“Two-Phase Continuum Models for Geophysical Particle-Fluid Flows”
GeoFlow 16, March 14-18, 2016
Max Planck Institute for the Physics of Complex Systems
Dresden, Germany

Entrainment Processes

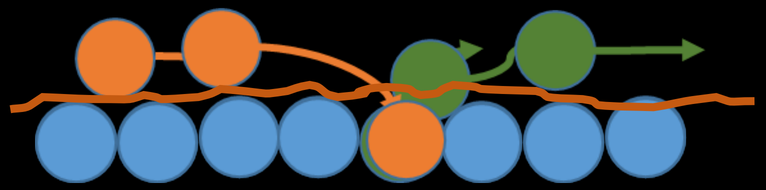
UNIVERSITÀ DEGLI STUDI DI TRENTO

DIPARTIMENTO DI INGEGNERIA CIVILE, AMBIENTALE E MECCANICA

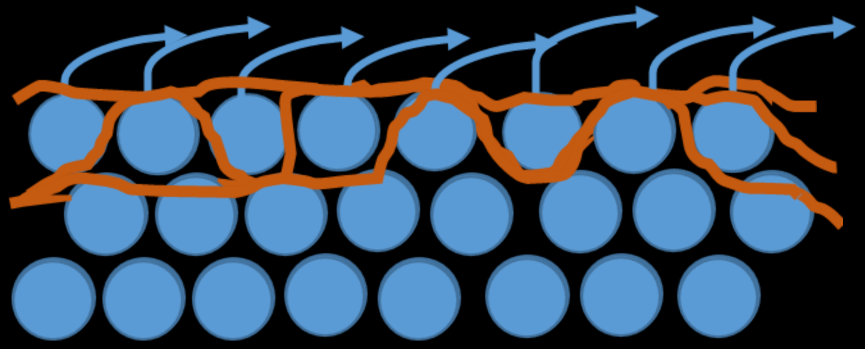
Luigi Fraccarollo

Entrainment Processes

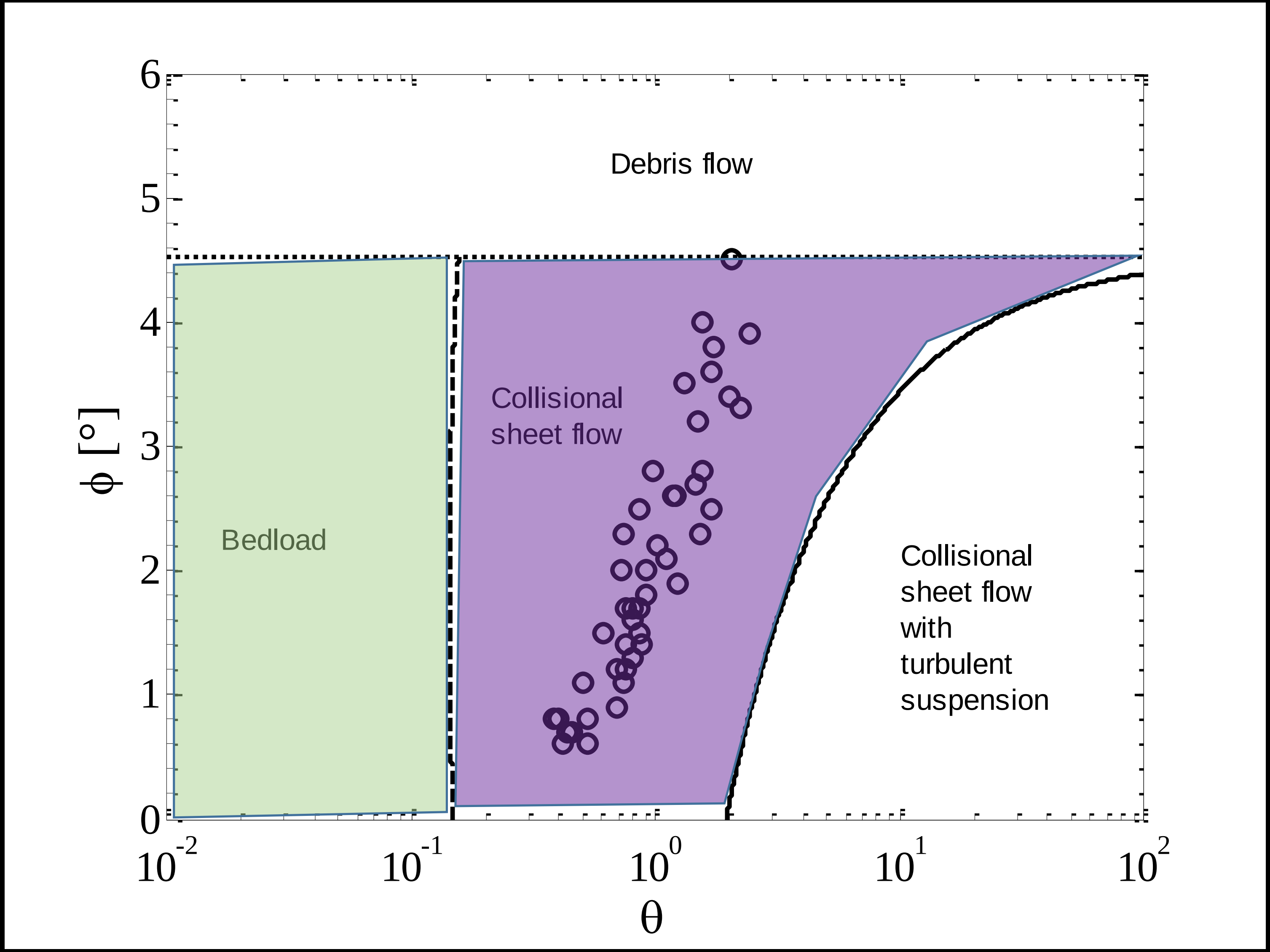
entrainment/disentrainment
with null total mass flux through bottom



entrainment or disentrainment
with net mass flux through bottom



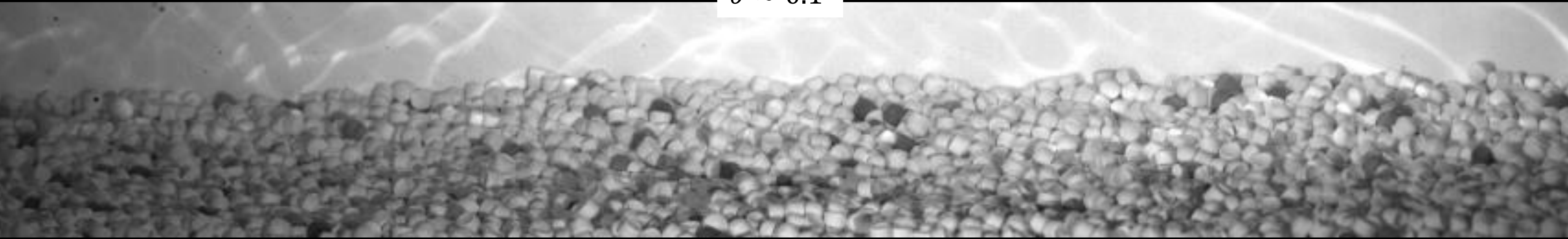
Regime Maps



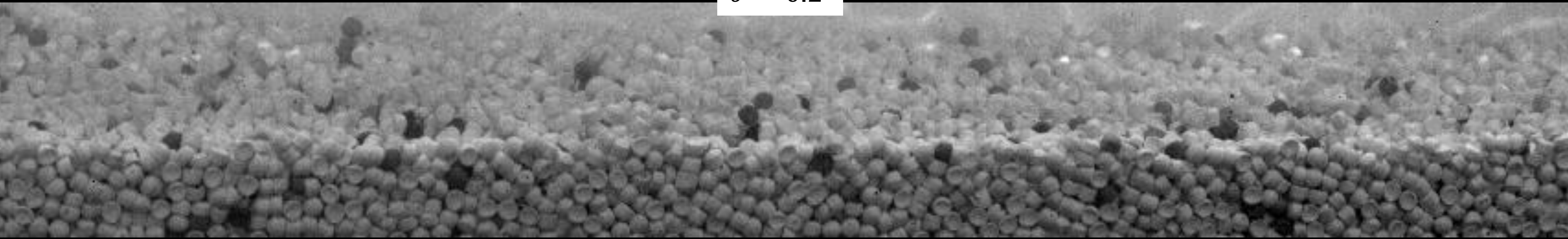
Berzi and Fraccarollo, PoF 2013

Ordinary Bedload

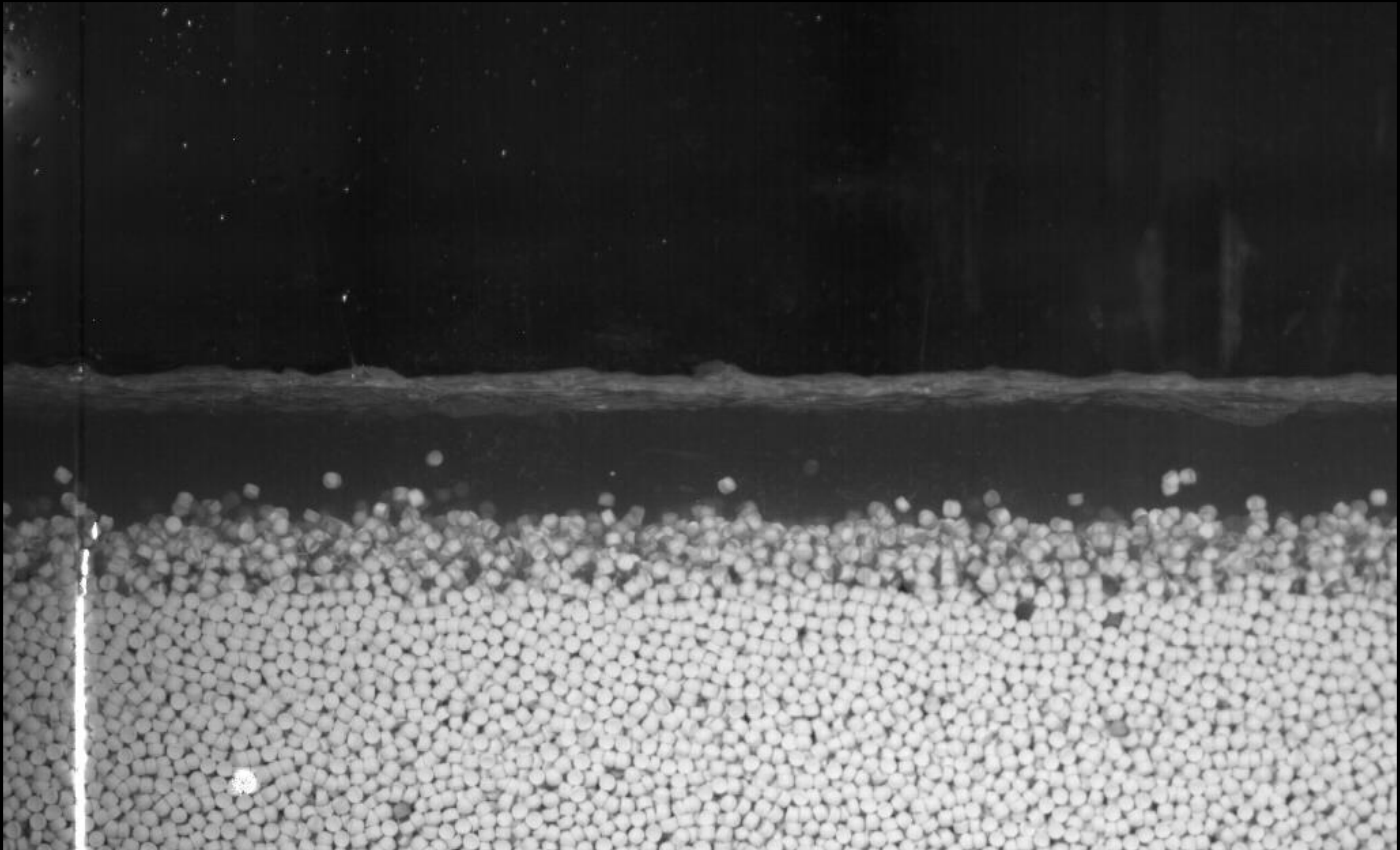
$\theta \approx 0.1$



$\theta \approx 0.2$

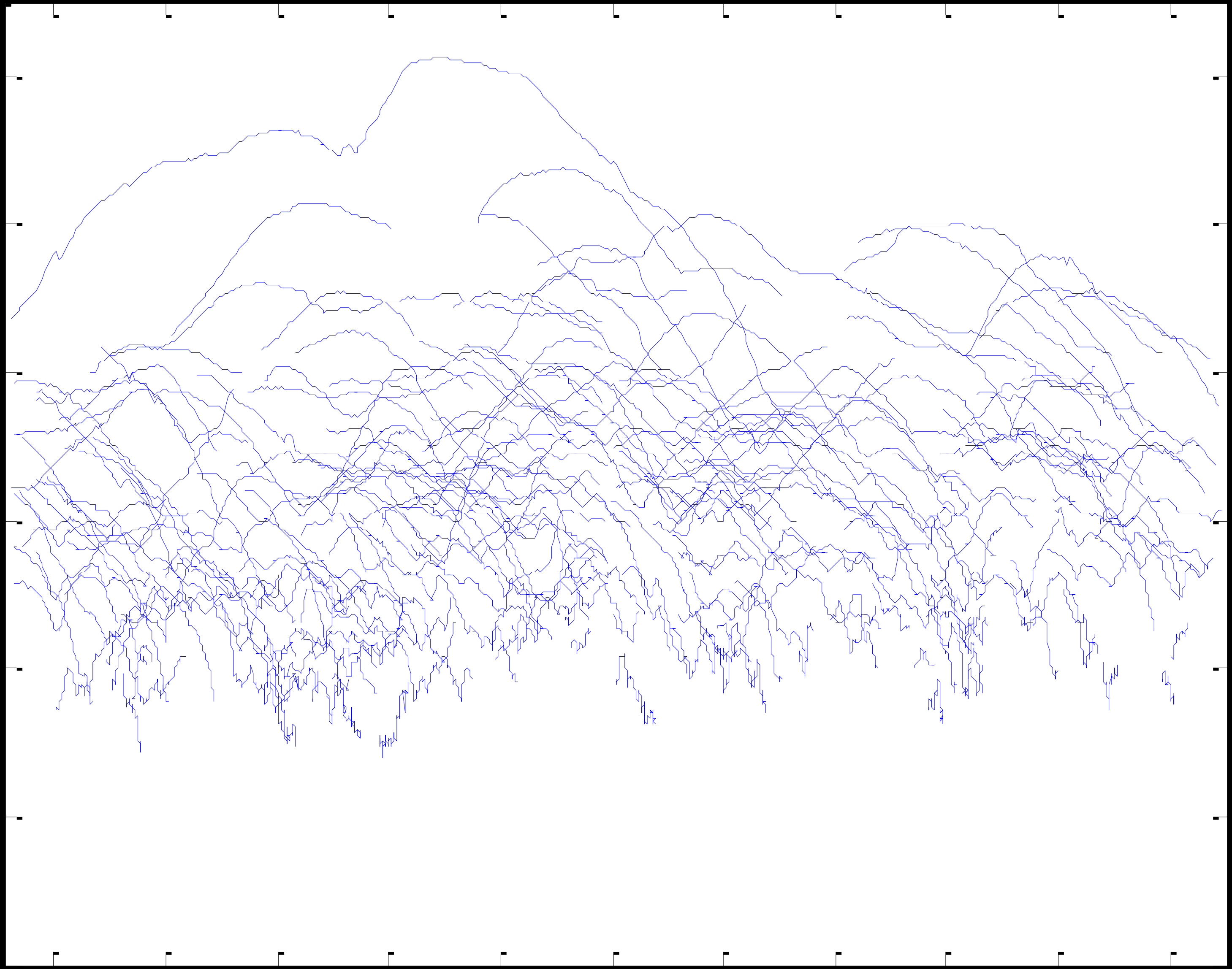


Shields ≈ 1



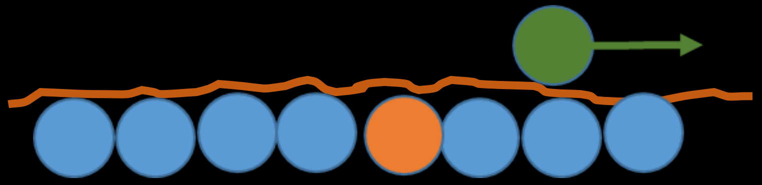
Collisional Bedload

Trajectories

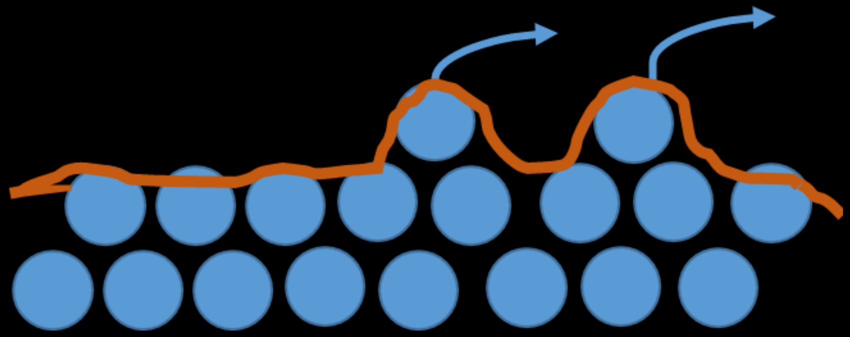


Entrainment Processes

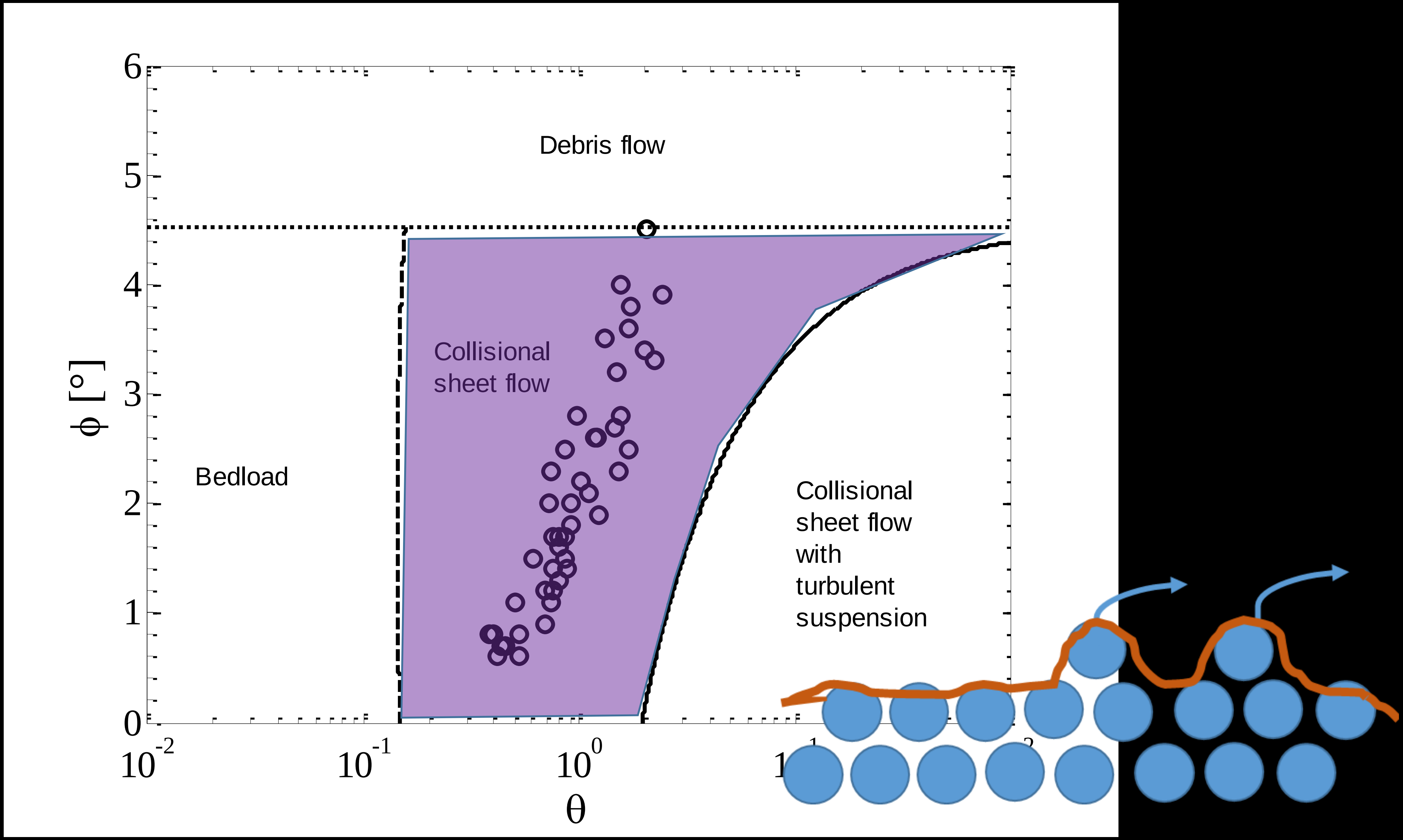
entrainment/disentrainment
with null total mass flux through bottom



entrainment or disentrainment
with net mass flux through bottom



Collisional BedLoad



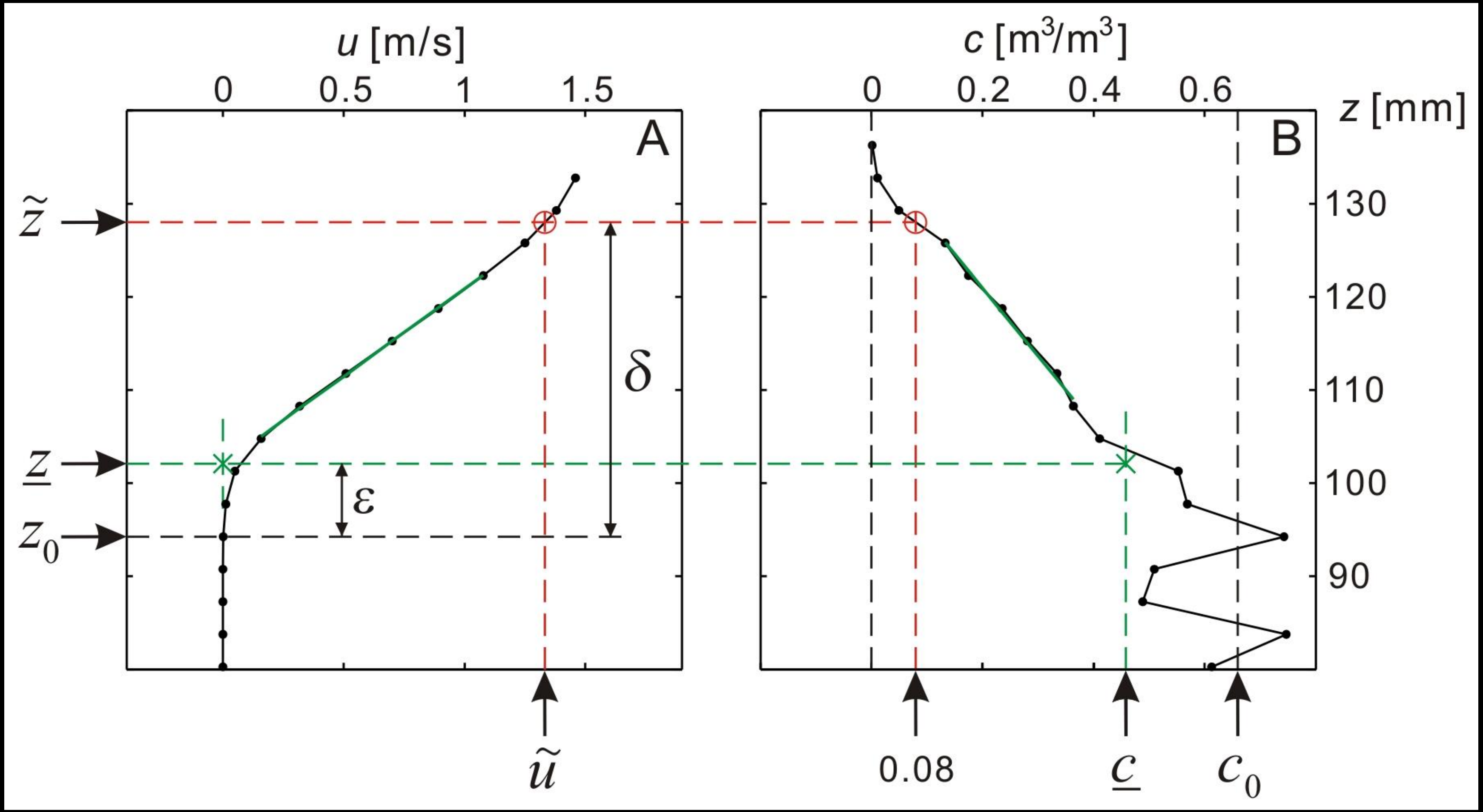
Collisional BedLoad

Available analytical theories (among others)

Capart H. & Fraccarollo L., 2011,
Geophysical Research Letters

Berzi D. & Fraccarollo L., 2013,
Physics of Fluids

Layer structure variables



Collisional BedLoad

Capart H. & Fraccarollo L., 2011,
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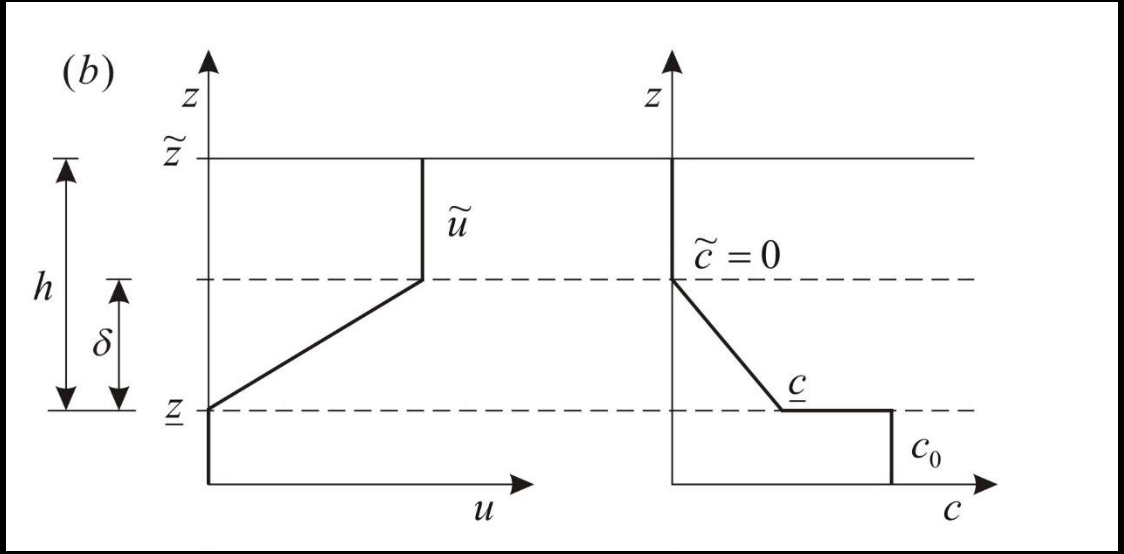
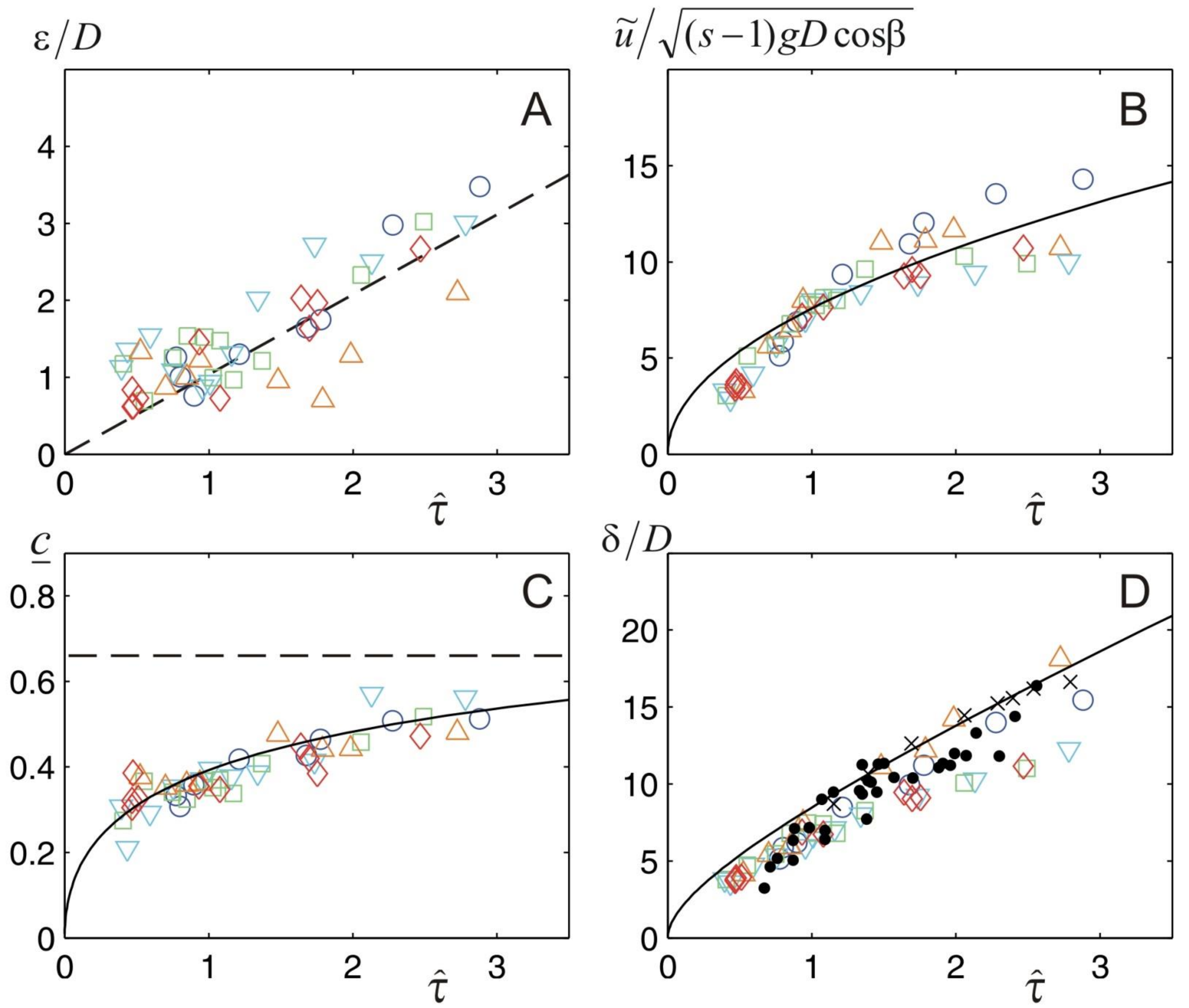


$$Ri = \frac{-g\cos\beta\,\mathrm{d}\rho/\mathrm{dz}}{\rho_W(\mathrm{d}u/\mathrm{dz})^2} = (s-1)\underline{c}\,g\,\delta\cos\beta/\tilde{u}^2$$

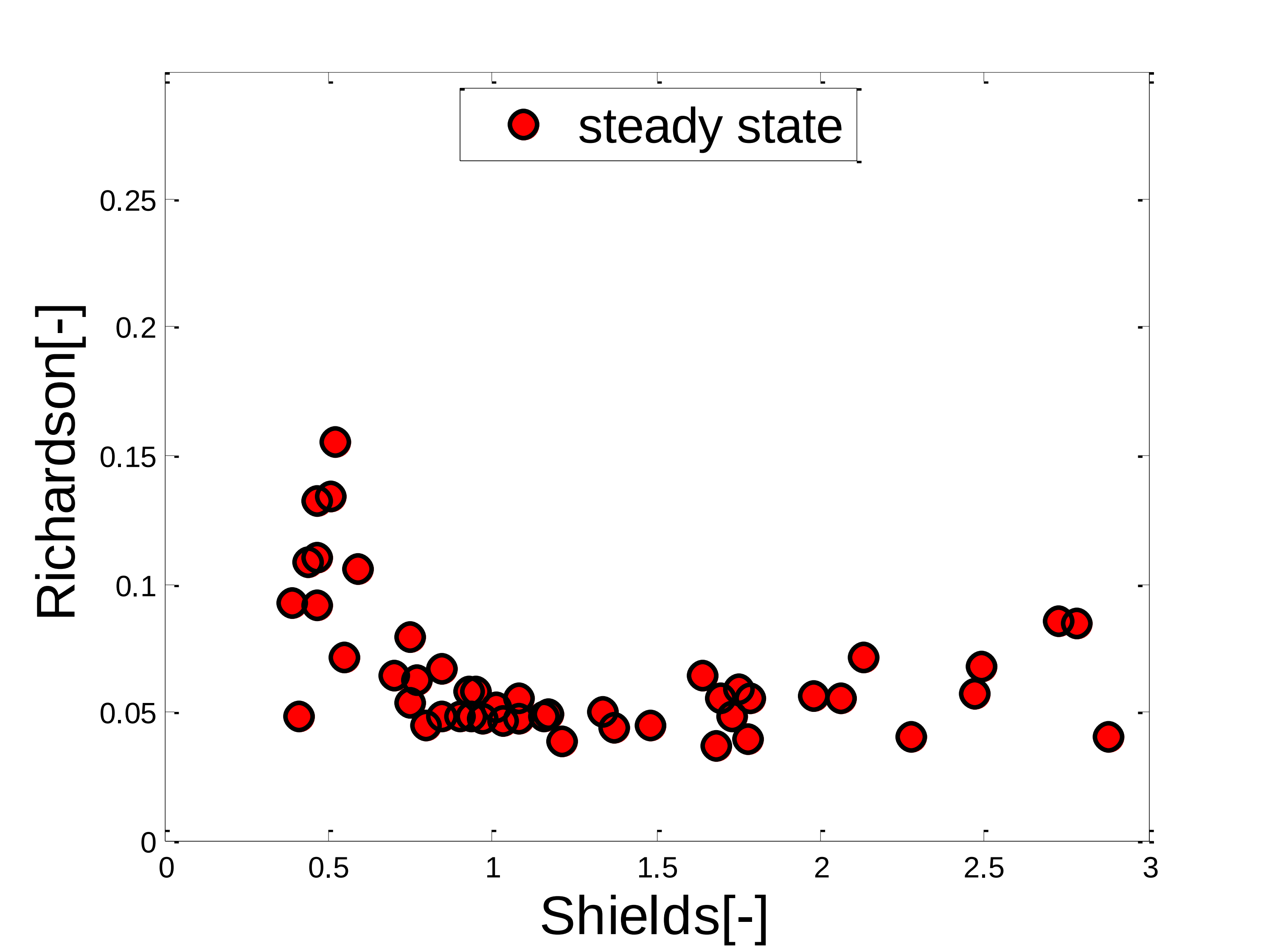
$$= 0.058$$

Collisional Bedload

Main outcomes of the theory



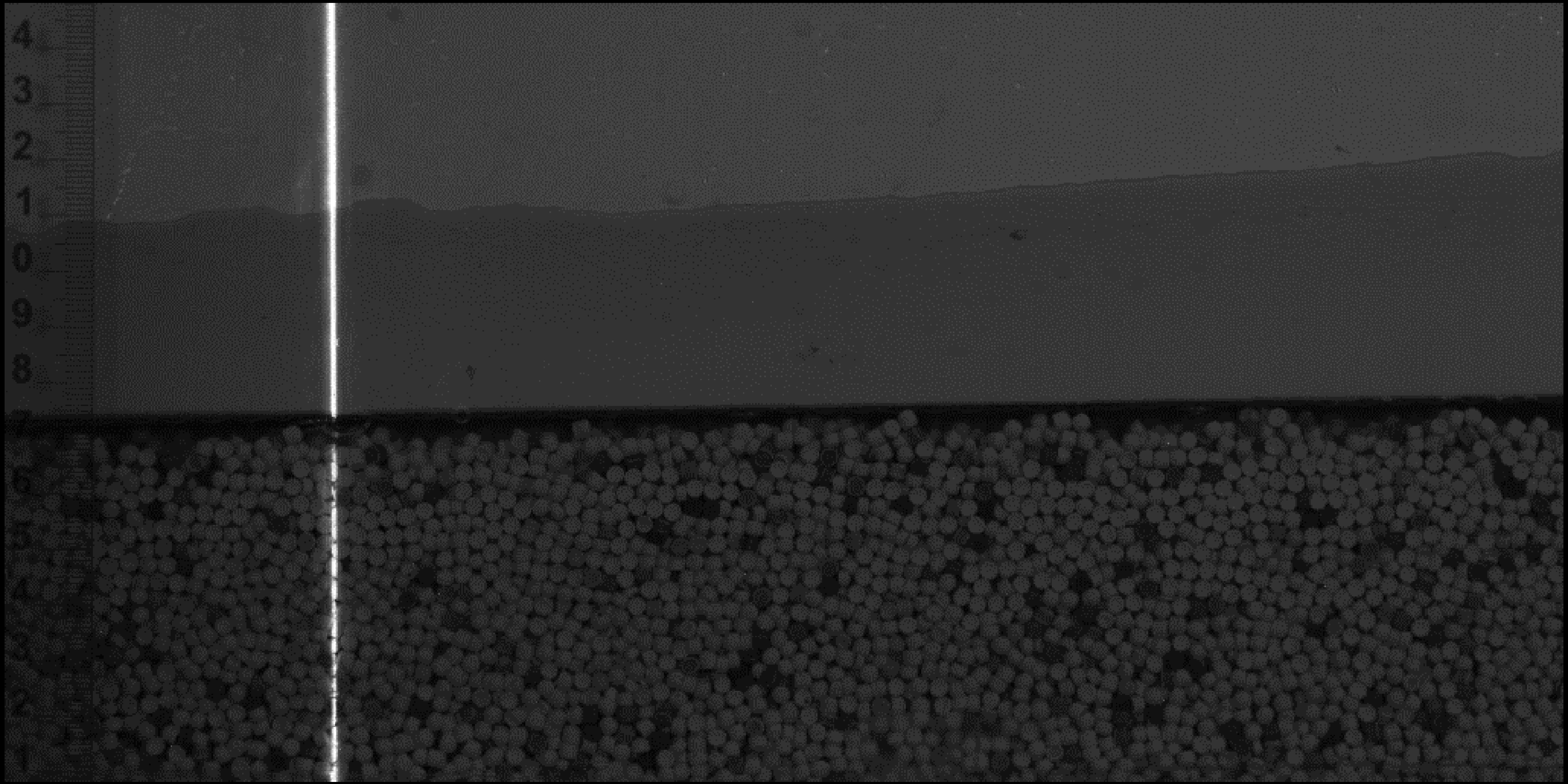
Collisional Bedload



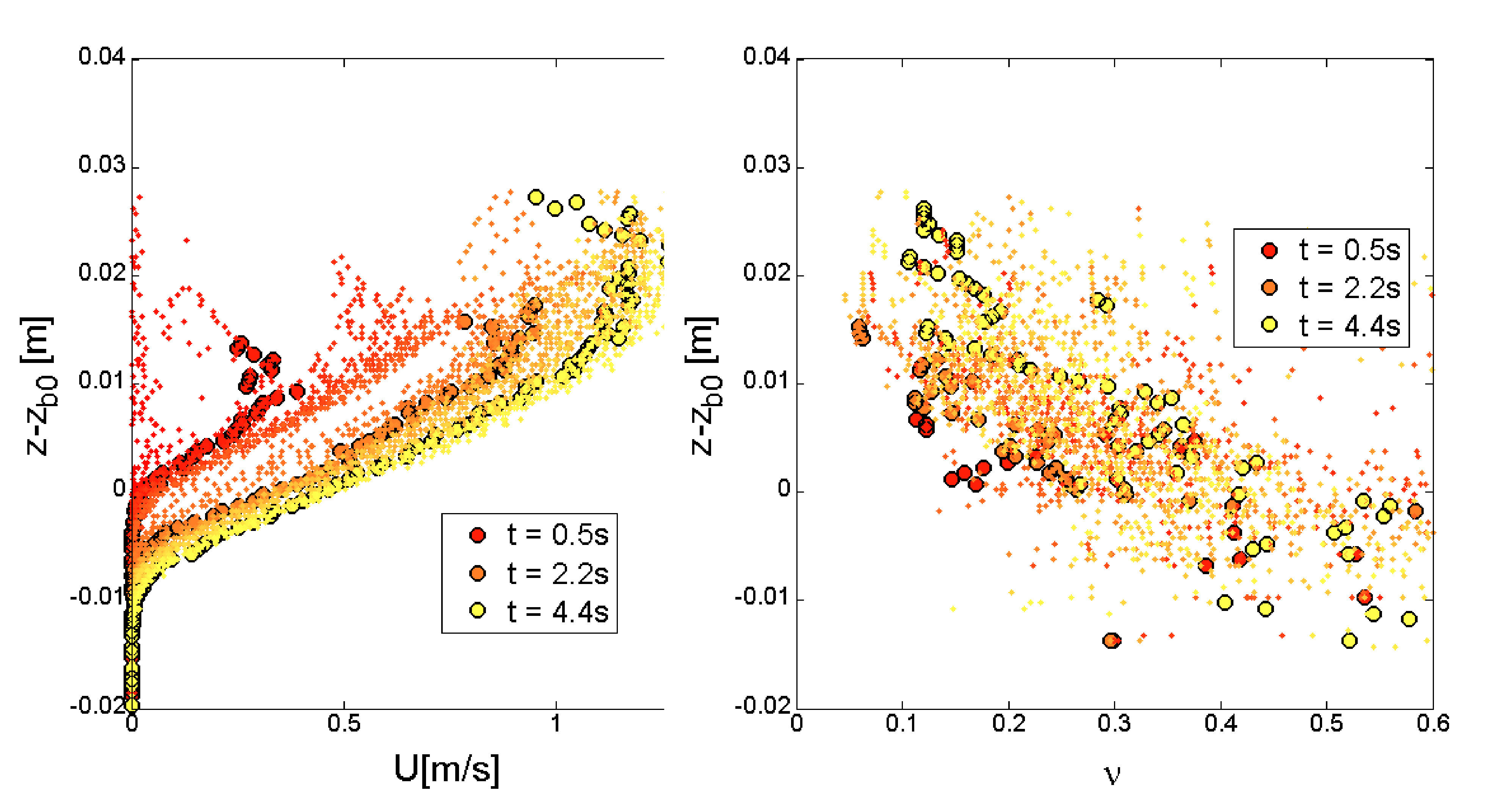
Uniform & Unsteady collisional bedload

MOVIE

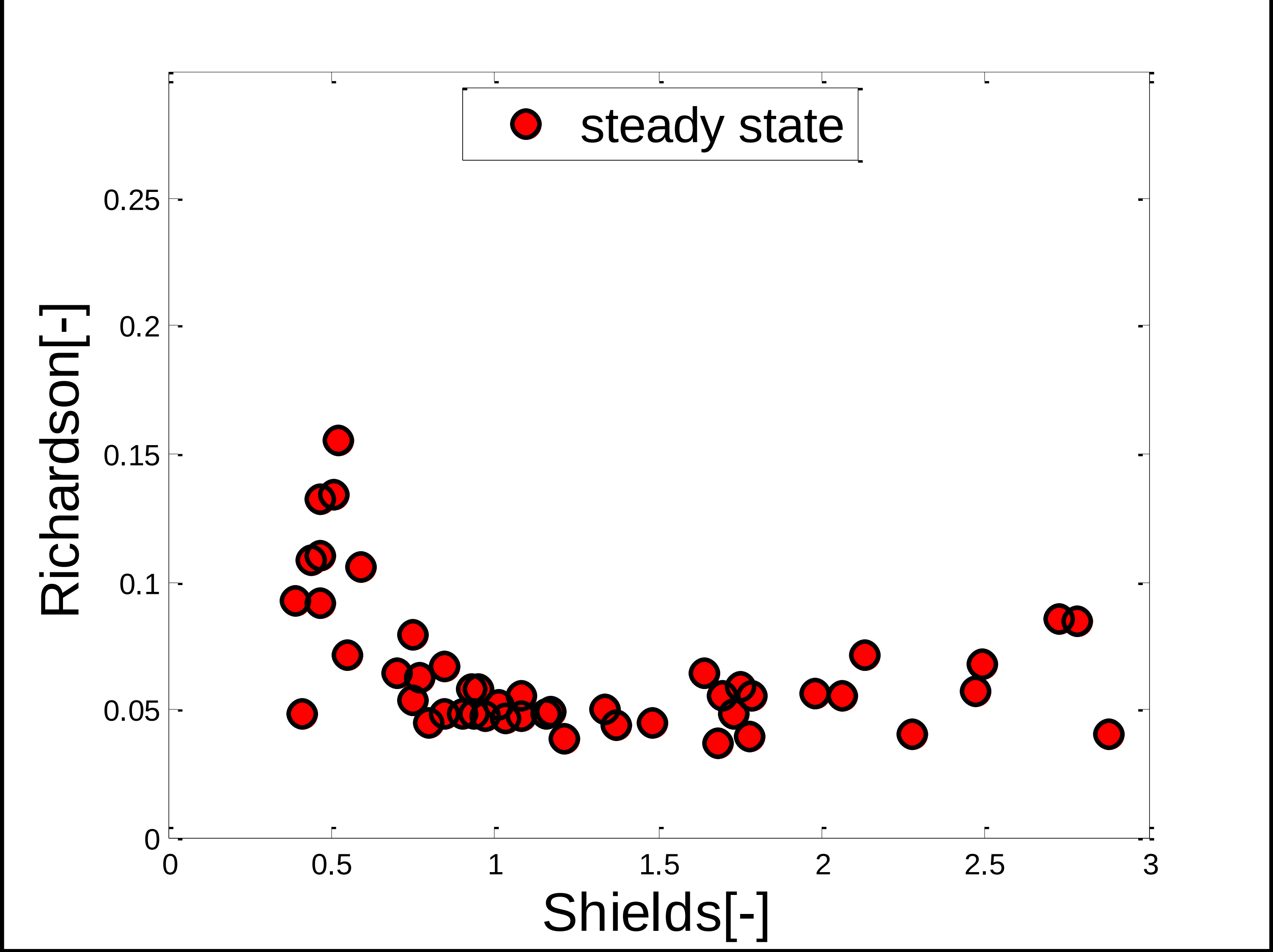
Uniform & Unsteady collisional bedload



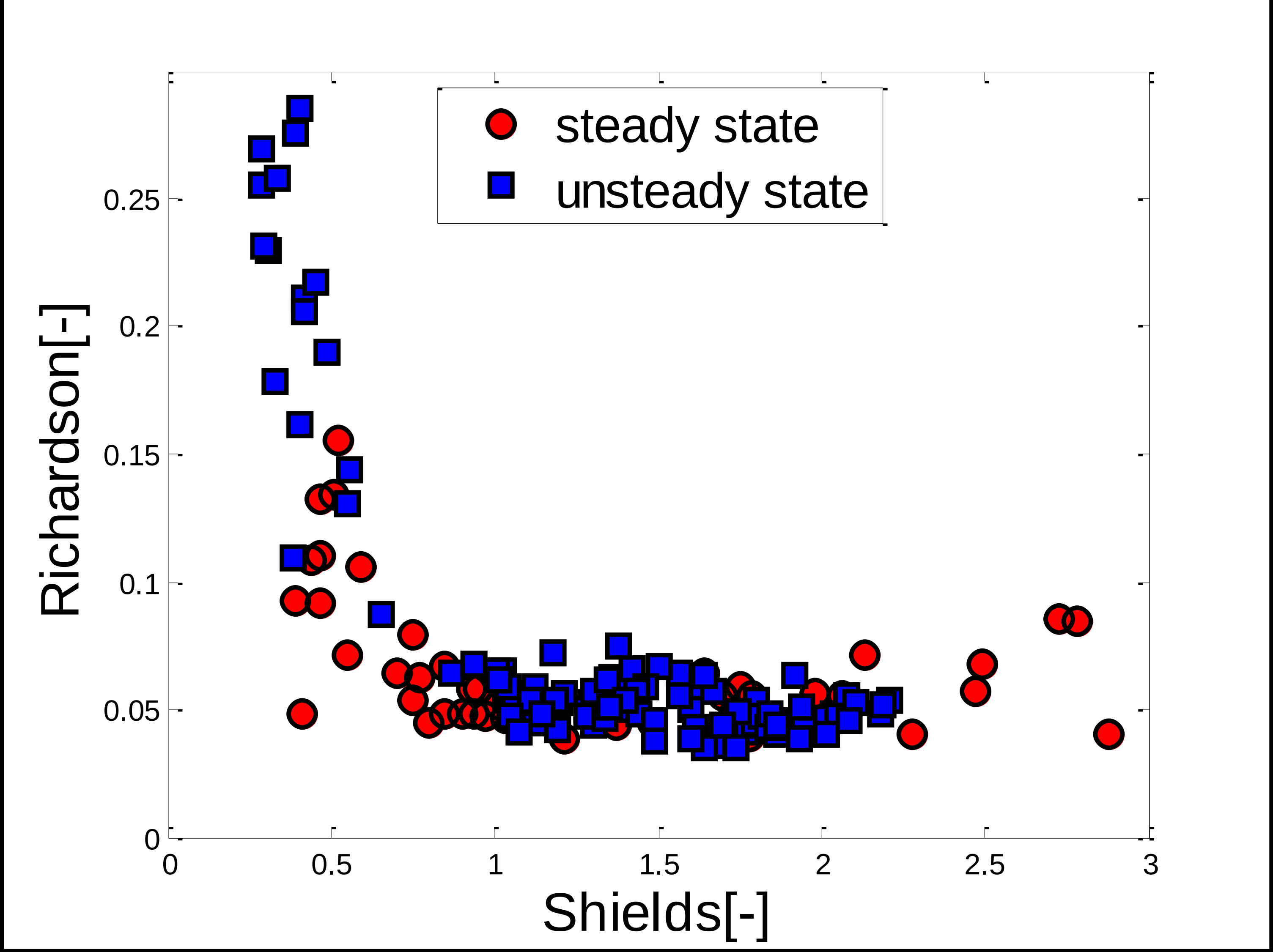
Collisional Bedload: uniform unsteady



Collisional Bedload: uniform unsteady



Collisional Bedload: uniform unsteady



Mathematical approach

GRL 2011 theory

Momentum equation in the y direction

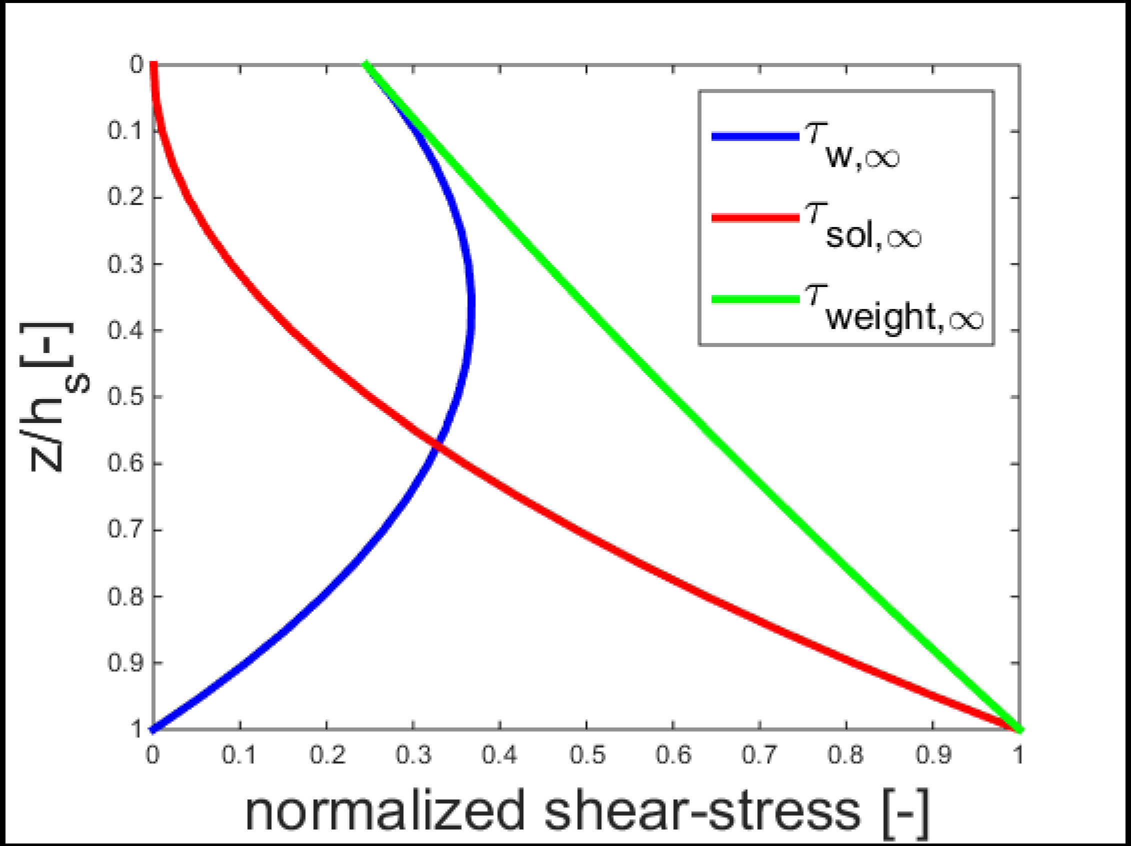
Energy equation

Continuity equations

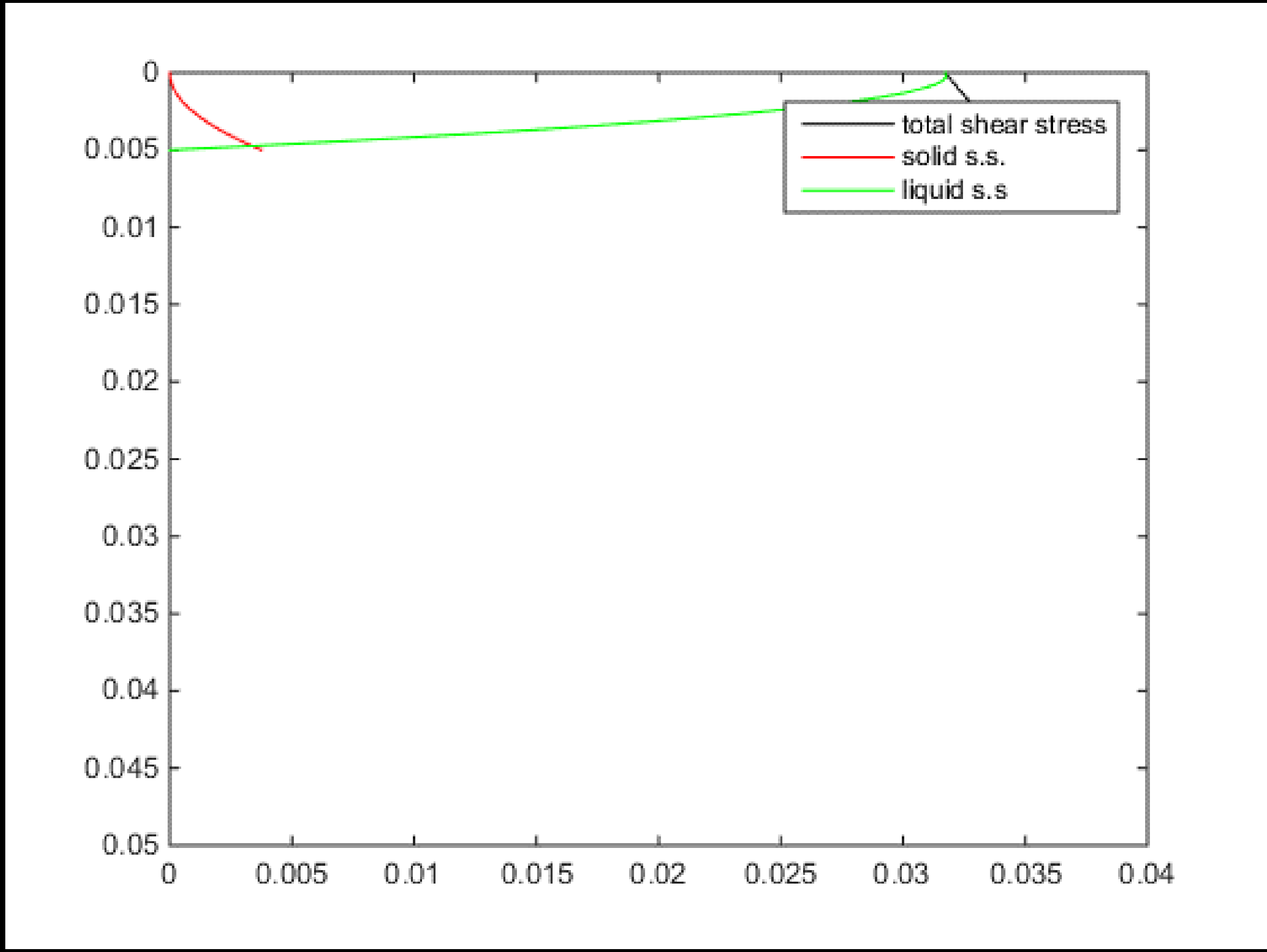
Similarity assumption for the variables involved: velocity, concentration

$$\begin{cases} f_1(v_s,h_s)\dfrac{dv_s}{dt}+f_2(v_s,h_s)\dfrac{dh_s}{dt}=f_3(v_s,h_s) \\ g_1(v_s,h_s)\dfrac{dv_s}{dt}+g_2(v_s,h_s)\dfrac{dh_s}{dt}=g_3(v_s,h_s) \end{cases}$$

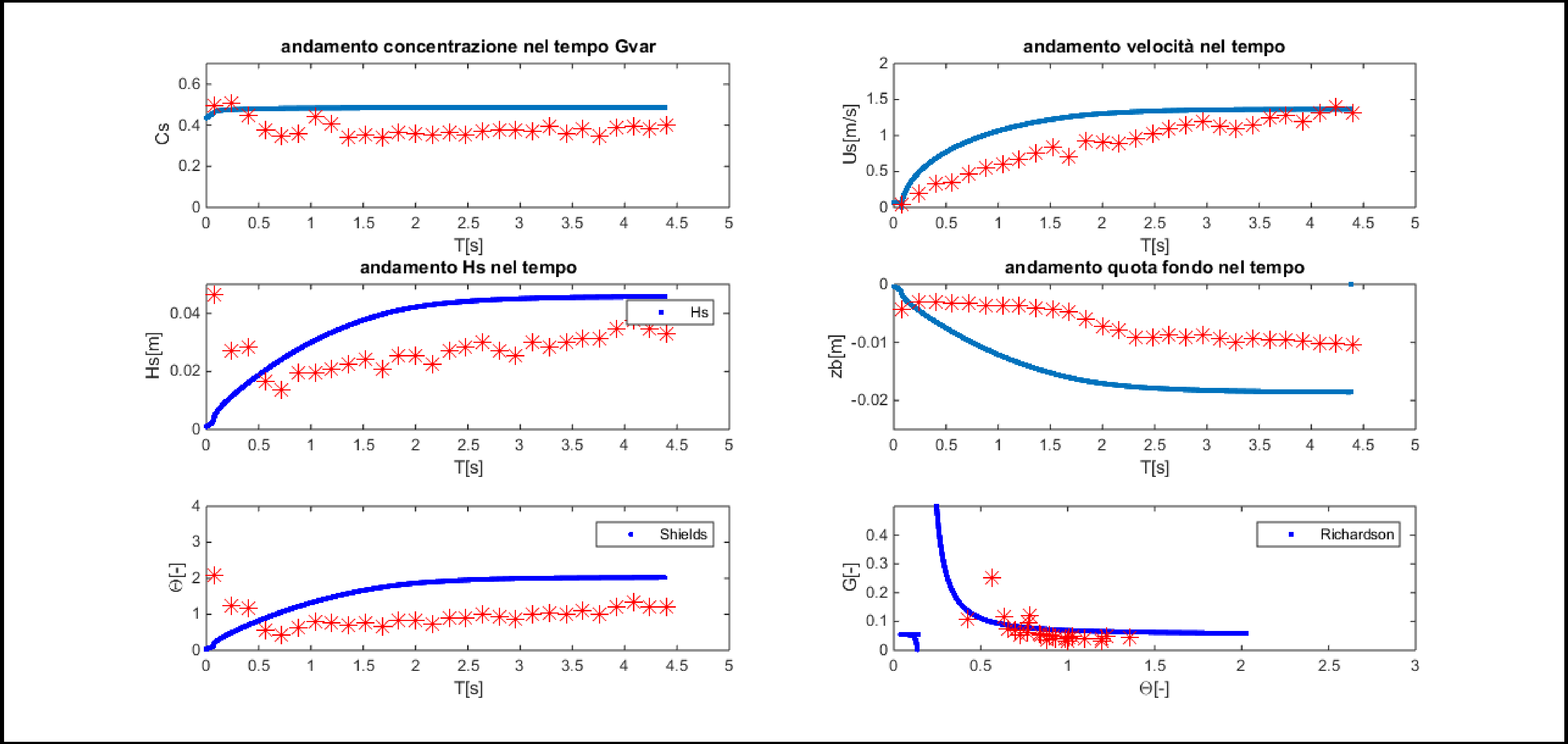
Results from the model



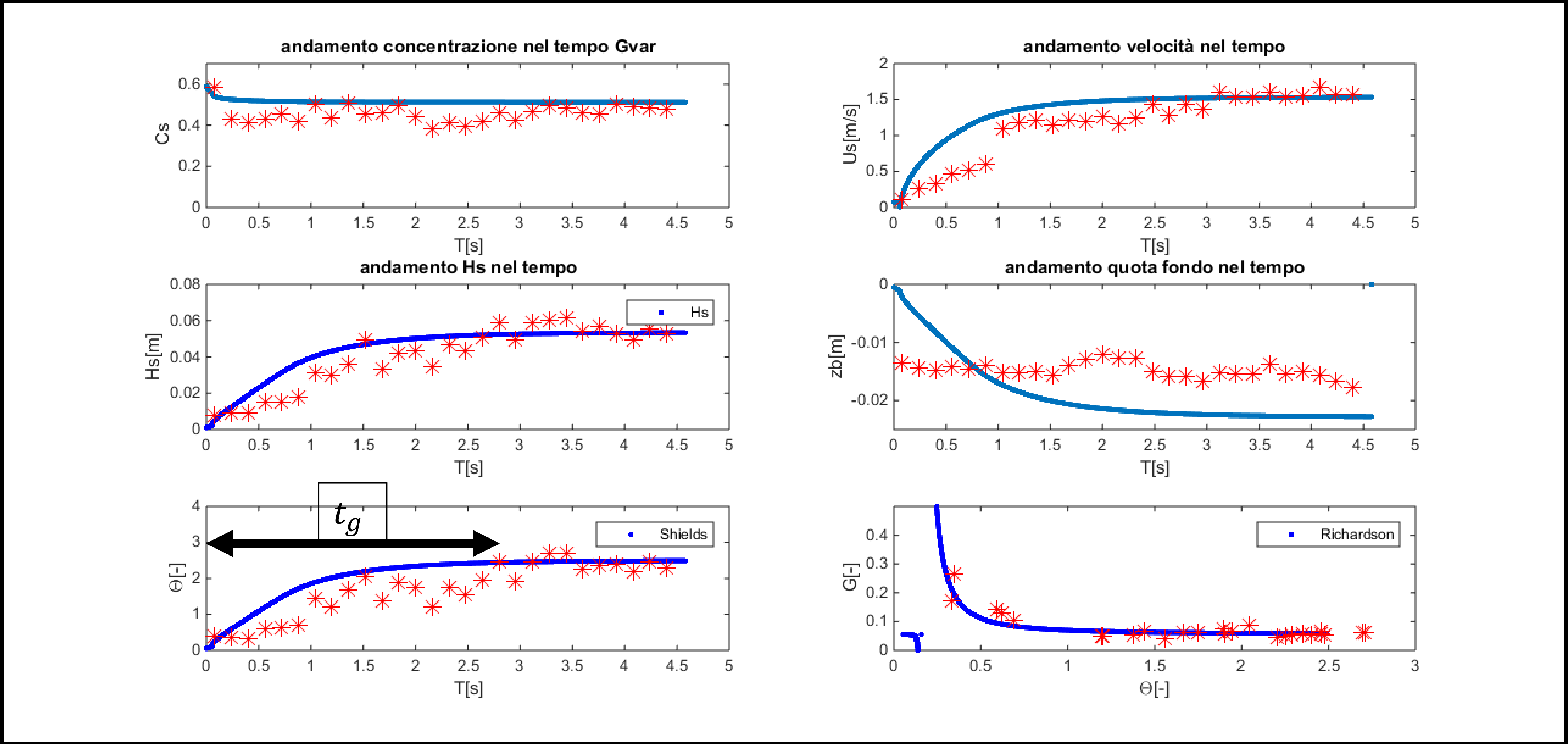
Results from the model



Comparison between model and experiment



Comparison between theory and experiment



t_g = geomorphic time scale

TWO-PHASE MODELING OF GRANULAR SEDIMENT FOR SHEET FLOWS

José María González Ondina, PhD Thesis

Cornell University 2015

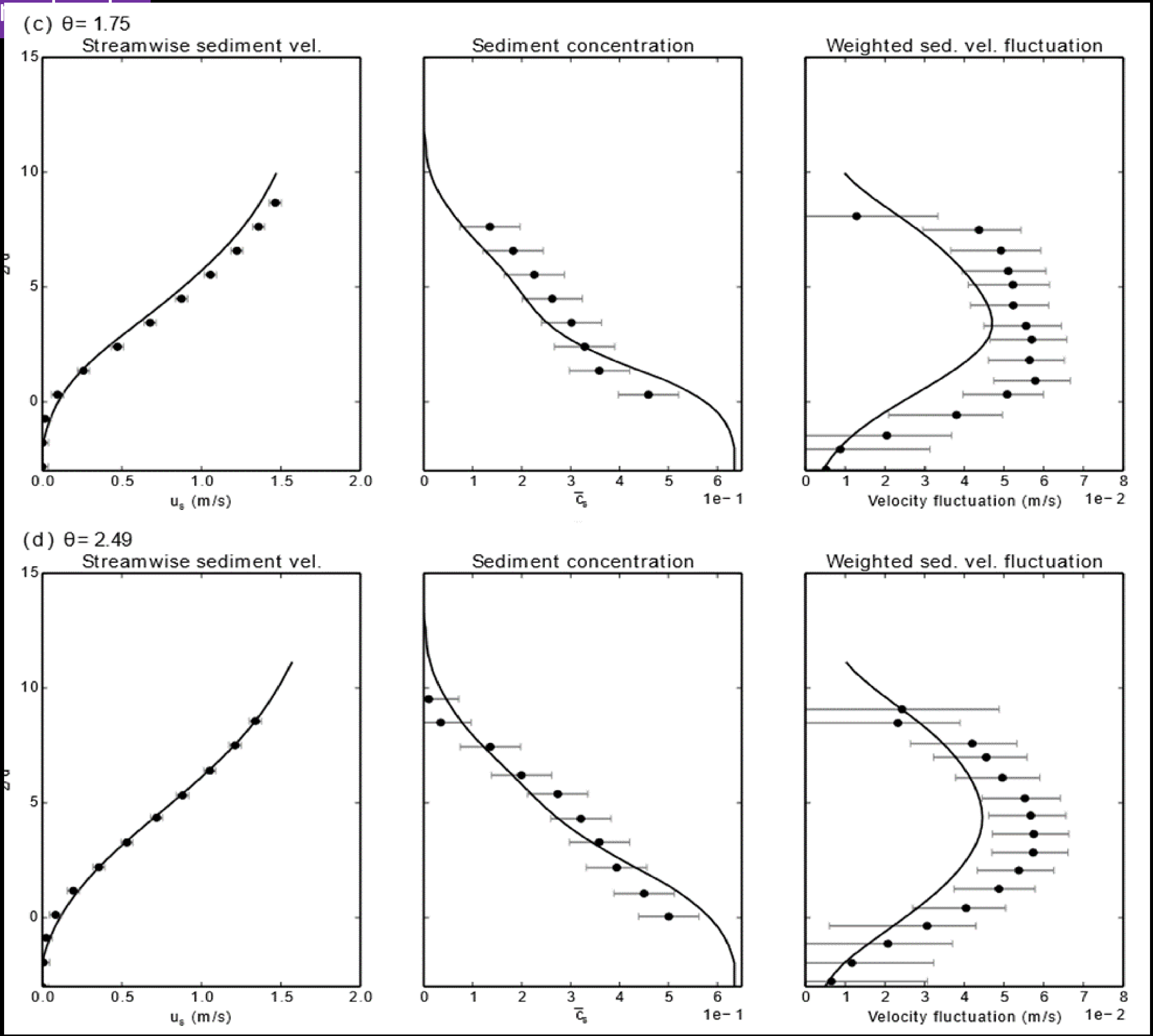
The thesis presents a description of sediment transport on two scale levels:

the large scale represents all physical processes that occur at length scales larger than the characteristic length scales of particles, where both phases can be seen as continuous

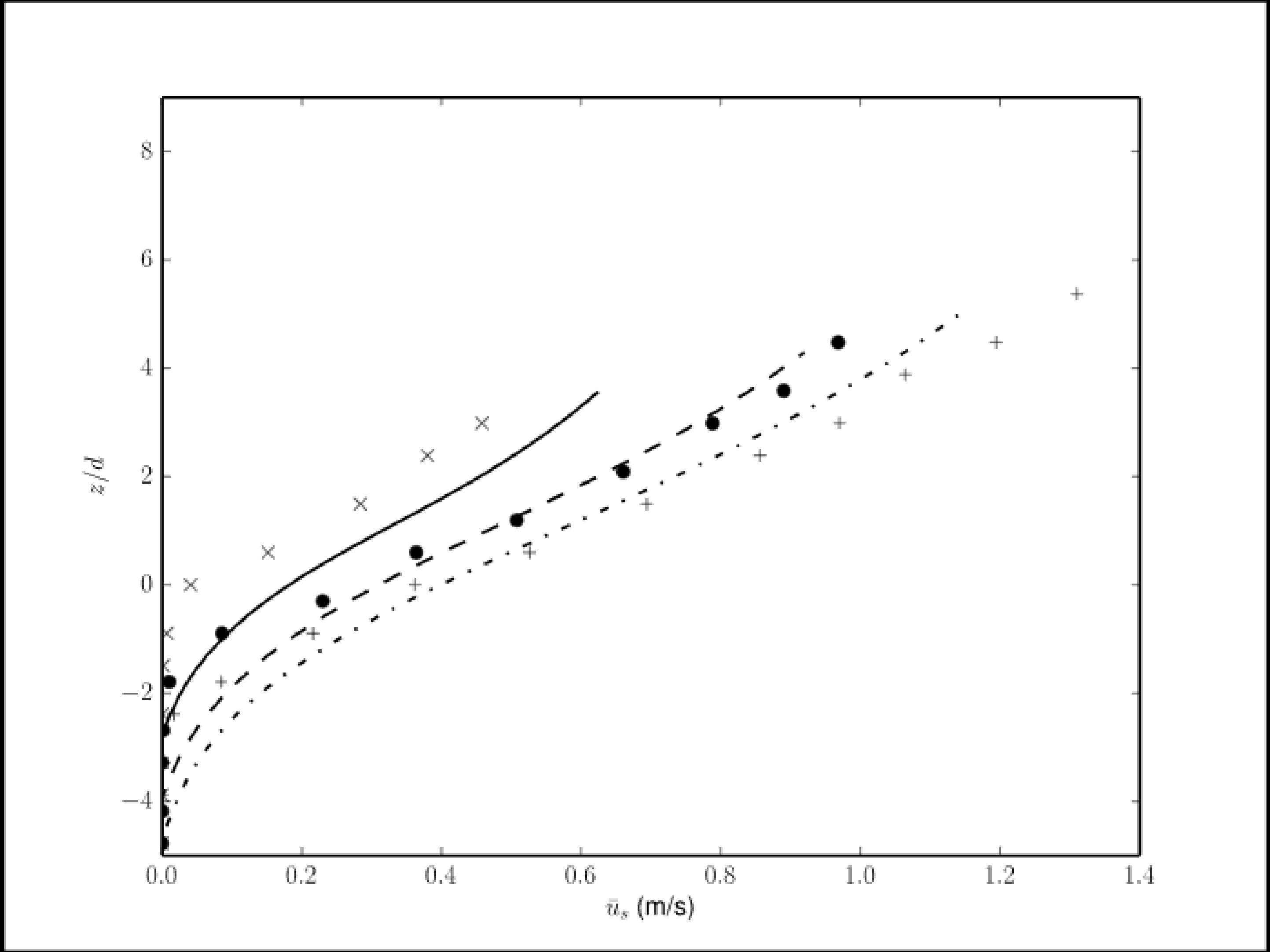
the small scale represents physical processes that occur at or below the length scale of particles including collisions and turbulence created or dissipated by them.

This approach results in more equations to be solved when compared to previous models this work is based on Amoudry, Hsu and Liu (2004); Hsu, Jenkins and Liu (2004).

Collisional Bedload: unifor



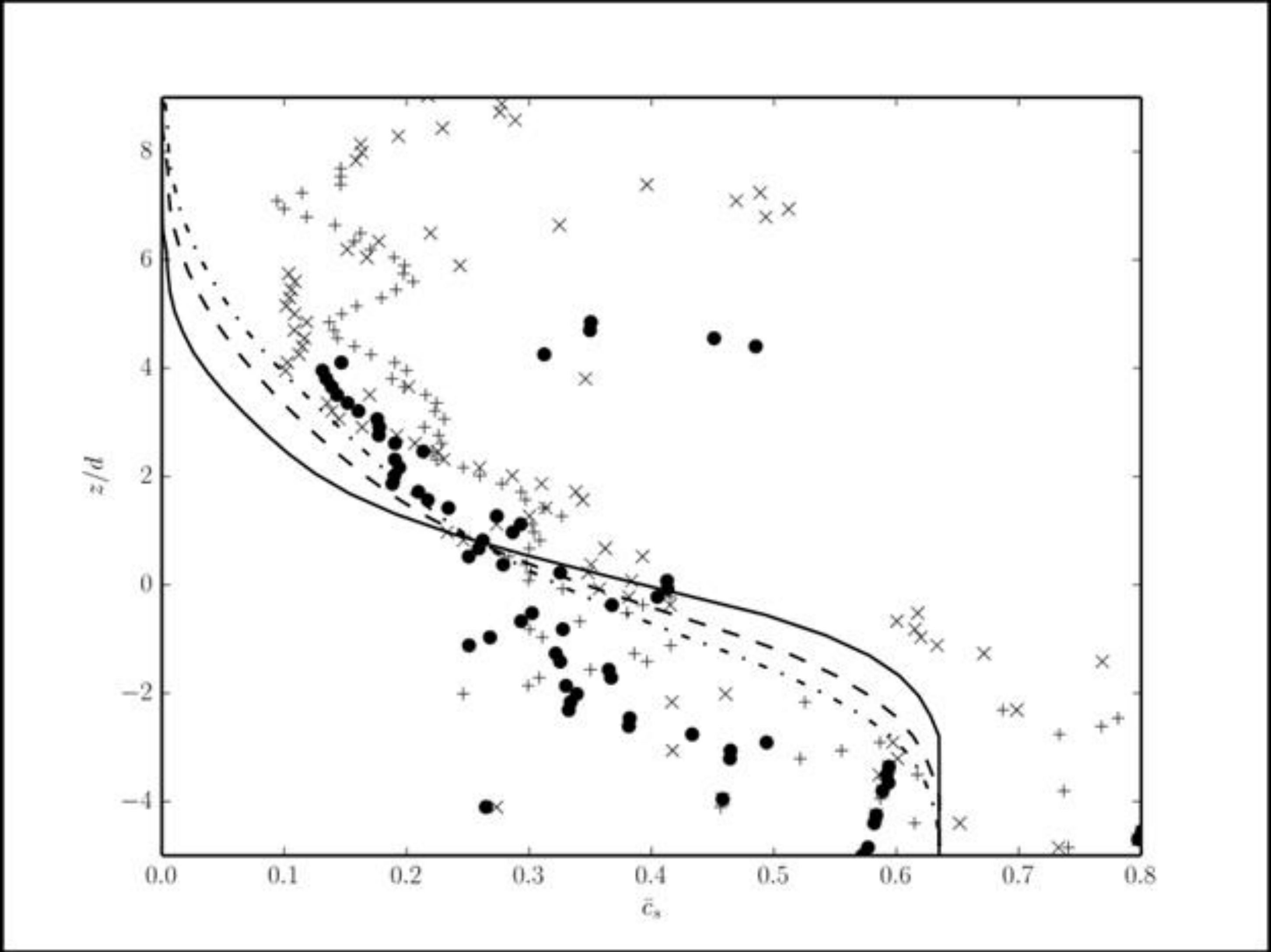
Collisional Bedload: uniform unsteady



Numerical model: Solid line is t=0.8 s, dashed line is t=2.72 s, dot-dashed line is t=4.8 s.

Experiments: (x) is t=0.8 s, (o) is t=2.72 s, (+) is t=4.8 s

Collisional Bedload: uniform unsteady



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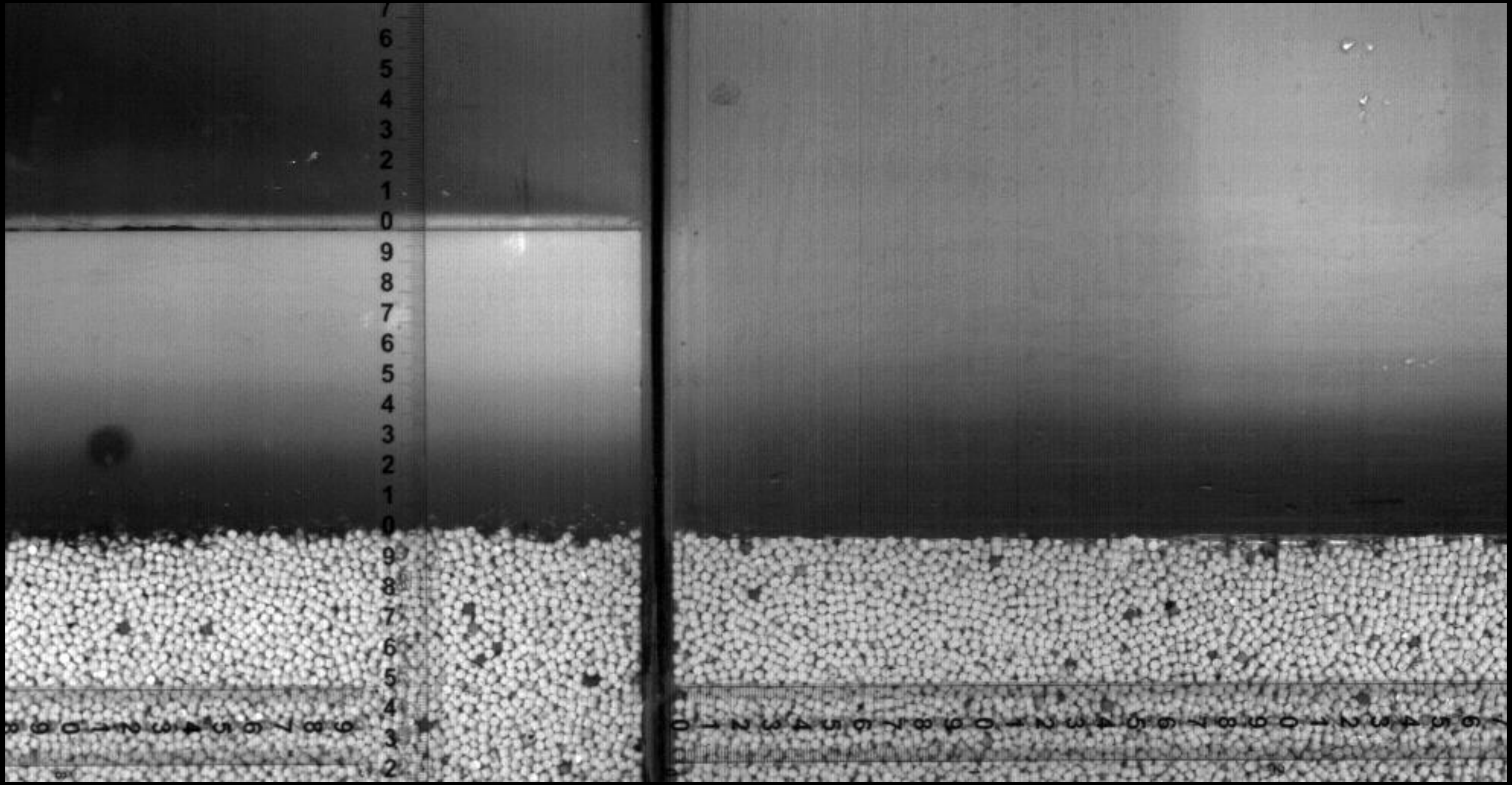
Experiments: (x) is t=0.8 s, (o) is t=2.72 s, (+) is t=4.8 s

Where do these physical aspects apply?

In processes involving morphological evolution under collisional bedload

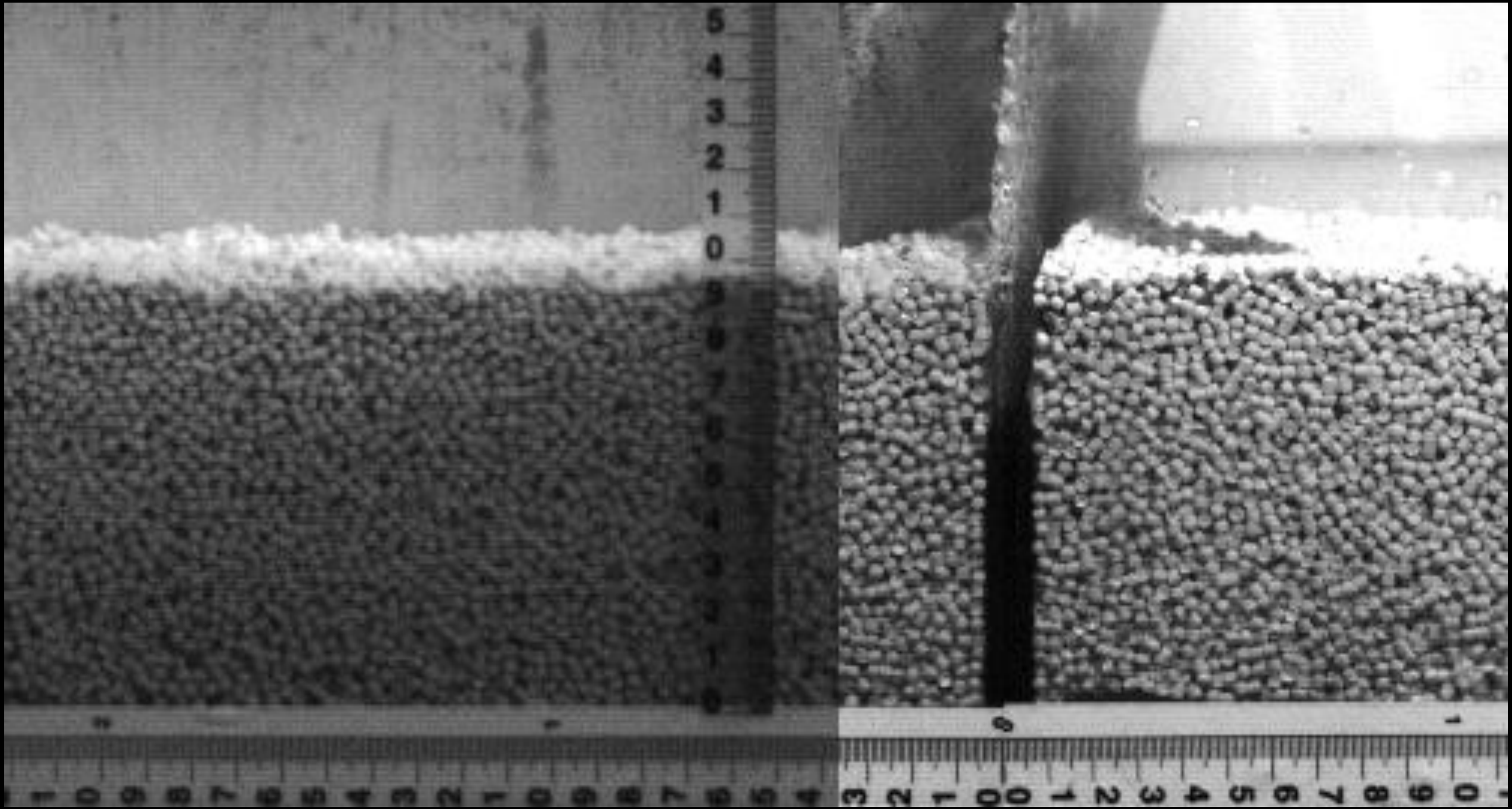
An example:

Erosional Dam Break



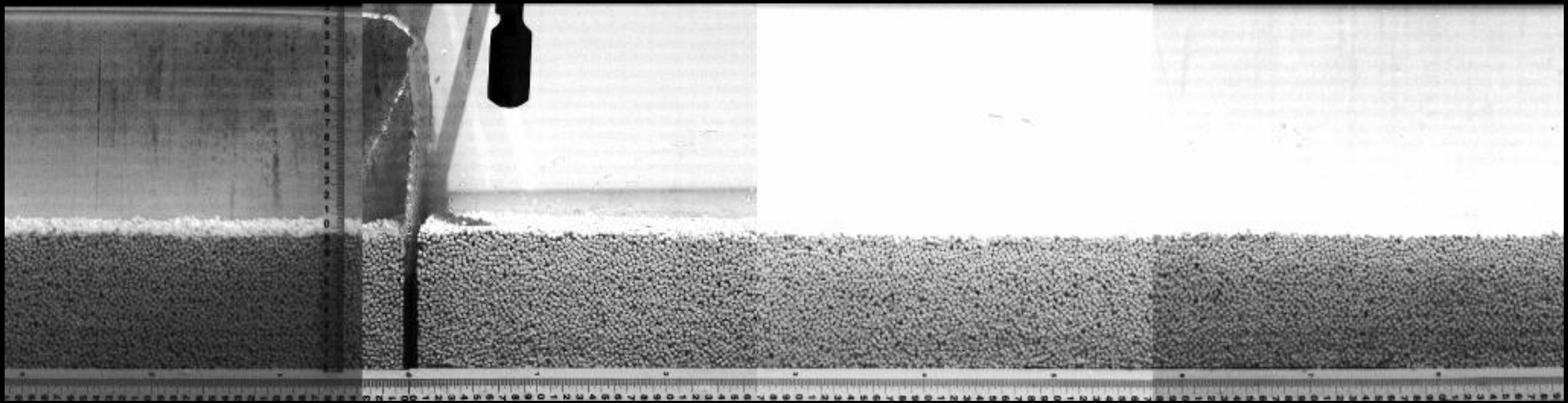
Erosional Dam Break

tracking the front

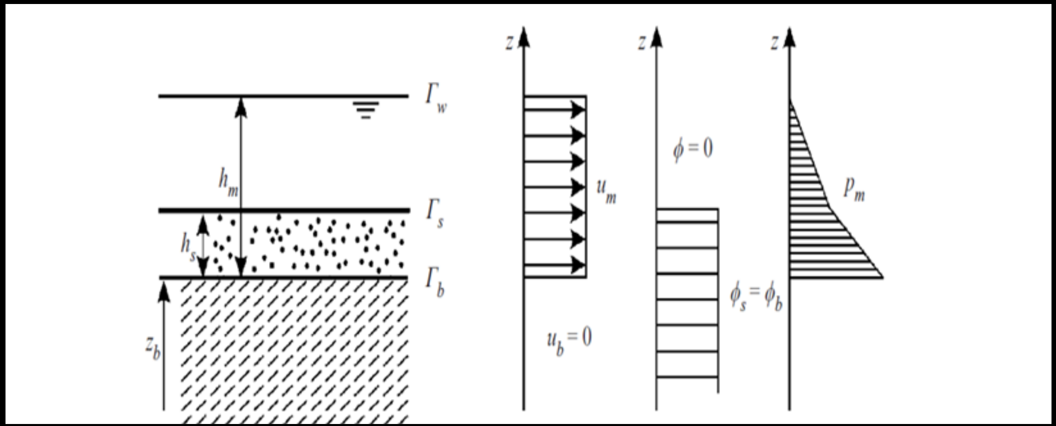


Erosional Dam Break

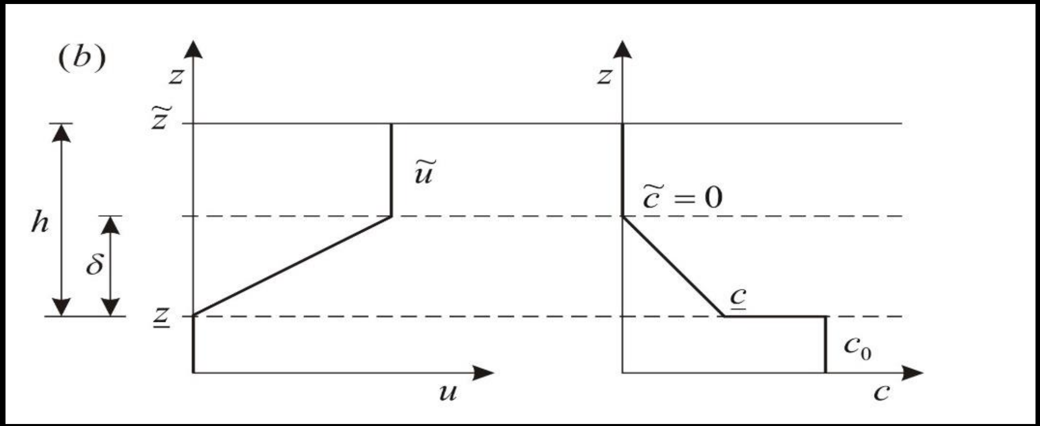
General view



JFM 2002



JFM 2013



Erosional Dam Break

Shallow water model based on four equations (with relaxation)

The system of divergence equations (28) and (29) can be written in vector form as

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{A}(\boldsymbol{U})\frac{\partial \boldsymbol{U}}{\partial x} = \boldsymbol{S}(\boldsymbol{U}), \tag{36}$$

where

$$e_b = \frac{\partial h_s}{\partial t} + \frac{\partial}{\partial x}(h_s u_m) = \frac{1}{t_l}(m u_m^2 - h_s), \tag{34}$$

where lag time t_l and mobility coefficient m are given by

$$t_l = \frac{1+r}{r}\frac{|u_m|}{\tan\varphi\, g}, \qquad m = \frac{s}{r}\frac{C_f}{\tan\varphi\, g}. \tag{35a,b}$$

$$\boldsymbol{S} = \begin{pmatrix} e_b \\ -e_b \\ -\dfrac{\tau_{bn}}{\rho_w} \end{pmatrix}. \tag{37a–c}$$

Symbol $\boldsymbol{U} = (h_m h_s z_b q_m)^T$ denotes the vector of dependent variables, $q_m = (h_m + r h_s)u_m$ is the momentum density of the heterogeneous mixture flow, $\boldsymbol{A}(\boldsymbol{U})$ is the Jacobian matrix and $\boldsymbol{S}(\boldsymbol{U})$ is the source term vector. Drawing from the operator splitting

Erosional Dam Break

Shallow water model based on three equations (without relaxation)

$$\frac{\partial W}{\partial t} + \boldsymbol{B}(W)\frac{\partial W}{\partial x} = \boldsymbol{0}, \tag{84}$$

where

$$W = \begin{pmatrix} h_w \\ z_s \\ u_m \end{pmatrix},$$

$$\boldsymbol{B}(W) = \begin{pmatrix} u_m & 0 & h_w \\ 0 & 0 & 3h_s \\ \dfrac{g(h_w + h_s)}{h_w + 3(1+r)h_s} & \dfrac{g(h_w + (1+r)h_s)}{h_w + 3(1+r)h_s} & \dfrac{h_w + 4(1+r)h_s}{h_w + 3(1+r)h_s}u_m \end{pmatrix}, \tag{85a, b}$$

in which again $h_s = h_s^{eq} = \mu u_m^2/g$.

System (84) forms a set of quasi-linear, first-order partial differential equations. The solution behaviour is controlled largely by the eigenstructure of matrix $\boldsymbol{B}$. Eigenvalues λ_i are the roots of the third-order polynomial resulting from

$$\det(\boldsymbol{B} - \lambda_i \boldsymbol{I}) = 0, \tag{86}$$

Erosional Dam Break

TIME SCALES

Froude similarity: $t_0 = \sqrt{\frac{h_0}{g}}$ hydrodynamic time scale

$t_g =$ geomorphic time scale

In the dam break flow, there is a geomorphic deceleration entirely due to bulking of the current with bed material of zero momentum, but finite inertia. Recent papers on it: Tai & Kuo 2008; Lê & Pitman 2010; Iverson 2012, Iverson and Ouyang, AGU, RevGeophysics, 2015.

For $t_0 < t$ shallow water assumptions apply

For $t_0 < t < t_g$ the adaptation model (4 eqs) applies

For $t_g < t$ the non adaptive model (3 eqs) applies

Erosional Dam Break

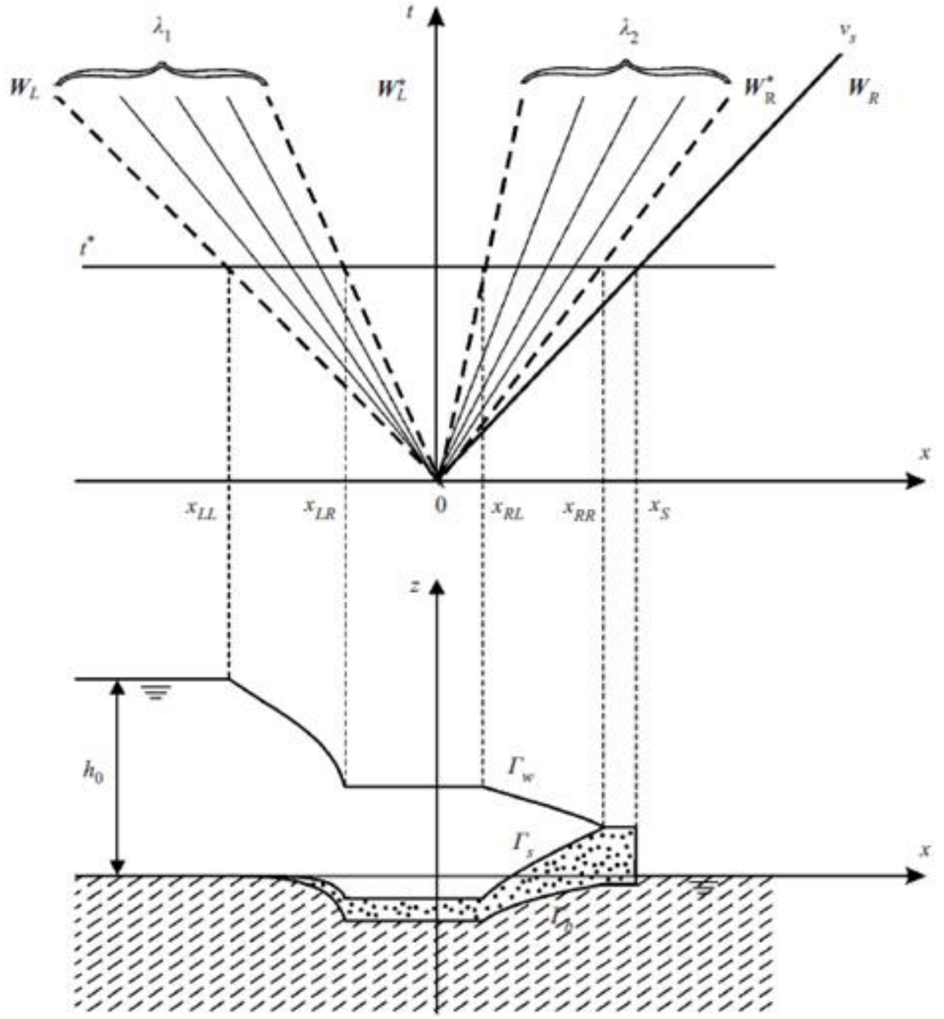


FIGURE 7. Wave structure of the erosional dam-break flow. Top: characteristic fans and shock path. Bottom: flow pattern as depicted by the three interface profiles Γ_w, Γ_s and Γ_b.

in §4.3. The postulated wave structure will be validated *a posteriori* by verifying that the compatibility equations (96) or (97) do indeed hold for the obtained waves.

Other scales:

$t_f =$ frictional time scale

$t_s =$ seepage time scale

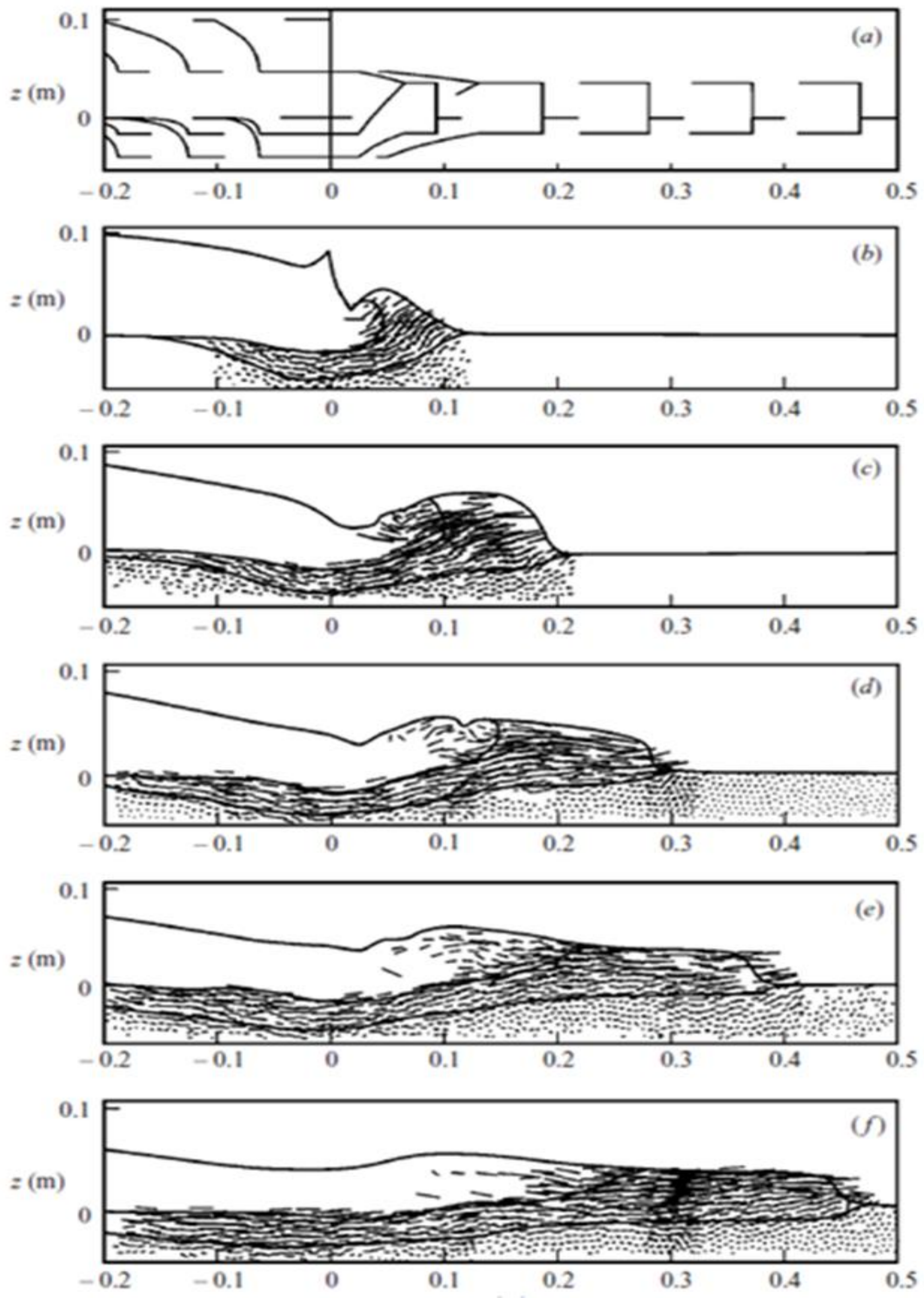
if

$$t_0 < t_g < t$$
$$t_s < t < t_f$$

n analytical solution is available

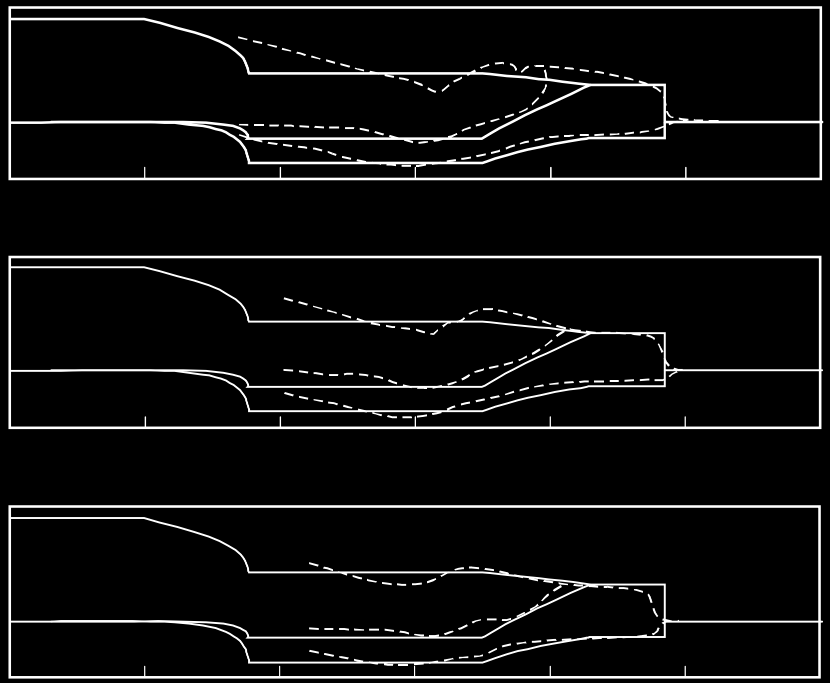
Fraccarollo and Capart, JFM, 2002
Spinewine and Capart, JFM, 2013

Erosional Dam Break



Erosional Dam Break

Theoretical and physical solutions



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Thank you for attention!

Luigi Fraccarollo