Counter-intuitive results from the energy current



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- Spectral current oscillates on energy scales $\hbar/\Delta\tau$ and $\hbar/\Delta t_d^{ee}$
 \Rightarrow exactly combines to $\Delta t_u^{ee} = \Delta\tau + \Delta t_d^{ee}$ in interference part.
- (Particle picture continues to hold for e-h absorption.)

Quantum optics with electrons:

- ▶ Single-particle sources allow to access single-particle coherence properties.
- ▶ Tunable few-particle effects.
- ▶ Proposals for on-chip fermionic single- and few-particle interference experiments.

- ▶ Time-dependent setup: Two-particle effects are visible in single-particle quantities.
- ▶ Charge/energy current separation.
- ▶ Particle-picture leads to paradoxical statements when considering the energy current.

G. Haack, M. Moskalets, J. Splettstoesser, and M. Büttiker, Phys. Rev. B **84**, 081303(R) (2011);
S. Juergens, J. Splettstoesser, and M. Moskalets, EPL **96**, 37011 (2011);
G. Rosselló, F. Battista, M. Moskalets, and J. Splettstoesser, Phys. Rev. B **91**, 115438 (2015).

Length scale estimates

Metallic (biased) source:

- Applied bias: $1\mu V$
 - Single-particle coherence time: $\hbar/eV \approx 4ns$
 - Drift velocity $\approx 10^4 m/s$
- $\Rightarrow \Lambda \approx 40\mu m$

Single-electron source:

- Driving frequency: $\Omega \approx GHz$
 - Transmission of the QOC, defining the source: $T \approx 0.1$
 - Pulse width - Single-particle coherence time: $T/\Omega \approx 10^{-10}s$
- $\Rightarrow \Lambda = v_D \Gamma \approx 1\mu m$

Single-particle coherence

$$C_{\text{SES}}(t, \Delta\tau) = \frac{-ie}{2\pi I_{\text{max}}} \frac{S_{\text{SES}}(t - \tau_{\text{u}}, \mu) S_{\text{SES}}^*(t - \tau_{\text{d}}, \mu) - 1}{\Delta\tau}$$

with

$$\Delta\tau = \tau_{\text{up}} - \tau_{\text{down}}$$

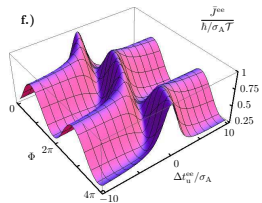
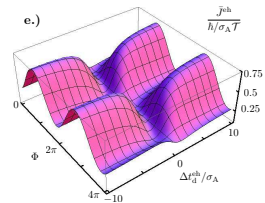
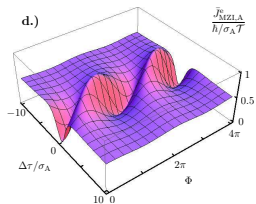
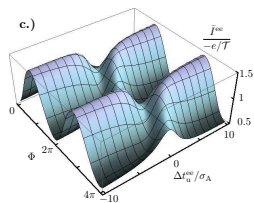
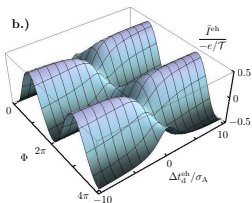
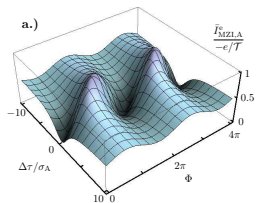
and

$$S_{\text{SES}} = S_{\text{SES}}(t) = \frac{\pm(t - t^{\pm}) - i\Gamma_{\text{SES}}}{\pm(t - t^{\pm}) + i\Gamma_{\text{SES}}}$$

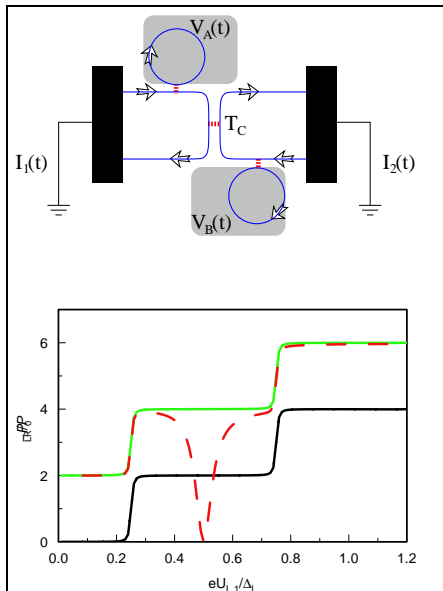
Spectral current with e-e collision

$$\begin{aligned}
 i^{ee}(E, \Phi) = & R_L R_R i_A^e(E) + T_L T_R i_A^e(E) \operatorname{Re} \left\{ 1 + \frac{4\sigma_A \sigma_B}{\Delta t_d^{ee2} + (\sigma_A - \sigma_B)^2} \right. \\
 & \left. - 2i\sigma_B \frac{\Delta t_d^{ee} - i(\sigma_A + \sigma_B)}{\Delta t_d^{ee2} + (\sigma_A - \sigma_B)^2} e^{-iE(\Delta t_d^{ee} + i(\sigma_A - \sigma_B))/\hbar} \right\} \\
 & + R_L T_R i_B^e(E) + T_L T_R i_B^e(E) \operatorname{Re} \left\{ 1 + \frac{4\sigma_A \sigma_B}{\Delta t_d^{ee2} + (\sigma_A - \sigma_B)^2} \right. \\
 & \left. - 2i\sigma_A \frac{\Delta t_d^{ee} - i(\sigma_A + \sigma_B)}{\Delta t_d^{ee2} + (\sigma_A - \sigma_B)^2} e^{-iE(\Delta t_d^{ee} + i(\sigma_B - \sigma_A))/\hbar} \right\} \\
 & - 2\gamma i_A^e(E) \operatorname{Re} \left\{ e^{-i\Phi} e^{-iE\Delta\tau} \left[1 + \frac{2i\sigma_B}{\Delta t_d^{ee} + i(\sigma_A - \sigma_B)} \right. \right. \\
 & \left. \left. (1 - e^{-iE(\Delta t_d^{ee} + i(\sigma_A - \sigma_B))/\hbar}) \right] \right\}
 \end{aligned}$$

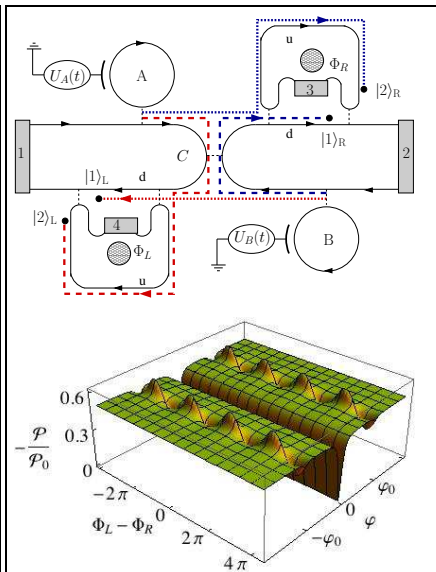
Currents in the presence of absorptions and collisions



Tunable two-particle effects



S. Ol'khovskaya, J. Splettstoesser, M. Moskalets, and M. Büttiker, Phys. Rev. Lett. **101**, 166802 (2008).



J. Splettstoesser, M. Moskalets, and M. Büttiker, Phys. Rev. Lett. **103**, 076804 (2009).