Dresden 2016-04-13

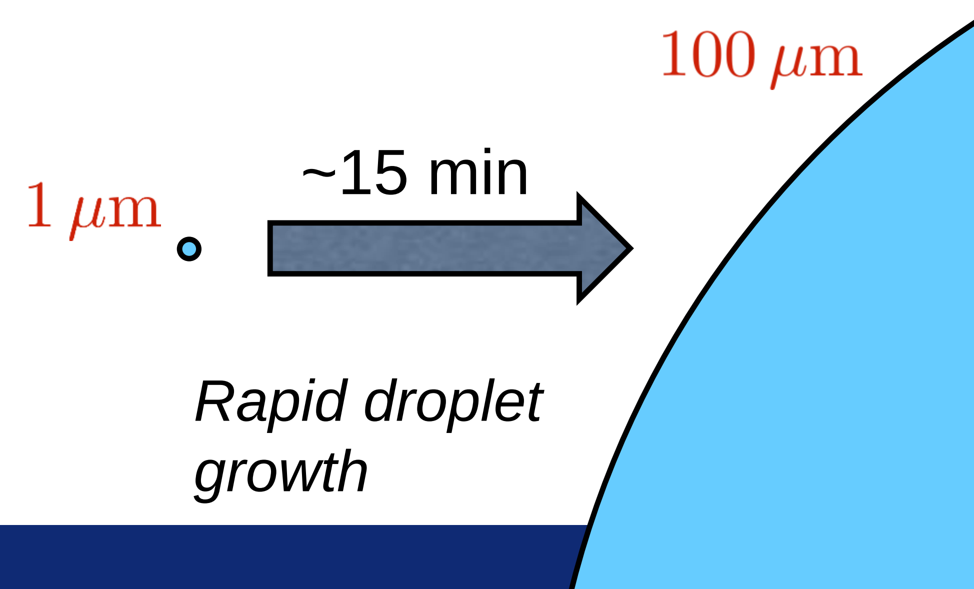
Collisions and relative velocities in turbulent aerosols

K. Gustavsson[1)], B. Mehlig[1)] , L. Biferale[2)] , S. Vajedi, G. Bewley

1) Department of Physics, University of Gothenburg, Sweden
2) Department of Physics, University of Tor Vergata, Italy

Small particles in mixing flows

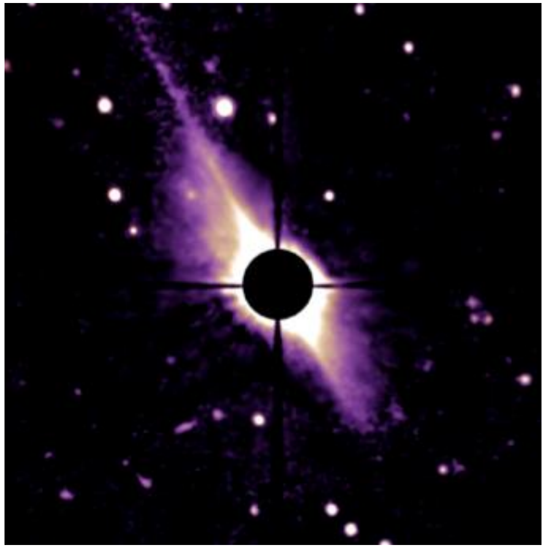
Rain droplets and ice crystals in rain clouds Pruppacher and Klett, (Springer 1997)



Small particles in complex flows

Rain droplets and ice crystals in rain clouds Pruppacher and Klett, (Springer 1997)

Dust grains in accretion disks
Praburam and Goree, Astrophys. J. **441** (1995)



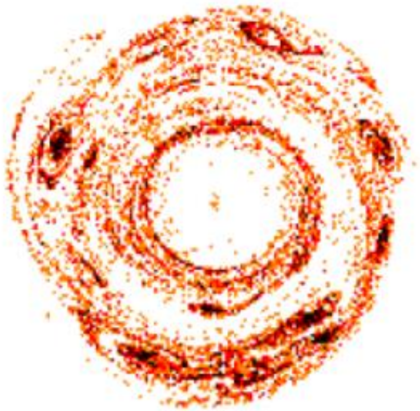


Photo and simulation of accretions disks

Small particles in complex flows

Rain droplets and ice crystals in rain clouds Pruppacher and Klett, (Springer 1997)

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Praburam and Goree, Astrophys. J. **441** (1995)

Climate models

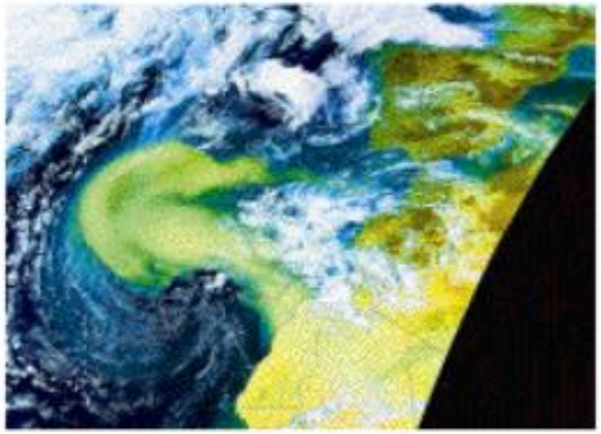
IPCC reports

Nature **432**, 962-963

Climate change: The cloud conundrum

Joyce E. Penner[1]

One of the great uncertainties in projecting global warming is accounting for the effects of small particles in Earth's atmosphere. Progress is nonetheless being made with this fiendishly complex problem.



Water and sand aerosols

Small particles in complex flows

Rain droplets and ice crystals in rain clouds Pruppacher and Klett, (Springer 1997)

Dust grains in accretion disks

Praburam and Goree, Astrophys. J. **441** (1995)

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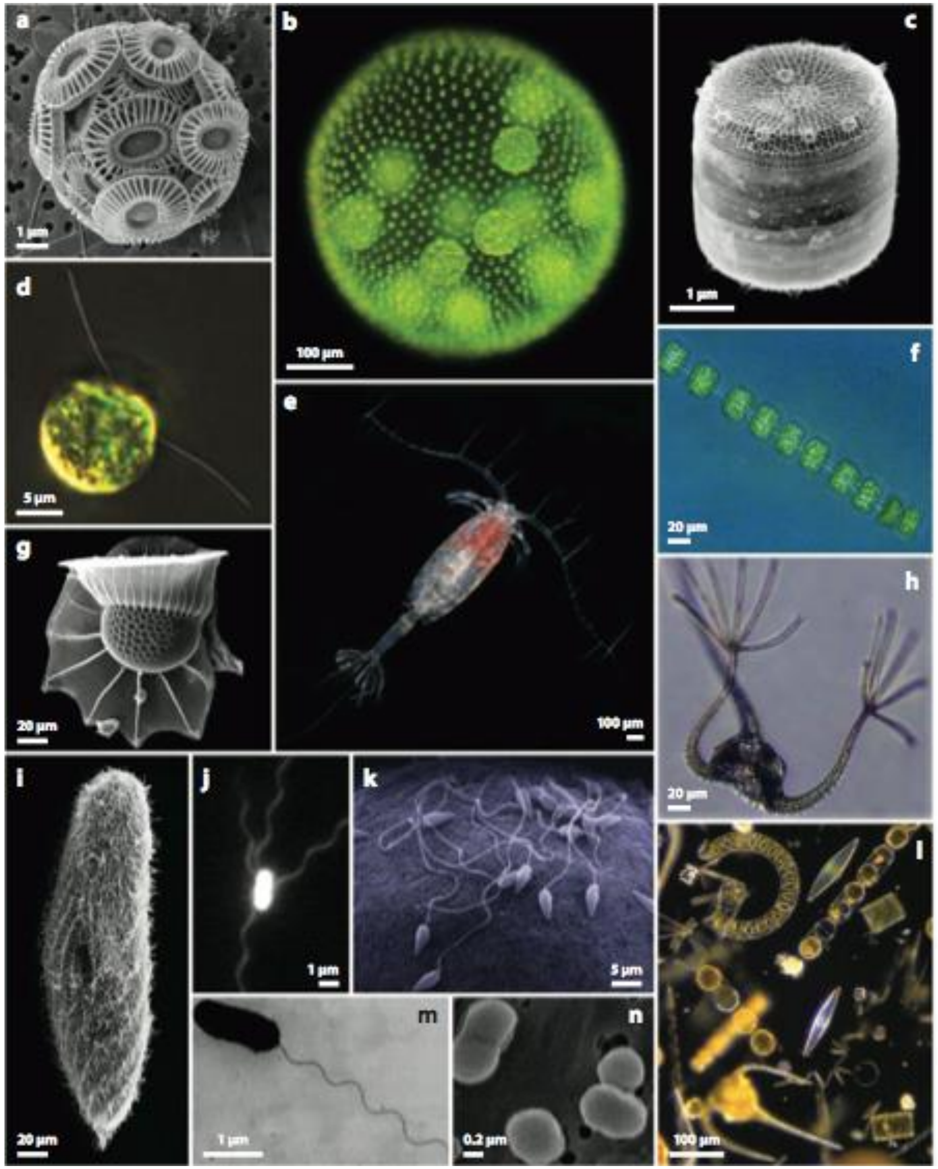
Microswimmers and bacteriae

Guasto et al, Annu. Rev. Fluid Mech. **44** (2012)



Plankton

Small particles in complex flows

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IPCC reports

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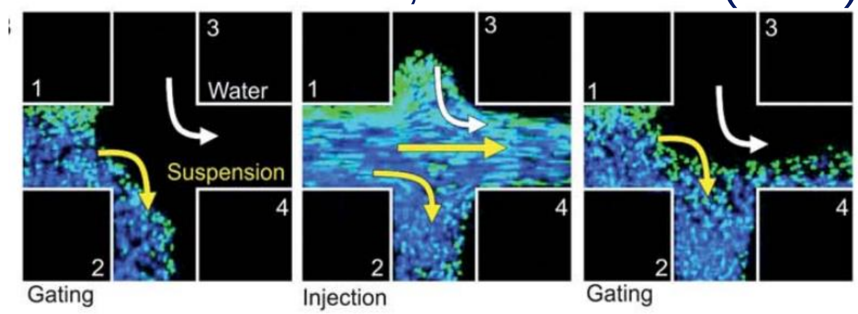
Guasto et al, Annu. Rev. Fluid Mech. **44** (2012)

Aristov et al, Soft Matter **9** (2013)



Separation of chiral particles using helical flow

Industrial applications

Mixing/separation of chemicals, colloidal solutions, sprays, combustion processes, pollution filtering, fibers in paper making, pipeline flows of slurries,…

SIAMUF

The Swedish Industrial Association for Multiphase Flows

Model

Spherical droplets move independently (until they collide)

Droplet equation of motion (small, heavy particles)

$$\ddot{\boldsymbol{r}} = (\boldsymbol{u}(\boldsymbol{r}, t) - \boldsymbol{v})/\tau_{\mathrm{p}} - g\hat{z}$$

τ_{p} particle response time (depends on droplet size and mass)
$\mathbf{r}$ particle position
$\mathbf{v}$ particle velocity
$\boldsymbol{g}$ gravitational acceleration

$\mathbf{u}(\mathbf{r}, t)$ stationary incompressible random velocity field
no preferred direction or position in either space or time
single scale flow with typical length scale η and time scale τ

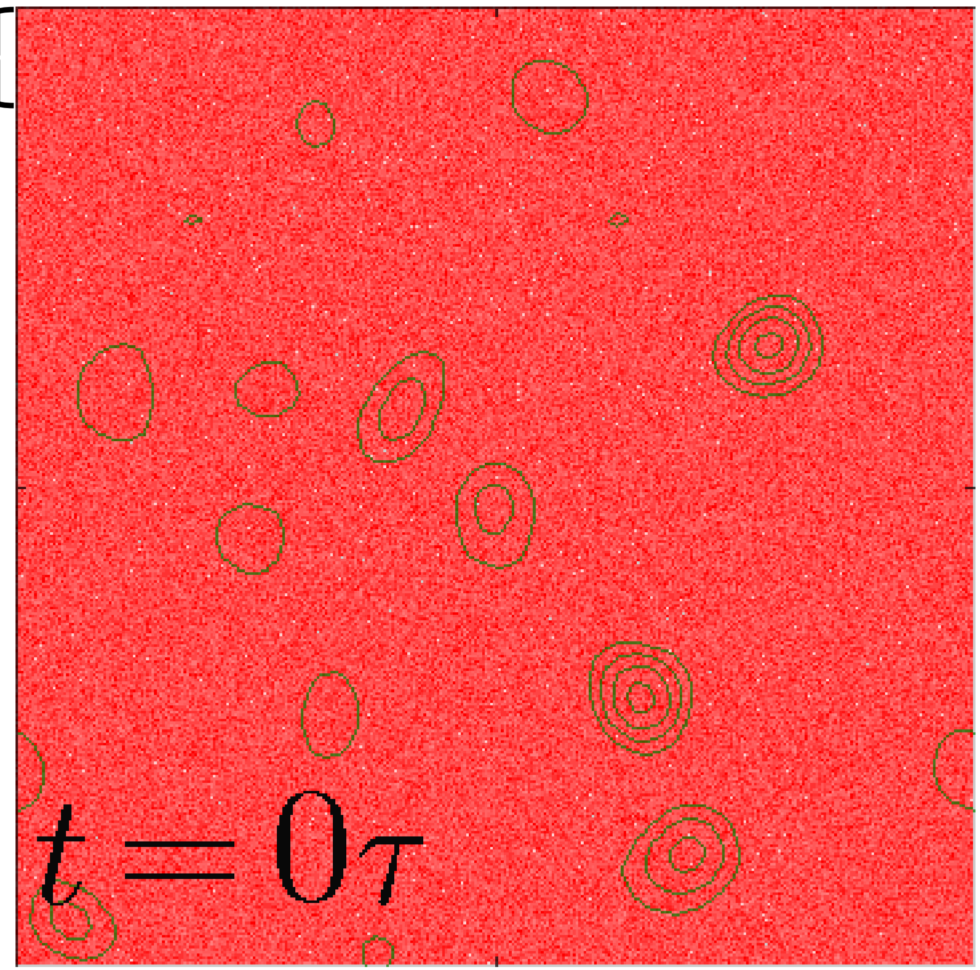
Aim: Try to understand relative motion between particles.
- What are the clustering mechanisms within this model?
- Which mechanisms make droplets collide?

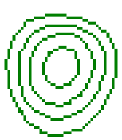
Random-flow model

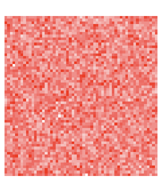
Non-interacting, non-colliding particles (red) follow a random flow

Length scale η

Time scale τ



 Region of high vorticity

 Particle density

Model parameters

- u_0 average flow speed
- η correlation length of flow
- τ correlation time of flow
- τ_p particle response time
- a particle radius
- n_0 particle density
- g gravitational acceleration

Dimensionless parameters

- $u_0\tau/\eta$ Kubo number (Ku)
- τ_p/τ Stokes number (St)
- $n_0 a^d$ packing fraction (small)
- a/η dimensionless size (small)
- $g\tau/u_0$ gravity parameter (F)

In rain cloud turbulence:

$$\mathrm{Ku} \sim 1$$
$$F \sim 1$$
$$\mathrm{St} \sim 10^9 a^2$$

(a particle size in meter)

R. Shaw, Annu. Rev. Fluid Mech **35** (2003)

Small droplet

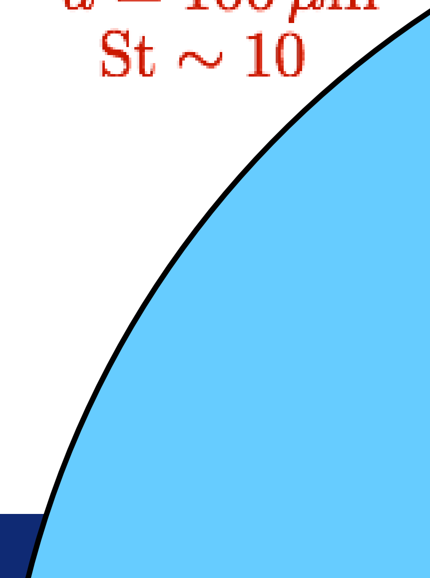
$a = 1\,\mu\mathrm{m}$

$\mathrm{St} \sim 10^{-3}$

Large droplet

$a = 100\,\mu\mathrm{m}$

$\mathrm{St} \sim 10$



Advantages of statistical model

Quick to simulate

Identifies universal properties at small scales

- Different flows as Ku changes

Allows for analytical solutions:

- Fokker-Planck description

  KG et al., New J. Phys. **10** (2008); Wilkinson et al., Europhys. Lett. **89** (2010)

- Matched asymptotics

  KG and Mehlig, Phys. Rev. E **84** (2011); J. Turbulence **15** (2014)

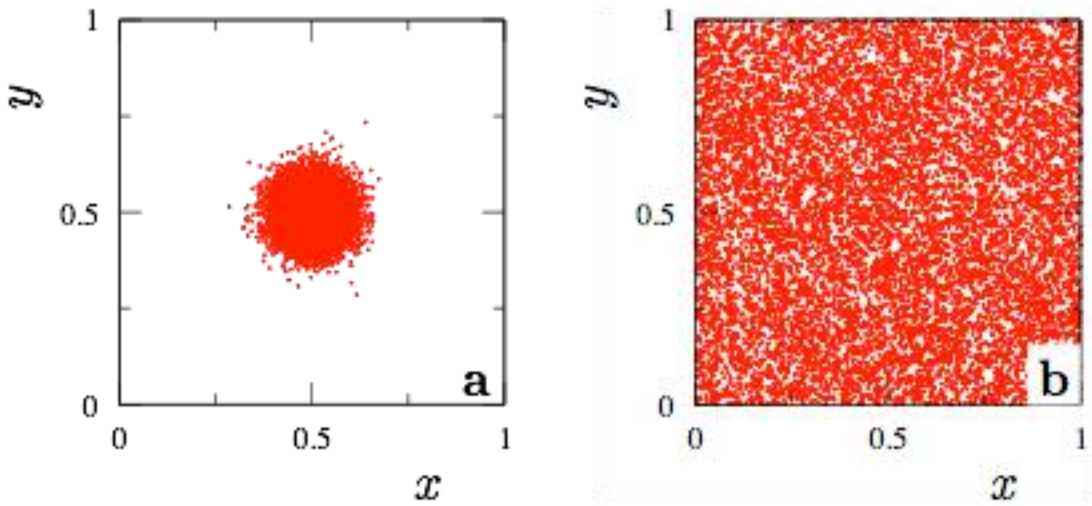
- Perturbation expansion (‘Kubo expansion’)

  KG and Mehlig, Europhys. Lett. E **96** (2011); Adv. Phys. (2016)

Distinguishes effects due to particle dynamics contra fluid properties

Mixing by random stirring

Computer simulation of 10^4 particles (red) in two-dimensional random flow (periodic boundary conditions in space)

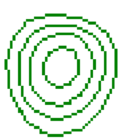


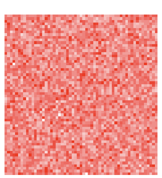
a initial distribution, **b** particle positions after random stirring.

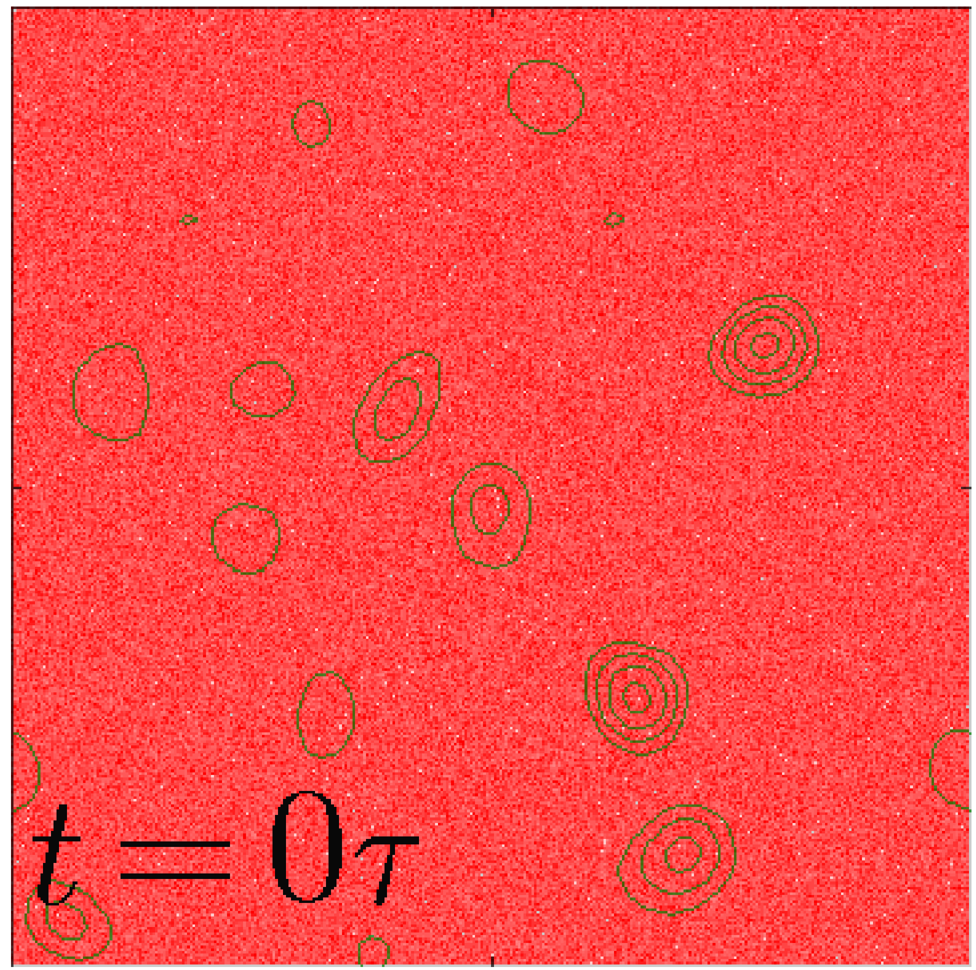
‘Unmixing’ of slightly inertial particles

Non-interacting, non-colliding particles (red) suspended in a random flow (gravity neglected)

$$\mathrm{St} = 0.1$$
$$\mathrm{Ku} = 1$$
$$F = 0$$

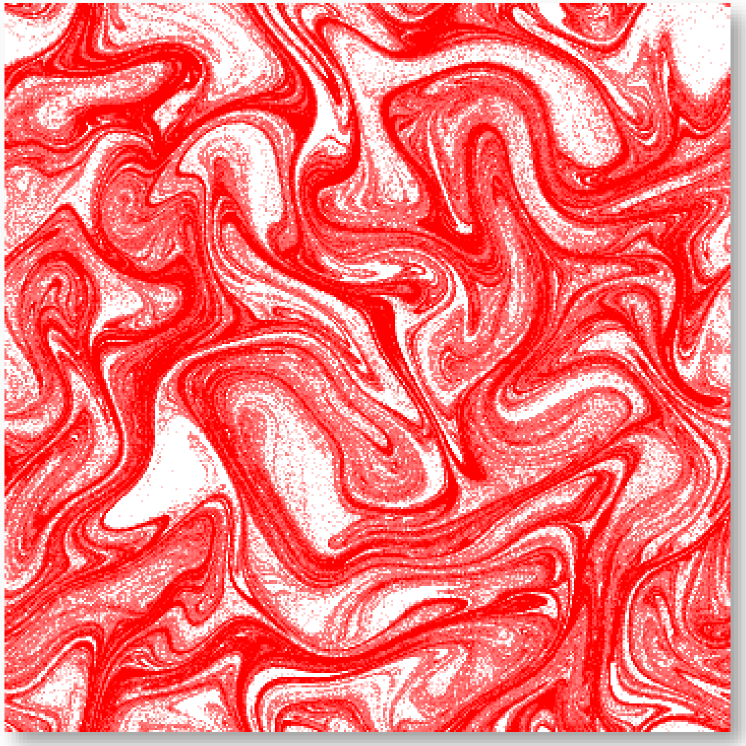

Region of high vorticity


Particle density

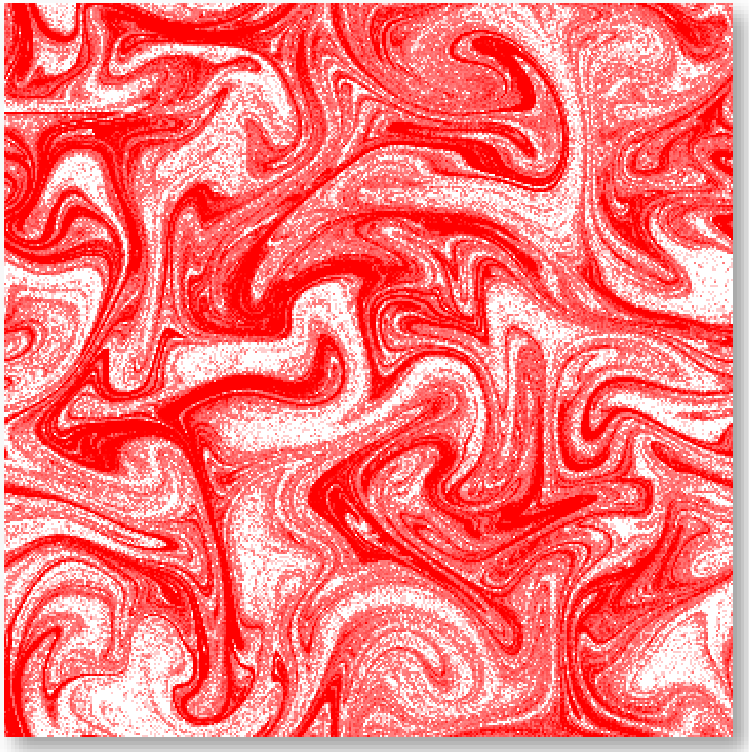


Comparison to compressible flows

A hint of what is going on...



Slightly inertial particles ($\mathrm{St} = 0.1$) suspended in an incompressible random flow.



Non-inertial particles ($\mathrm{St} = 0$) suspended in a compressible random flow.

Centrifuge mechanism

Maxey, J. Fluid Mech. **174**, 441, (1987)

Inertial droplets are centrifuged out of vortices

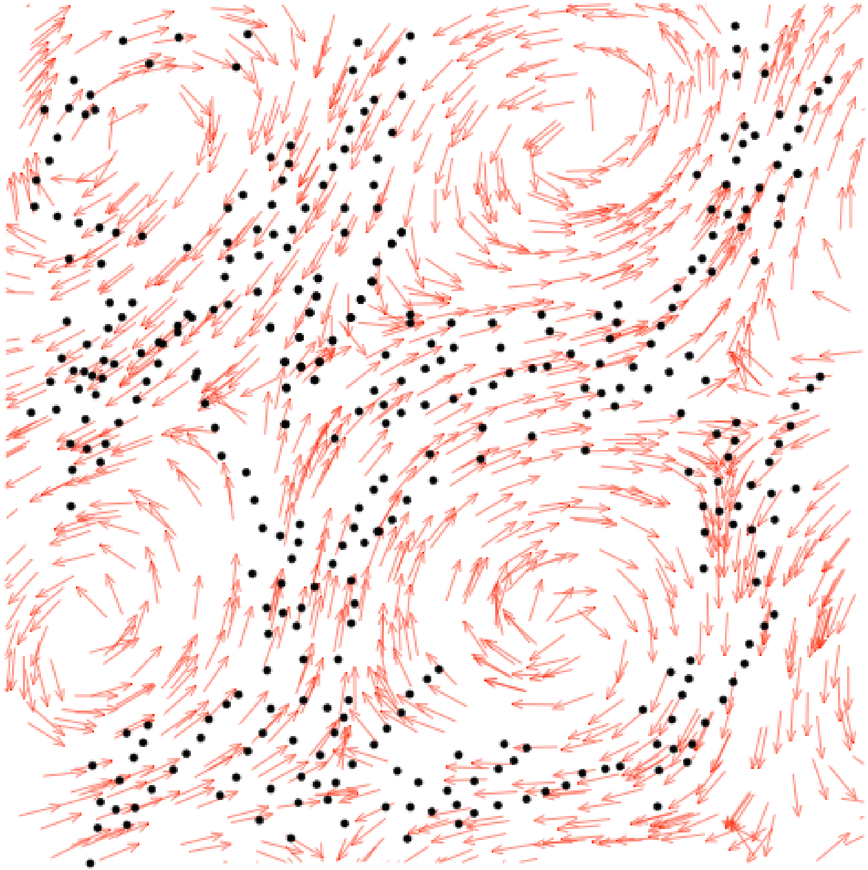
For slightly inertial particles ($\mathrm{St} \approx 0$)

$$\boldsymbol{v} = \boldsymbol{u} - \tau_{\mathrm{p}} \left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla}) \boldsymbol{u} \right]$$

Particles follows effective velocity field $\mathbf{v}$, which is compressible

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = -\tau_{\mathrm{p}} \mathrm{Tr} \left[\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} \right)^2 \right]$$

Droplets cluster due to long-lived flow structures. This is an example of ‘preferential sampling’.

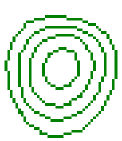


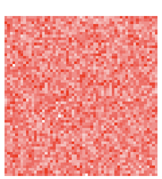
Particles avoid regions of high vorticity

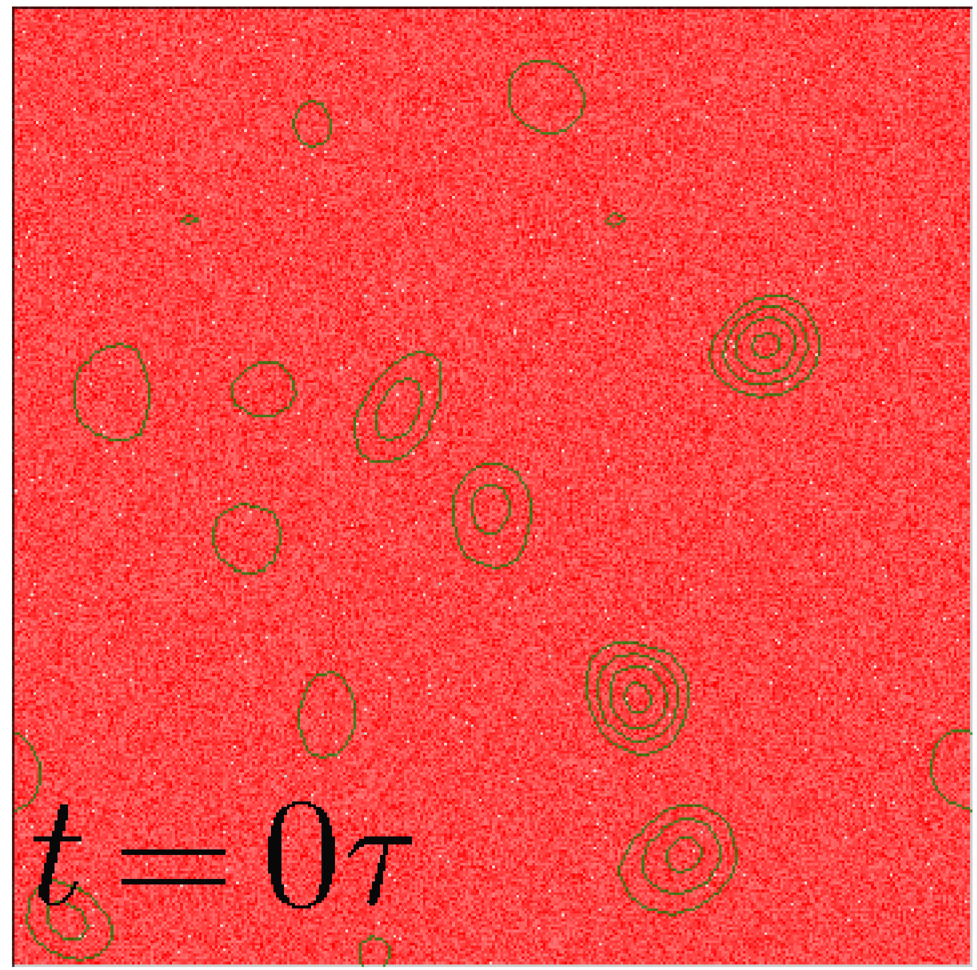
‘Unmixing’ of very inertial particles

Non-interacting, non-colliding particles (red) suspended in a random flow (gravity neglected)

$$\mathrm{St} = 10$$
$$\mathrm{Ku} = 0.1$$
$$F = 0$$


Region of high vorticity


Particle density



Multiplicative amplification

The motion of heavy particles ($\mathrm{St} \gg 1$) is independent of the instantaneous value of the fluid if Ku is small enough ($\mathrm{Ku} \ll \sqrt{\mathrm{St}}$).

Replace the position-dependent flow by ‘random kicks’:

$$\mathbf{u}(\mathbf{r}_t, t) \to \mathbf{u}(t)$$

Langevin/Fokker-Planck treatment possible.
Dynamics described by single parameter: $\epsilon^2 \sim \mathrm{Ku}^2\mathrm{St}$

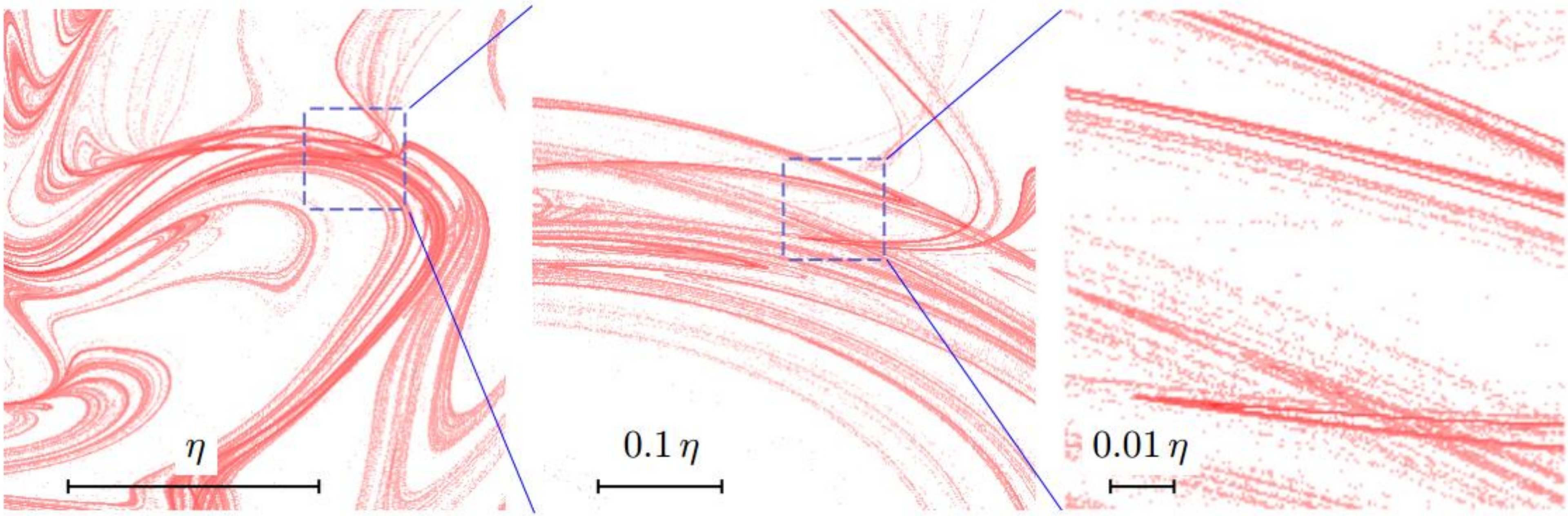
Clustering results as the net effect of many small deformations of small cloud of close-by particles, uncorrelated from any instantaneous structures in the flow.

Mehlig & Wilkinson, Phys. Rev. Lett. **92** (2004) 250602
Duncan et al., Phys. Rev. Lett. **95** (2005)
Wilkinson et al., Phys. Fluids **19** (2007) 113303

Small-scale fractal clustering

Inertial particles cluster on self-similar structures, 'fractals'

Sommerer & Ott, Science **259** , 334, (1993)



$\mathrm{St} = 10$

$\mathrm{Ku} = 0.1$

Quantification of fractal clustering I

Lyapunov exponents $\lambda_1 \geq \lambda_2 \geq \ldots$ describe contraction/expansion rate of separations $\delta\mathcal{R}_t$, areas $\delta\mathcal{A}_t$ etc. in a cloud of closeby particles

$$\lambda_1 = \lim_{t\to\infty} t^{-1} \ln(\delta\mathcal{R}_t)$$
$$\lambda_1 + \lambda_2 = \lim_{t\to\infty} t^{-1} \ln(\delta\mathcal{A}_t)$$

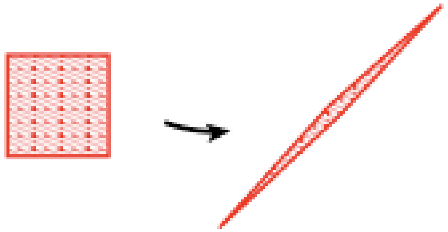
J. Sommerer & E. Ott, Science 259 (1993) 351

When $\mathrm{St} > 0$ and not too large, the ($d = 2$) dynamics is:

- chaotic (positive maximal Lyapunov exponent)

$$\lambda_1 > 0$$

- compressible (areas contract)



$$\lambda_1 + \lambda_2 < 0$$

Fractal dimension $d_{\mathrm{L}} \equiv 2 - \dfrac{\lambda_1 + \lambda_2}{\lambda_2}$

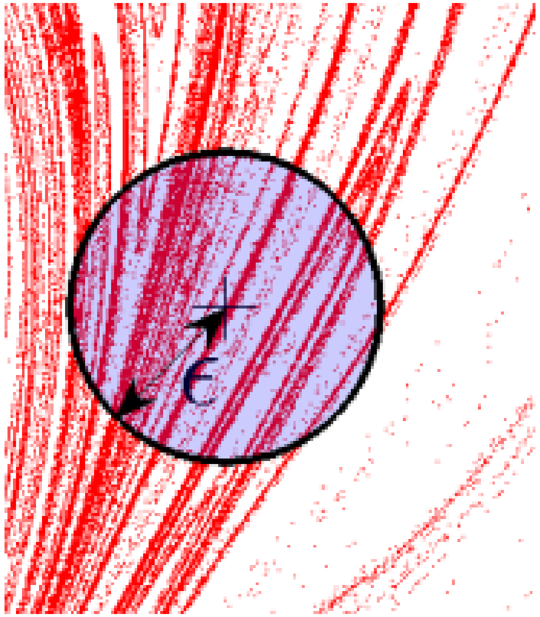
Kaplan & Yorke, Springer Lecture Notes in Mathematics **730**, 204, (1979)

Quantification of fractal clustering II

The number of N uniformly distributed particles in a sphere of radius ϵ is proportional to the volume, i.e. $N \sim \epsilon^d$ (d is the spatial dimension).

On a fractal, the number of droplets in the sphere is proportional to $N \sim \epsilon^{d_2}$, where d_2 is the 'correlation dimension'.

E. Ott, *Chaos in dynamical systems*, 478p (2002)

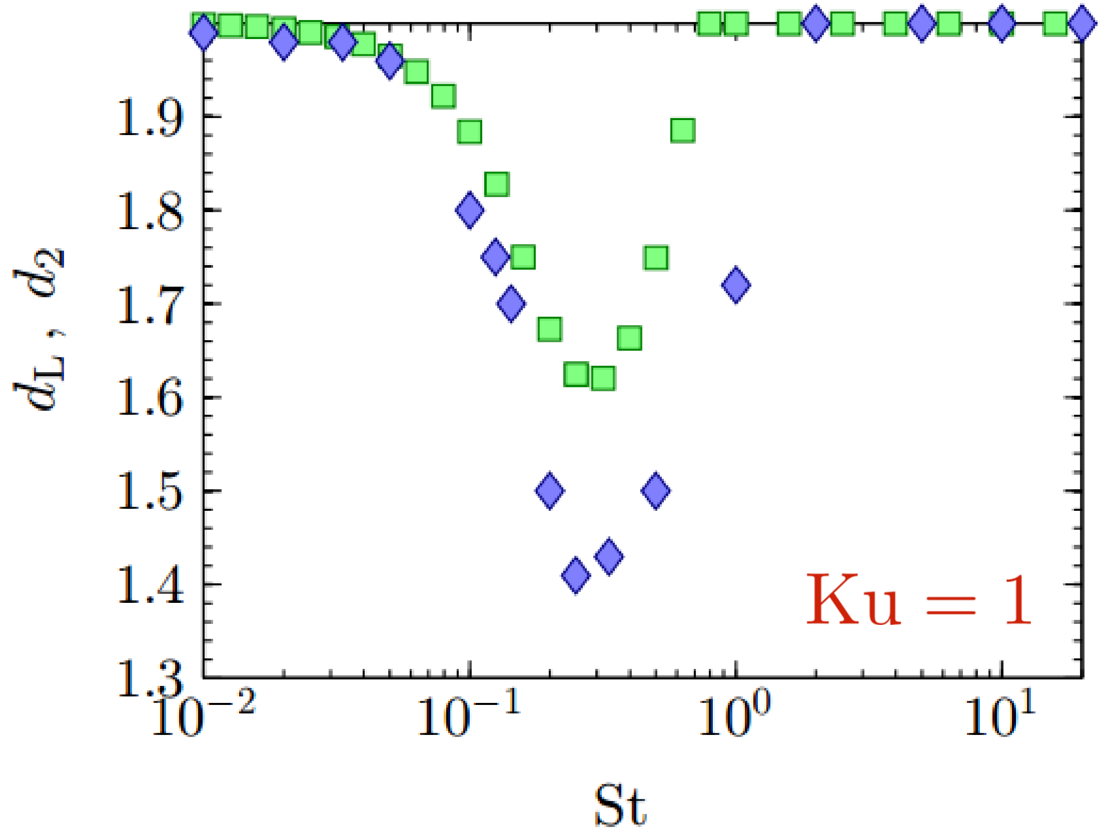


A small d_2 corresponds to large fractal clustering.

Numerical results ($d = 2$)

For clustering in random and turbulent flows, different measures of the clustering does not give the same result (‘multifractal’).

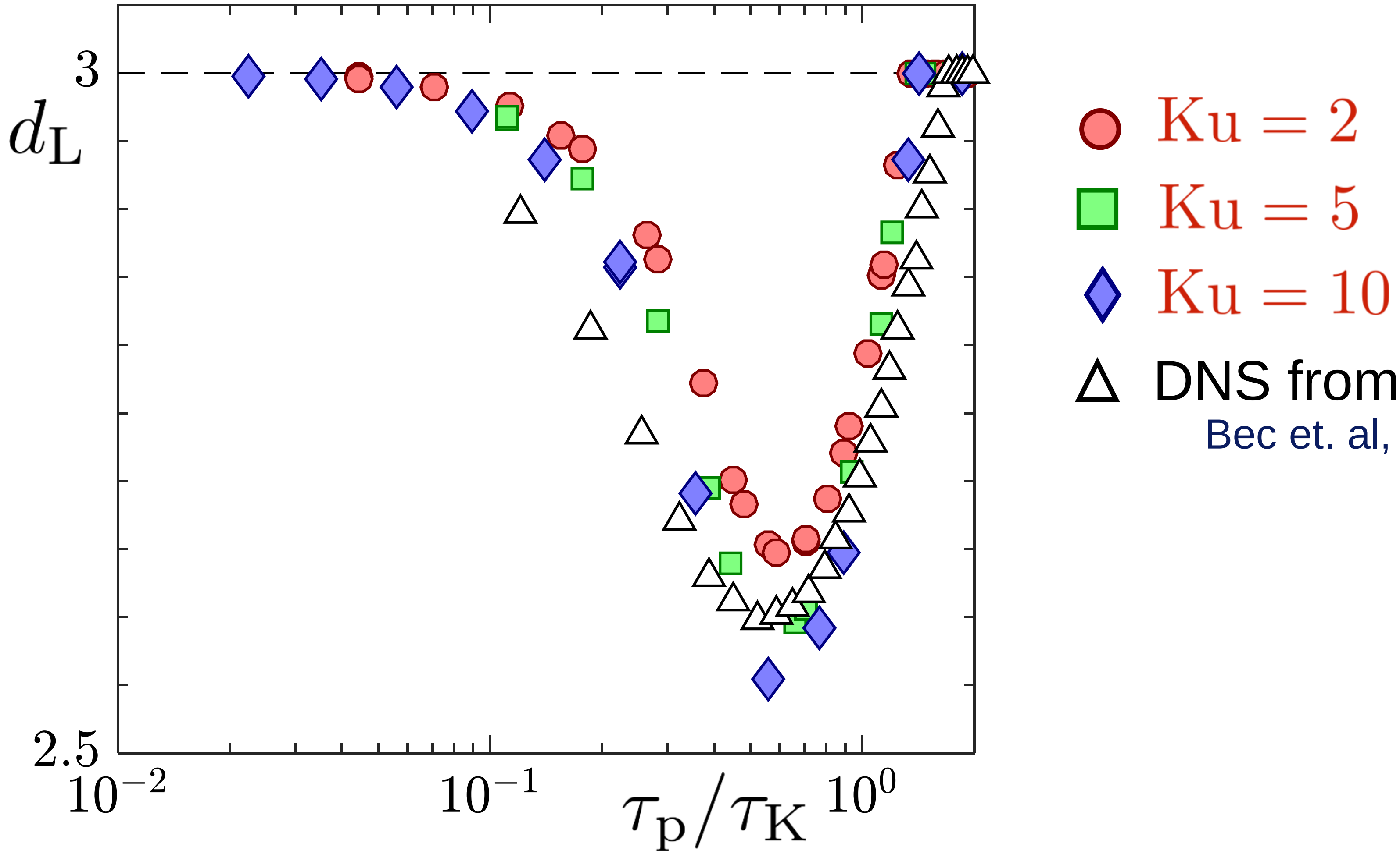
Bec et. al, Phys. Rev. Lett. **92**, 224501, (2004)



Lyapunov dimension (d_L, ☐) *and Correlation dimension* (d_2, ◇)

Comparison to DNS

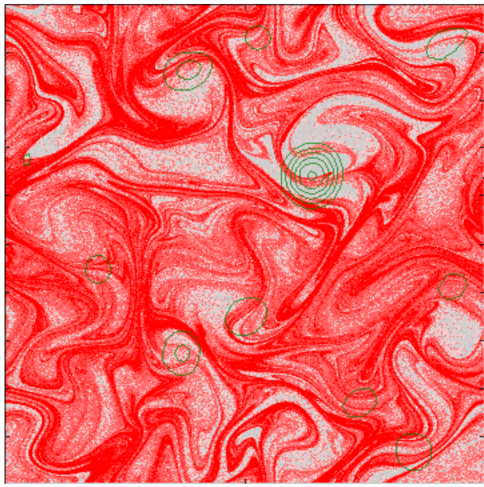
Clustering in statistical model show qualitative agreement with DNS of turbulence in persistent-flow limit (large Ku).



Bec et. al, Phys. Fluids **18**, 091702, (2006)

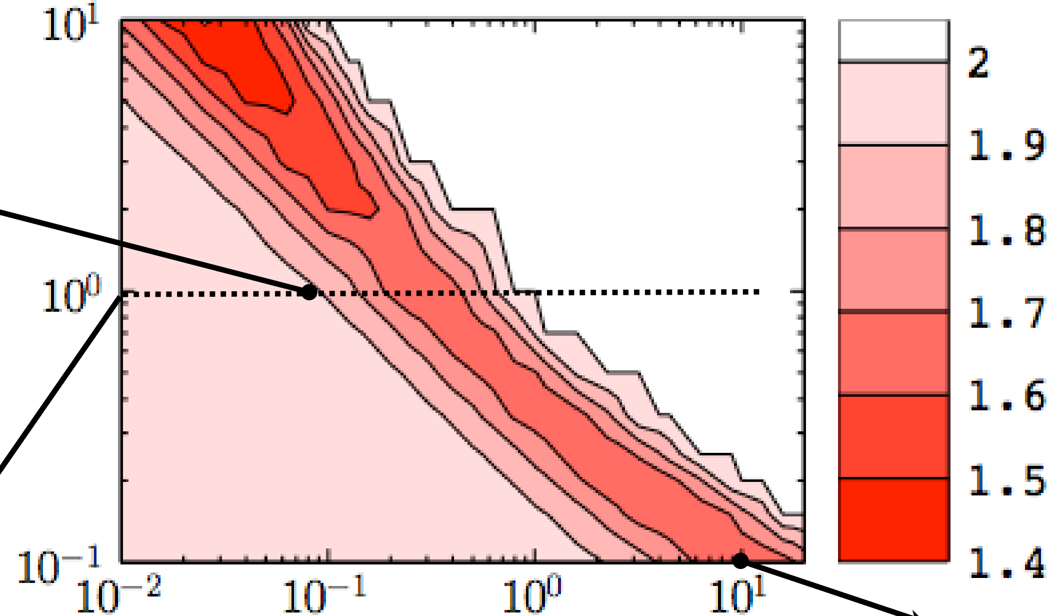
Clustering without gravity

$$d_{\mathrm{L}} \equiv 2 - \frac{\lambda_1 + \lambda_2}{\lambda_2}$$

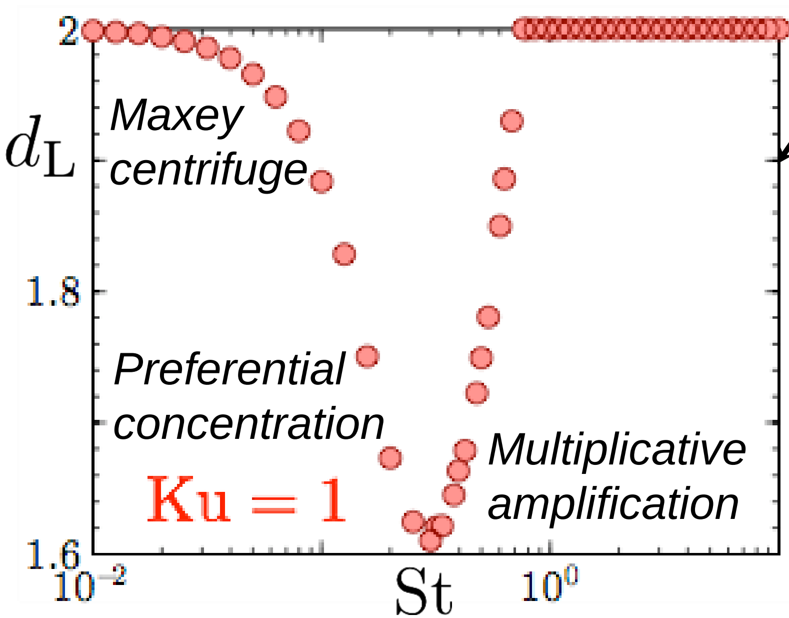


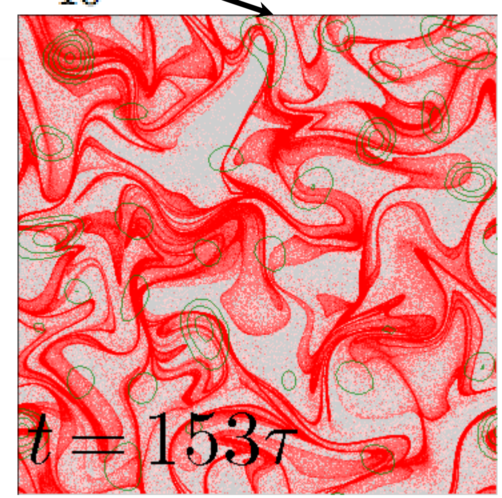
Maxey centrifuge effect

Flow intensity Ku Fractal dimension d_{L}



Inertia St





Multiplicative amplification

Deterministic dynamics with gravity

Dynamics in the absence of $\boldsymbol{u}$

$$\dot{\boldsymbol{r}} = \boldsymbol{v}$$
$$\dot{\boldsymbol{v}} = (\cancel{\boldsymbol{u}(\boldsymbol{r},t)} - \boldsymbol{v})/\tau_{\mathrm{p}} - g\hat{z}$$

Particles reach a terminal 'settling velocity' (from $\dot{\boldsymbol{v}} = 0$)

$$\boldsymbol{v}_{\mathrm{s}} \equiv -\tau_{\mathrm{p}} g\hat{z}$$

The flow velocity typically lead to increased settling speeds

Wang & Maxey, J. Fluid. Mech. **256** (1993)

Relative motion between two particles is only affected by gravity through the $\boldsymbol{r}$-dependence in $\boldsymbol{u}(\boldsymbol{r},t)$. Gravity is expected to alter correlations between flow and particle trajectories.

'Unmixing' of falling inertial particles

Non-interacting, non-colliding particles (red) suspended in a random flow (no clustering when $F = 0$)

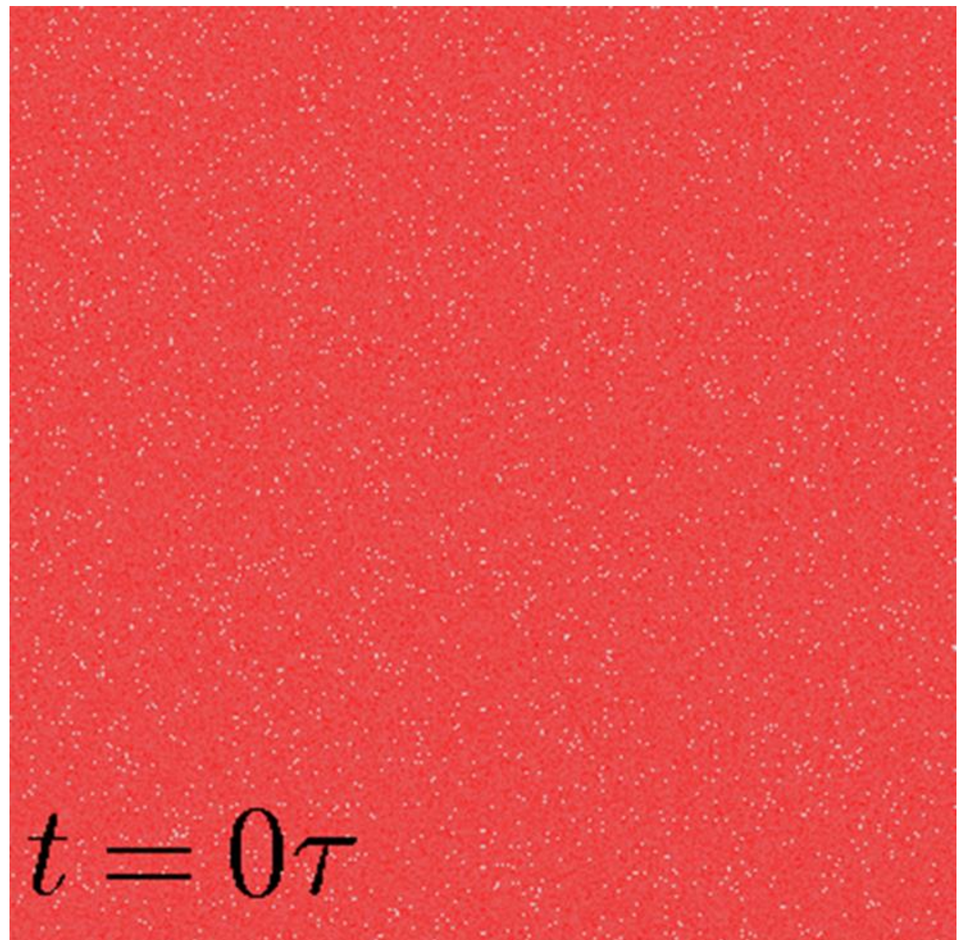
$\mathrm{St} = 10$

$\mathrm{Ku} = 1$

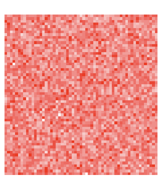
$F = 1$

Frame moving with settling velocity $\boldsymbol{v}_\mathrm{s}$

$\boldsymbol{v}_\mathrm{s}$



Large-St gravitational clustering



Particle density

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$\mathrm{Ku}=1$

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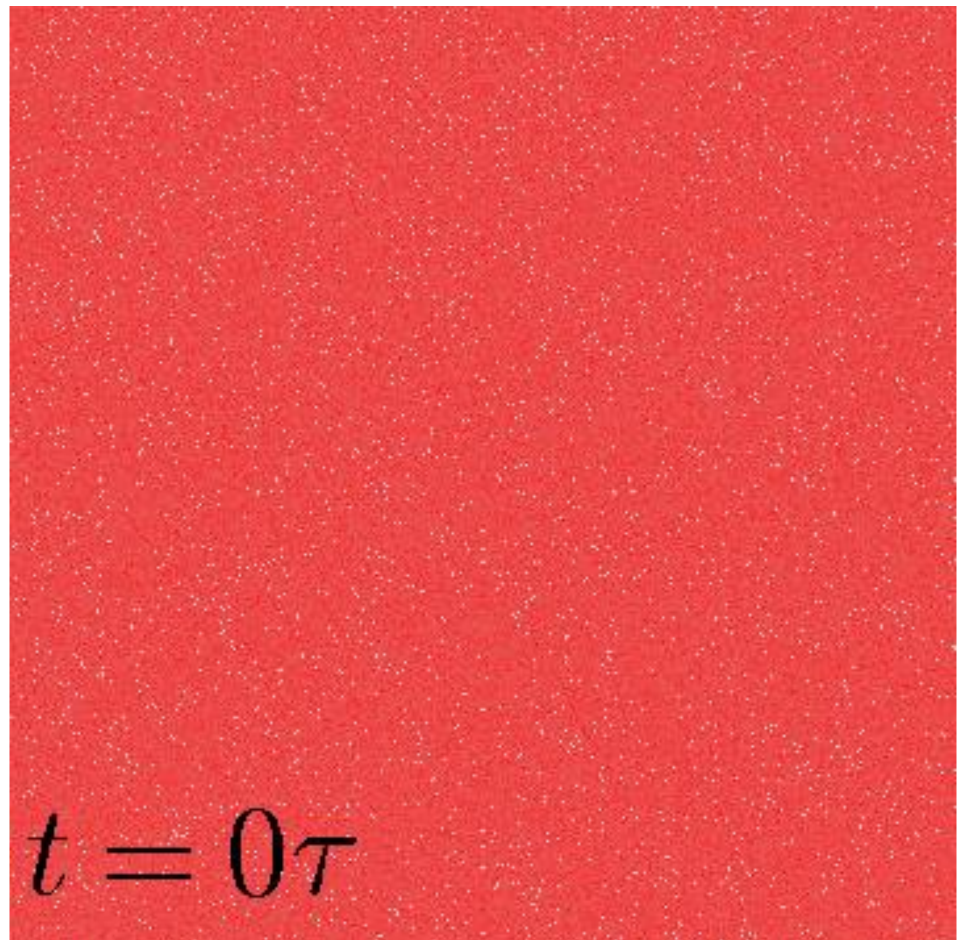
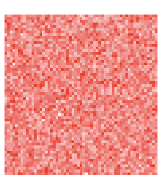
Frame moving with settling velocity $\boldsymbol{v}_\mathrm{s}$

$\boldsymbol{v}_\mathrm{s}$



Large-St gravitational clustering



Particle density

Large-St dynamics

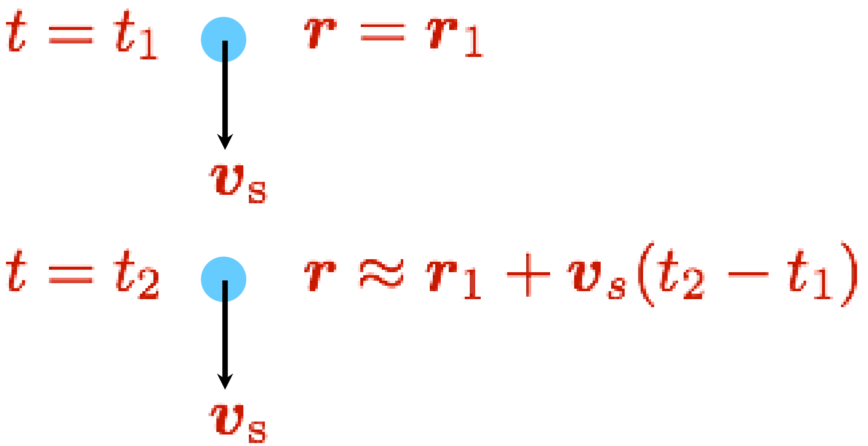
Deterministic solution $\boldsymbol{r} \approx \boldsymbol{r}_0 + \boldsymbol{v}_s t$ with settling velocity $\boldsymbol{v}_{\rm s} \equiv -\tau_{\rm p} g \hat{z}$

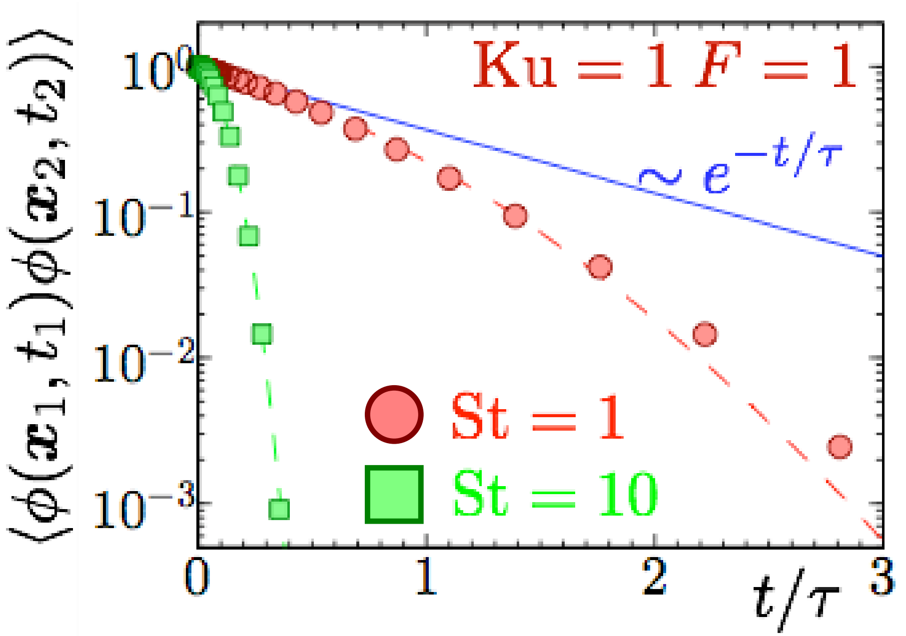
Spatial decorrelation becomes faster than time decorrelation.

Single-particle correlation function at two different times

$$\langle u(\boldsymbol{x}_1, t_1) u(\boldsymbol{x}_2, t_2)\rangle$$
$$\sim u_0^2 e^{-|t_1-t_2|/\tau - (\boldsymbol{x}_1-\boldsymbol{x}_2)^2/(2\eta^2)}$$
$$\sim u_0^2 e^{-|t_1-t_2|/\tau - v_{\rm s}^2 (t_1-t_2)^2/(2\eta^2)}$$

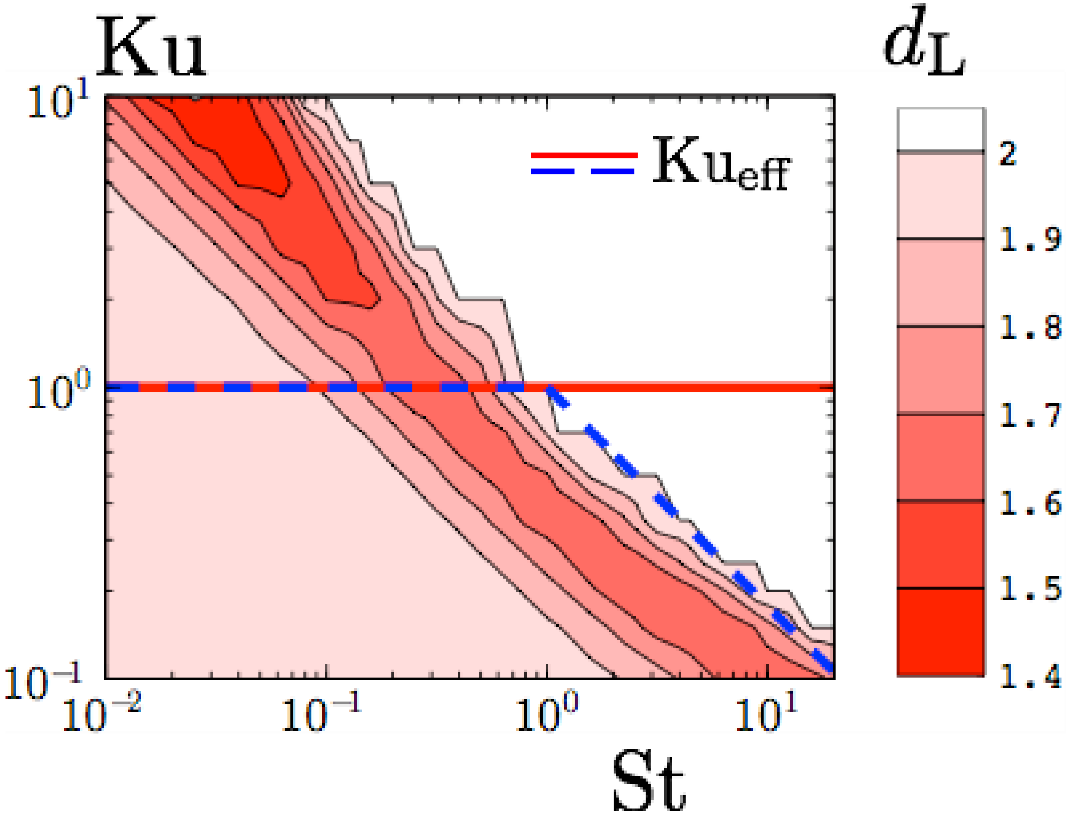
When $G \equiv v_{\rm s}\tau/\eta = \mathrm{Ku}\,\mathrm{St}\,F$ is large the effective correlation time approaches white noise.

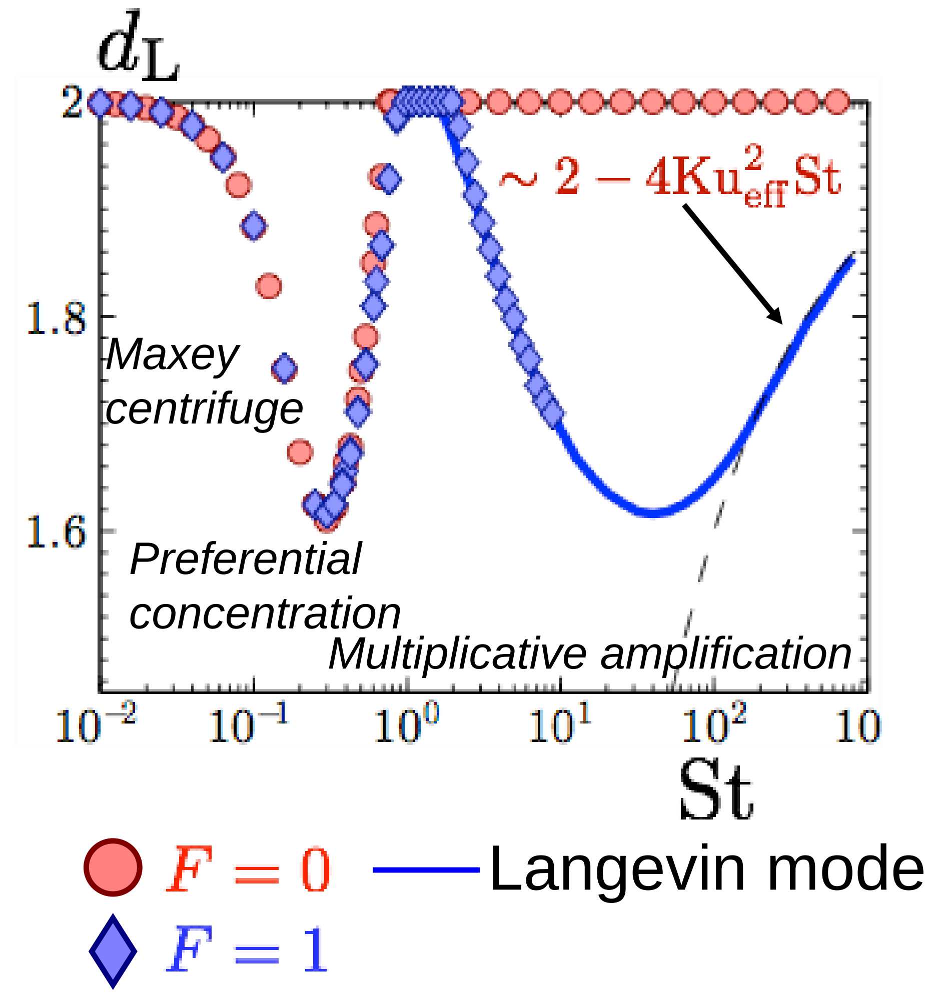




Large-St clustering due to settling

The effective correlation time τ_{eff} results in $\mathrm{Ku}_{\mathrm{eff}} = u_0\tau_{\mathrm{eff}}/\eta$.
The dynamics with $F > 0$ and $\mathrm{Ku} = 1$ can be roughly mapped onto the dynamics with $F = 0$.

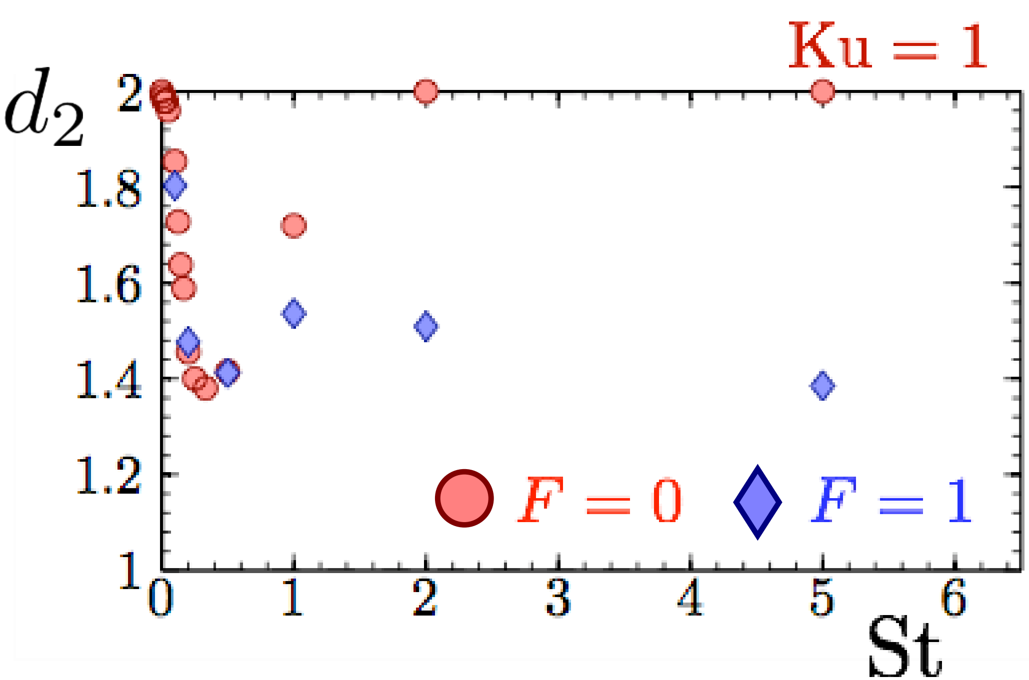




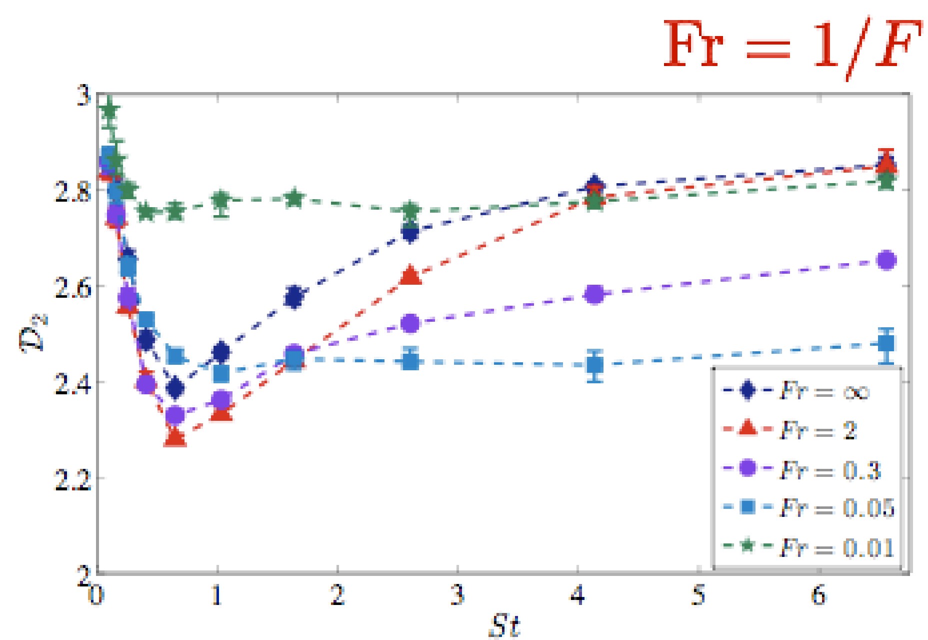
KG Mehlig & Vajedi, Phys. Rev. Lett. **112** (2014)

Comparison to turbulence

Comparison of correlation dimension in statistical model ($d = 2$) to results from DNS.



Statistical model



DNS

Bec et al, Phys. Rev. Lett. **112** 184501 (2014)

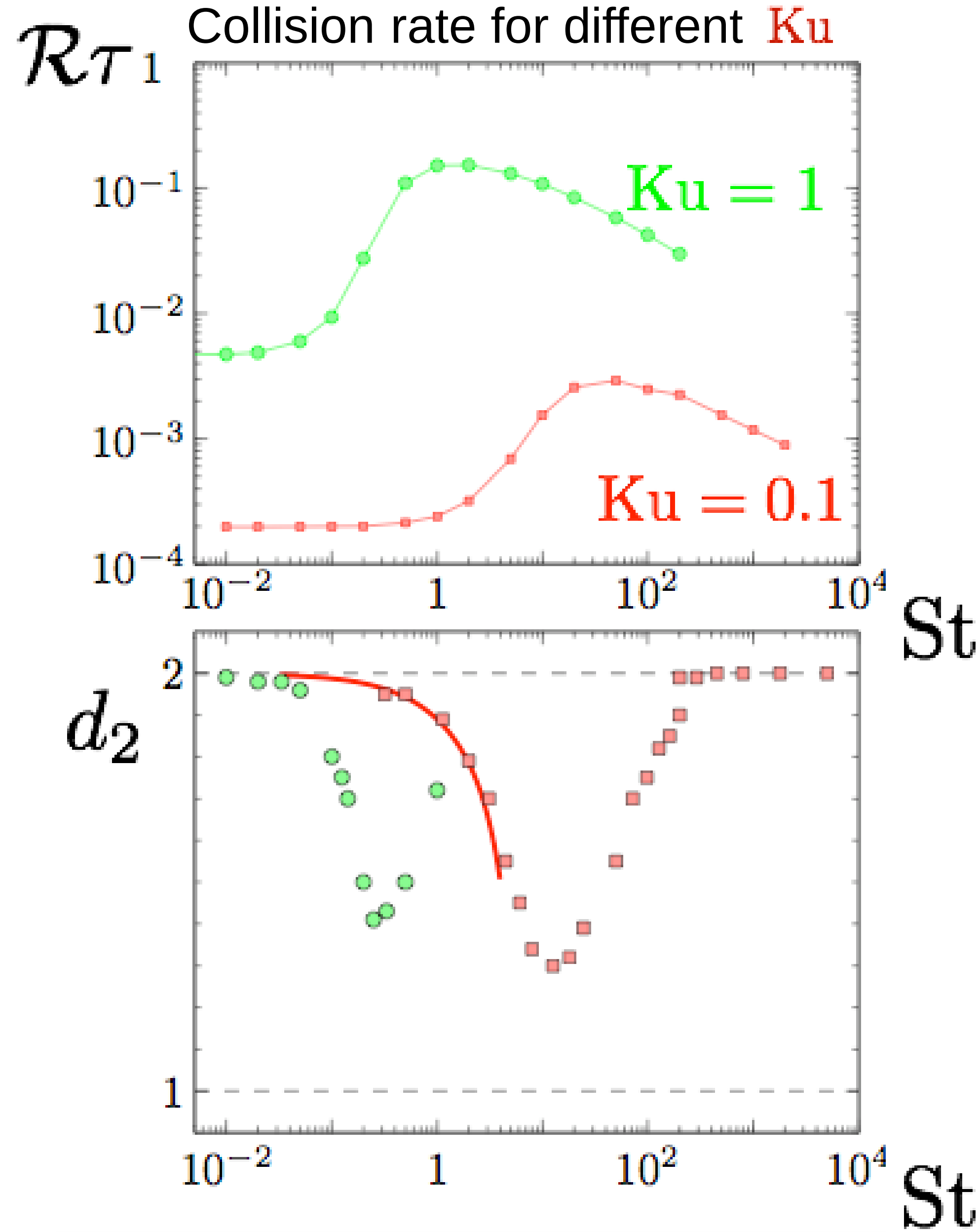
The rate of collisions ($d = 2$)

Collision rate $\mathcal{R}$ for a test particle suspended with n_0 particles.

When St is small $\mathcal{R}$ increases as correlation dimension d_2 increases as expected.

The clustering peaks before the collision rate peaks.

The collision rate only drops slowly for large St where there is negligible clustering.

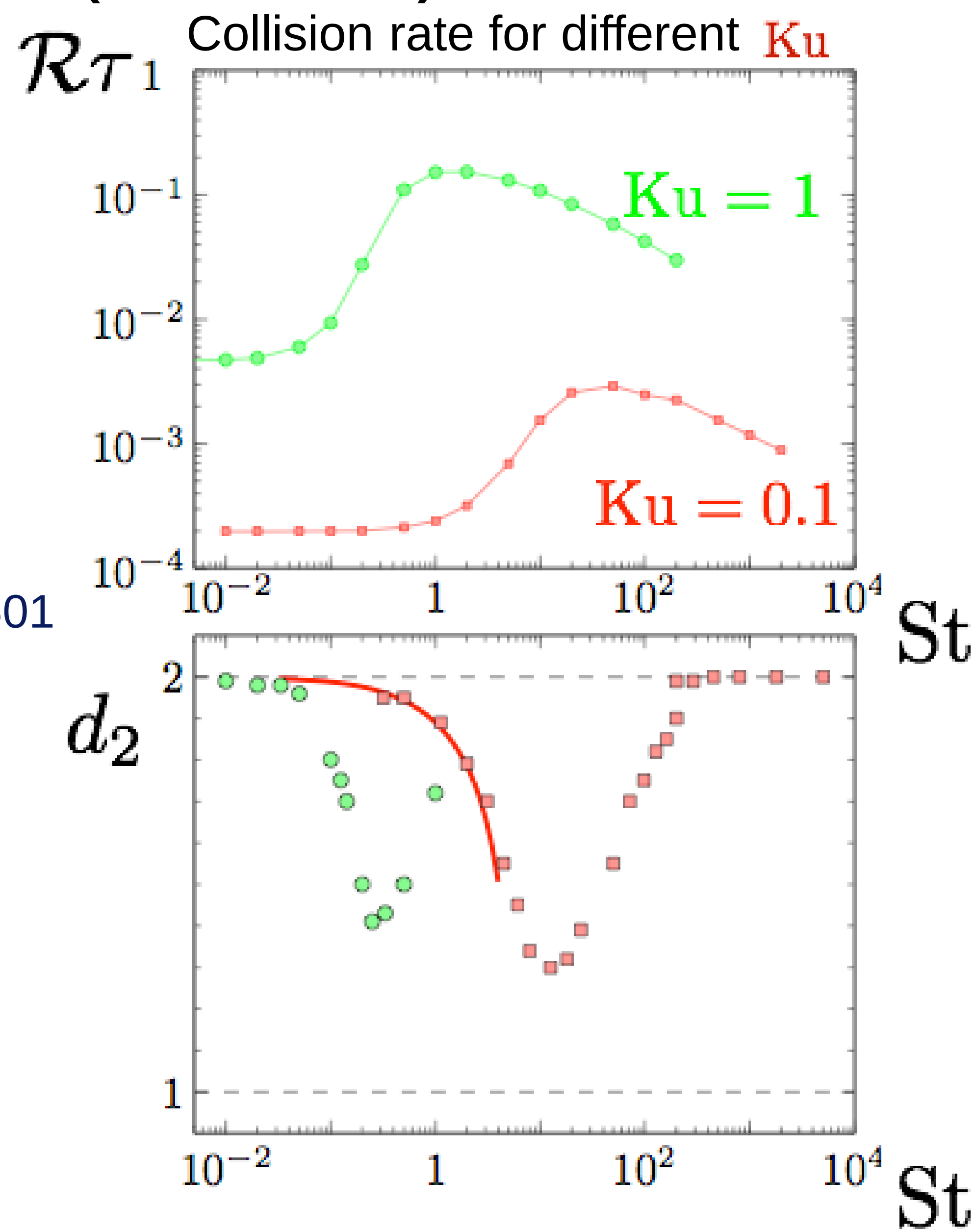


The rate of collisions ($d = 2$)

Explanation:

'Caustics'

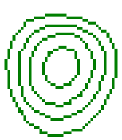
Falkovich et. al, Nature **419**, 151, (2002)
Wilkinson et. al, Phys. Rev. Lett. **97** 048501 (2006)

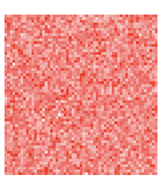


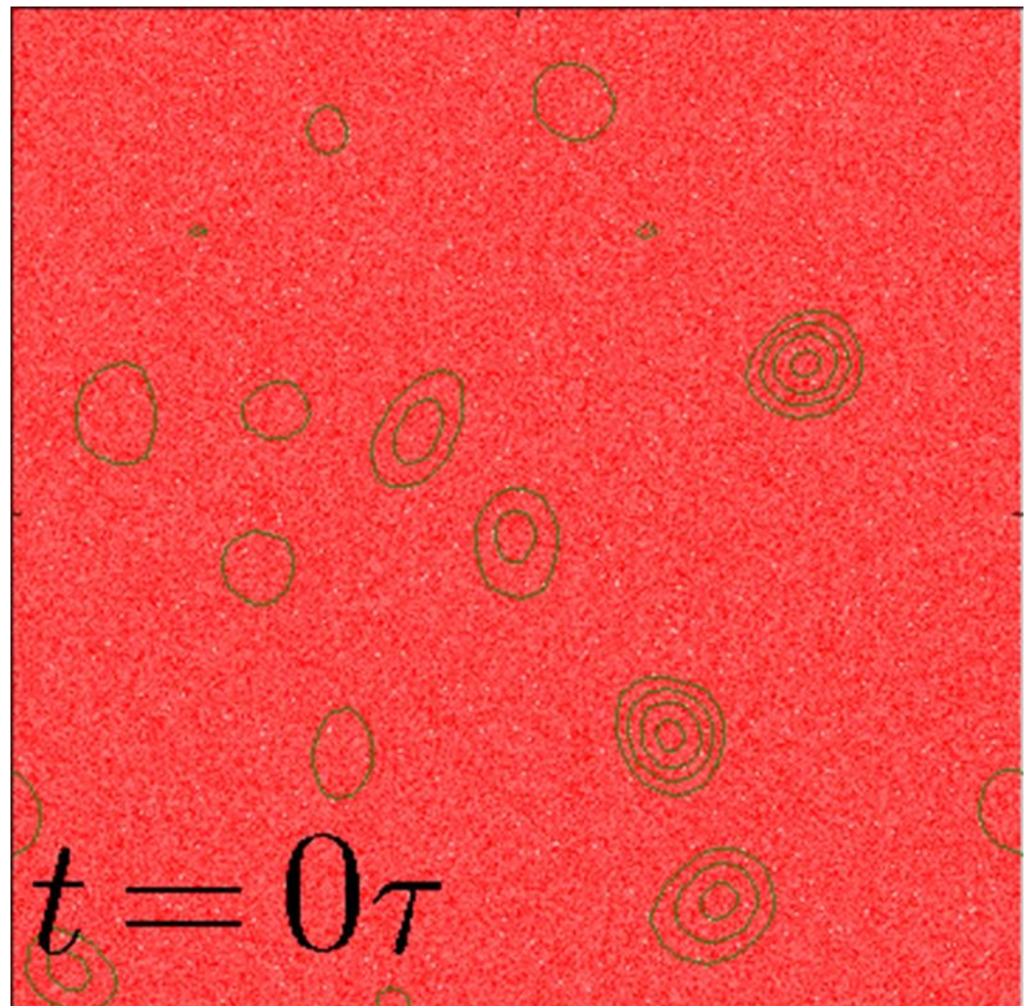
Motion of very inertial particles

Non-interacting, non-colliding particles (red) suspended in a random flow (gravity neglected)

$$\mathrm{St} = 10$$
$$\mathrm{Ku} = 1$$
$$F = 0$$

 Region of high vorticity

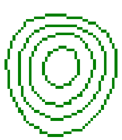
 Particle density

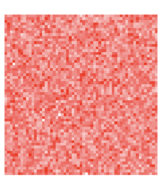


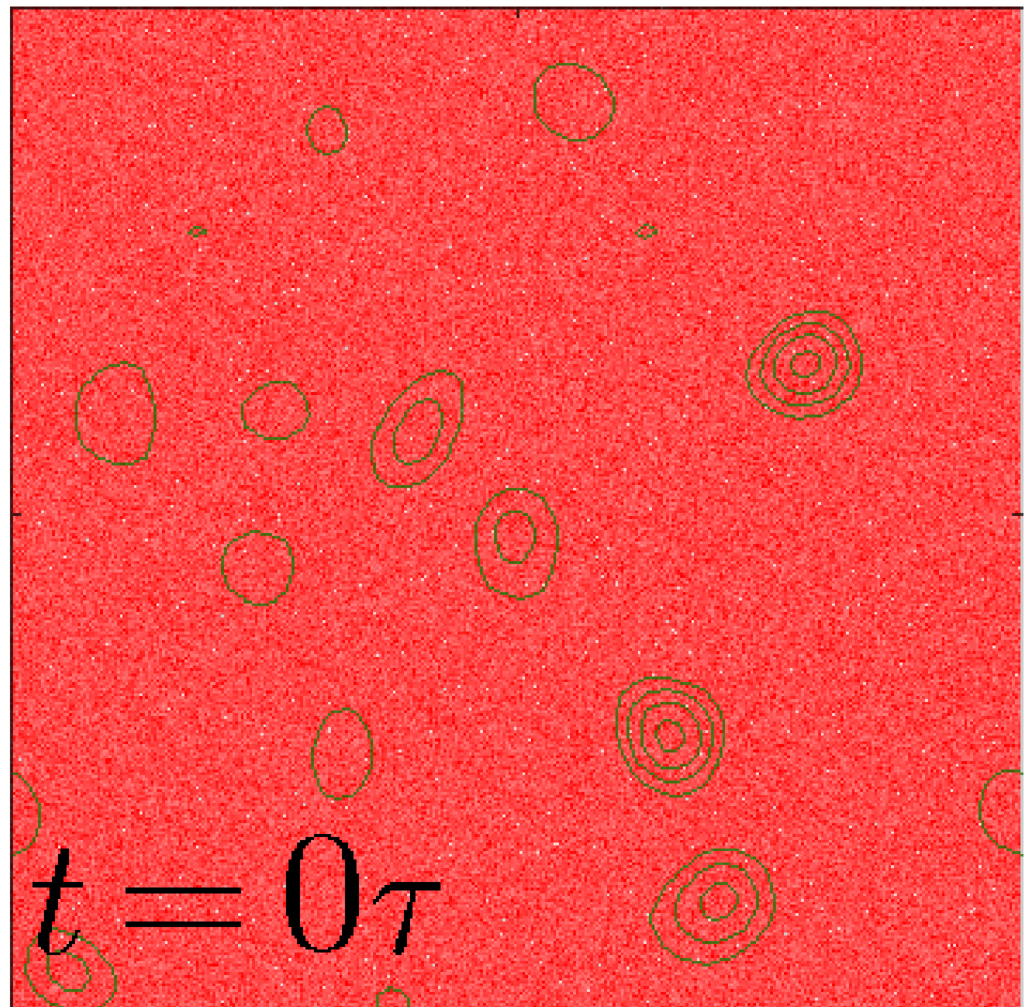
Motion of very inertial particles

Non-interacting, non-colliding particles (red) suspended in a random flow (gravity neglected)

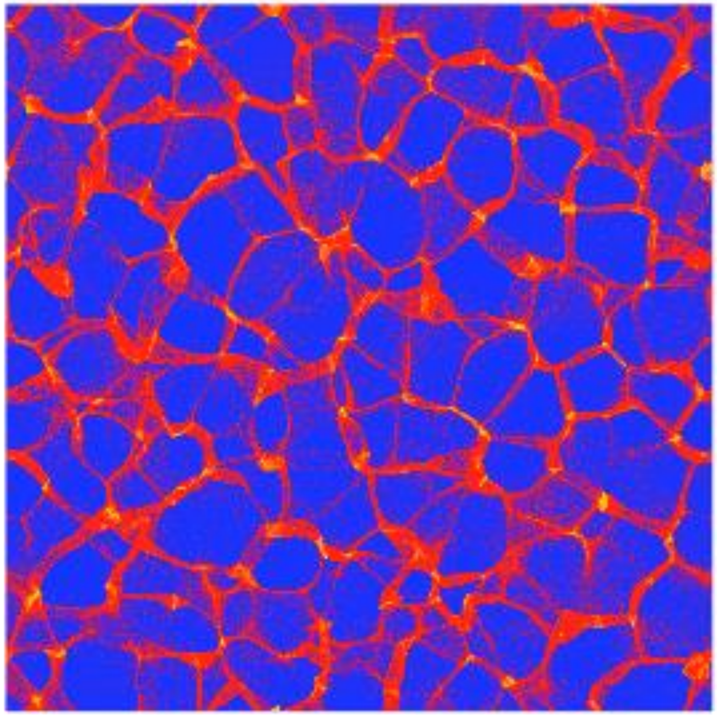
$$\mathrm{St} = 10$$
$$\mathrm{Ku} = 1$$
$$F = 0$$


Region of high vorticity


Particle density



Caustics

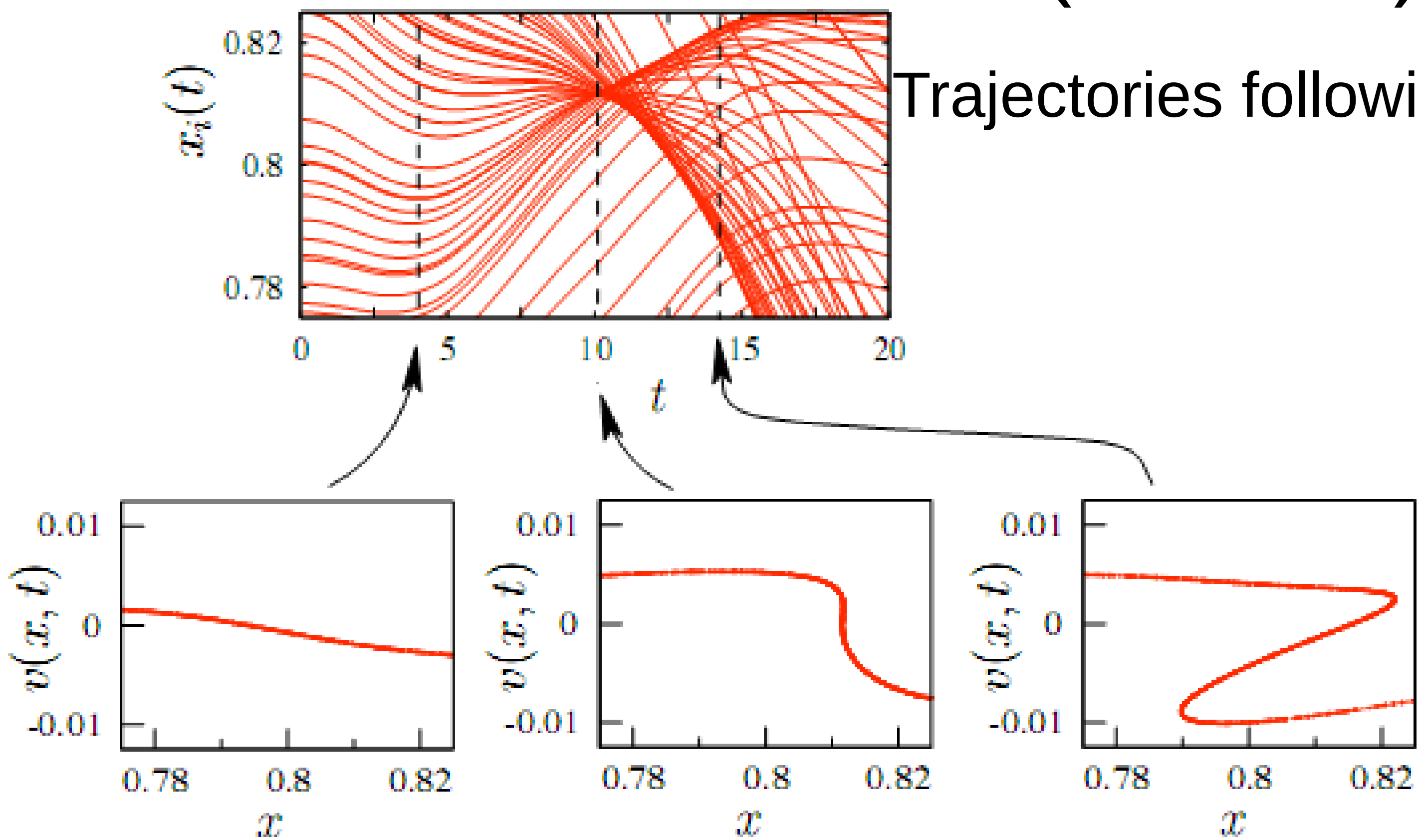


Density of particles suspended in a (compressible) random flow.

Caustics of sun light in water

http://www.physics.utoronto.ca/~peet/

Formation of a caustic ($d = 1$)



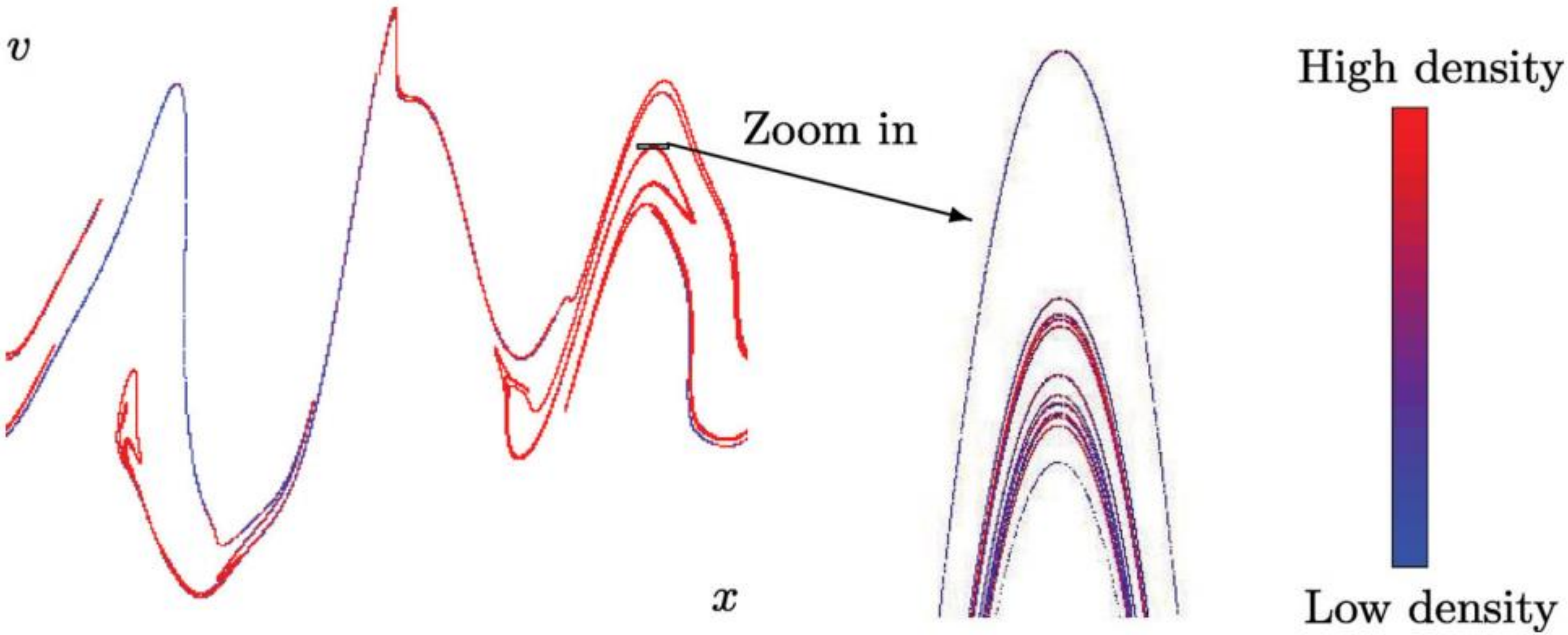
Trajectories following Stokes e.q.m.

Initially the velocity v is a single valued function of position x.
Faster particles overtake the slower ones.
Caustics are formed where the slope is infinite, i.e. when $z = \dfrac{\delta v}{\delta x} = -\infty$
z is local and fail to describe the multi-valuedness between caustic pairs. Phase-space description of separations (Δx, Δv) necessary.

Particle positions at large times ($d = 1$)

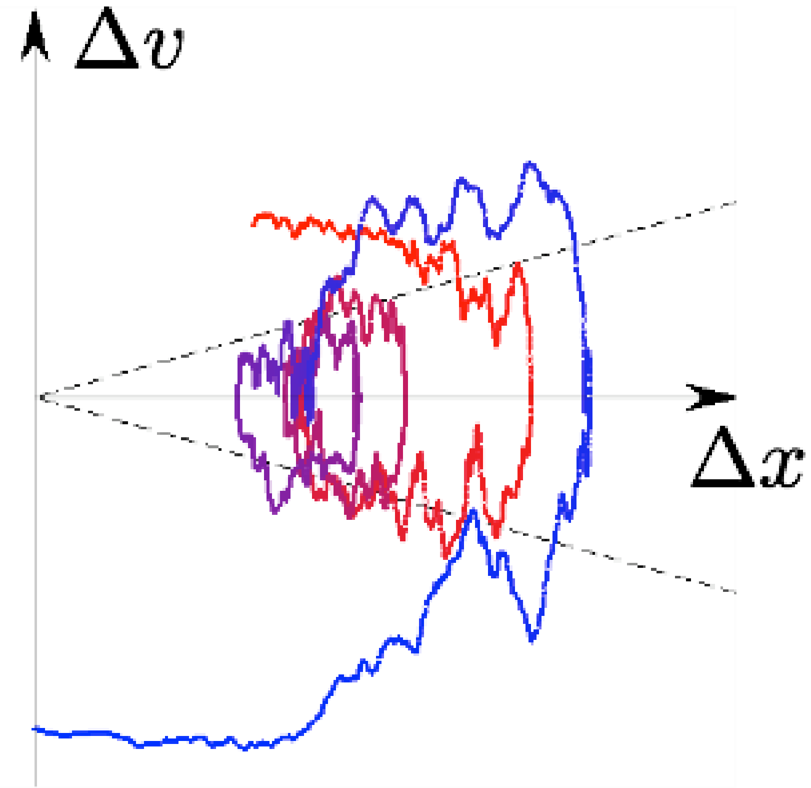
Plot particle positions x and velocities v moving according to Stokes' law with $\mathrm{Ku} = 0.1$ and $\mathrm{St} = 100$ (so $\lambda_1 > 0$) at a large time.

Particles distribute on fractal in phase-space with phase-space fractal dimension D_2. Here $D_2 \approx 0.24$.



Trajectories of separations ($d = 1$)

Consider the relative motion between two particles with separation Δx and relative velocity Δv .



Example of relative trajectory between two droplets.

KG & B. Mehlig, J. Turbulence **15** (2014); Phys. Rev. E **84** (2011)

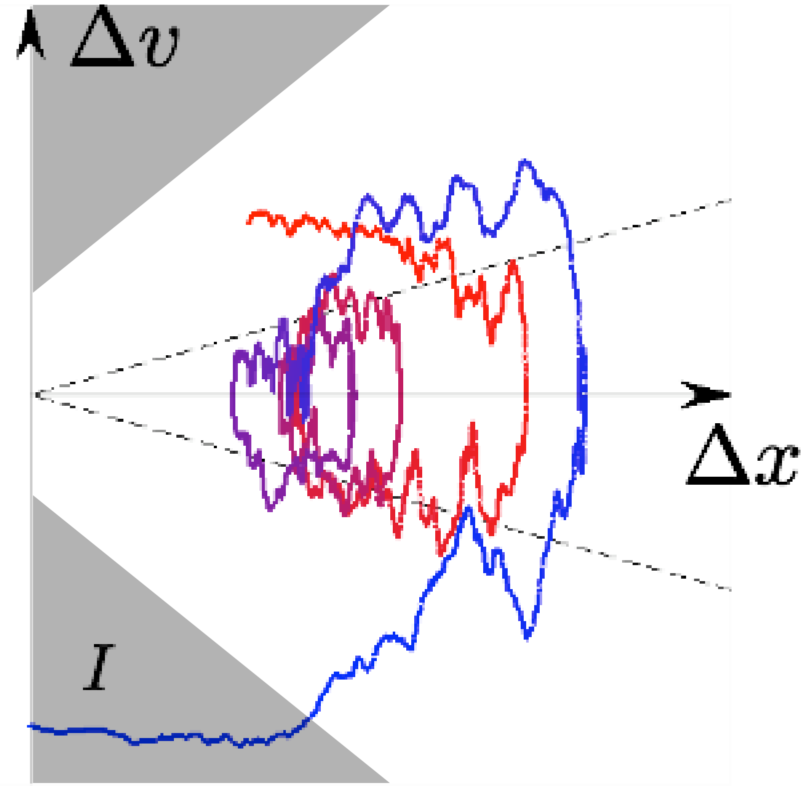
Trajectories of separations ($d = 1$)

Consider the relative motion between two particles with separation Δx and relative velocity Δv .

Case I :
When $|\Delta v| \gg |\Delta x|$ and particles approach each other, a caustic occurs. The motion can be approximated by uniform, i.e. $\Delta v = \mathrm{const.}$ universally.

For a given Δv , all Δx are equally likely, i.e. the probability distribution of Δx and Δv behaves as

$$\rho(\Delta v, \Delta x) \sim f_I(\Delta v)$$



Example of relative trajectory between two droplets.

KG & B. Mehlig, J. Turbulence **15** (2014); Phys. Rev. E **84** (2011)

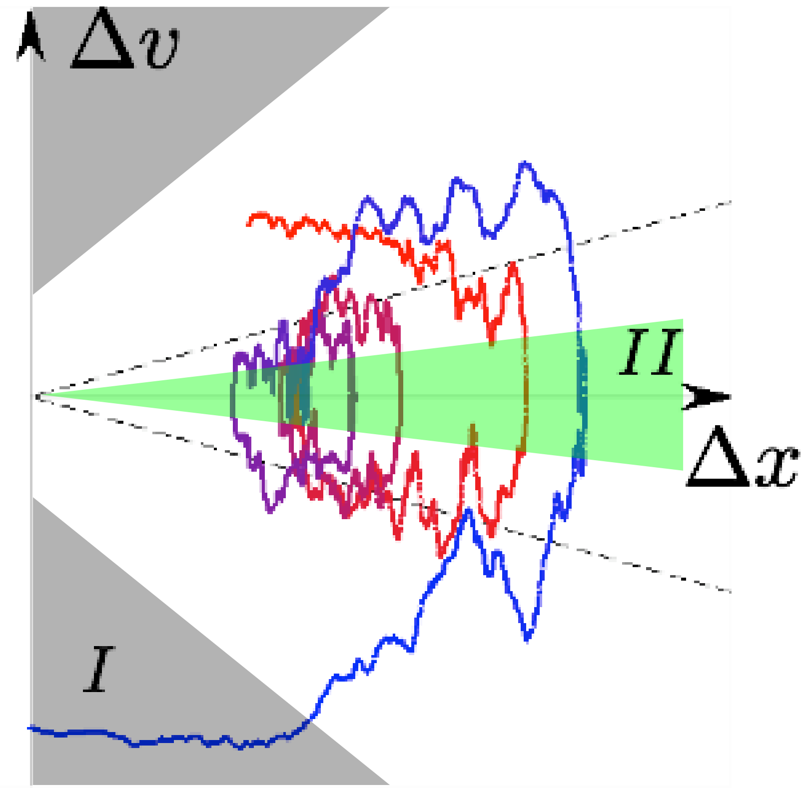
Trajectories of separations ($d = 1$)

Consider the relative motion between two particles with separation Δx and relative velocity Δv .

Case II :
When $|\Delta v| \ll |\Delta x|$, changes in the relative amplitude of Δx are small compared to changes in the relative amplitude of Δv .

Use $\Delta x \approx \text{const.}$ to get

$$\rho(\Delta v, \Delta x) \sim f_{II}(\Delta x)$$



Example of relative trajectory between two droplets.

KG & B. Mehlig, J. Turbulence **15** (2014); Phys. Rev. E **84** (2011)

Distribution of separations ($d = 1$)

Match these asymptotic limits along a line $\Delta v = z^* \Delta x$ (small Δx)

$$\rho(\Delta v, \Delta x) \sim \begin{cases} f_I(\Delta v) & \text{for } |\Delta v| \geq z^* |\Delta x| \\ f_{II}(\Delta x) & \text{for } |\Delta v| < z^* |\Delta x| \end{cases}$$

Determine $f_I(\Delta v)$ and $f_{II}(\Delta x)$ from the definition of the phase-space correlation dimension D_2 , $P(w) \sim w^{D_2 - 1}$, for small phase-space separations $w = \sqrt{\Delta x^2 + (\Delta v / z^*)^2} \ll 1$:

$$P(w) = \int_{-z^* w}^{z^* w} \mathrm{d}\Delta v \rho(w, \Delta v) \frac{\partial w}{\partial \Delta x}$$

Put $\left(\Delta v = w z^* \sin(\gamma)\right)$

$$\sim 2z^* w \int_0^{\pi/4} \mathrm{d}\gamma f_{II}(w \cos\gamma) + 2z^* w \int_{\pi/4}^{\pi/2} \mathrm{d}\gamma f_I(w z^* \sin\gamma)$$

$$\sim w^{D_2 - 1}$$

This gives $f_I(\Delta v) \sim |\Delta v|^{D_2 - 2}$ and $f_{II}(\Delta x) \sim |\Delta x|^{D_2 - 2}$.

Distribution of separations ($d = 1$)

The matching gives

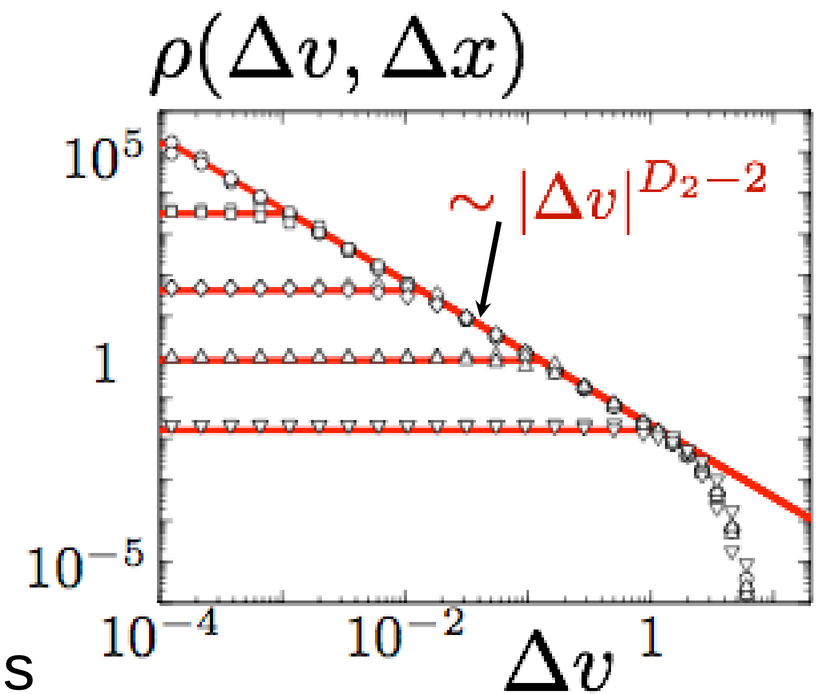
$$f_I(\Delta v) \sim |\Delta v|^{D_2-2}$$
$$f_{II}(\Delta x) \sim |\Delta x|^{D_2-2}$$

and thus

$\rho(\Delta v, \Delta x)$

$$\sim \begin{cases} |\Delta v/z^*|^{D_2-2} & \text{for } |\Delta v| \geq z^*|\Delta x| \\ |\Delta x|^{D_2-2} & \text{for } |\Delta v| < z^*|\Delta x| \end{cases}$$

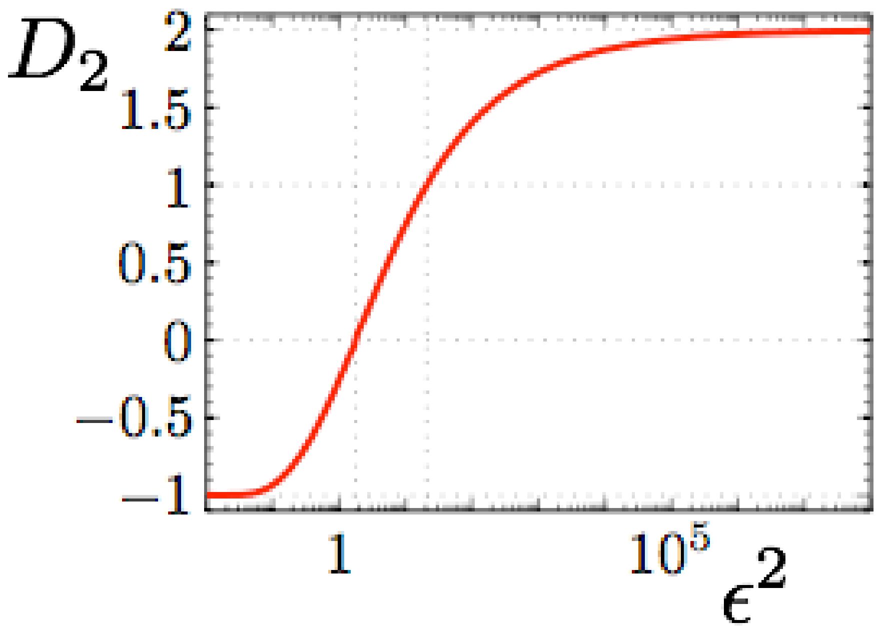
This result is valid for many systems with caustics and fractal clustering and non-singular force at $\Delta x = 0$.

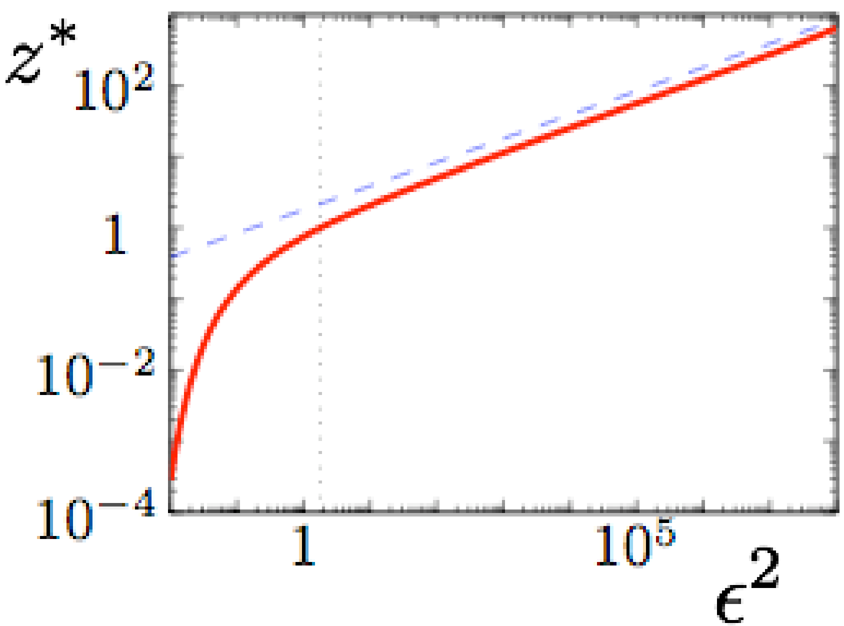


Distribution of Δx and Δv at $\Delta x = \{10^{-4}, 10^{-3}, 0.01, 0.1, 1\}$

Results in white-noise limit ($d = 1$)

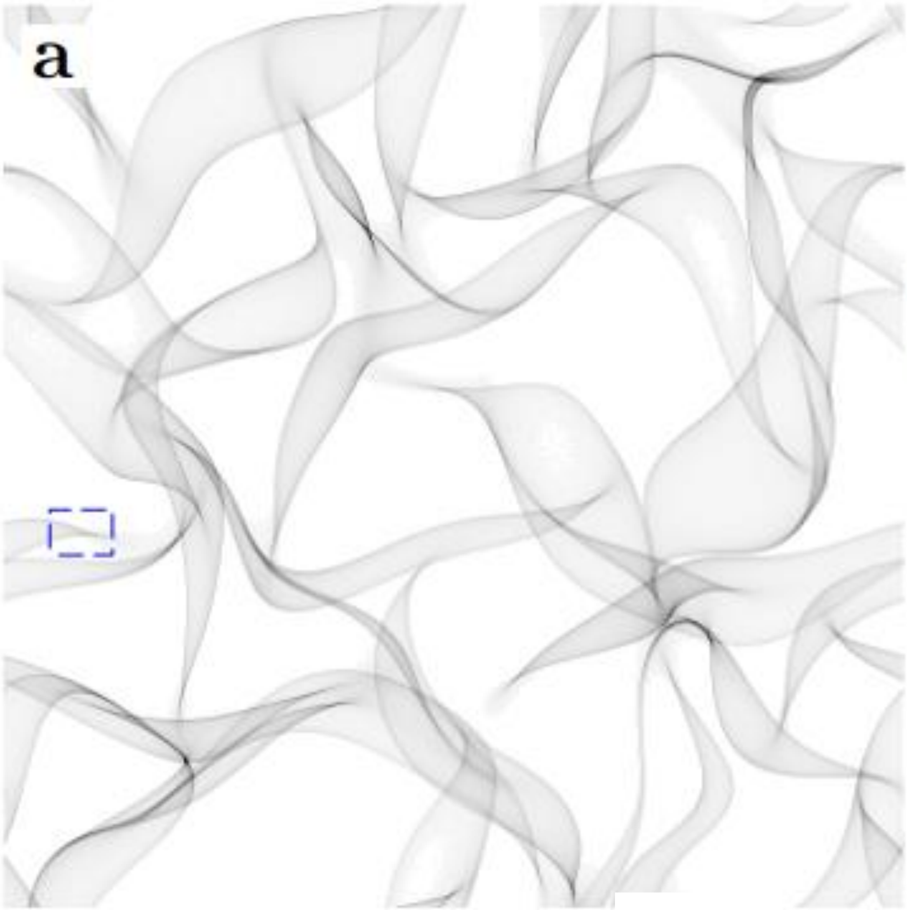
The analytic solution for small $|\Delta x|$ gives the phase-space correlation dimension D_2 and the matching scale z^*.



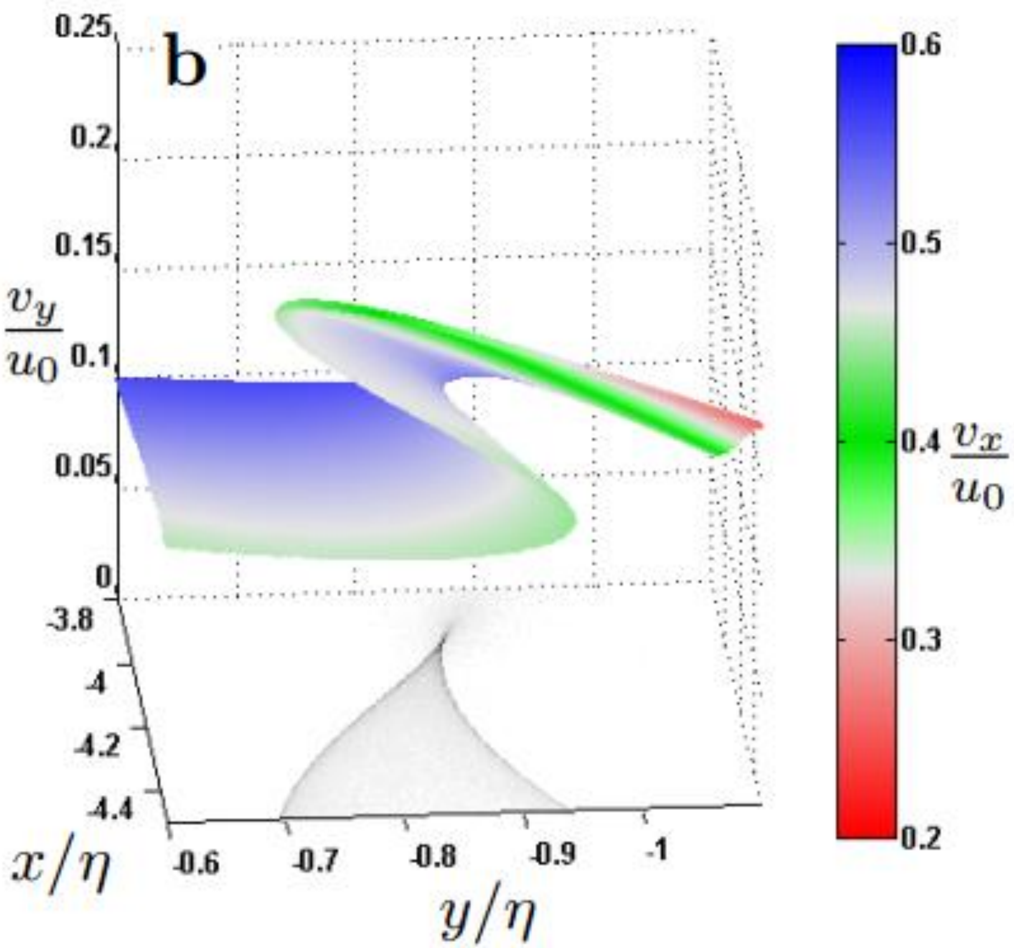


Caustics in two spatial dimensions

$\mathrm{Ku} = 1 \quad \mathrm{St} = 10 \quad F = 0$



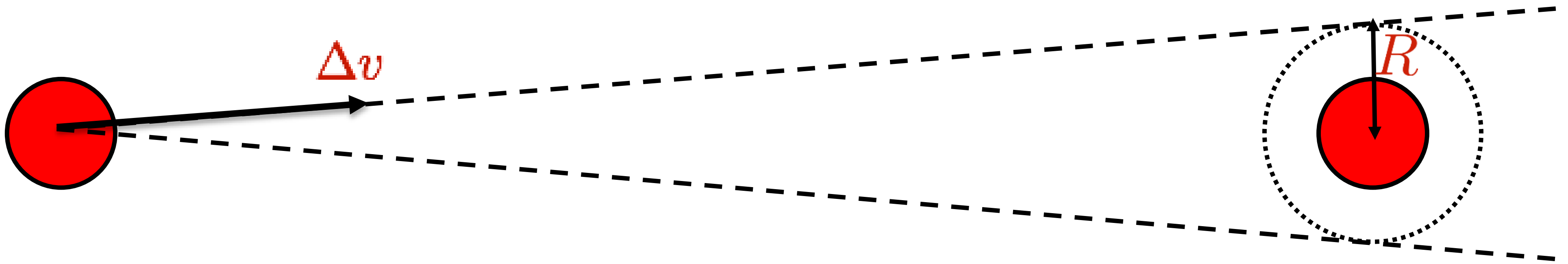
Particle density in x, y -plane



Phase-space picture of zoomed rectangle.

Relative velocities (general d)

Match the two asymptotes as before. When $|\Delta v| \gg R \equiv |\Delta x|$ the uniform relative motion only gives a contribution to the distribution at small separations if its angular component is small enough:



This results in a geometrical factor R^{d-1}.

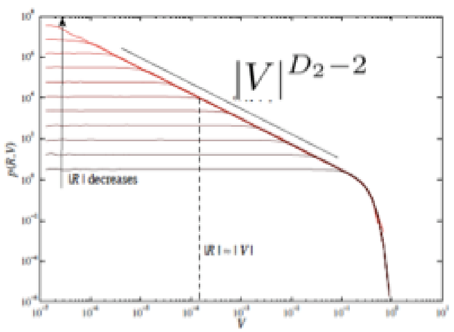
Comparison to distribution of phase-space separations gives:

$$\rho(R, \Delta \mathbf{v}) \sim R^{d-1} |\Delta \mathbf{v}|^{D_2 - 2d} \quad \text{if} \quad |\Delta \mathbf{v}| \gg R$$

$$\rho(R, \Delta \mathbf{v}) \sim R^{D_2 - d - 1} \quad \text{if} \quad |\Delta \mathbf{v}| \ll R$$

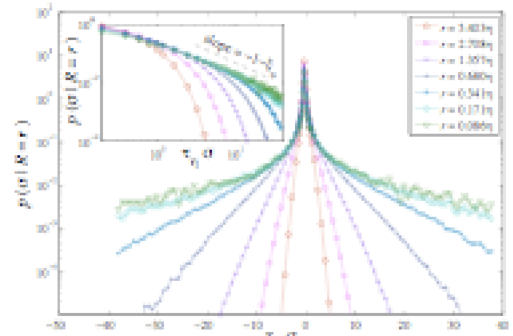
Universality

We find that the distribution of relative velocities show universal power-law tails, independent of the driving flow.



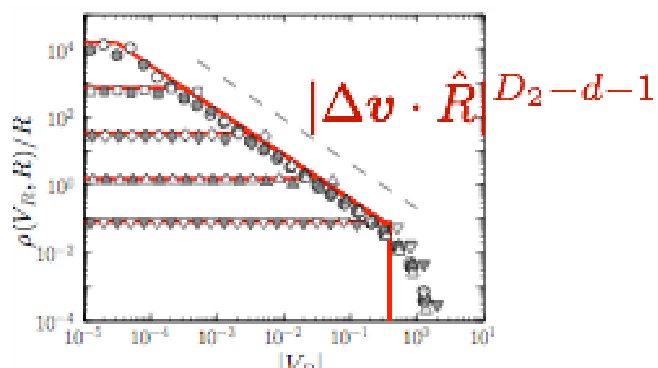
Smooth 'Kraichnan flow' ($d = 1$)

M. Cencini, Talk: MP0806_CG3.pdf (2009)



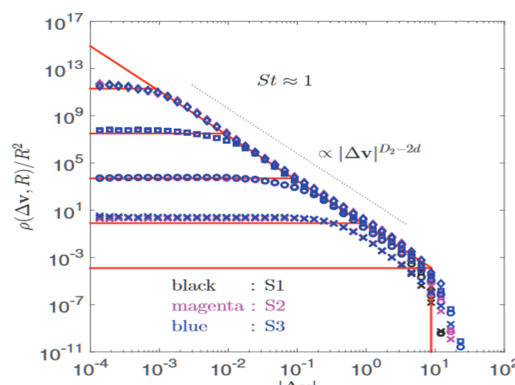
Numerical simulations of turbulence

J. Bec et. al, J. Fluid Mech. **646** (2010)



Distribution of radial velocities

KG & B. Mehlig J. turbulence **15** (2014)



DNS at different Reynolds numbers

V. Perrin & H. Jonker, Phys. Rev. E **92** (2015)

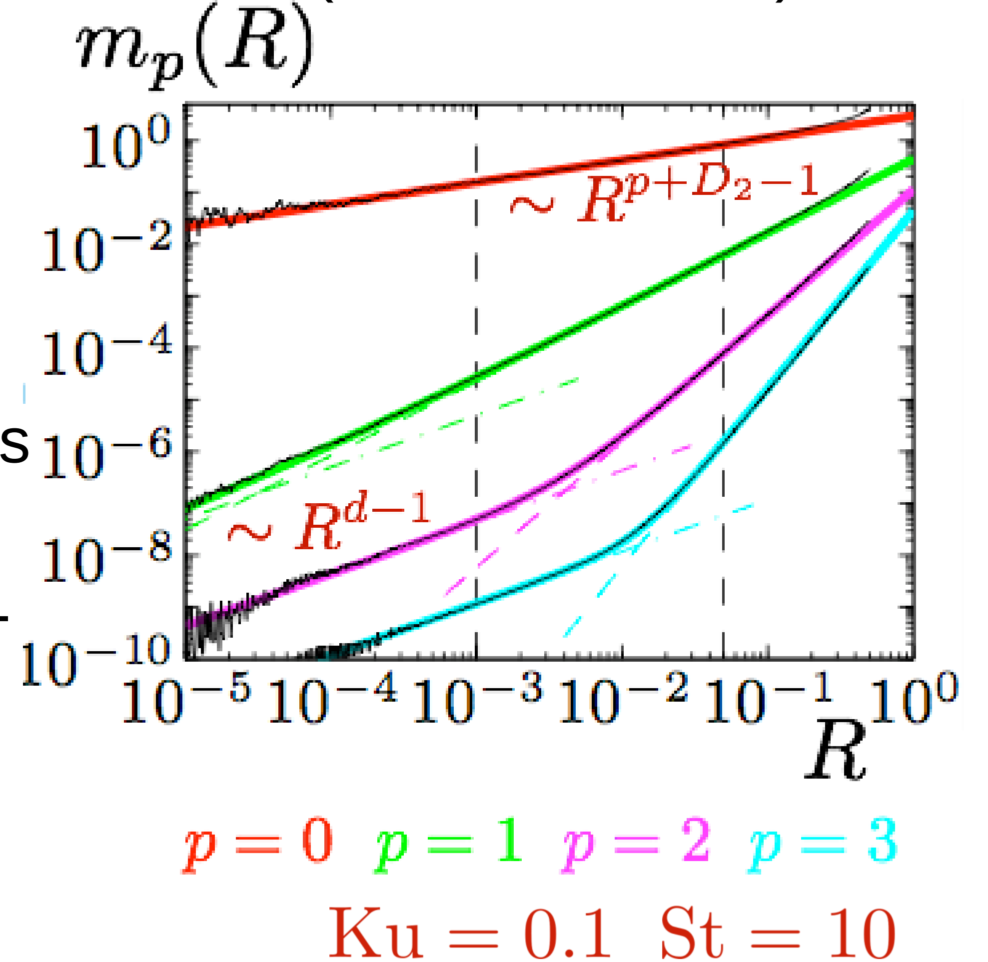
Consequences

Distribution of relative velocities govern collision rates.
Introduce cut-off at typical maximal relative velocities and integrate over $\Delta\boldsymbol{v}$ to get moments m_p of radial velocities ($v_r \equiv \Delta\boldsymbol{v}\cdot\Delta\boldsymbol{r}$)

$$m_p \equiv \int \mathrm{d}\boldsymbol{v}|v_r|^p \rho(\Delta\boldsymbol{r}, \Delta\boldsymbol{v})$$

$$\sim B_p R^{p+D_2-1} + C_p R^{d-1}$$

Smooth dynamics due to caustics

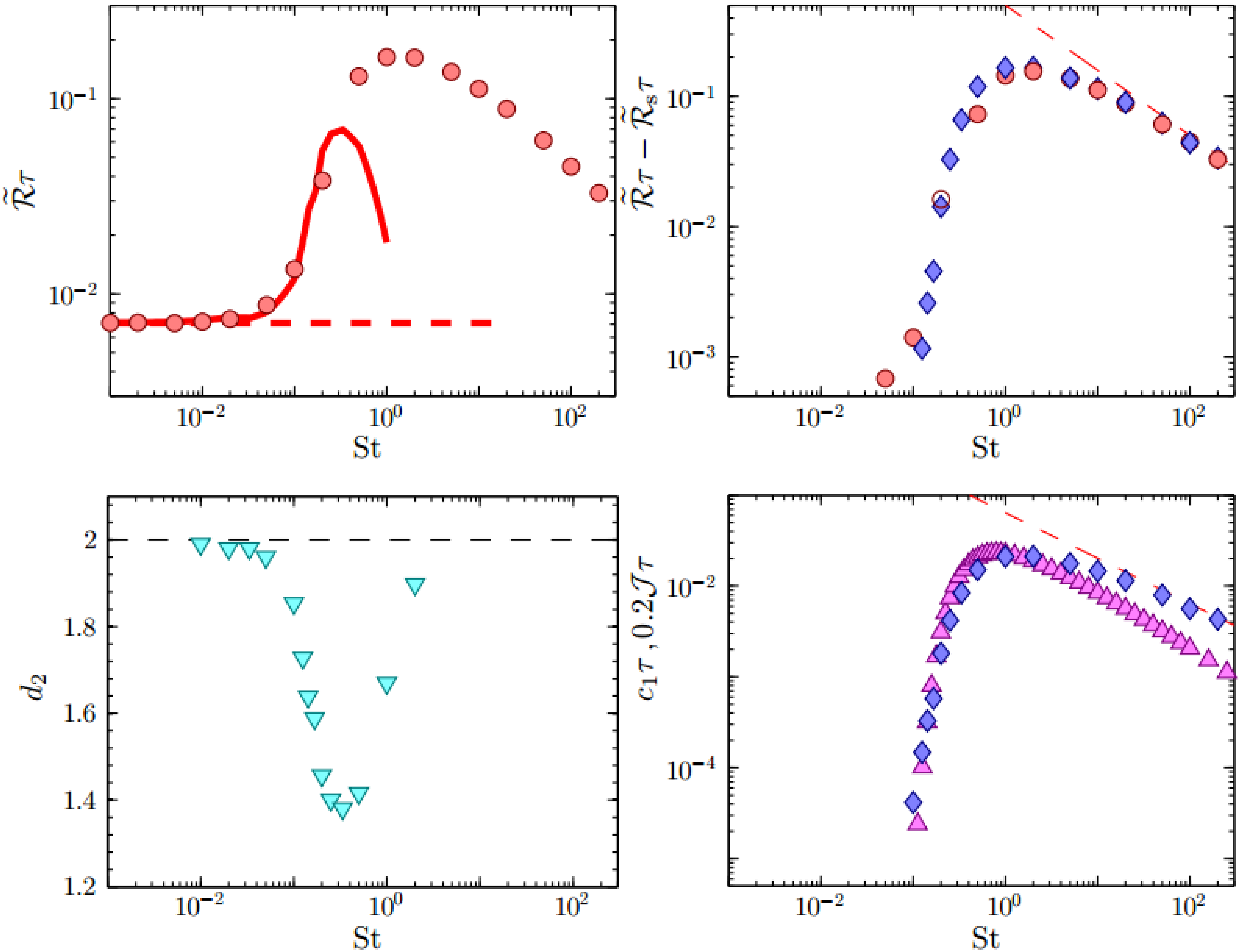


Case $p=0$ gives the spatial correlation dimension $d_2 = \min(D_2, 1)$.

Case $p=1$ gives an estimate of the 'recollision rate'

Higher p more affected by caustics

Ghost-particle collision rate ($m_1(2\overline{a})$)



Simulations of collisions $\mathcal{R} = m_1(2\overline{a})$

Smooth contribution $\sim b_1(2\overline{a})^{d_2}$

Caustic contribution $\sim c_1(2\overline{a})^{d-1}$

Caustic formation rate (arbitrary prefactor)

$a = 0.01\eta\,,\ d = 2$
$\mathrm{Ku} = 1\,,\ F = 0$

Conclusions, relative velocities

We find that the distribution of relative velocities show universal power-law tails, independent of the underlying flow.

The power-law tails corresponds to collisions with large relative velocities.

The collision rate has two contributions: smooth + due to caustics

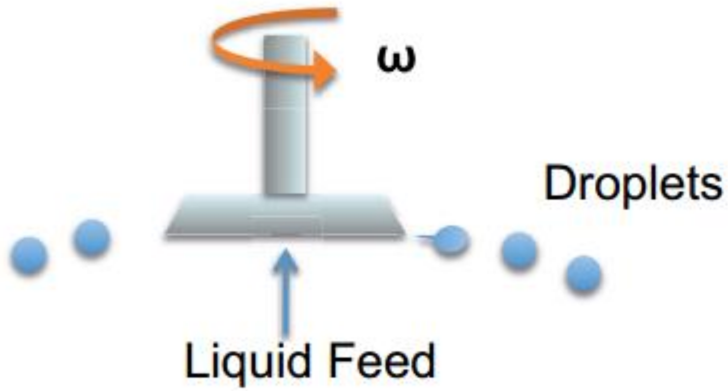
The contribution from caustics dominates the collision rate for small particles.

The smooth contribution is increased due to fractal clustering.

Crystal ball experiment

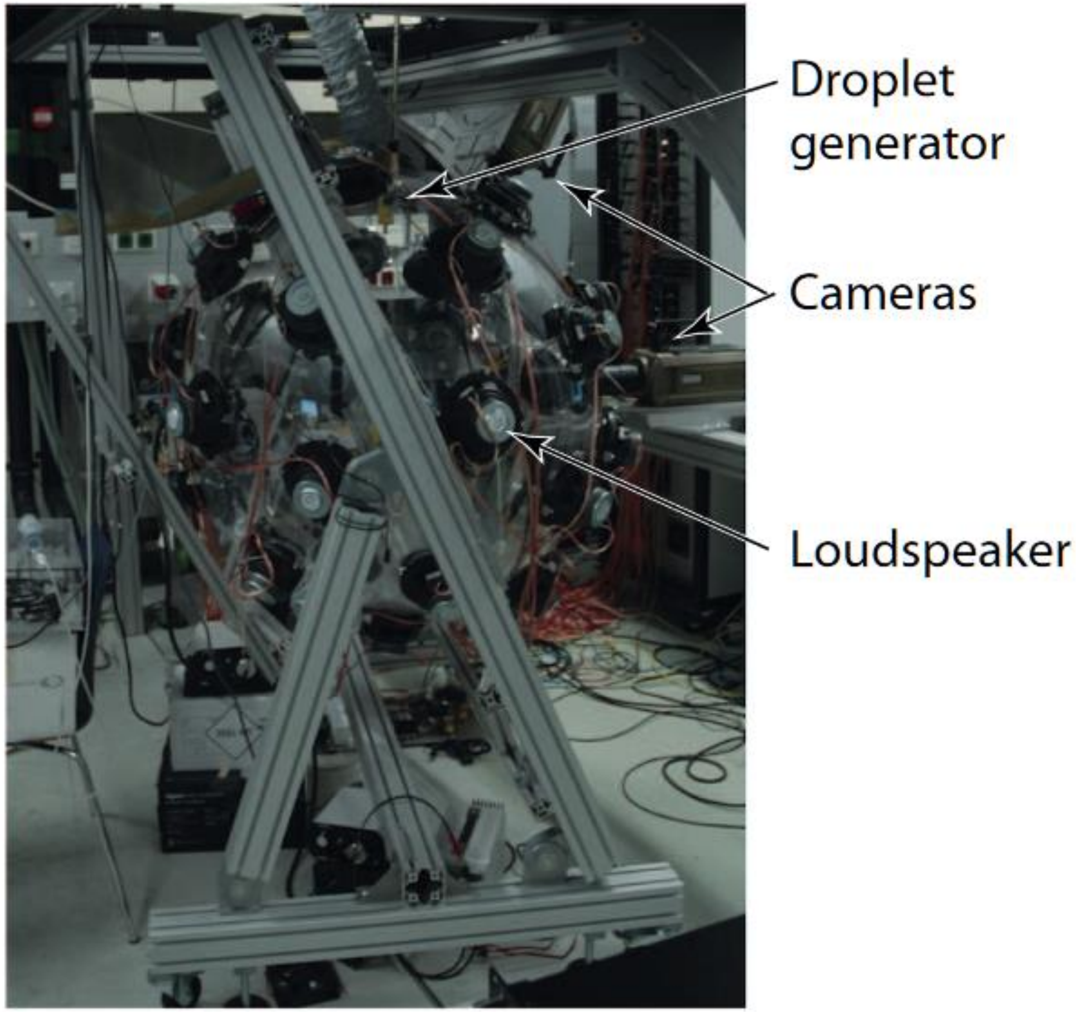
G. Bewley, E-W. Saw and E. Bodenschatz, New Journal of Physics **15** (2013)

Droplets injected in meter-size plexiglass ball.



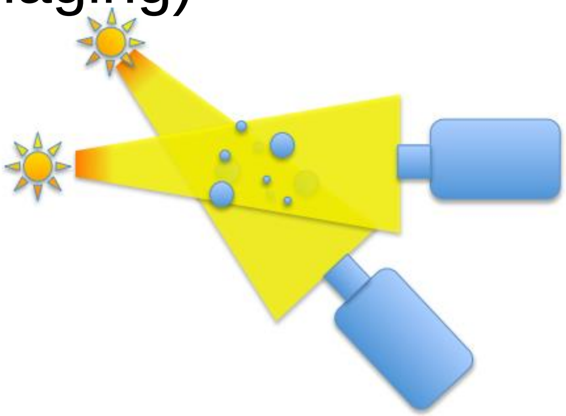
Turbulence driven by 32 randomly pulsating loudspeakers.

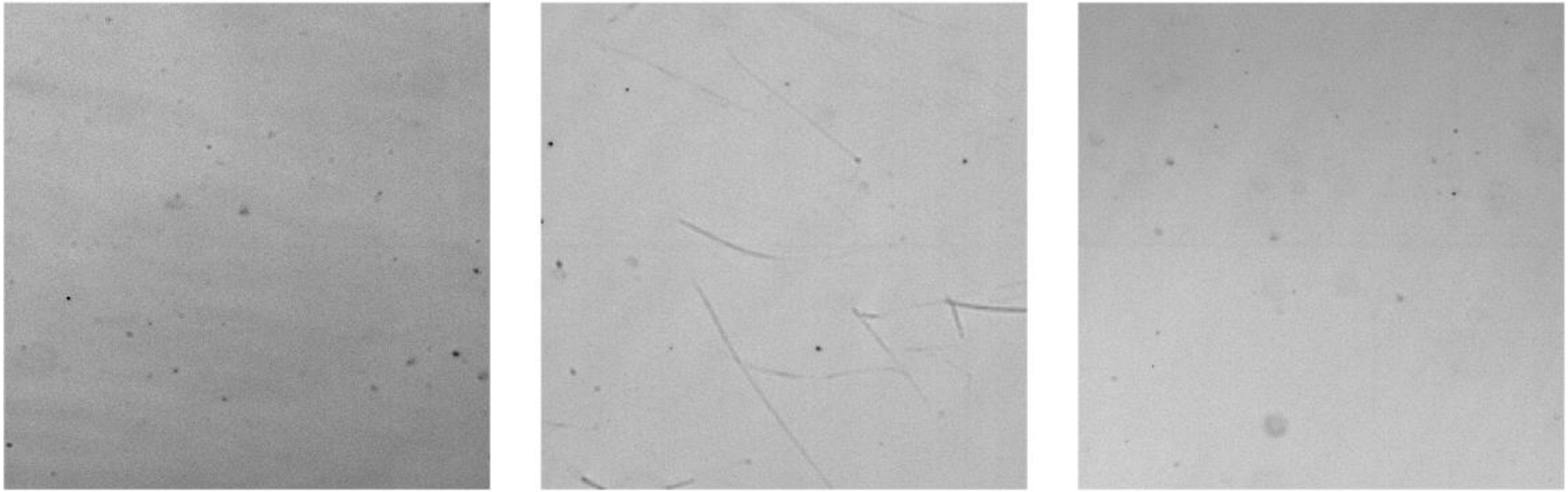
A millimeter-sized cube in the center is monitored by three high-speed cameras.



Particle tracking: Raw data

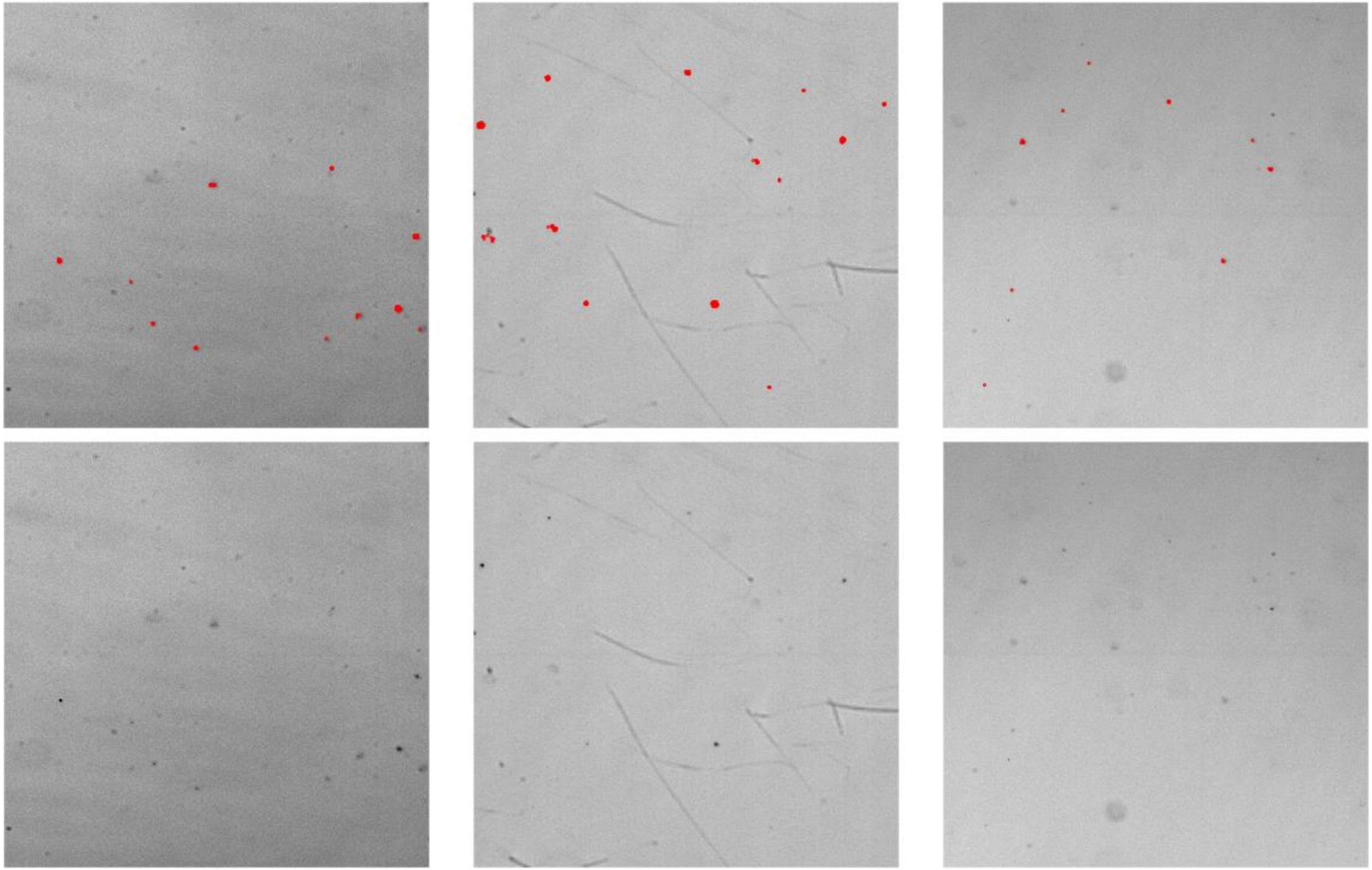
Recorded video from three cameras (shadow imaging)





t=0.1 ms

Particle identification

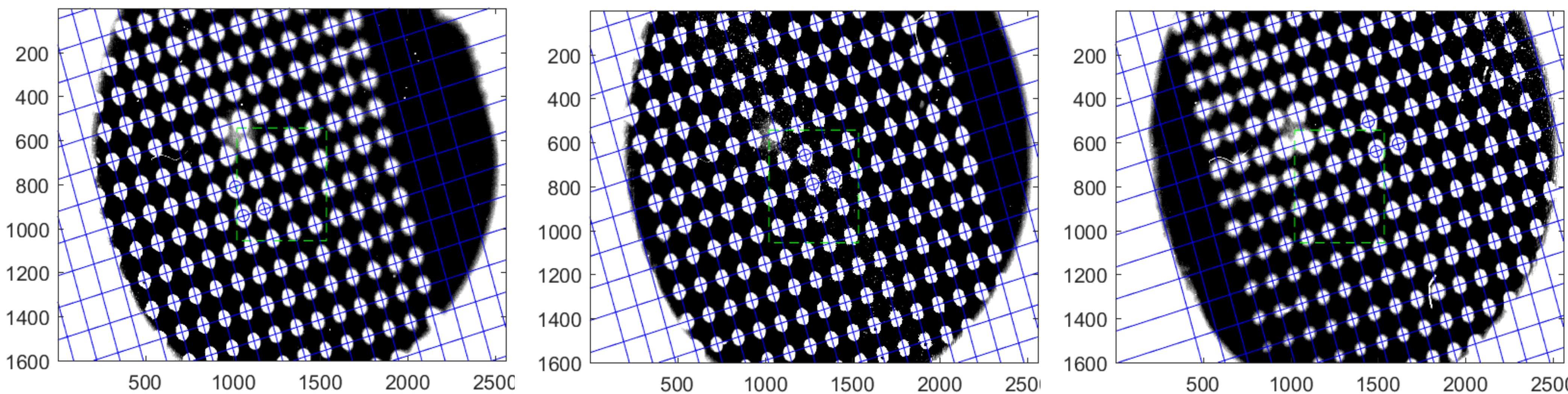


t=0.1 ms

Setting up a coordinate system

A three-dimensional coordinate system is set up by moving a punch-hole mask (separation $0.5\,\mathrm{mm}$)in steps of $0.5\,\mathrm{mm}$ in the z-direction. Find linear mapping between world coordinate $\boldsymbol{r}^{(\mathrm{world})}$ and image plane $\boldsymbol{r}^{(\mathrm{image})}$ for each camera: $\boldsymbol{r}^{(\mathrm{image})} = \mathbb{A}\boldsymbol{r}^{(\mathrm{world})} + \boldsymbol{T}$

$z = -2\,\mathrm{mm}$ $z = 0$ $z = +2\,\mathrm{mm}$



— Projection of Eulerian grid onto the image plane after calibration

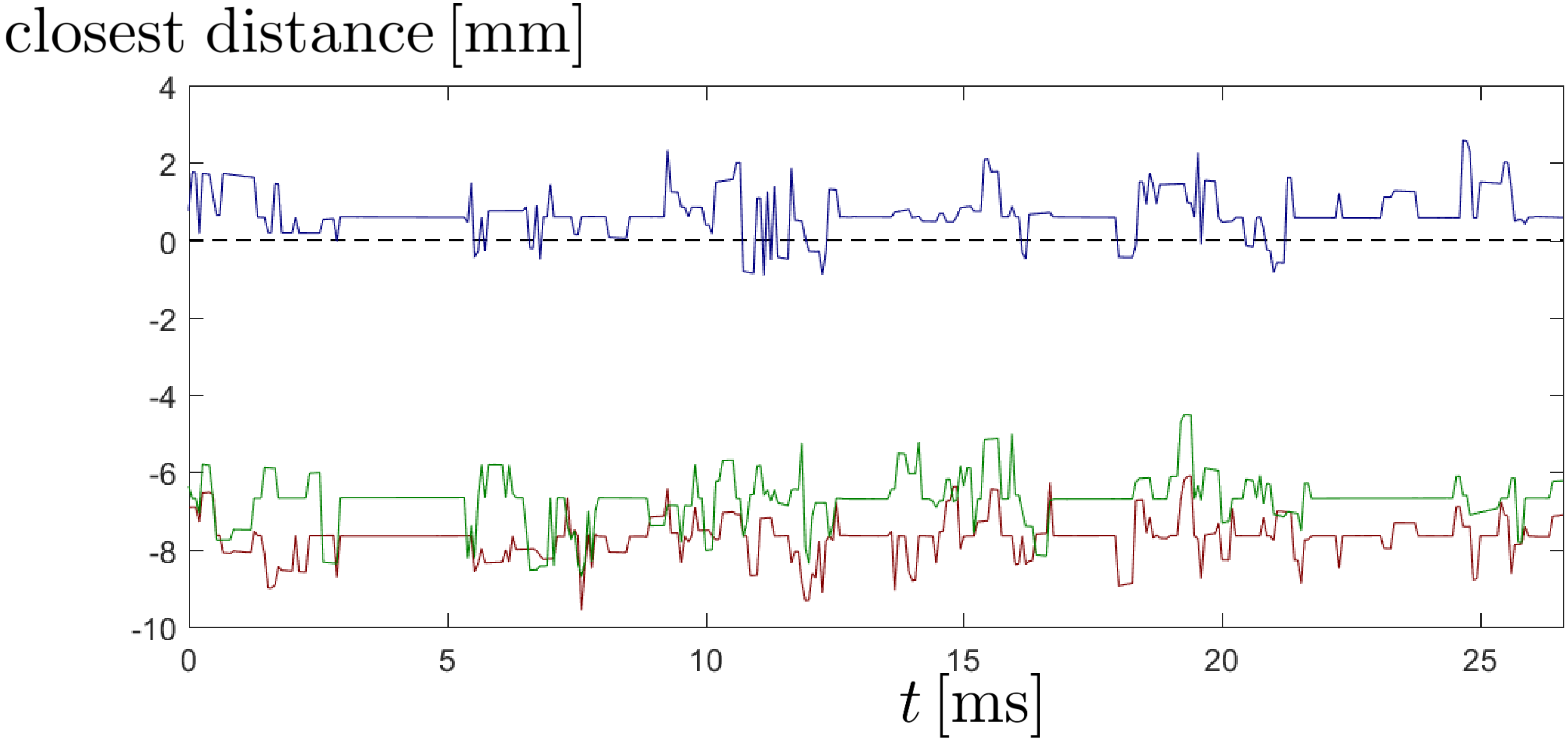
····· Measurement area

After calibration image positions correspond to a ray in the real space. The intersection point of the three rays gives particle position.

Recalibration

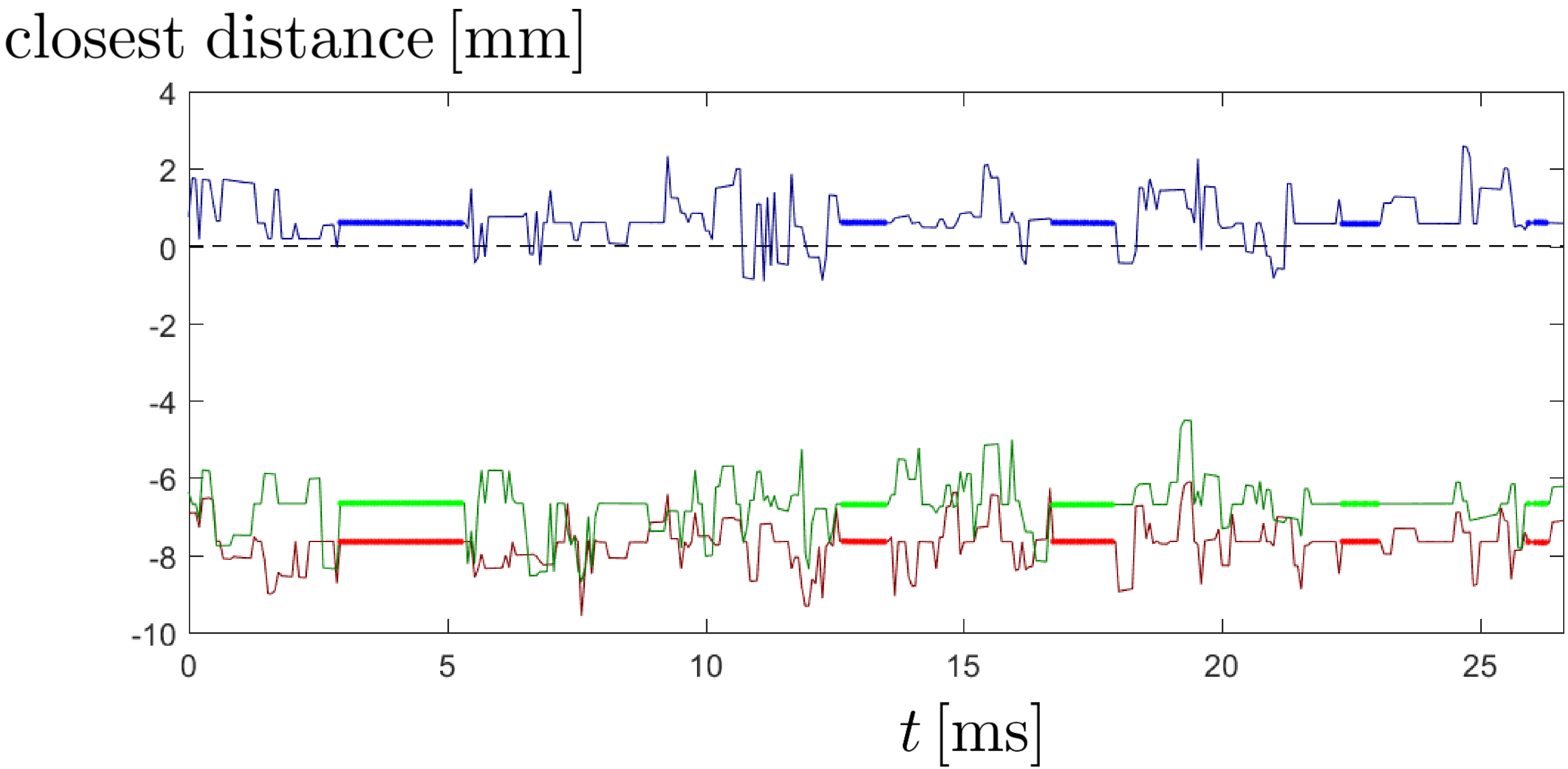
In experimental runs the camera system has shifted compared to the calibration due to temperature variations etc.

Figure shows closest distance between rays corresponding to the darkest shadow in each camera.



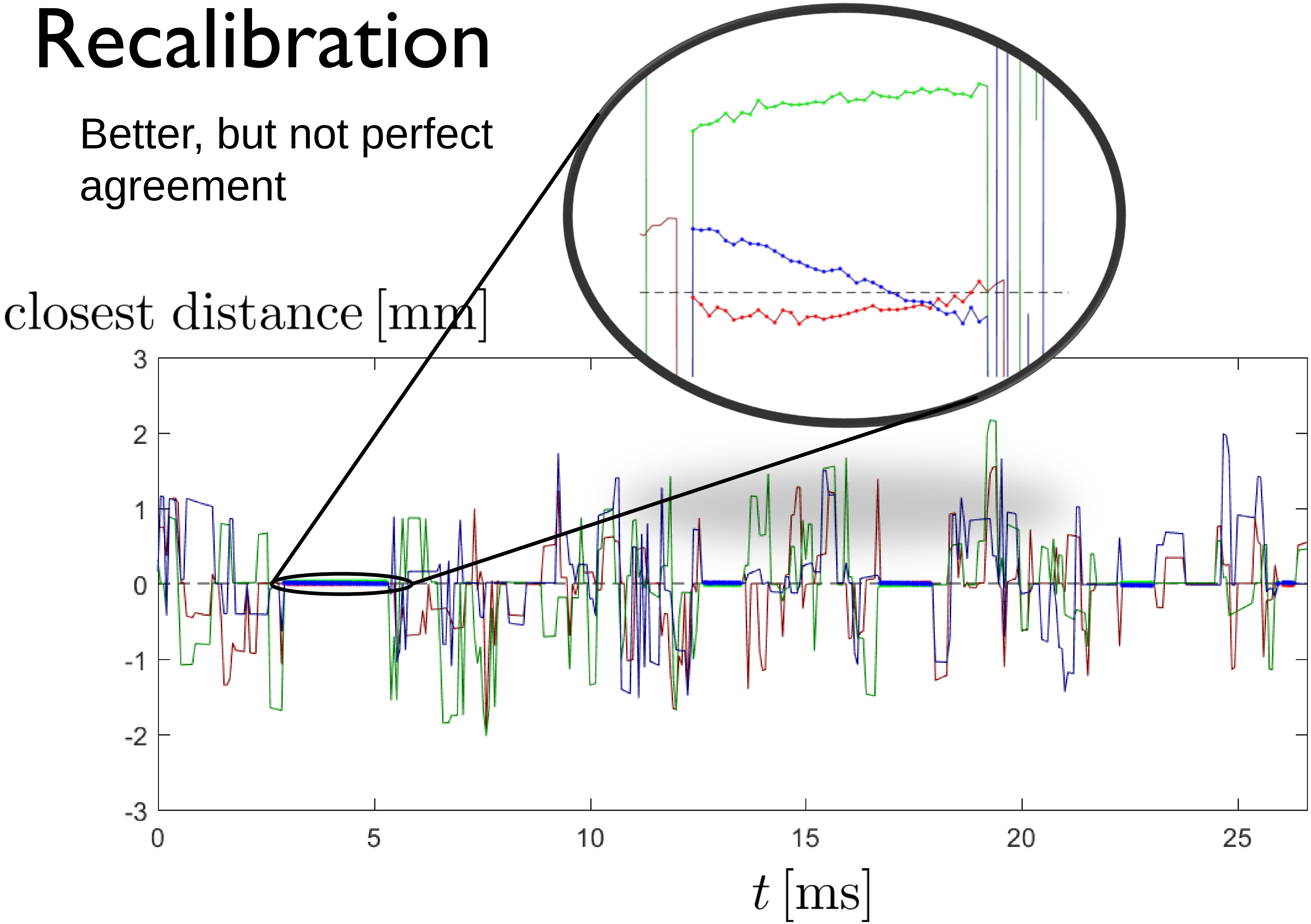
Recalibration

Solution: Select regions where it is clear that the rays originates from the same particle and readjust the calibration.



Recalibration

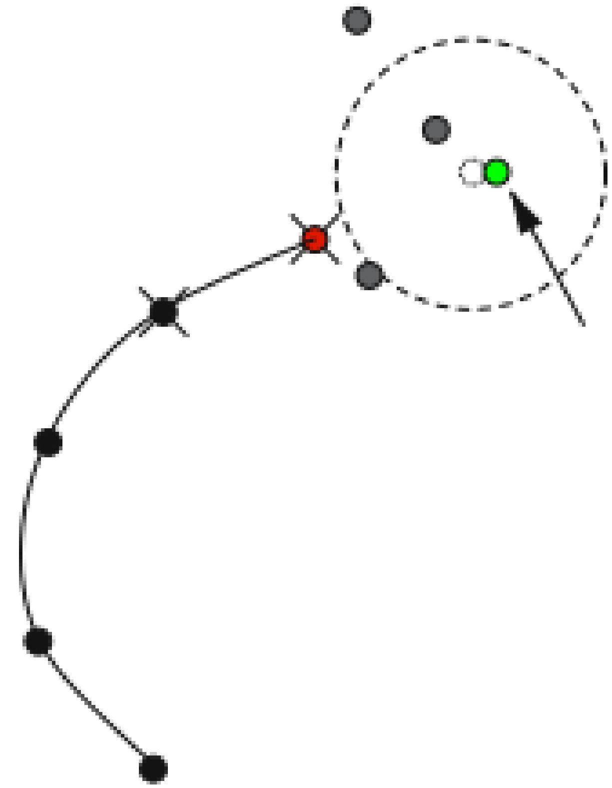
Better, but not perfect agreement



Forming trajectories

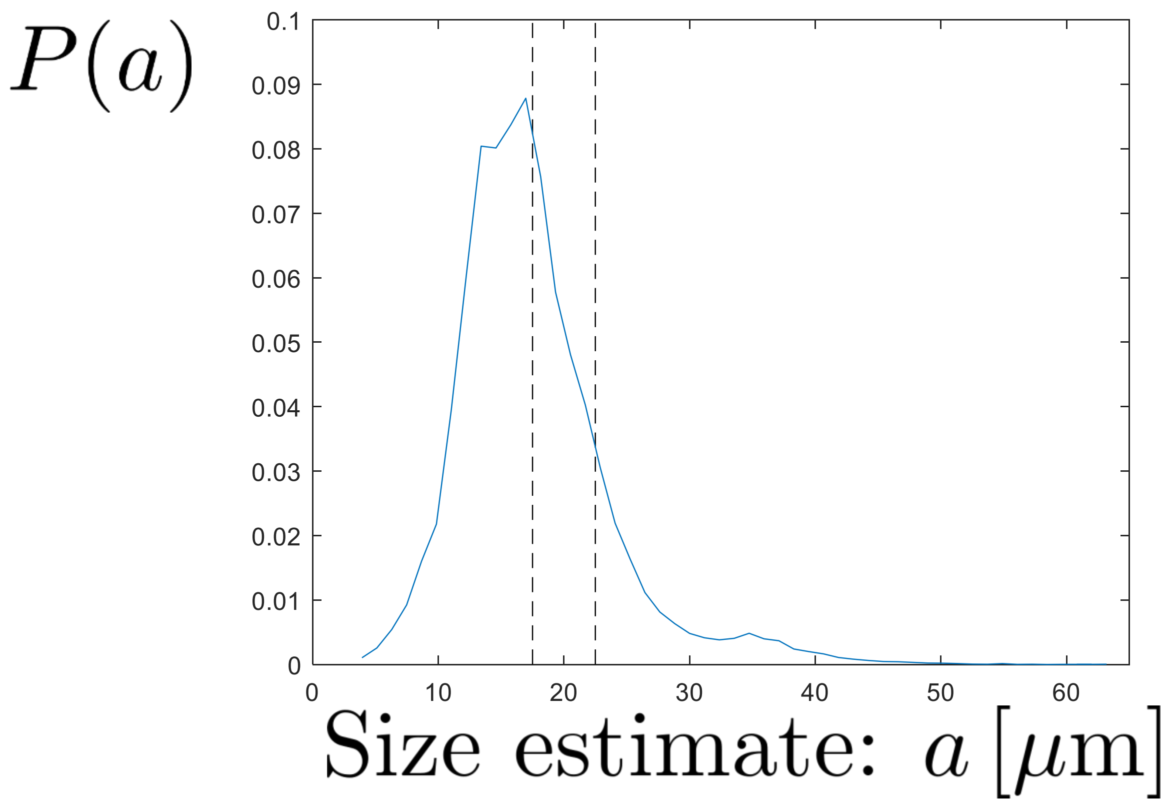
After the calibration images of the three cameras are identified to form world coordinated of candidate particles.

Using particle tracking methods particle trajectories are assembled and analyzed.



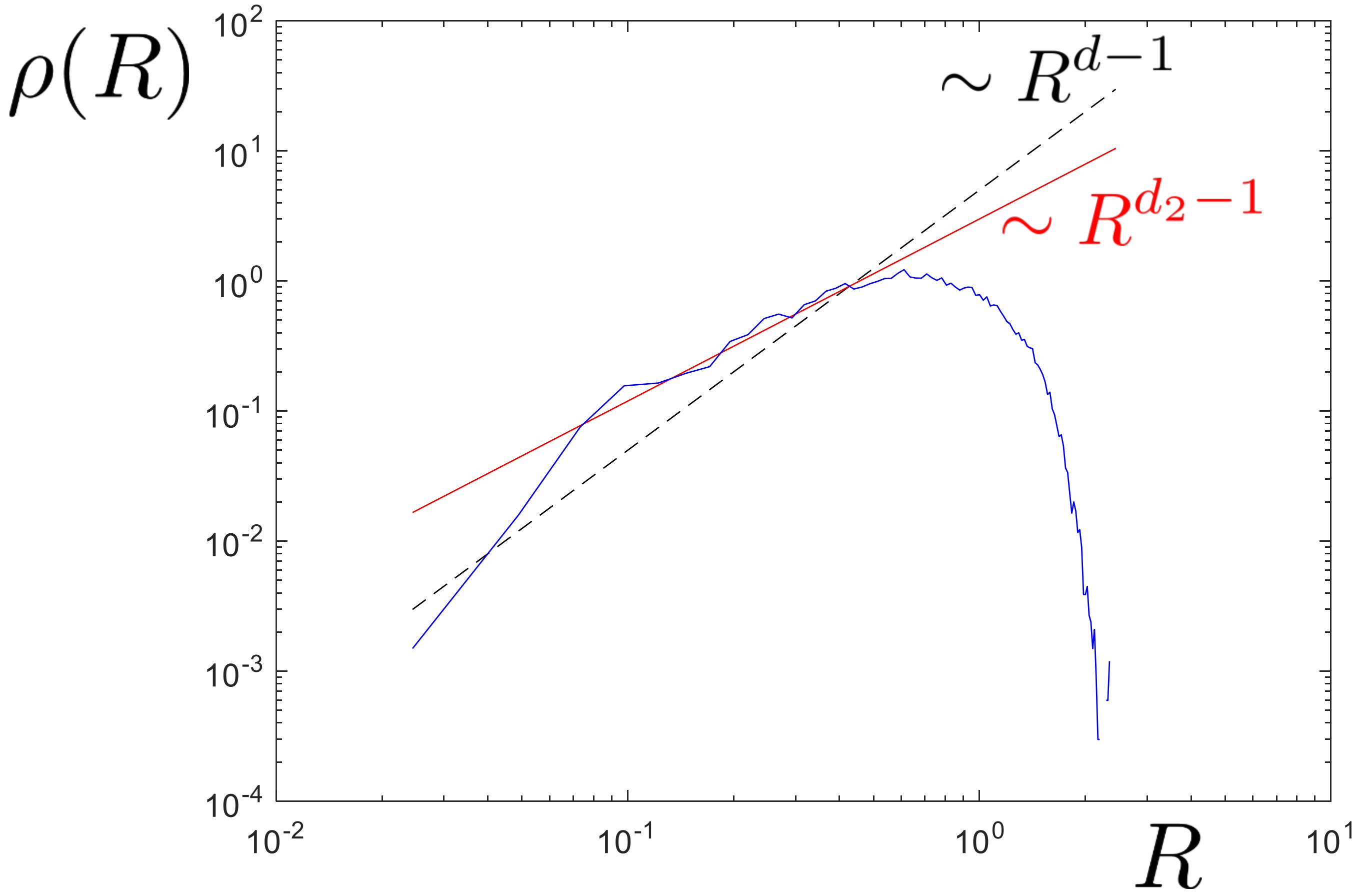
Result: Droplet size distribution

Distributed of estimated particle sizes for the accepted trajectories



Result: Correlation dimension

Relative motion of particles of size ~20 micrometers.
Distribution of separation seems to show power-laws for small separations

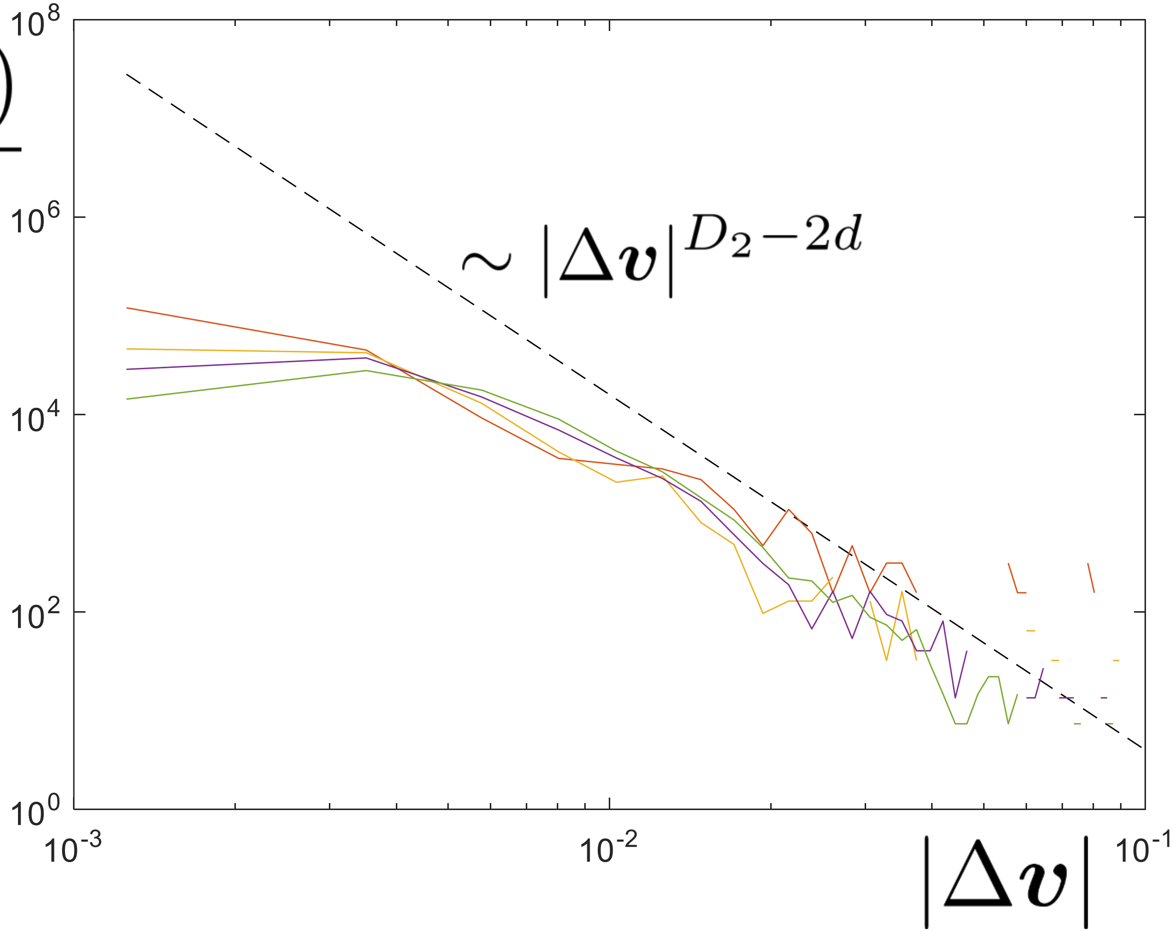


$d_2 \approx 2.4$

Result: Power-law tails

Relative motion of particles of size ~20 micrometers.
Distribution of relative velocities is consistent with predicted power-law tails

$$\frac{\rho(R, \Delta \boldsymbol{v})}{R^{d-1}}$$



$D_2 \approx 2.4$

Values of R chosen below 0.5