Analytical Mesoscale Modeling of Aeolian Sand Transport

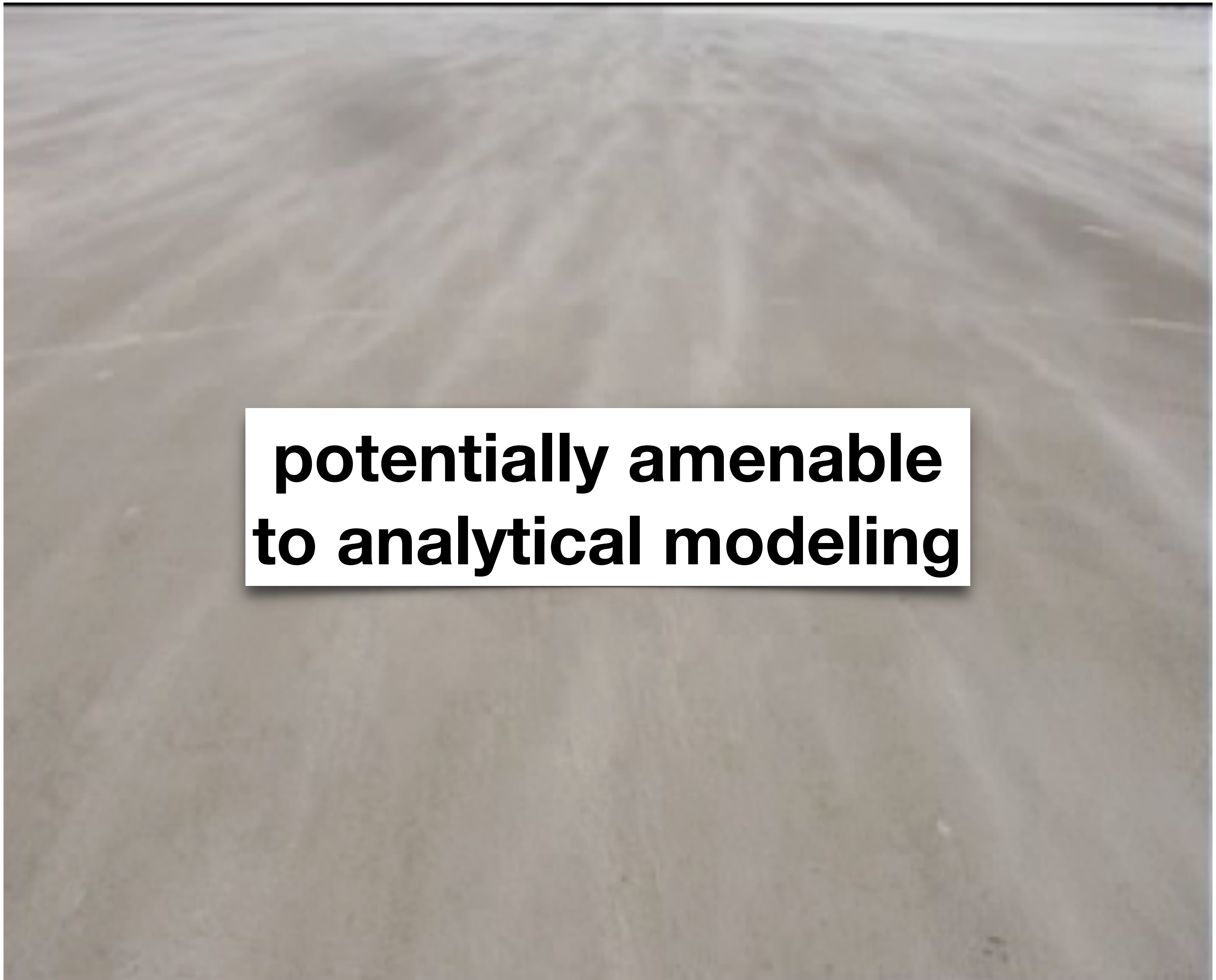
Marc Lämmel, Anne Meiwald, Klaus Kroy

UNIVERSITÄT LEIPZIG

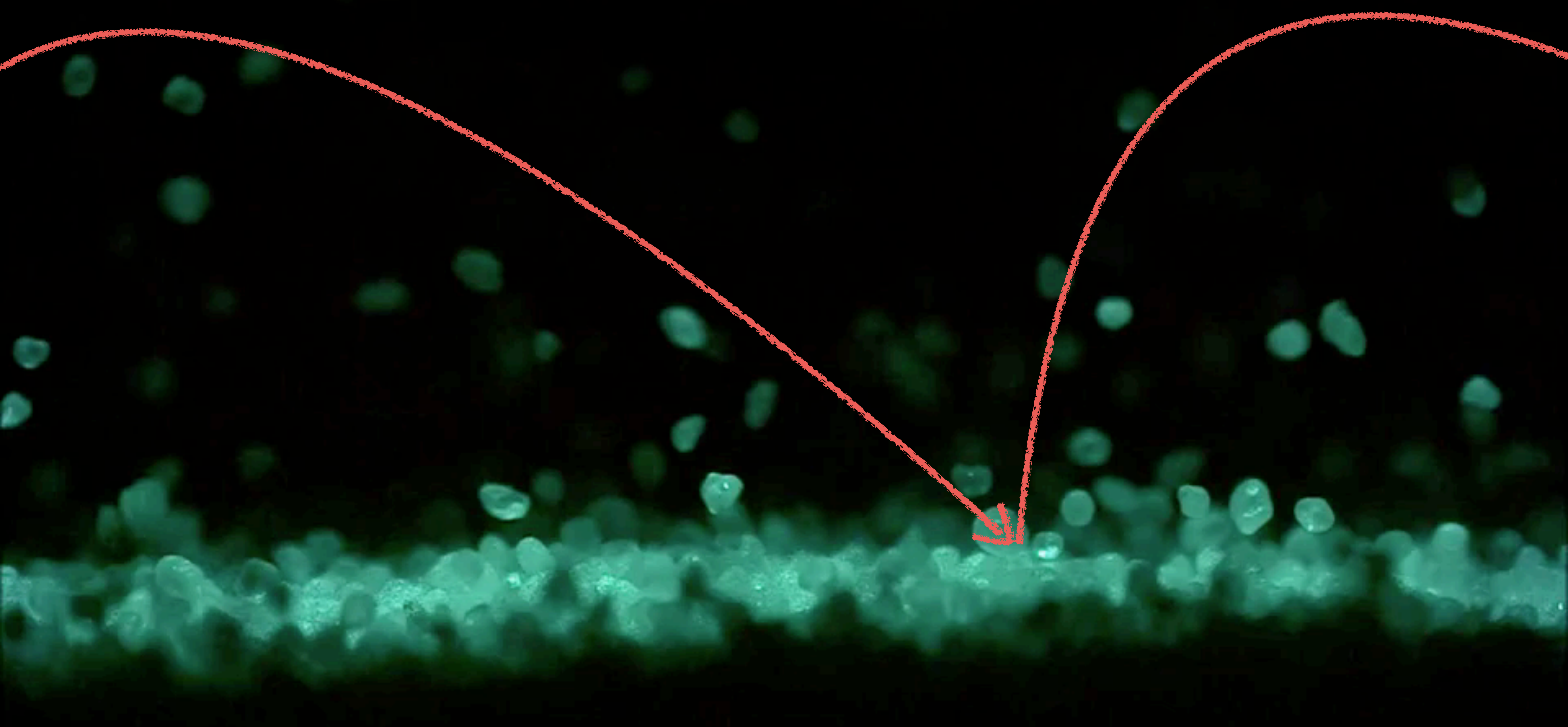
GeoFlo16 — Dresden

Aeolian Sand Transport

Mesoscale Process

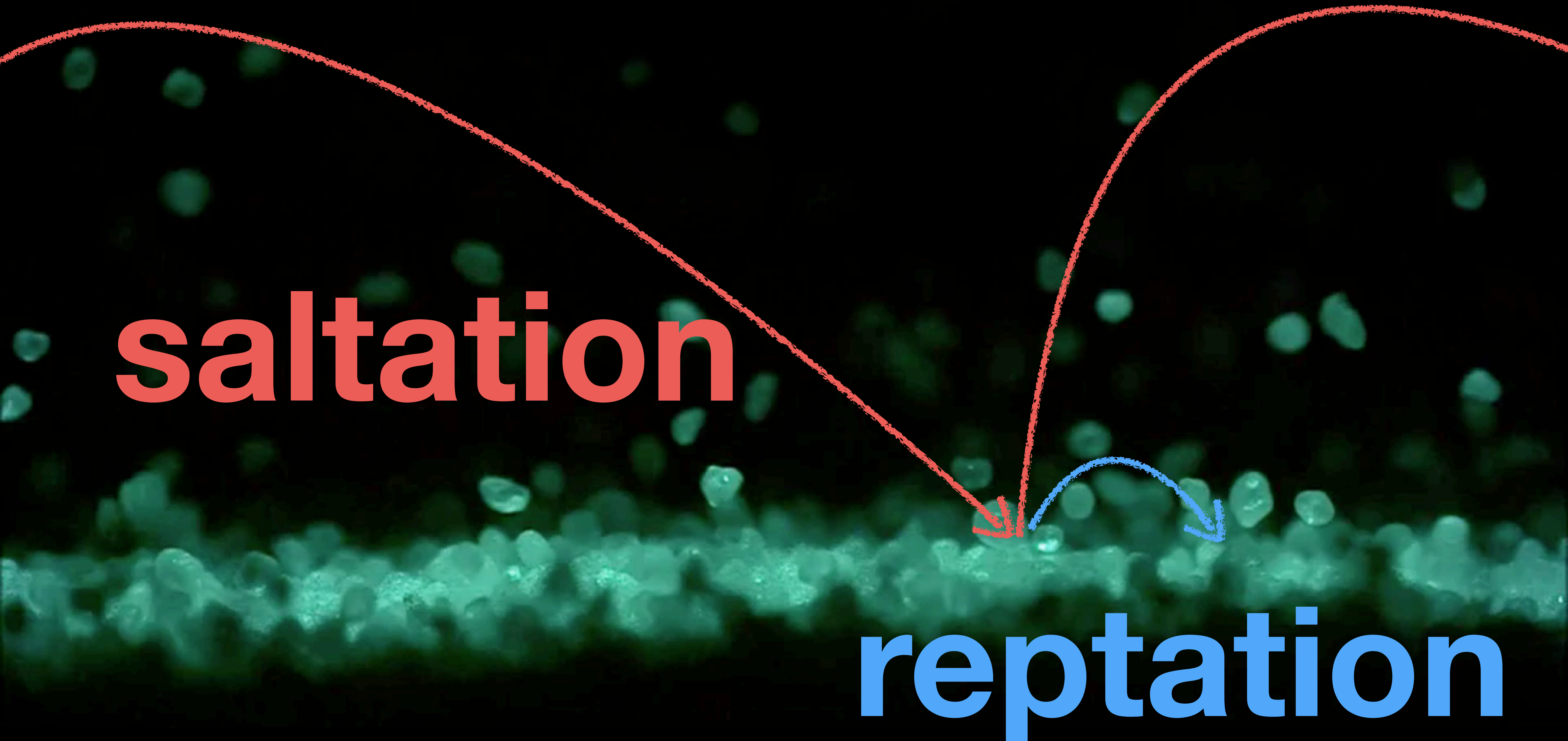


Mean-Field approach



movie @ A. Valance & BBC

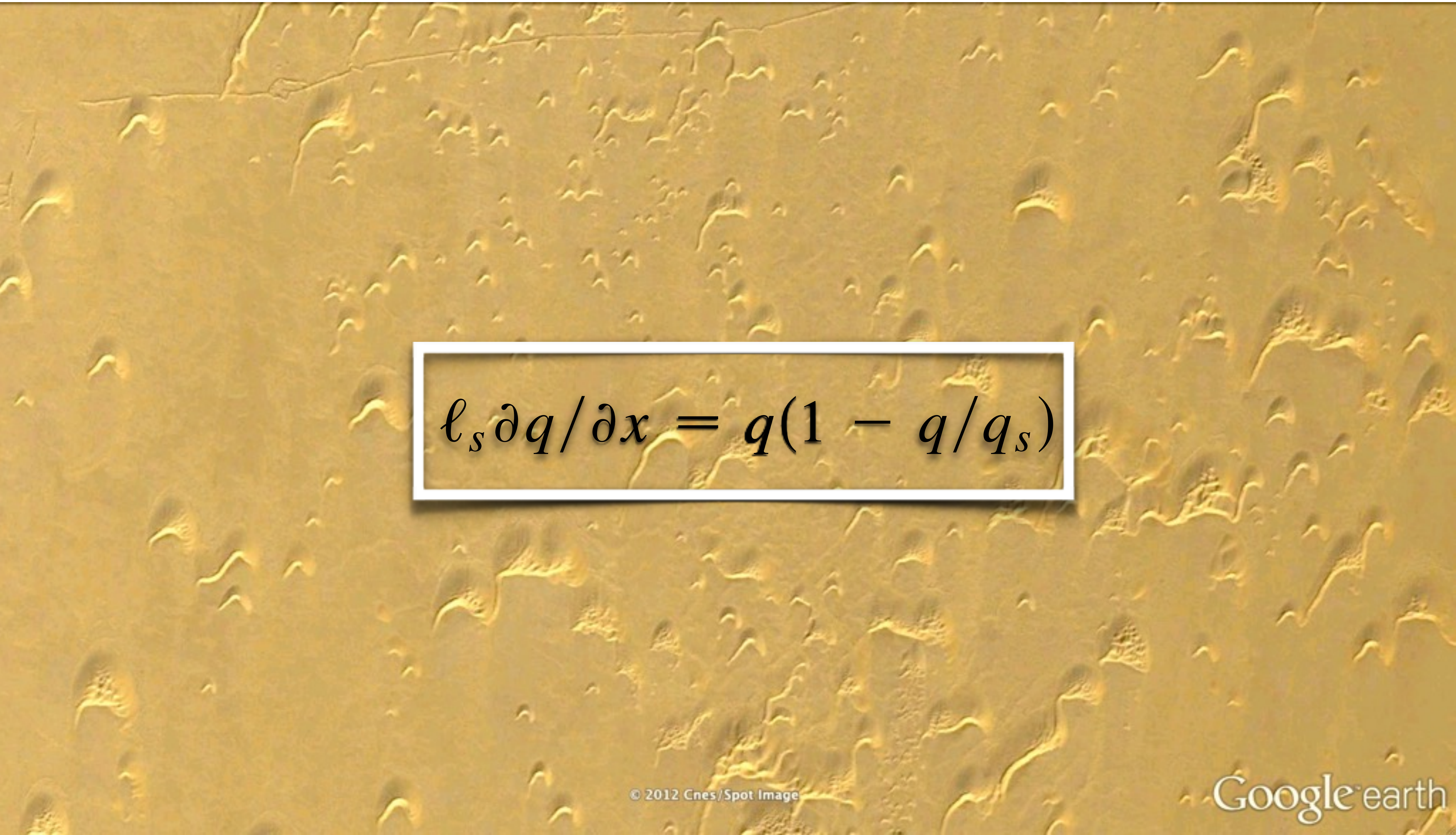
Two-species approach



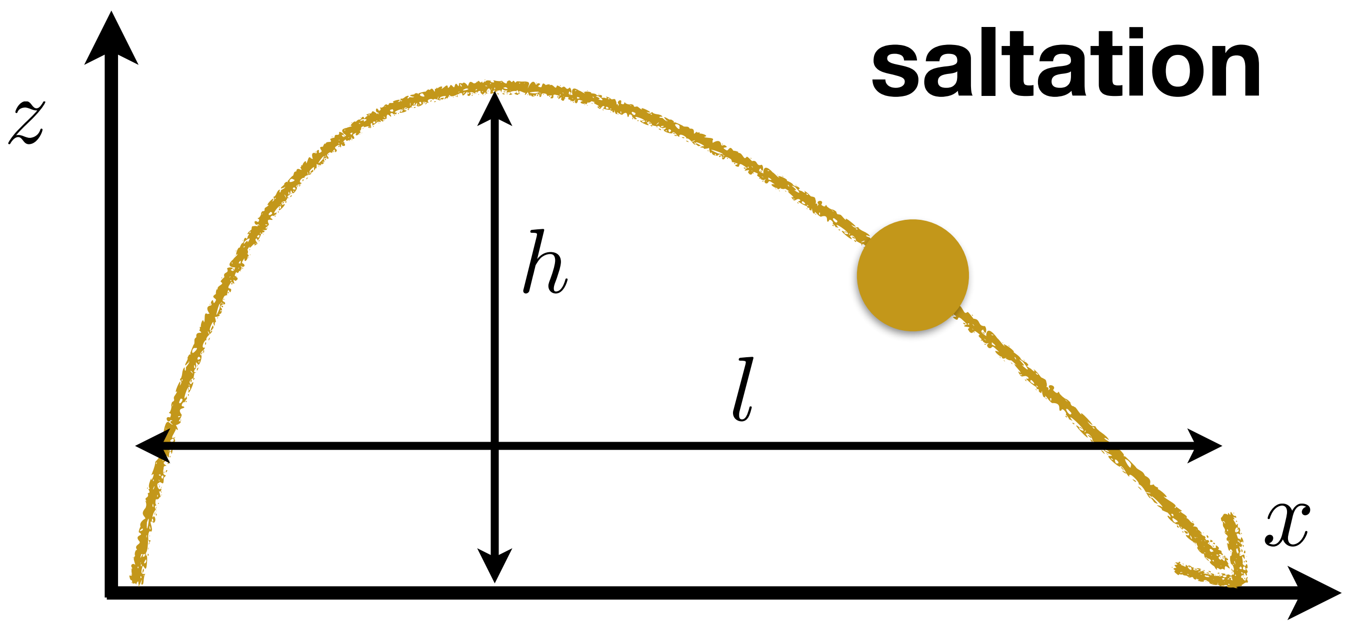
movie @ A. Valance & BBC

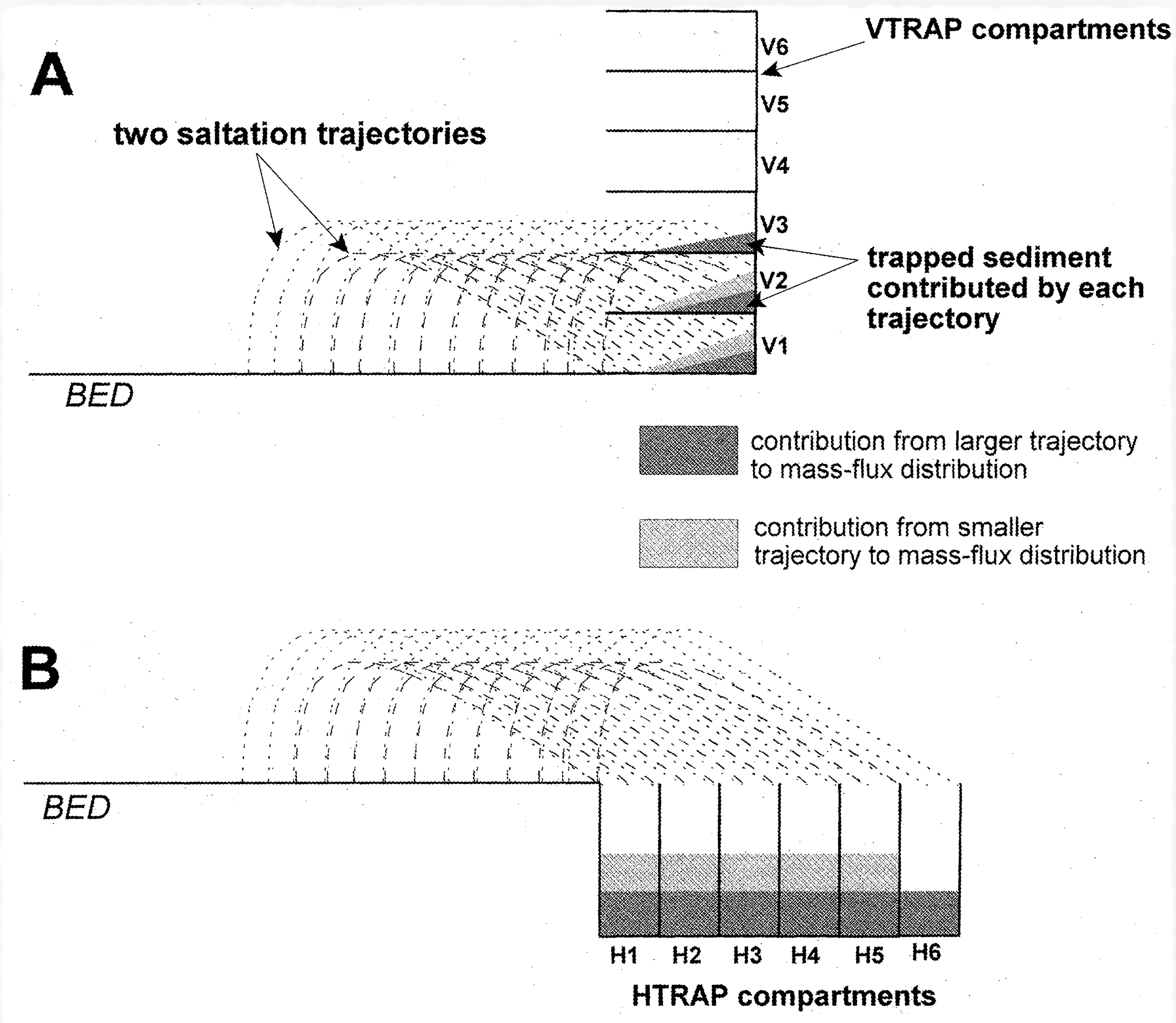
Aeolian Structure Formation

$$\ell_s \partial q / \partial x = q(1 - q/q_s)$$



Mesoscale Phenomena





J. Fluid Mech. (1964), *vol.* 20, *part* 2, *pp.* 225–242 225
Printed in Great Britain

Saltation of uniform grains in air

By P. R. OWEN
Department of Aeronautics, Imperial College, London

(Received 14 April 1964)

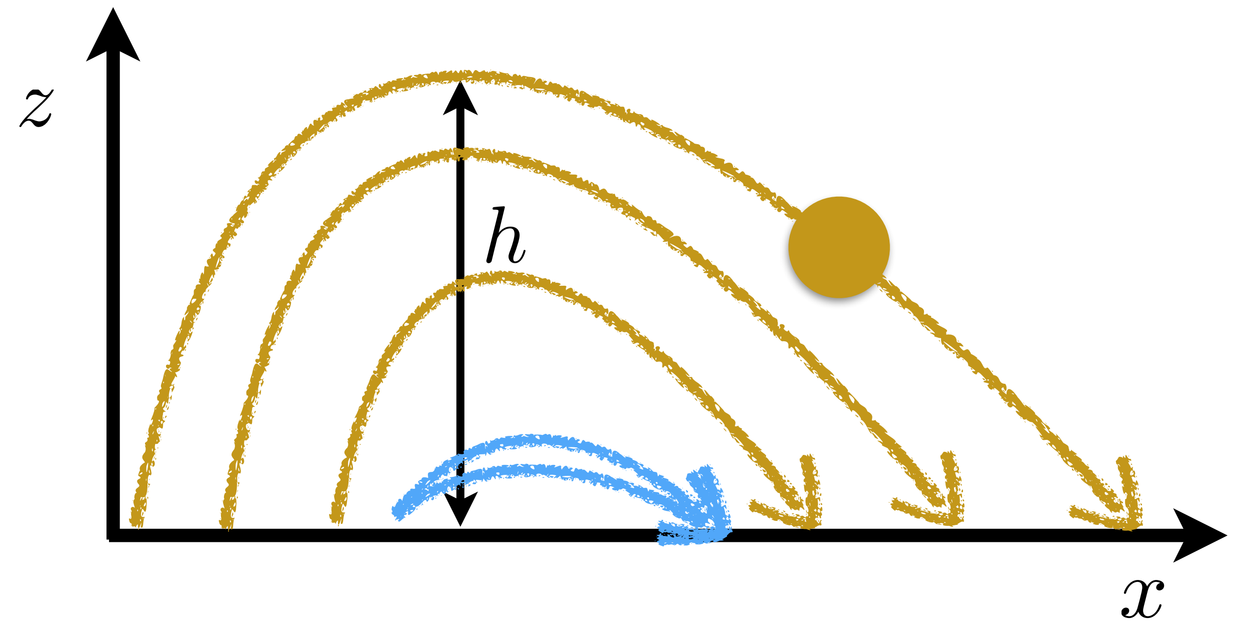
HEIGHT RESOLVED

- **particle concentration**
- **particle velocity**
- **particle flux**
- **hop length & height**
- **wind speed**

→ DISTRIBUTIONS

of grain trajectories

Analytically Feasible Tasks



Hierarchical Approach

- Single Trajectory
- Distribution of Trajectories
- 2 Species
- Compare to Experiments & Simulation

J. Fluid Mech. (2004), *vol.* 510, *pp.* 47–70. © 2004 Cambridge University Press
DOI: 10.1017/S0022112004009073 Printed in the United Kingdom

A two-species model of aeolian sand transport

By BRUNO ANDREOTTI

New Journal of Physics
The open-access journal for physics

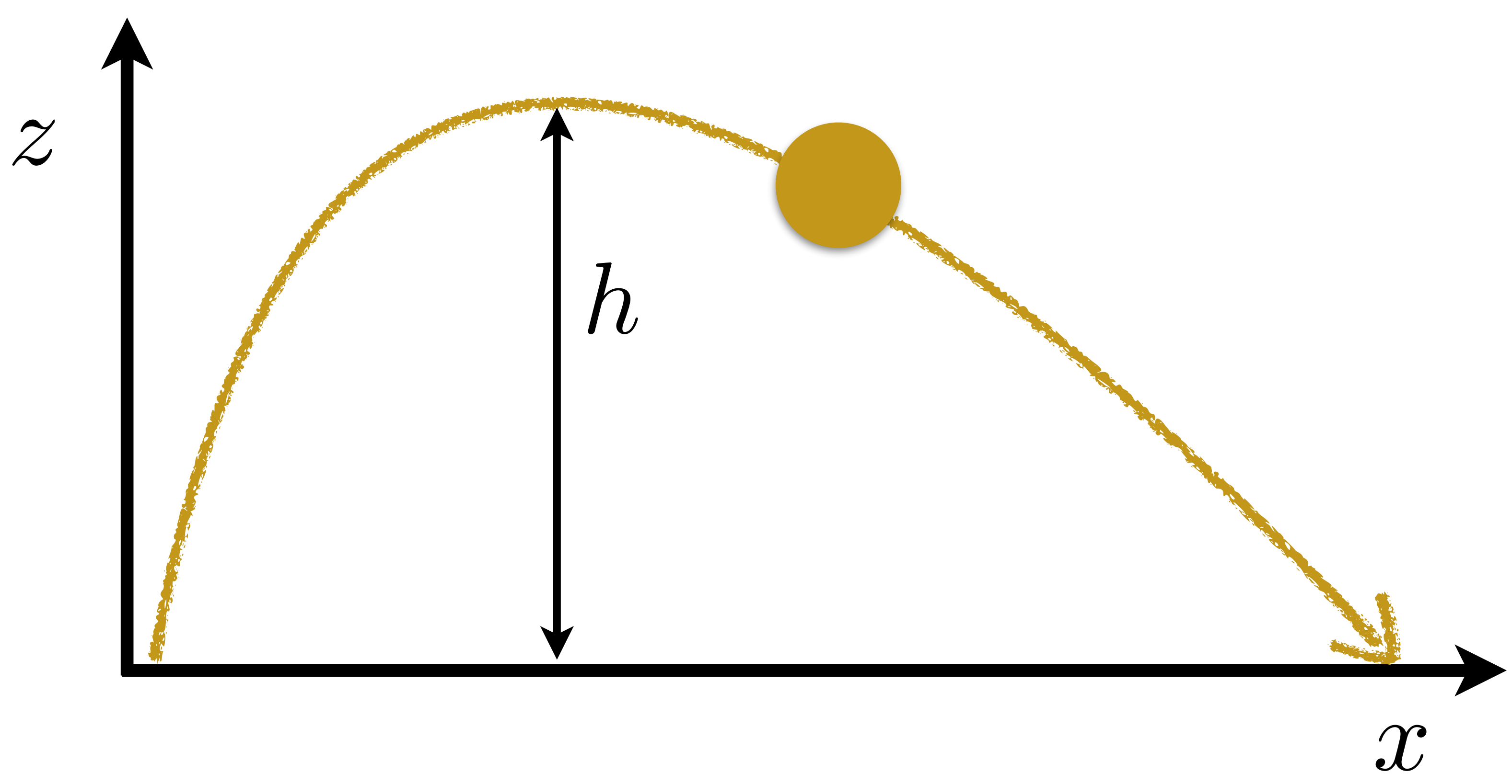
A two-species continuum model for aeolian sand transport

M Lämmel, D Rings and K Kroy[1]

particle distribution

$$P(z, h)$$

Prob to observe particle
on trajectory of height h at z



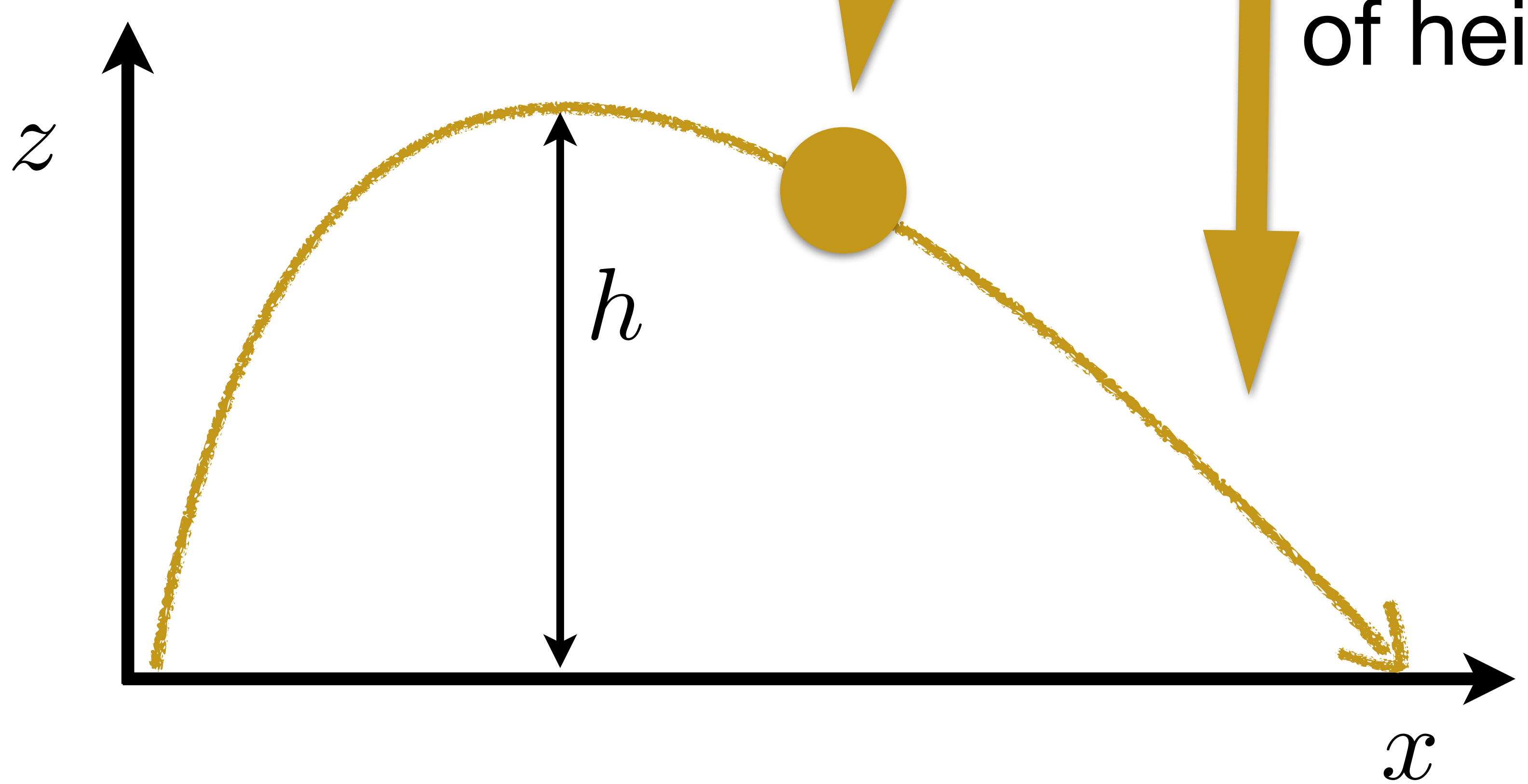
particle distribution

$$P_{\bar{h}}(z, h) = P(z|h) P_{\bar{h}}(h)$$

wind 2-spec

Prob for particle at z if on this trajectory

Prob for trajectory of height h



$$P_{\bar{h}}(h) \not\propto ?\, e^{-h/\bar{h}}$$

Reptation/Splash

~ barometer formula with granular temperature

J. Fluid Mech. (1983), vol. 130, pp. 187–202 187
Printed in Great Britain

A theory for the rapid flow of identical, smooth, nearly elastic, spherical particles

By J. T. JENKINS
Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York
AND S. B. SAVAGE

Saltation

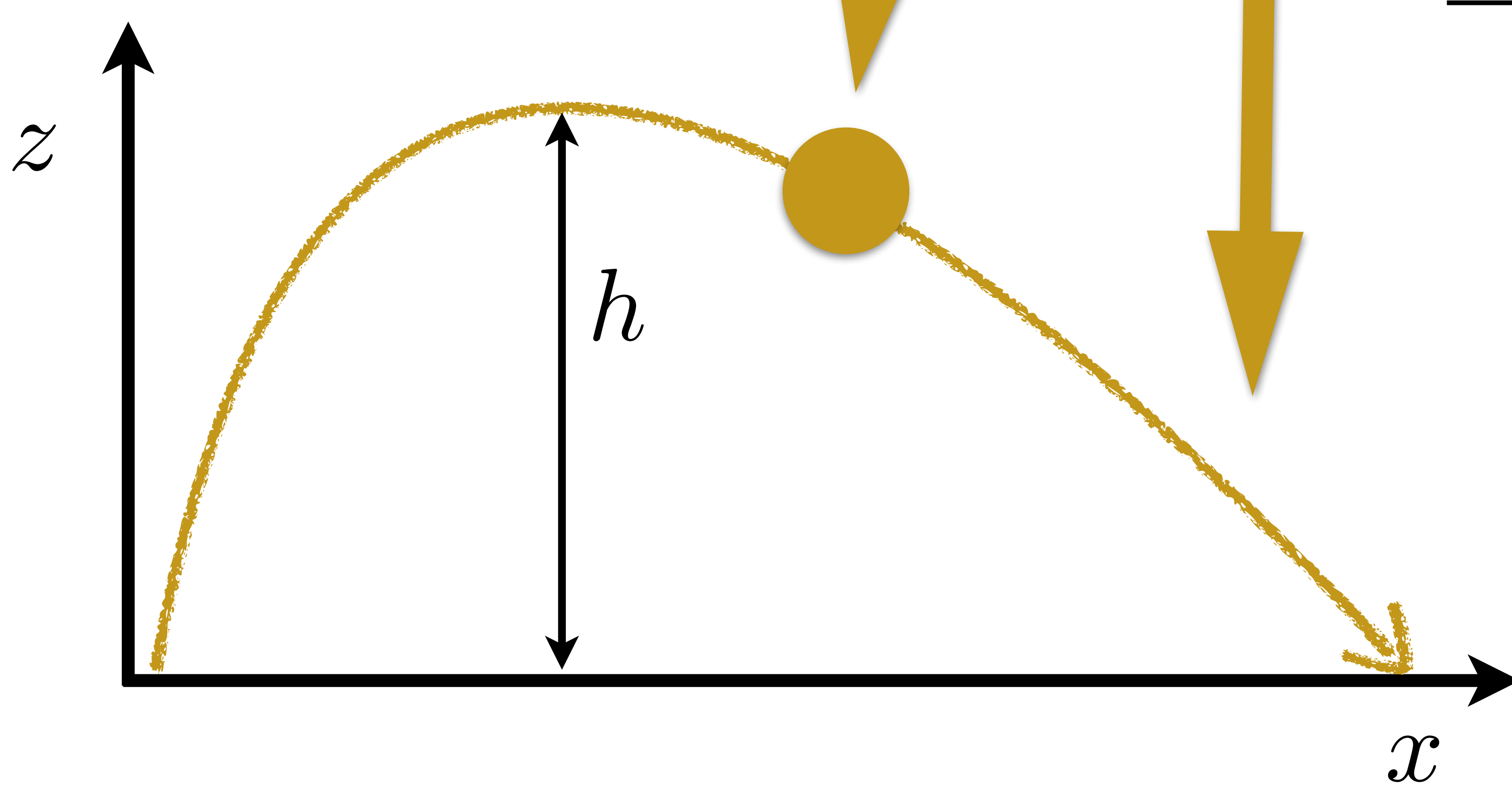
$$h \propto N\,; \quad P(\text{survival}) \propto e^{-N}$$

particle distribution

$$P_{\bar{h}}(z, h) = P(z|h) P_{\bar{h}}(h)$$

Prob for particle on this trajectory at z

$$\frac{e^{-h/\bar{h}}}{\bar{h}}$$

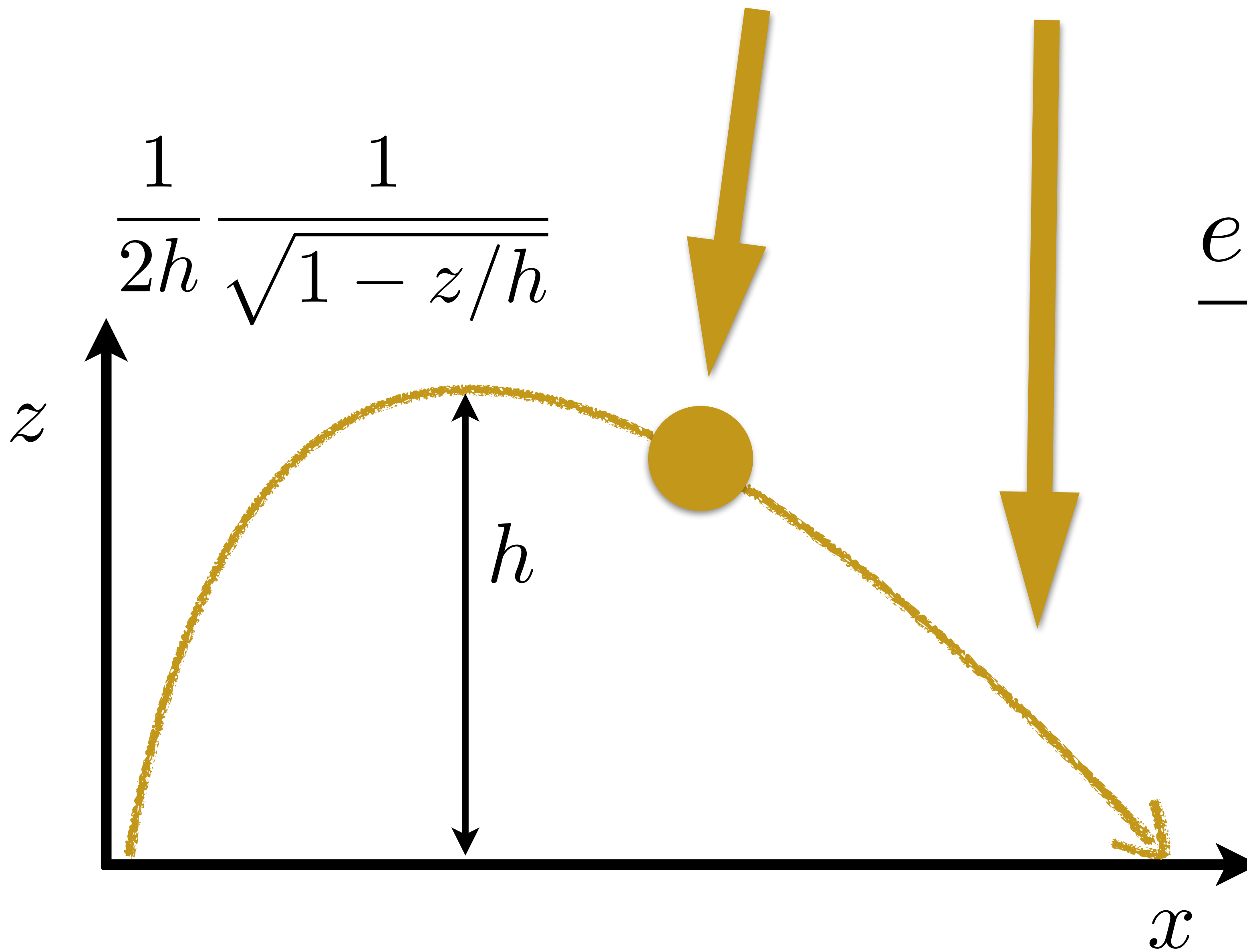


particle distribution

$$P_{\bar{h}}(z,h) = P(z|h)P_{\bar{h}}(h)$$

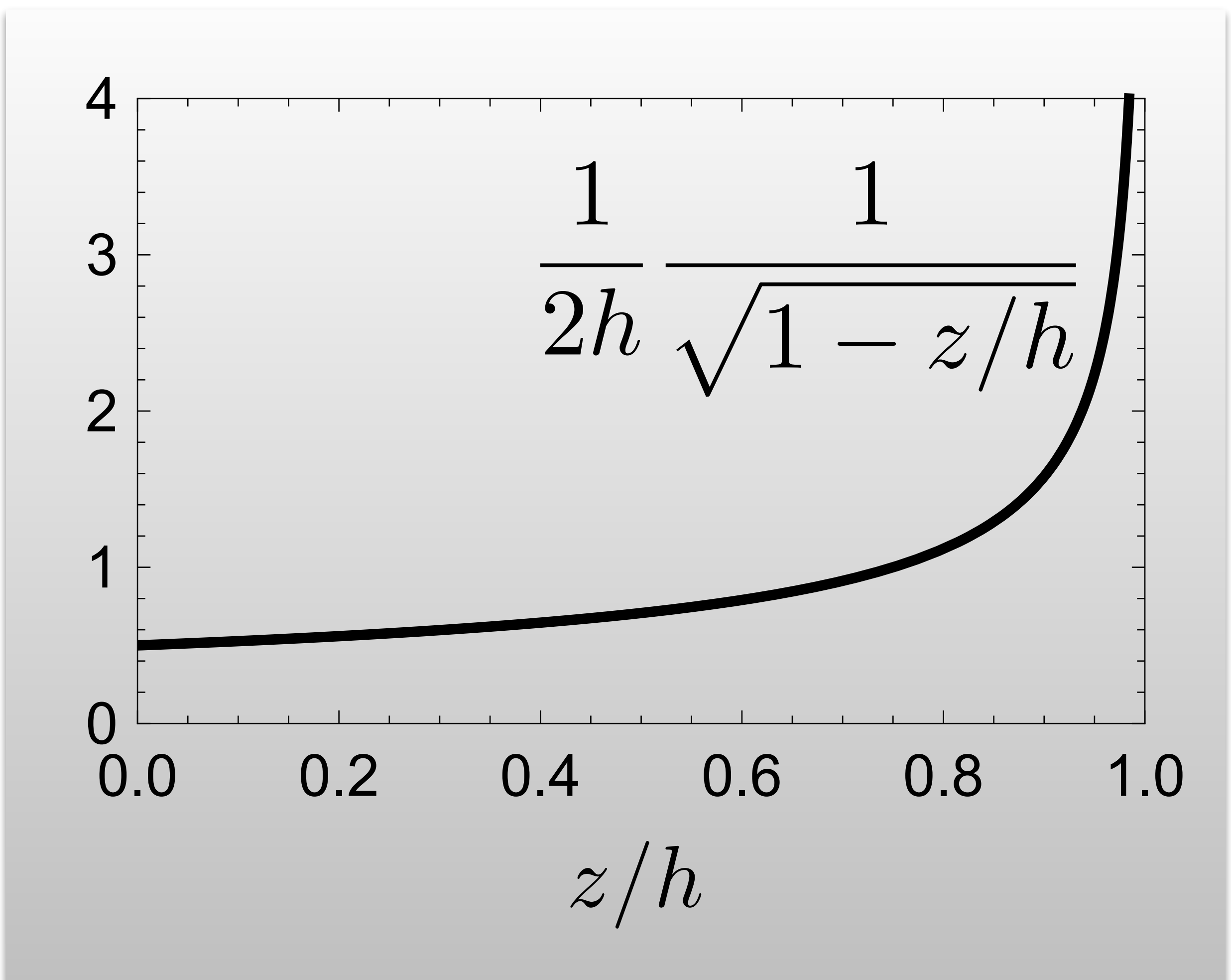
$$\frac{1}{2h}\frac{1}{\sqrt{1-z/h}}$$

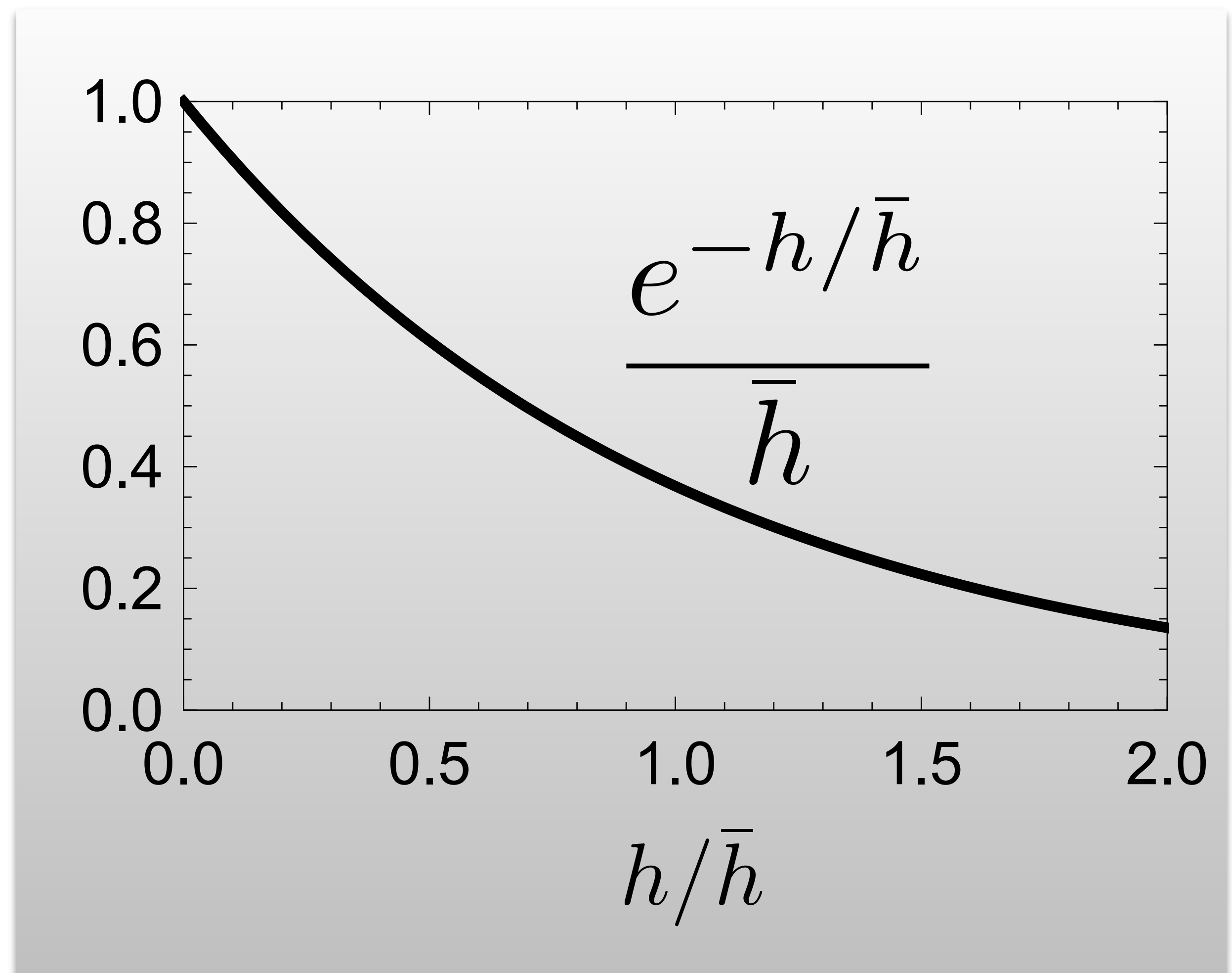
$$\frac{e^{-h/\bar{h}}}{\bar{h}}$$



particle distribution

$$P_{\bar{h}}(z,h) = P(z|h)P_{\bar{h}}(h)$$





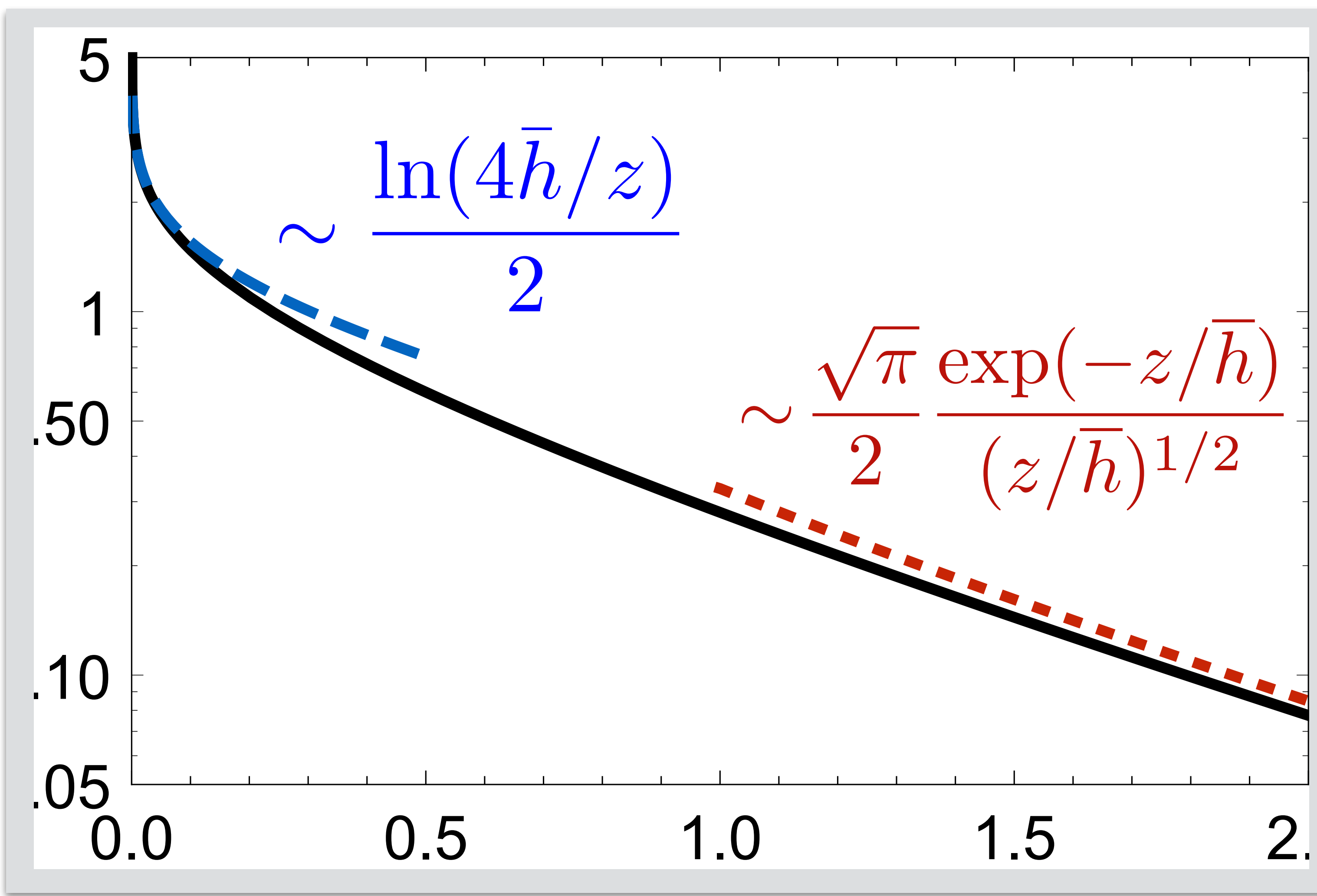
height-resolved observables

- **grain density** $\rho(z)$
- **horizontal flux** $j(z)$
- **vertical flux** $\phi(z)$
- **grain-borne stress** $\tau_{\mathrm{g}}(z)$
- **hop length distribution** $P(z, l)$
- **hop length distribution** $P(\ell) \propto -\partial_\ell \phi_\ell(0)$

particle distribution

$$\rho_{\bar{h}}(z) = \int \mathrm{d}h \, P_{\bar{h}}(z, h)$$

Prob for particle at z
for **any** trajectory



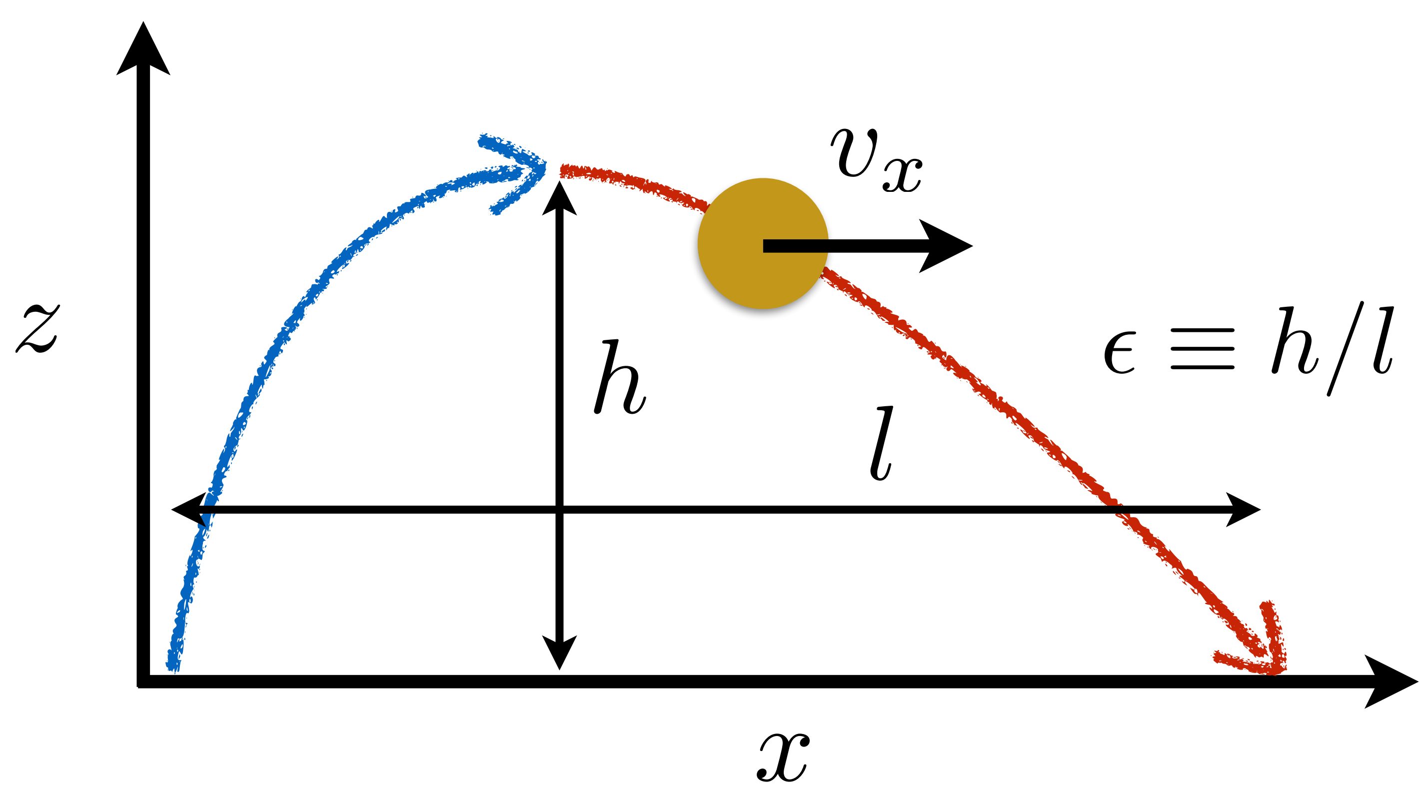
$z/\bar{h}$

wind, 2 spec

particle velocity

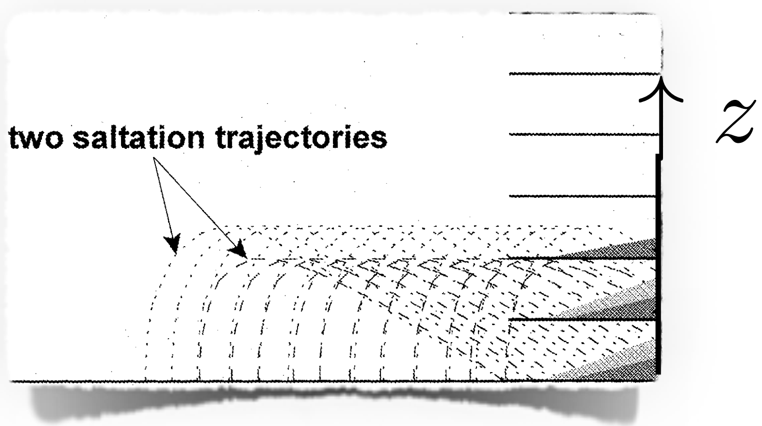
$v_x(\cancel{z}, \cancel{l}, h) \approx \sqrt{2gh}/4\epsilon$ hop-length/flight-time

~ free-fall/const.-ϵ approx



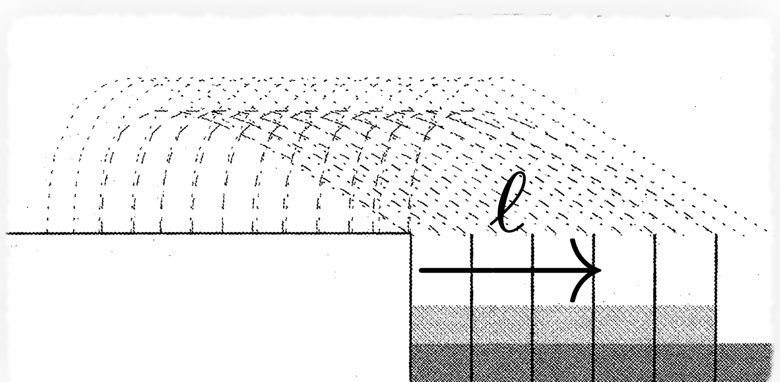
~ constant

horizontal sand flux



$$j_{\bar{h}}(z) = \int \mathrm{d}h\, v_x(h)\, P_{\bar{h}}(z,h) = q\frac{e^{-z/\bar{h}}}{\bar{h}}$$

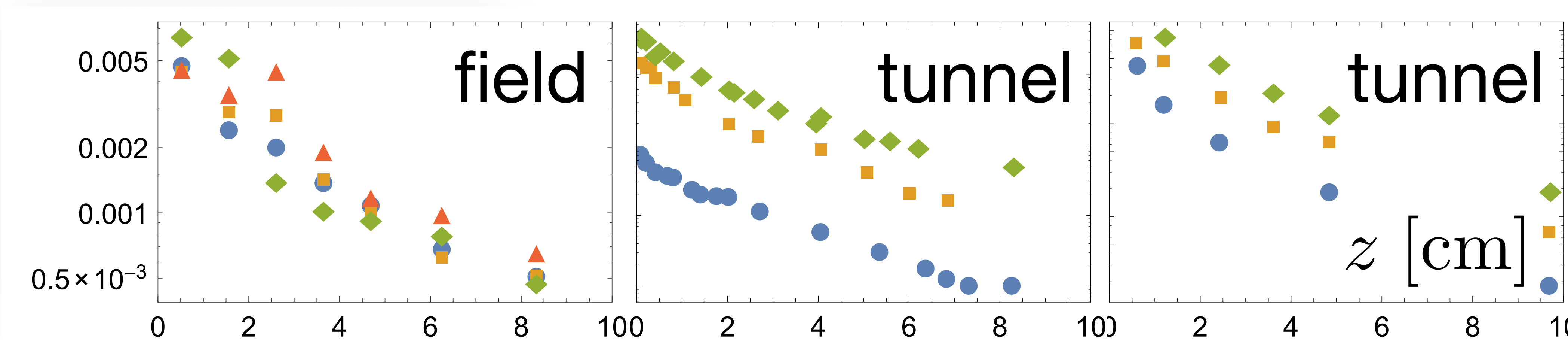
vertical sand flux



$$\phi_\ell(z) = \int_{l(h)>\ell} \mathrm{d}h\, v_z(z,h)\, P_{\bar{h}}(z,h)$$

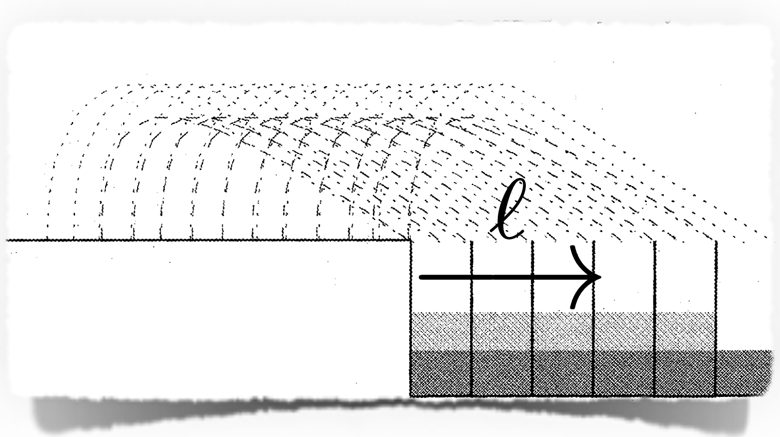
$$v_z(z,h) = \sqrt{2g(h-z)} \qquad \phi_\ell(z=0) = q\frac{\mathrm{erfc}\sqrt{\ell\epsilon/\bar{h}}}{\bar{h}/\epsilon}$$

horizontal sand flux



$$q\frac{e^{-z/\bar{h}}}{\bar{h}}$$

vertical sand flux



$$\phi_\ell(z) = \int_{l(h)>\ell} \mathrm{d}h\, v_z(z,h)\, P_{\bar{h}}(z,h)$$

$$v_z(z,h) = \sqrt{2g(h-z)} \qquad \phi_\ell(z=0) = q\frac{\operatorname{erfc}\sqrt{\ell\epsilon/\bar{h}}}{\bar{h}/\epsilon}$$

grain-scale experiments

Rasmussen, Mikkelsen, Sedimentology (1998)

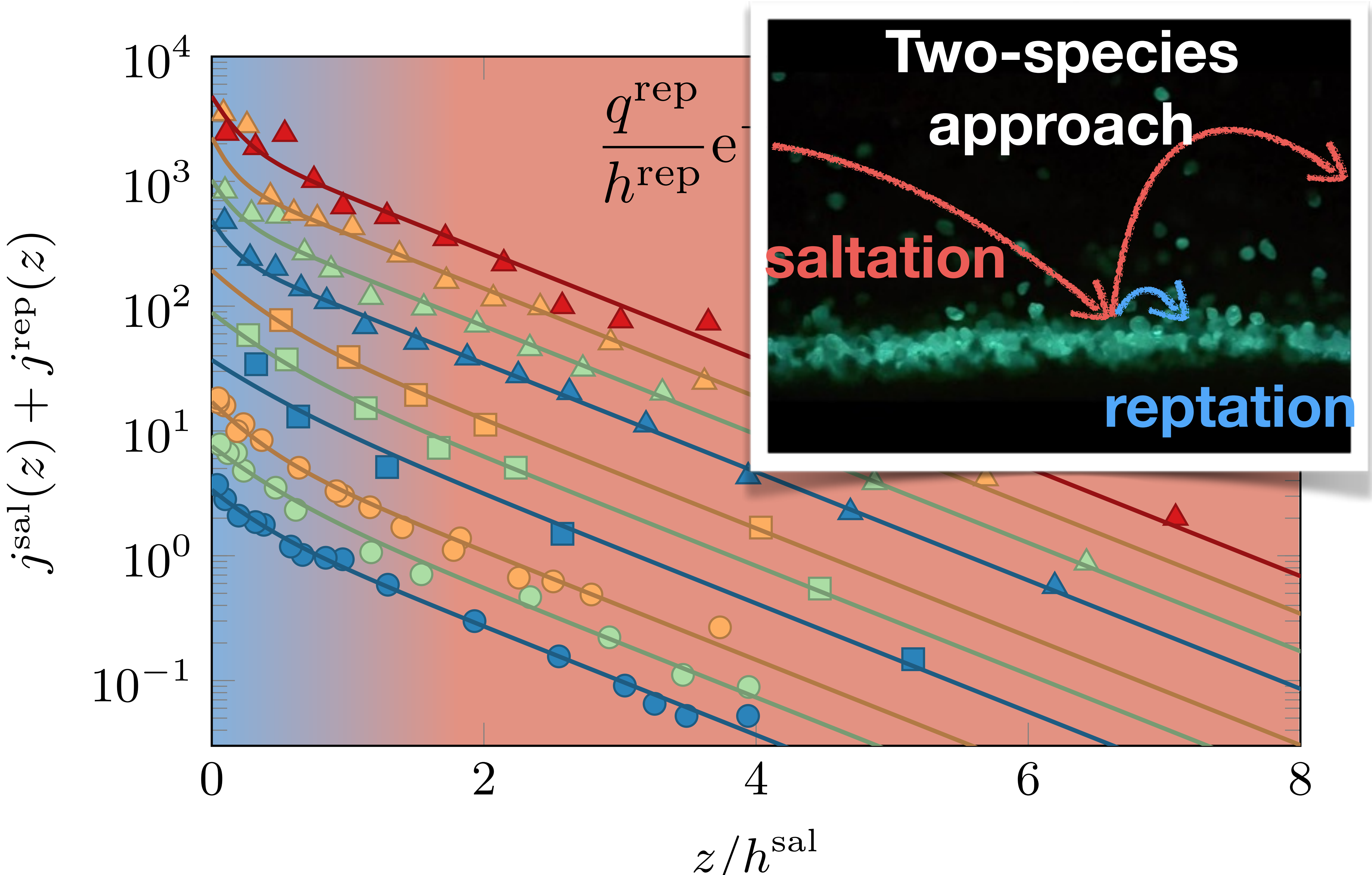
Namikas, Sedimentology (2003)

Rasmussen, Sørensen, J. Geophys. Res. (2008)

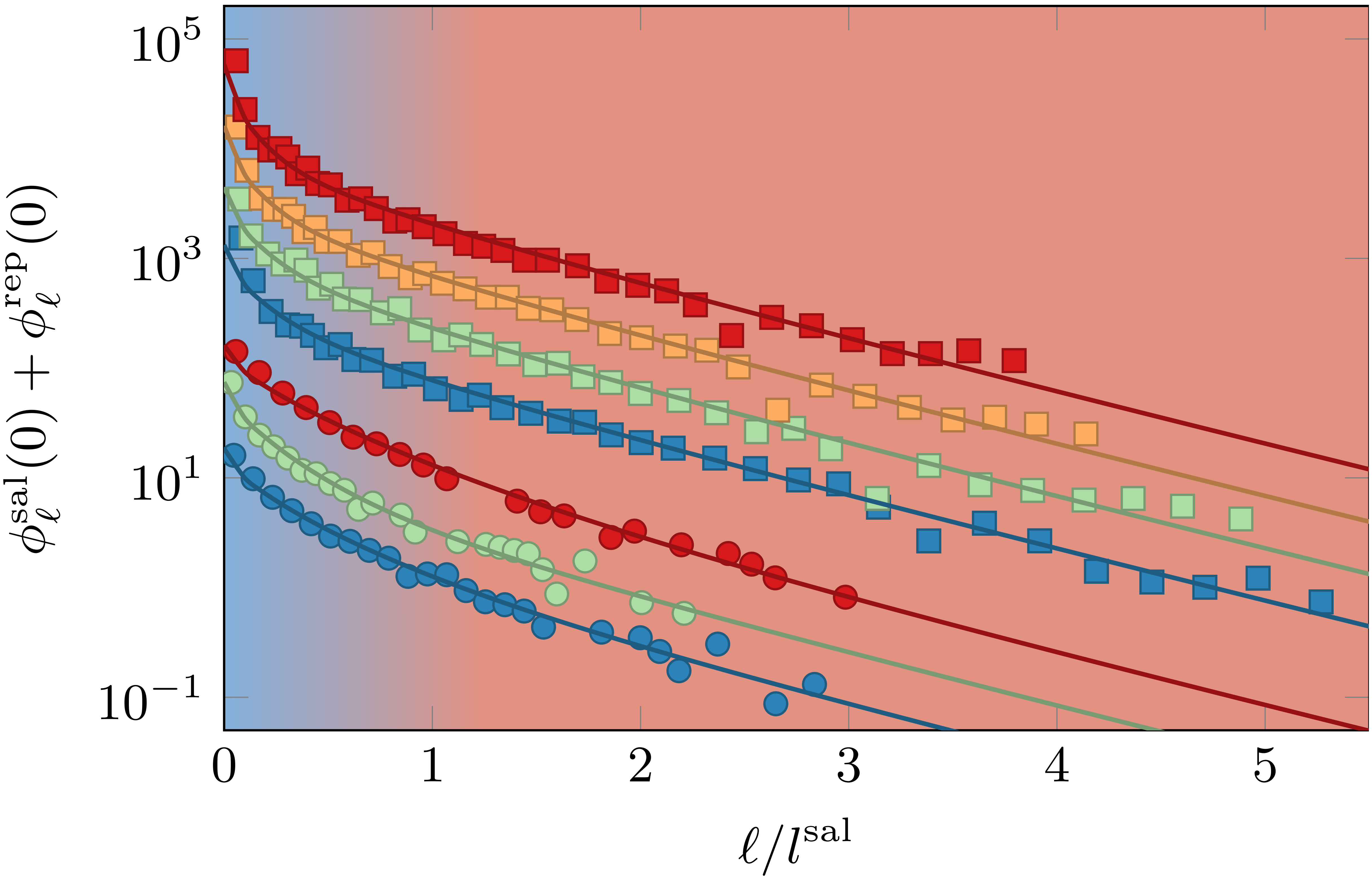
Ho, Valance, Dupont, Moctar, Aeolian Research (2014)

Durand, Claudin, Andreotti, PNAS (2014)

horizontal sand flux

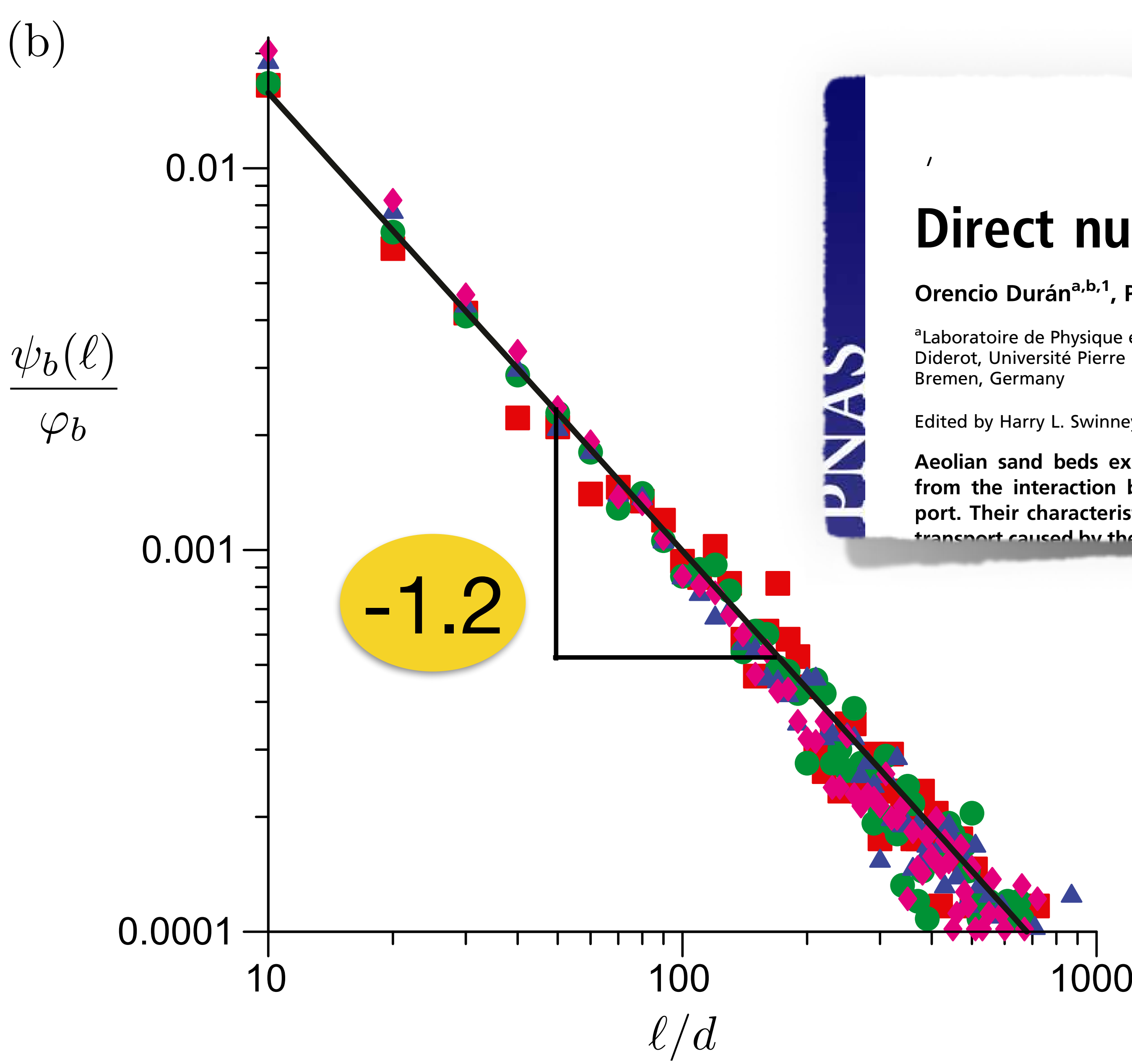


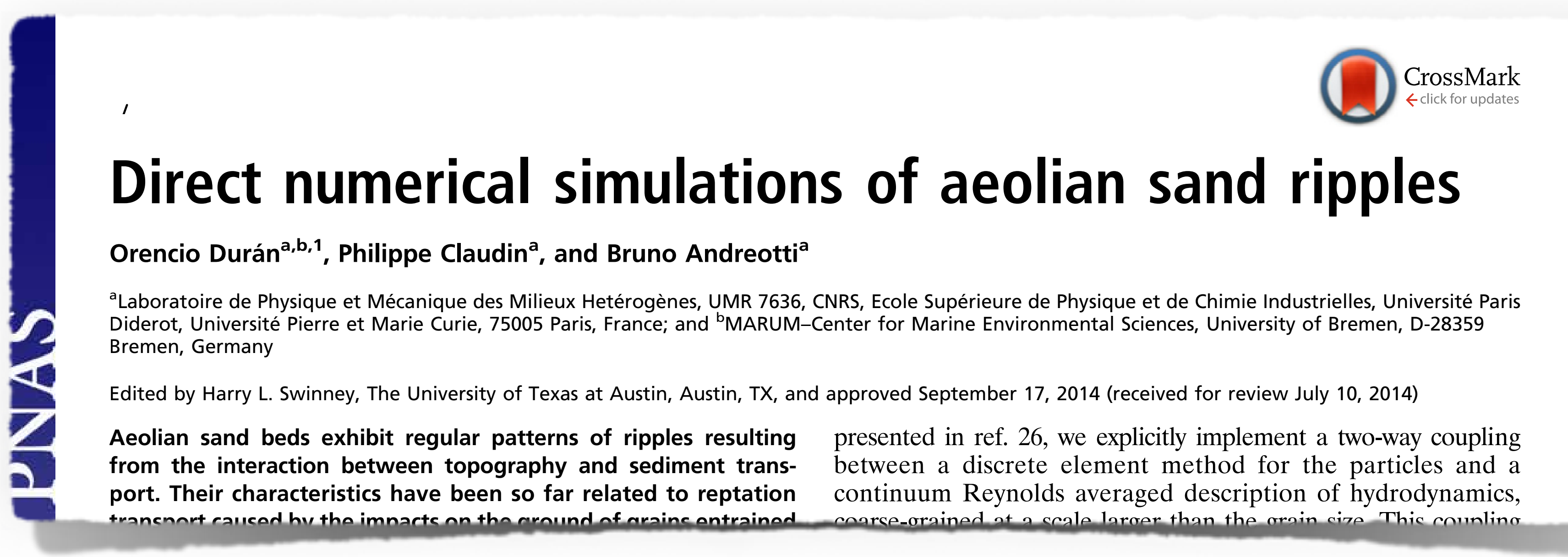
vertical sand flux



hop length distribution

$$P(l|z) \propto P[z, h(l)]\partial_l h(l) \propto \frac{e^{-\epsilon l/\bar{h}}}{l}$$





Direct numerical simulations of aeolian sand ripples

Orencio Durán[a,b,1], Philippe Claudin[a], and Bruno Andreotti[a]

[a]Laboratoire de Physique et Mécanique des Milieux Hetérogènes, UMR 7636, CNRS, Ecole Supérieure de Physique et de Chimie Industrielles, Université Paris Diderot, Université Pierre et Marie Curie, 75005 Paris, France; and [b]MARUM–Center for Marine Environmental Sciences, University of Bremen, D-28359 Bremen, Germany

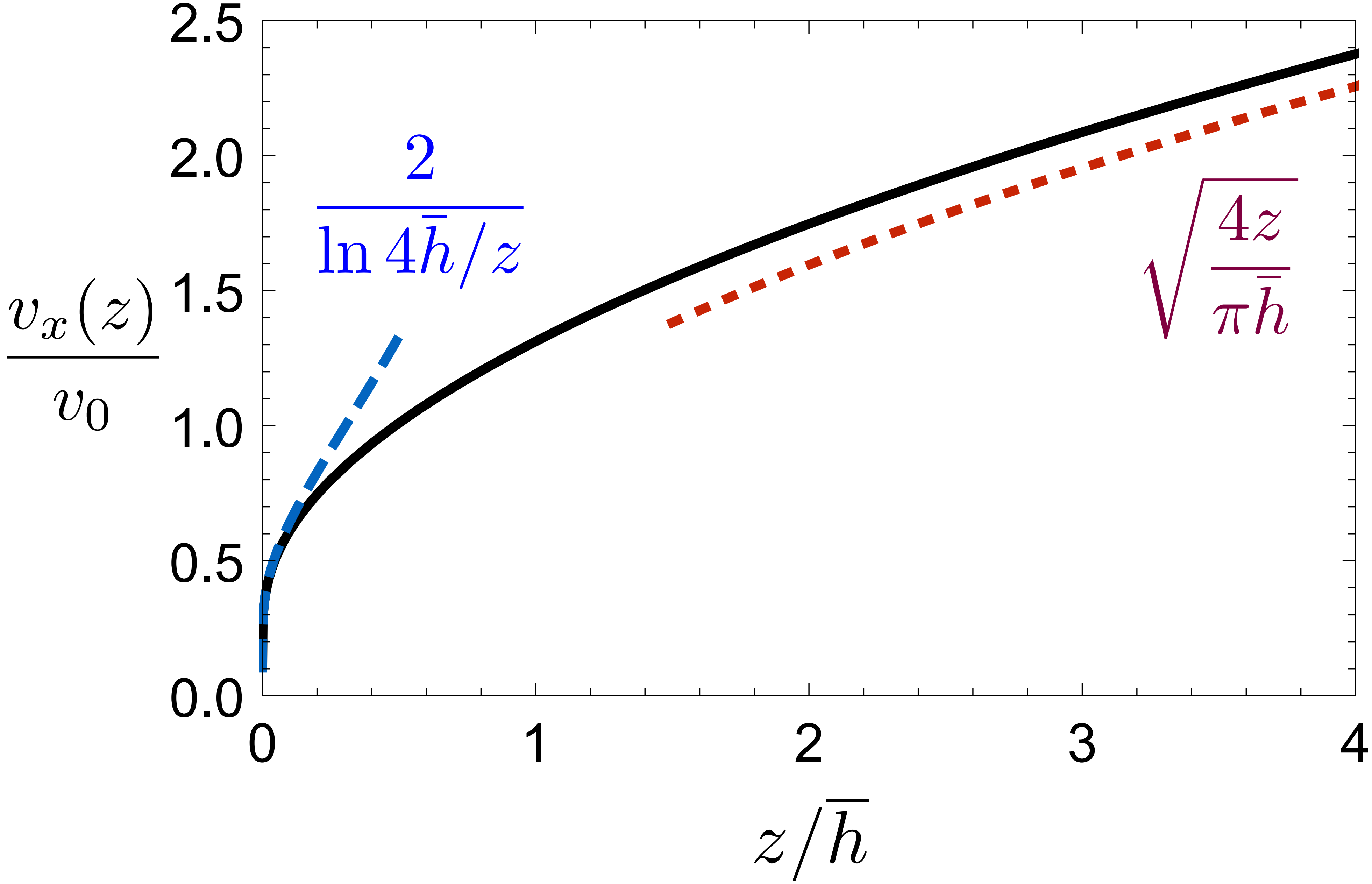
Edited by Harry L. Swinney, The University of Texas at Austin, Austin, TX, and approved September 17, 2014 (received for review July 10, 2014)

Aeolian sand beds exhibit regular patterns of ripples resulting from the interaction between topography and sediment transport. Their characteristics have been so far related to reptation transport caused by the impacts on the ground of grains entrained

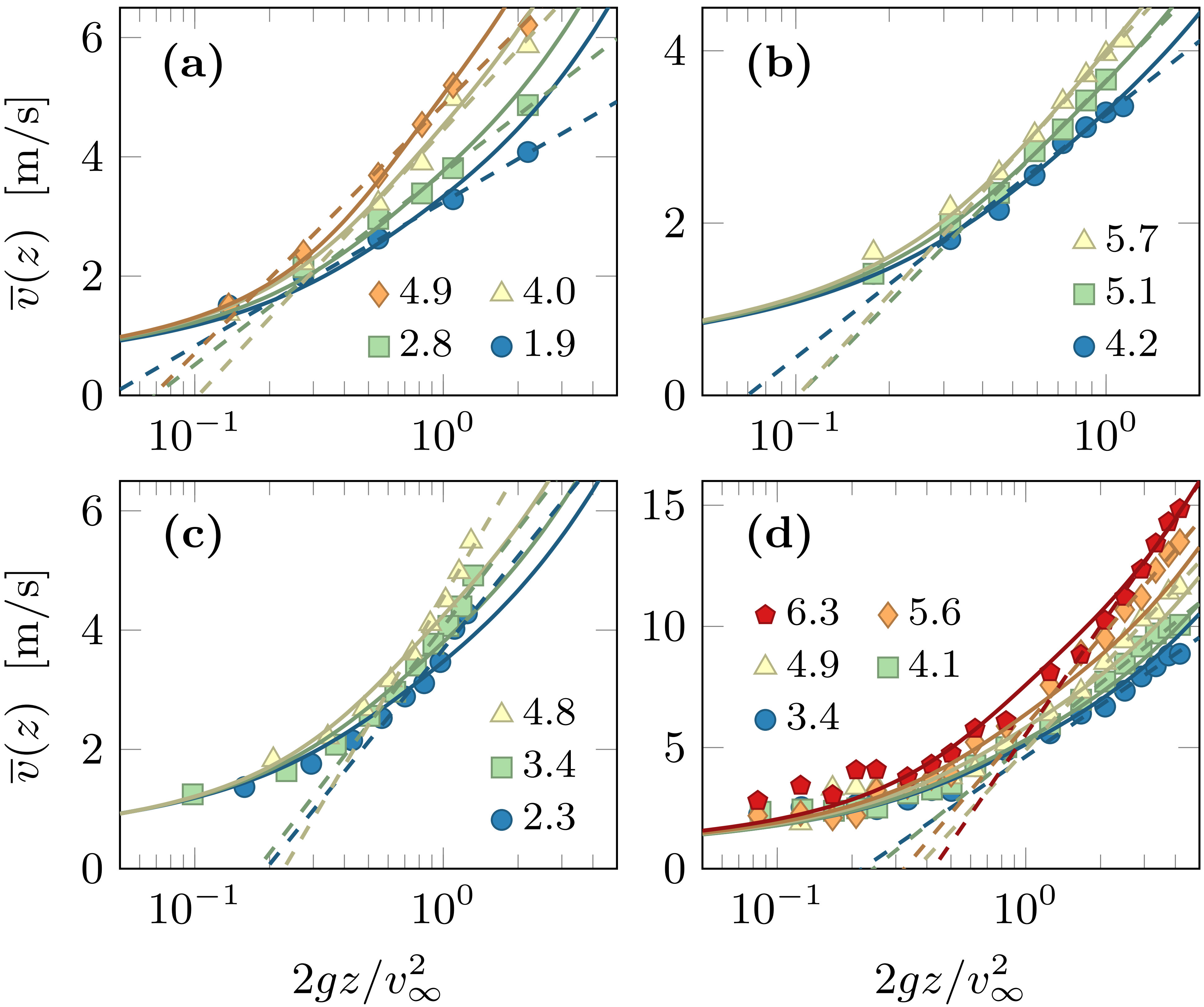
presented in ref. 26, we explicitly implement a two-way coupling between a discrete element method for the particles and a continuum Reynolds averaged description of hydrodynamics, coarse-grained at a scale larger than the grain size. This coupling

particle velocity

$$v_x(z) = j(z)/\rho(z)$$



particle velocity

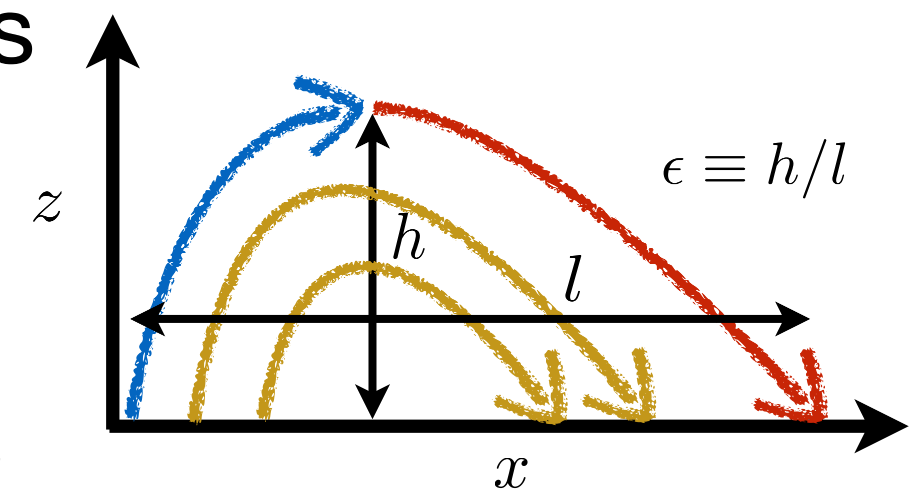


Summary

▸ **analytical mesoscale model of aeolian transport**

- based on grain scale physics
- ensemble of trajectories
- & two-species
- height-resolved observables
- applications to turbulent closure

  & data analysis & various mesoscale phenomena



thank you!