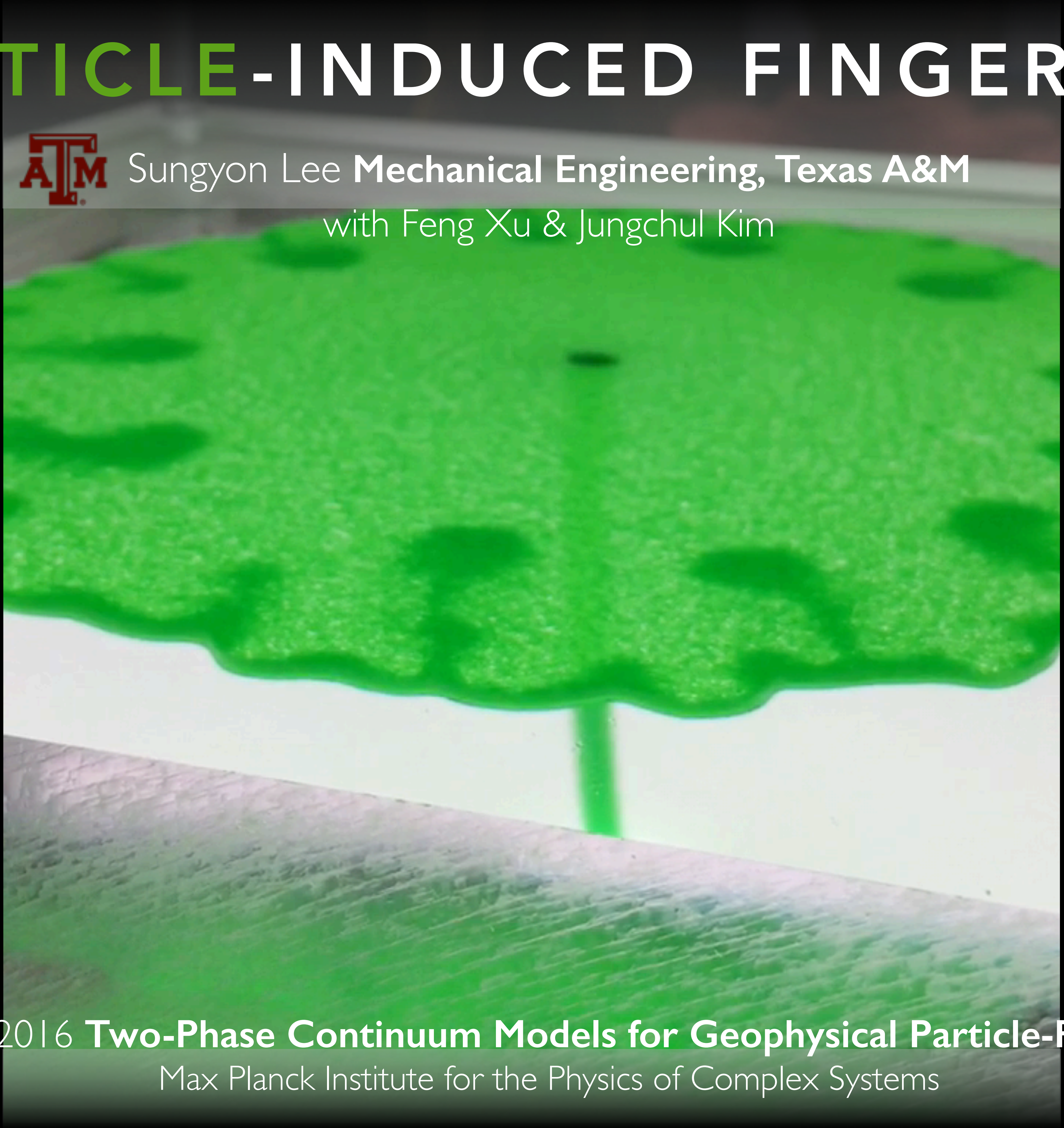
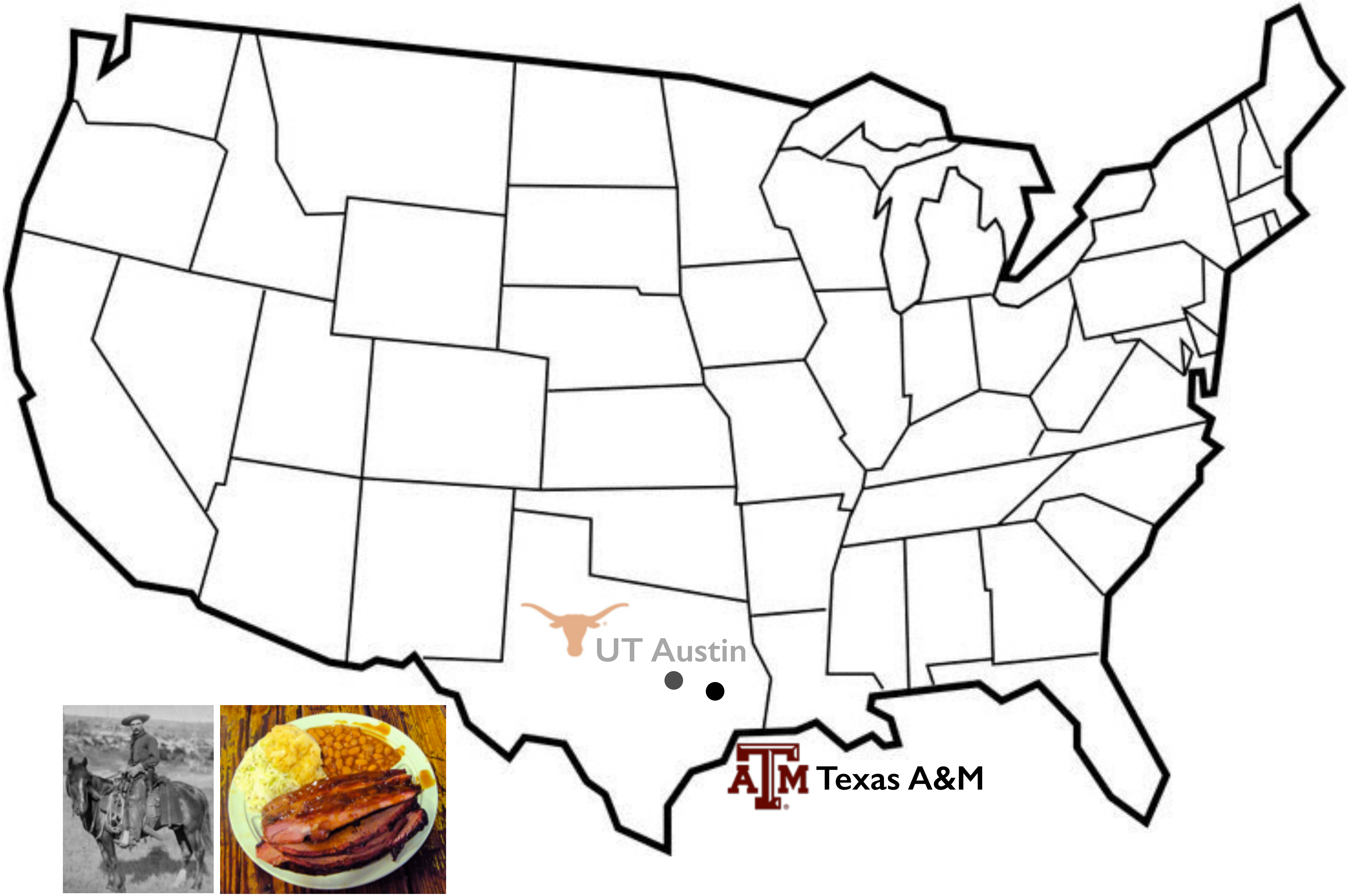
PARTICLE-INDUCED FINGERING



Sungyon Lee **Mechanical Engineering, Texas A&M**
with Feng Xu & Jungchul Kim

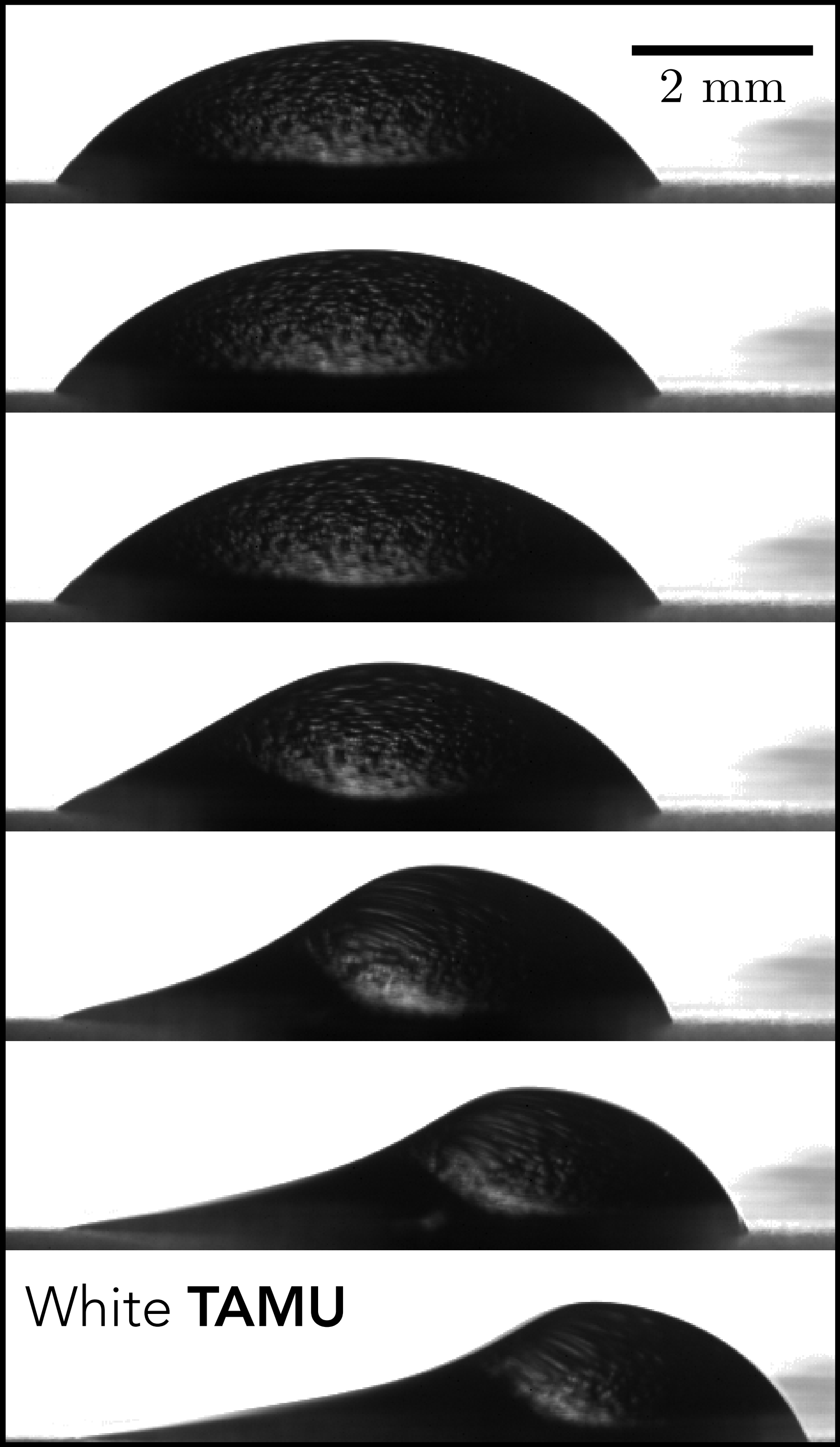
03.14-18.2016 **Two-Phase Continuum Models for Geophysical Particle-Fluid Flows**
Max Planck Institute for the Physics of Complex Systems

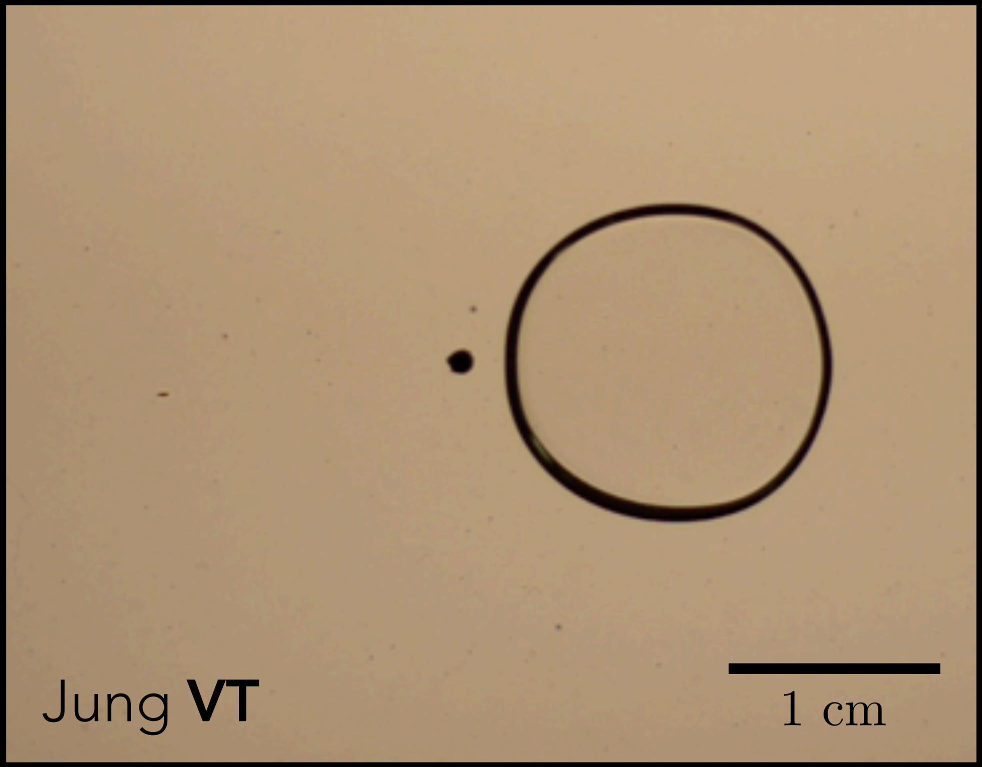
BUT FIRST….

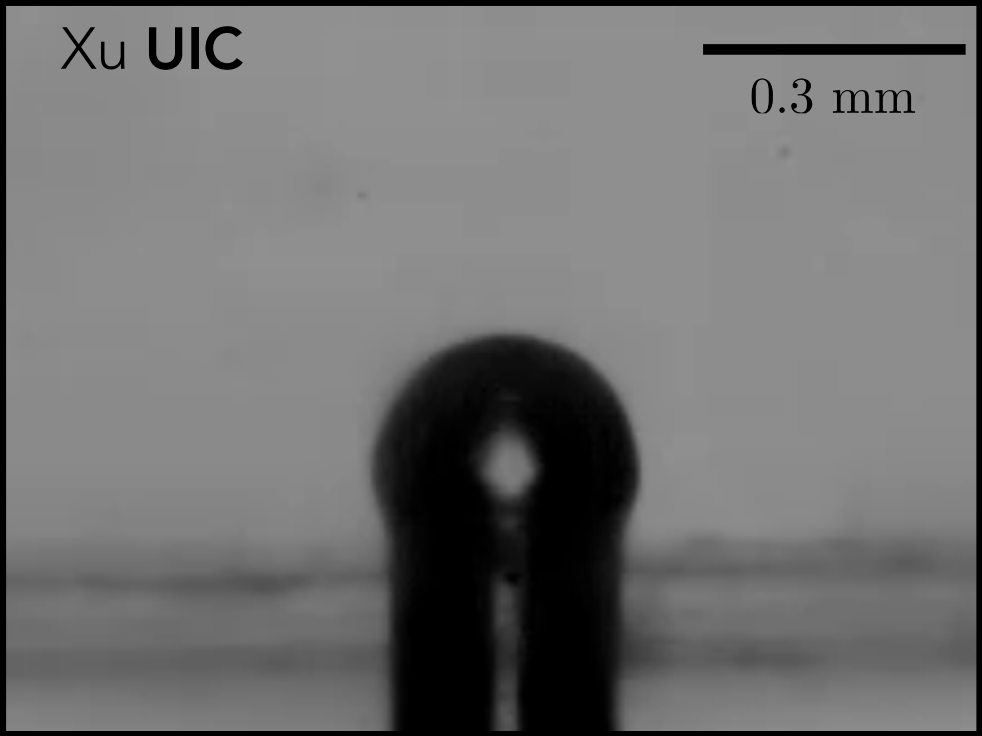


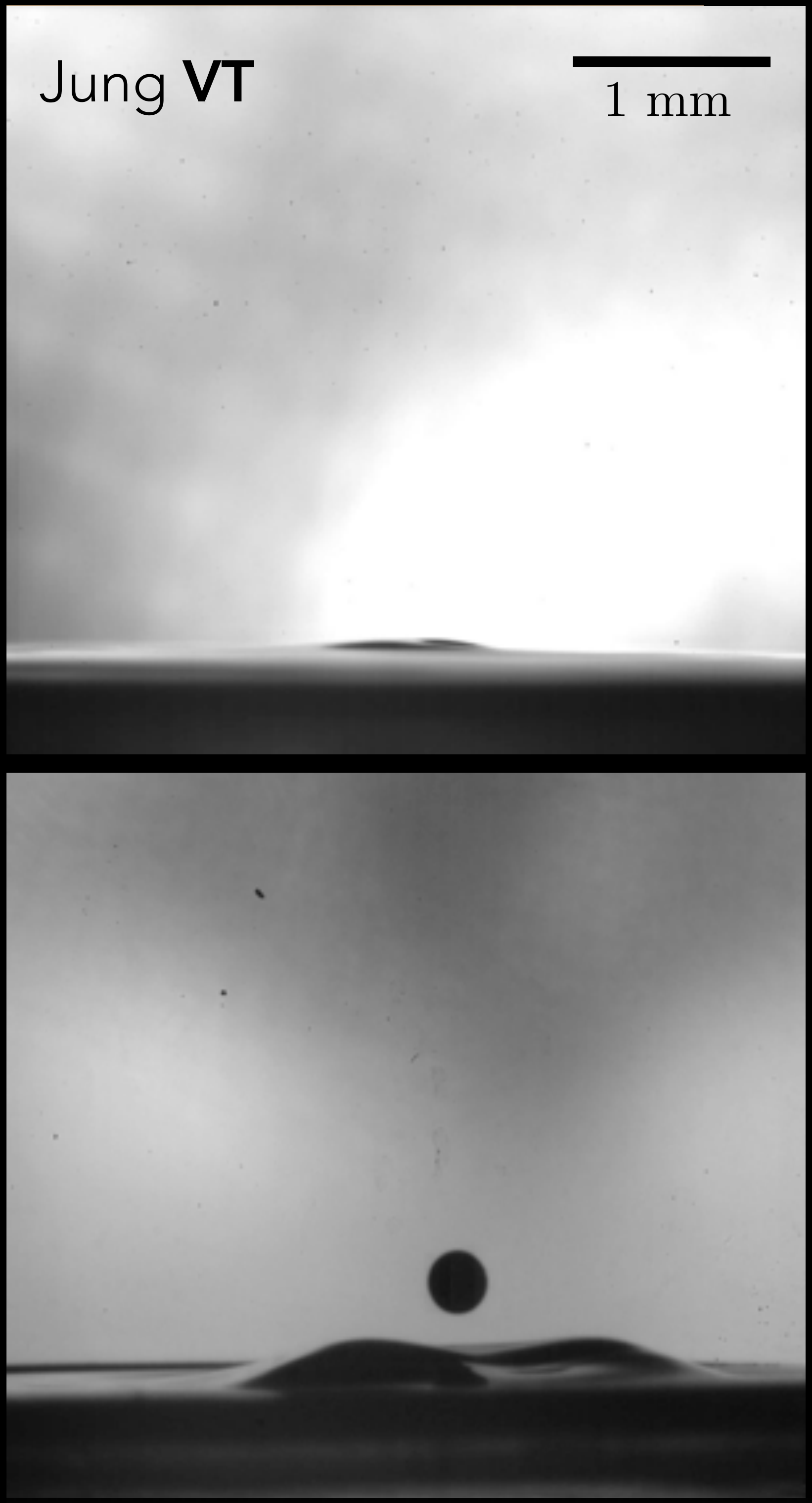
DrIPs

DR**O**PS, **INTERFACES** & **P**ARTICULATE SYSTEMS





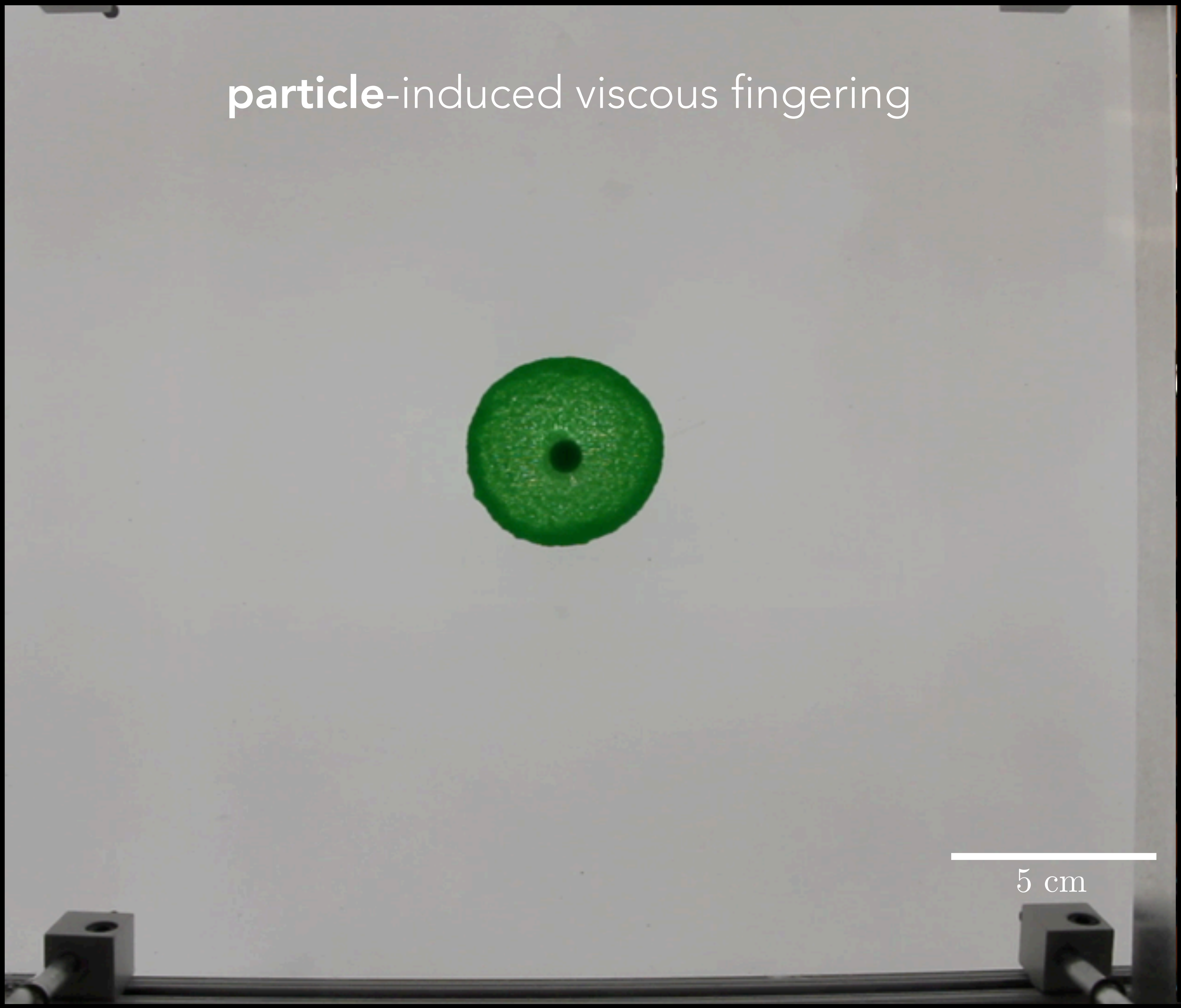




We study **interfacial** dynamics.

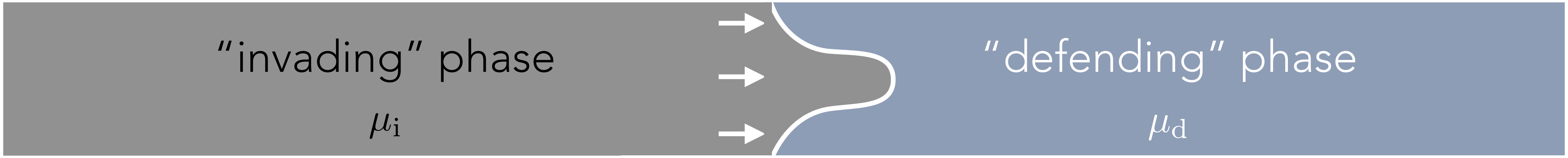
DrIPs

DROPS, INTERFACES & PARTICULATE SYSTEMS



We also like **particles**…

VISCOUS FINGERING

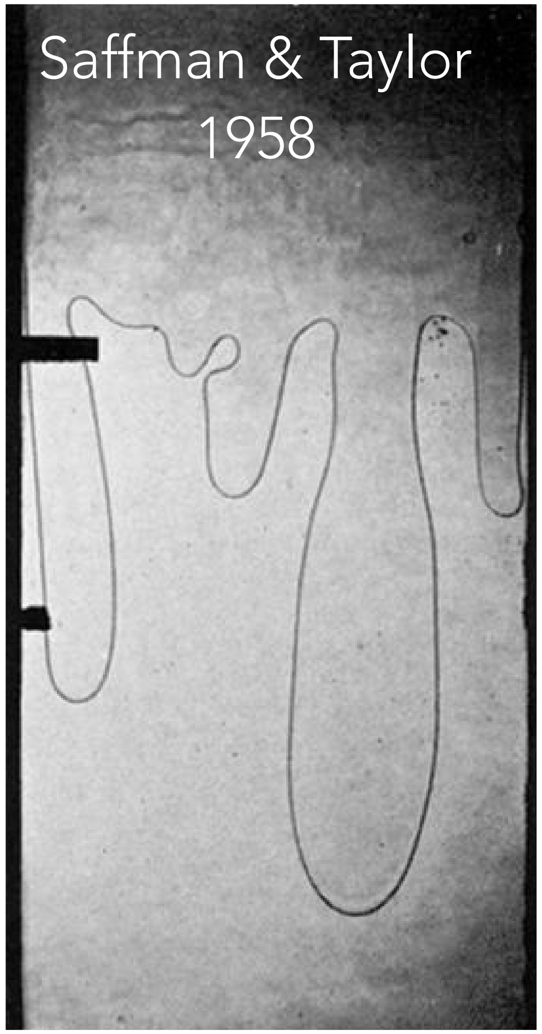

"invading" phase

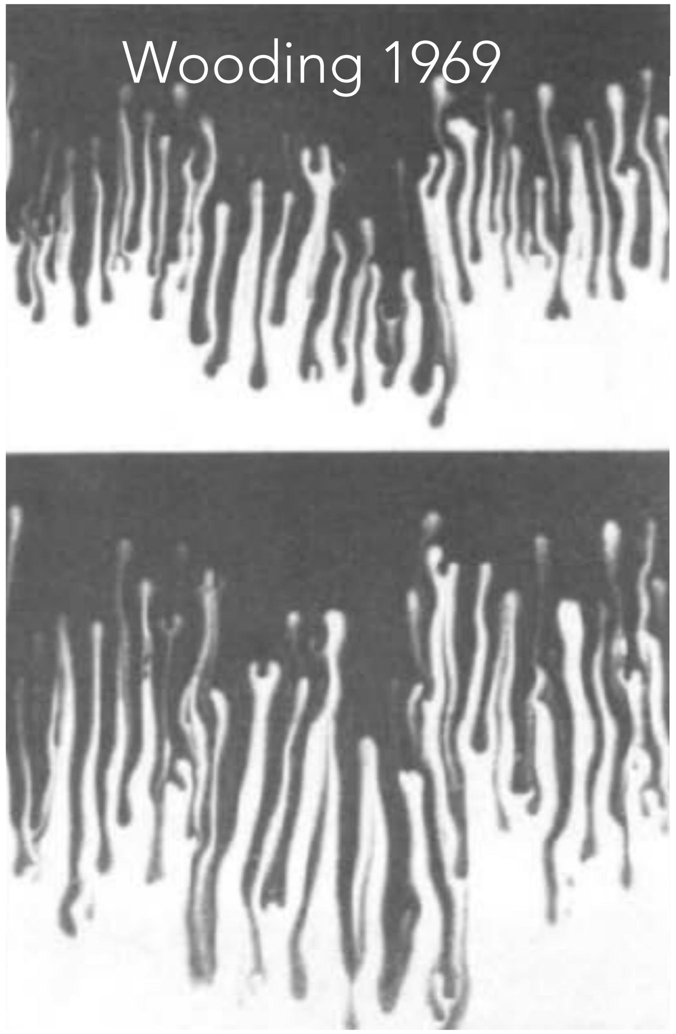
μ_{i}

"defending" phase

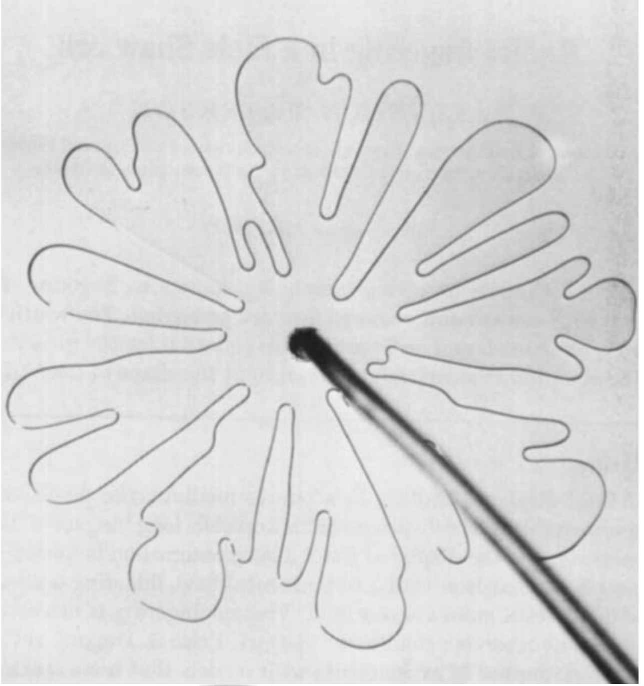
μ_{d}

Interface is **unstable** when $\mu_{\mathrm{i}} < \mu_{\mathrm{d}}$.


Saffman & Taylor 1958


Wooding 1969

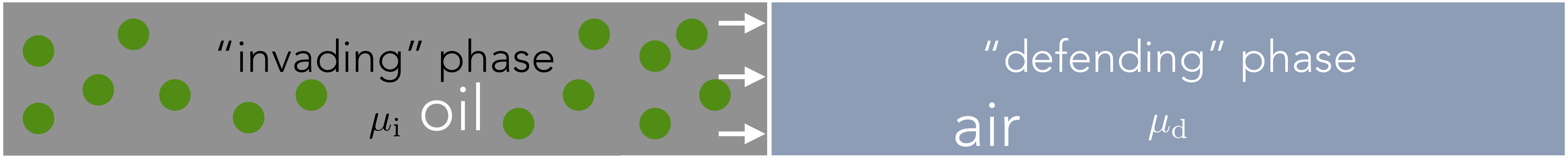
Paterson 1981

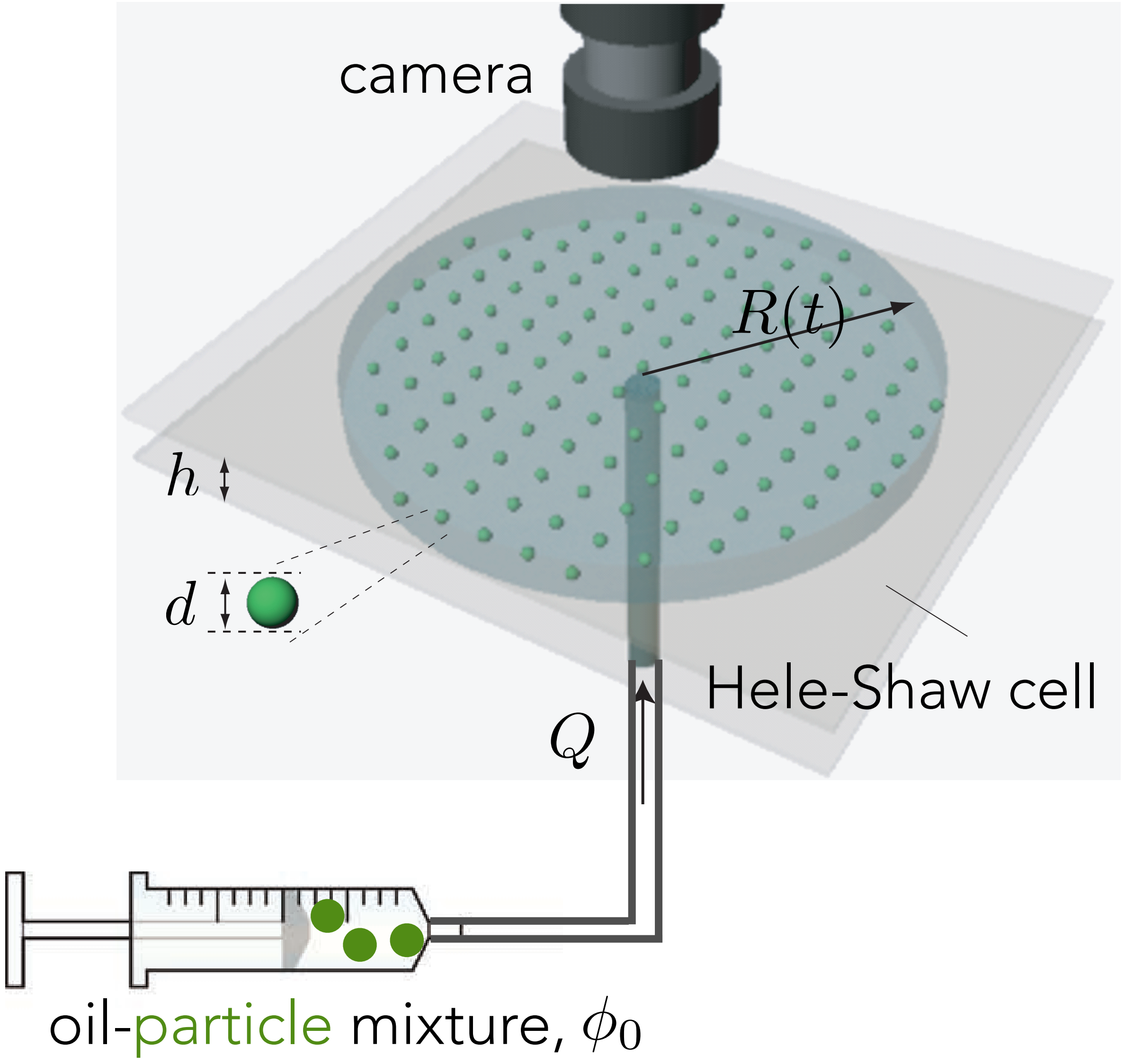


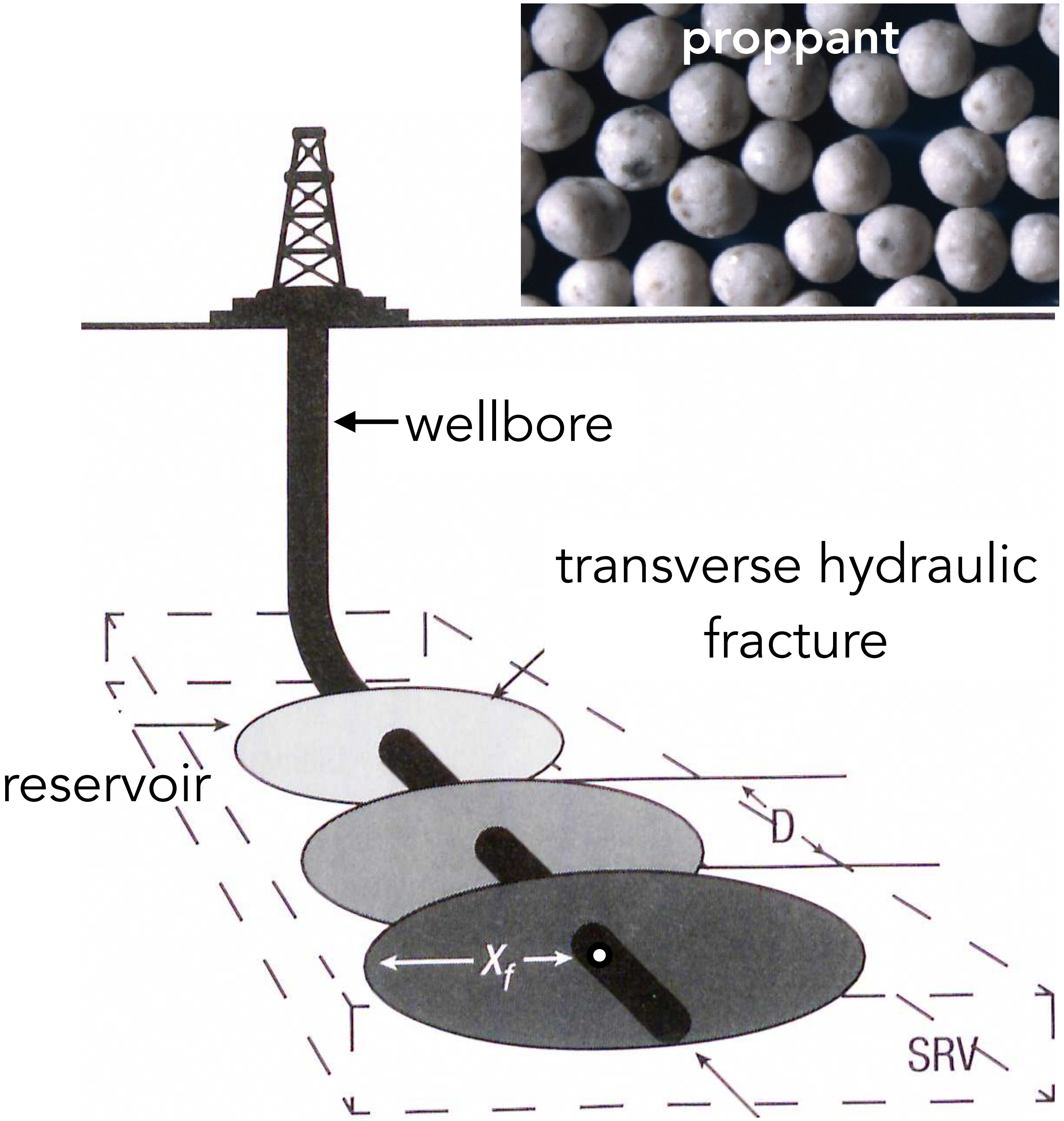
Praud & Swinney 2005

Pihler-Puzovic et al 2012

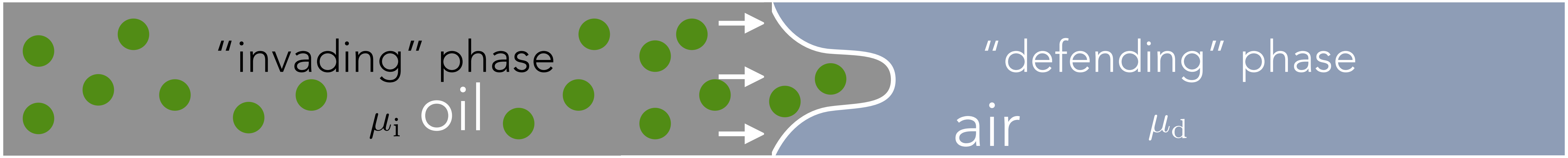
VISCOUS FINGERING + PARTICLES

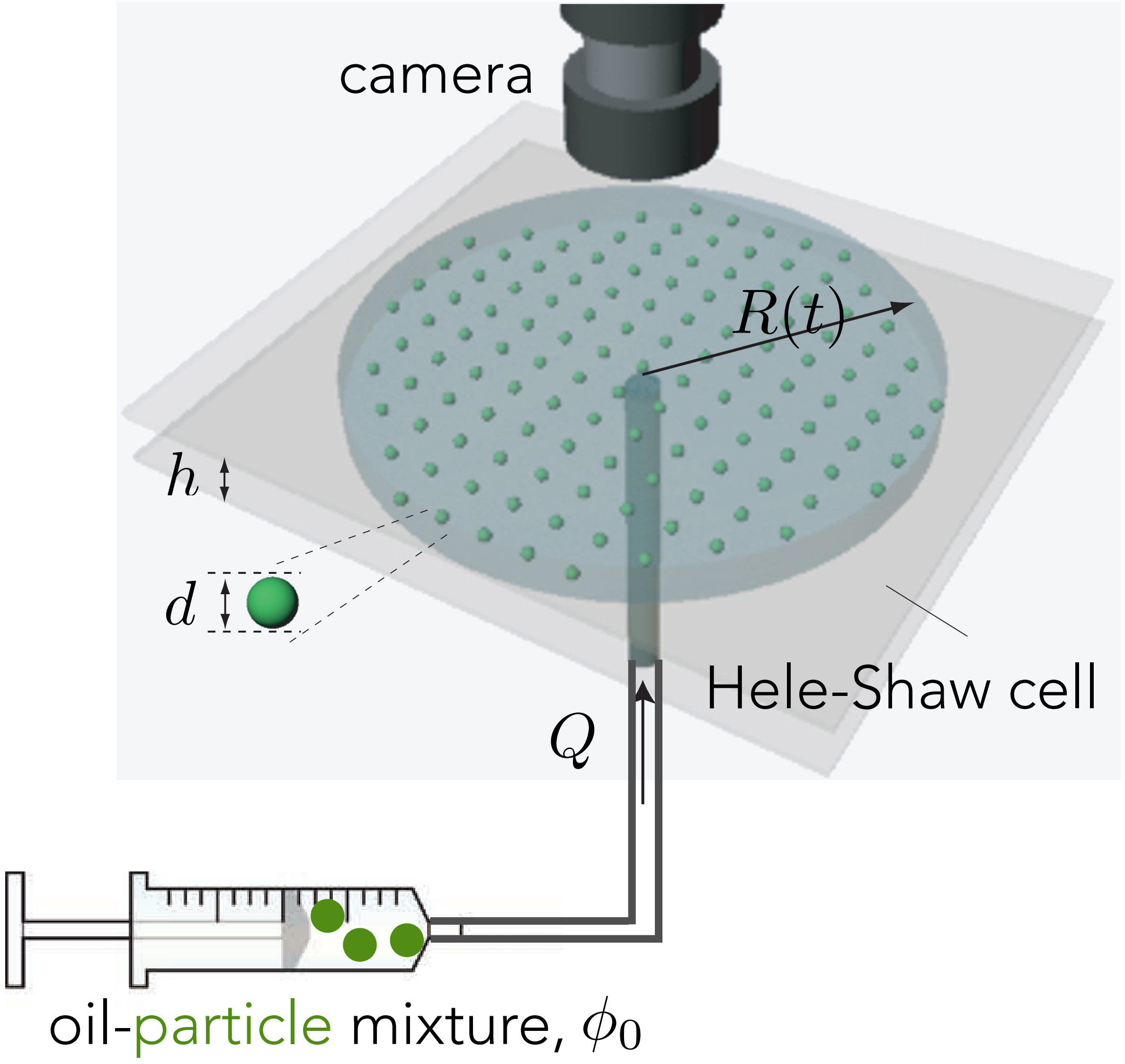




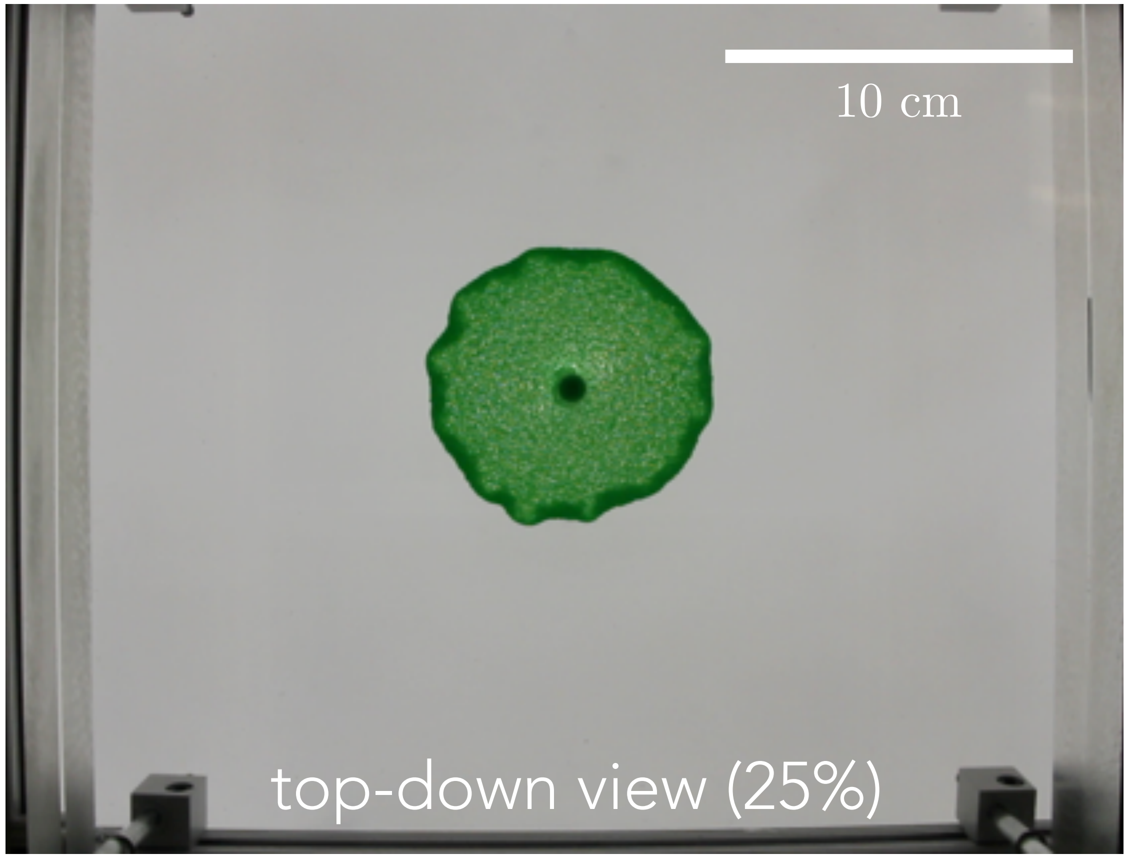


VISCOUS FINGERING + PARTICLES



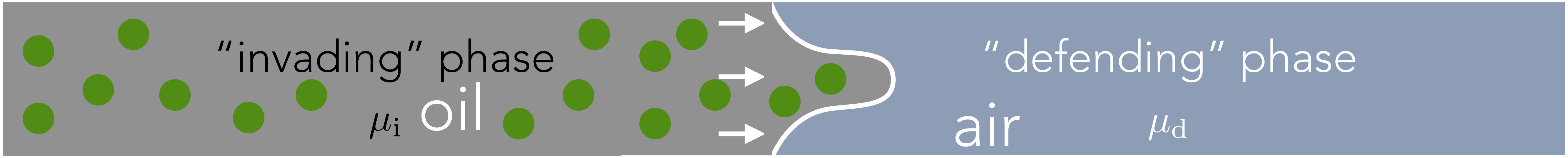


Particles **destabilize** the interface!



Tang et al 2000; Ramachandran & Leighton 2010

VISCOUS FINGERING + PARTICLES



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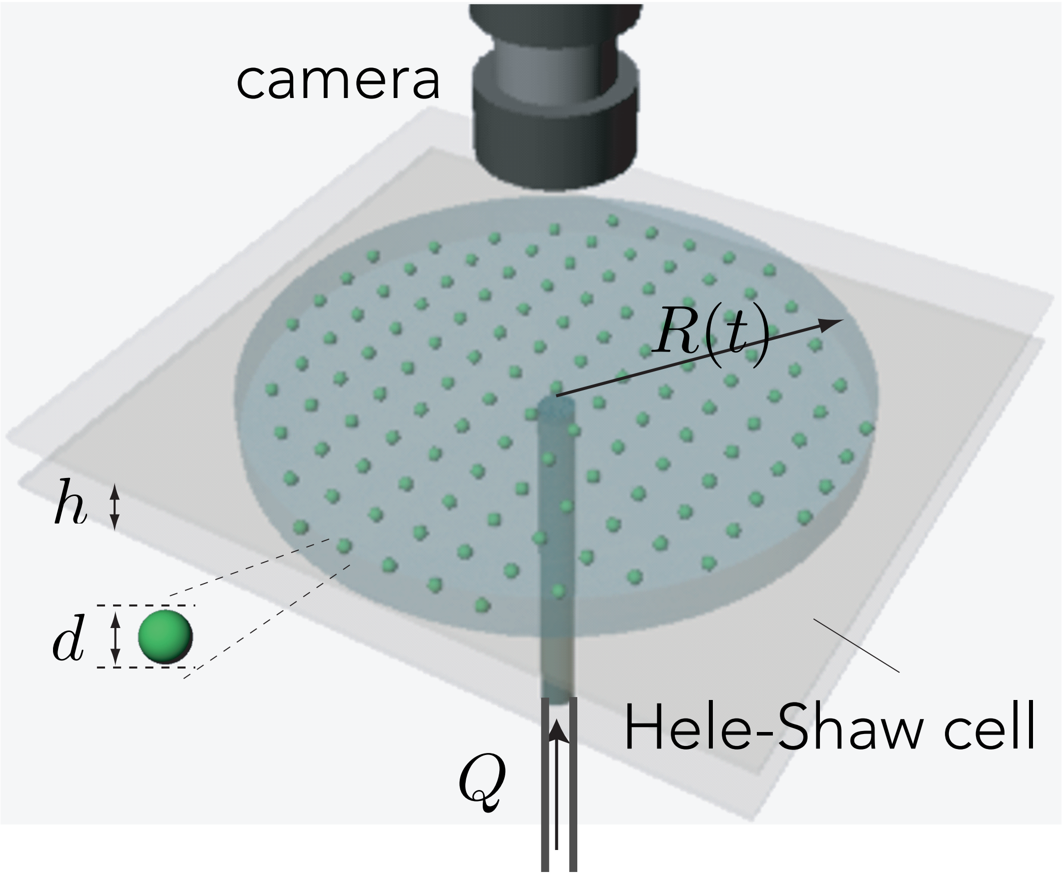
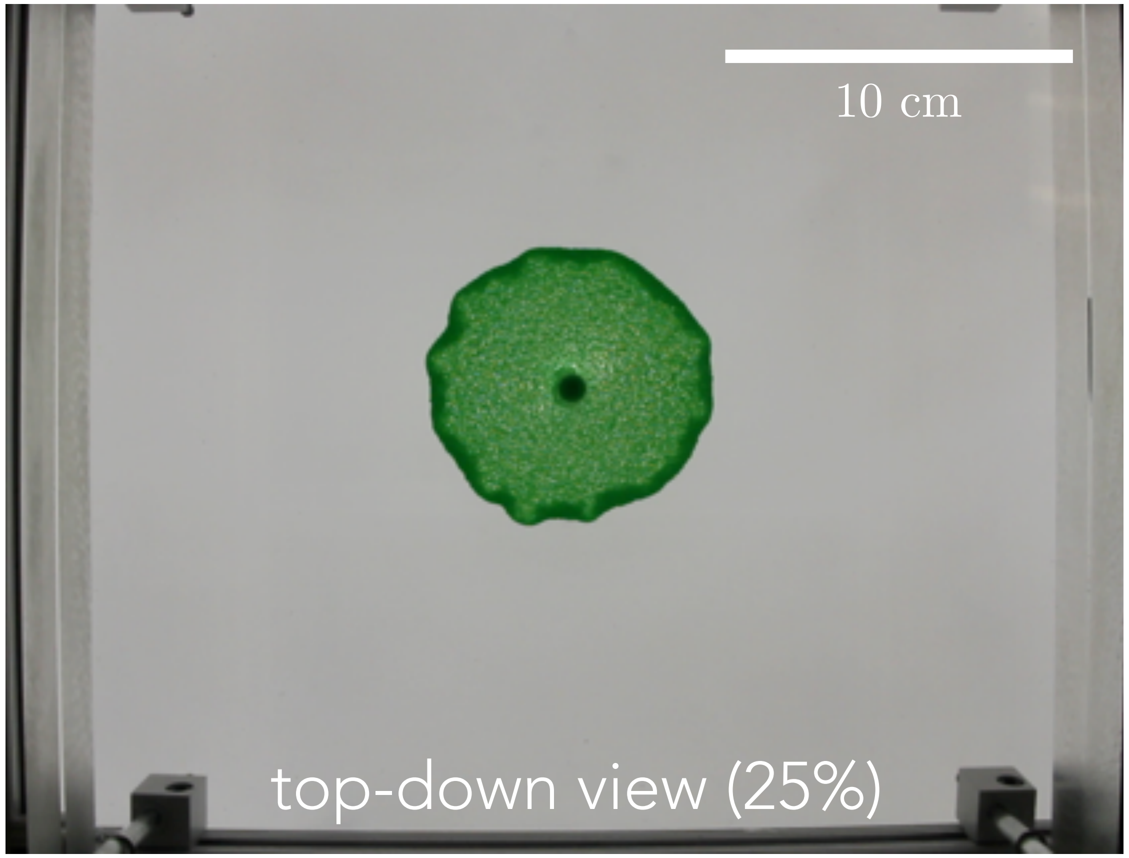
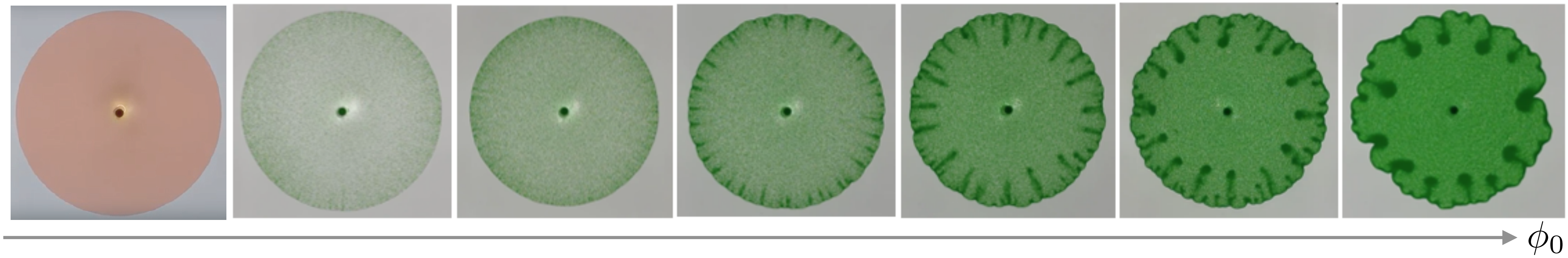
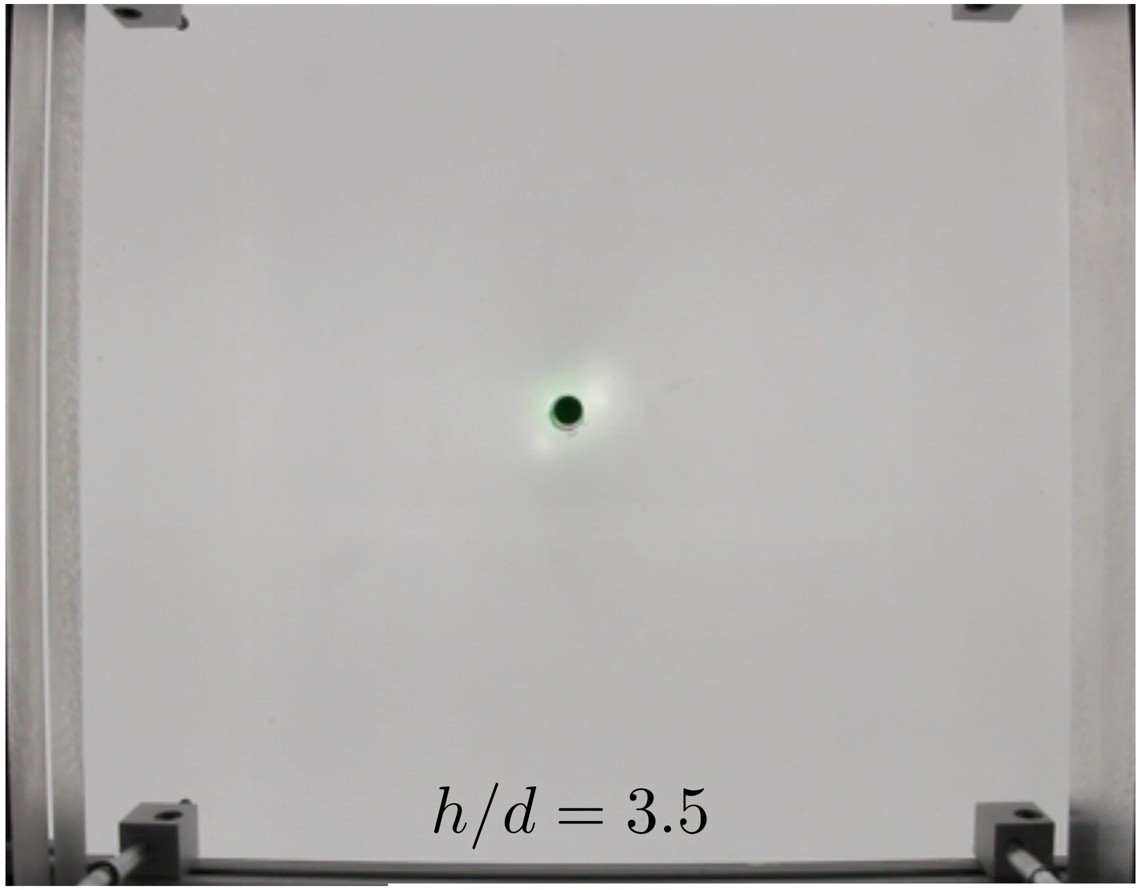






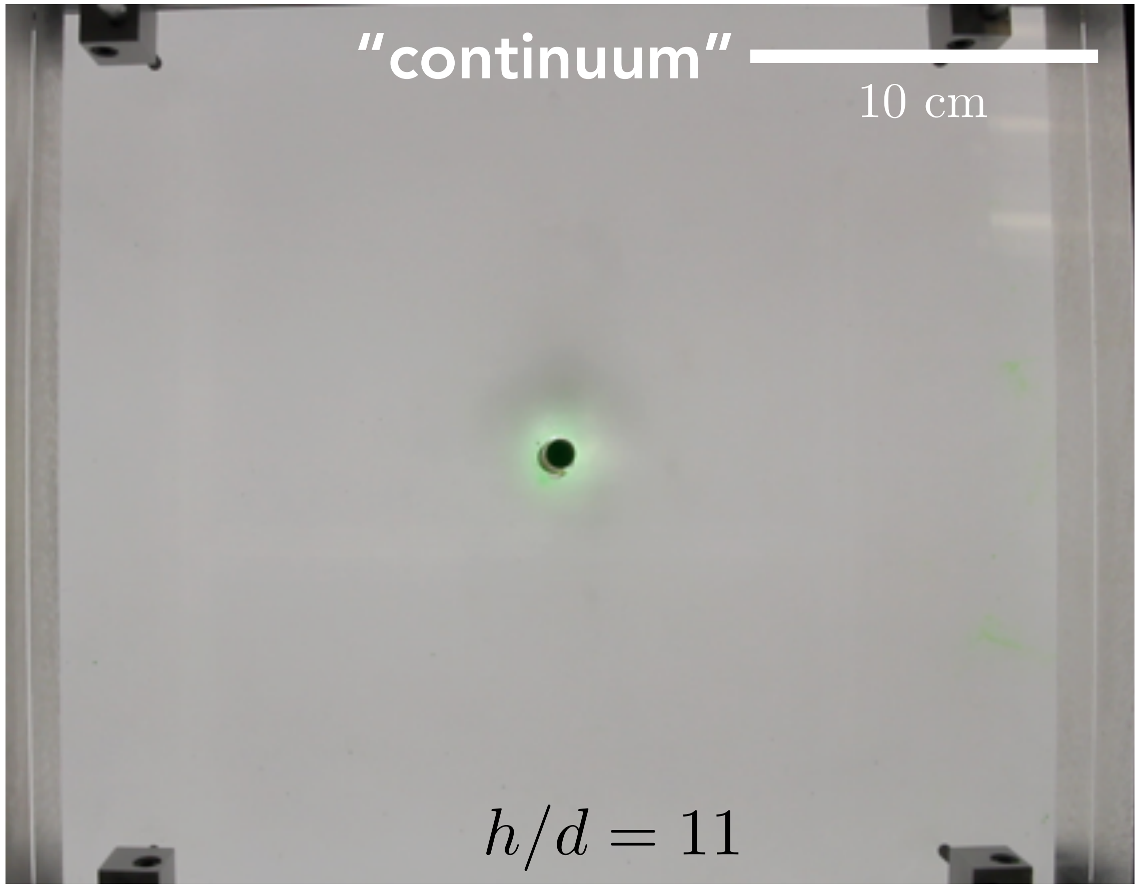
PARTICLE-INDUCED FINGERING

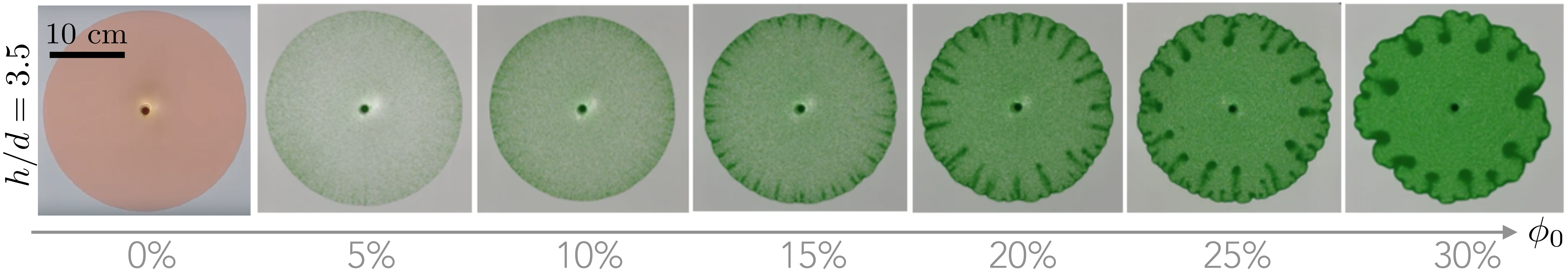


$\phi_0 = 20\%$

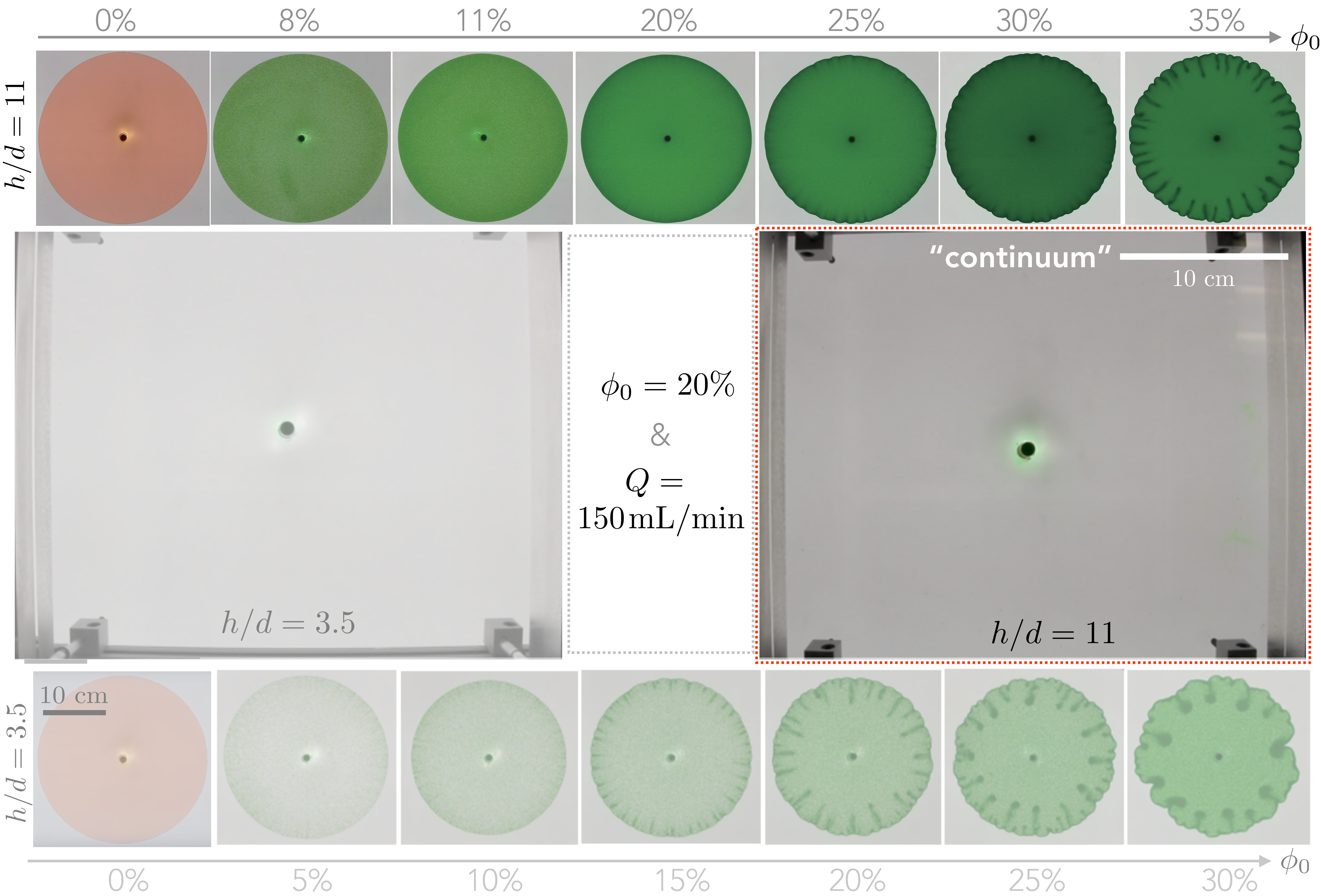
&

$Q = 150\,\mathrm{mL/min}$





PARTICLE-INDUCED FINGERING



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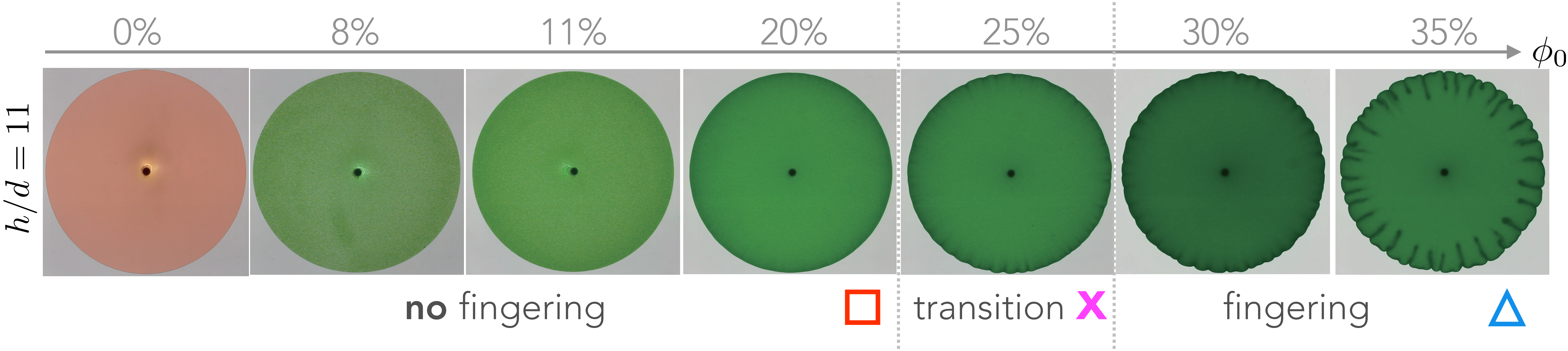
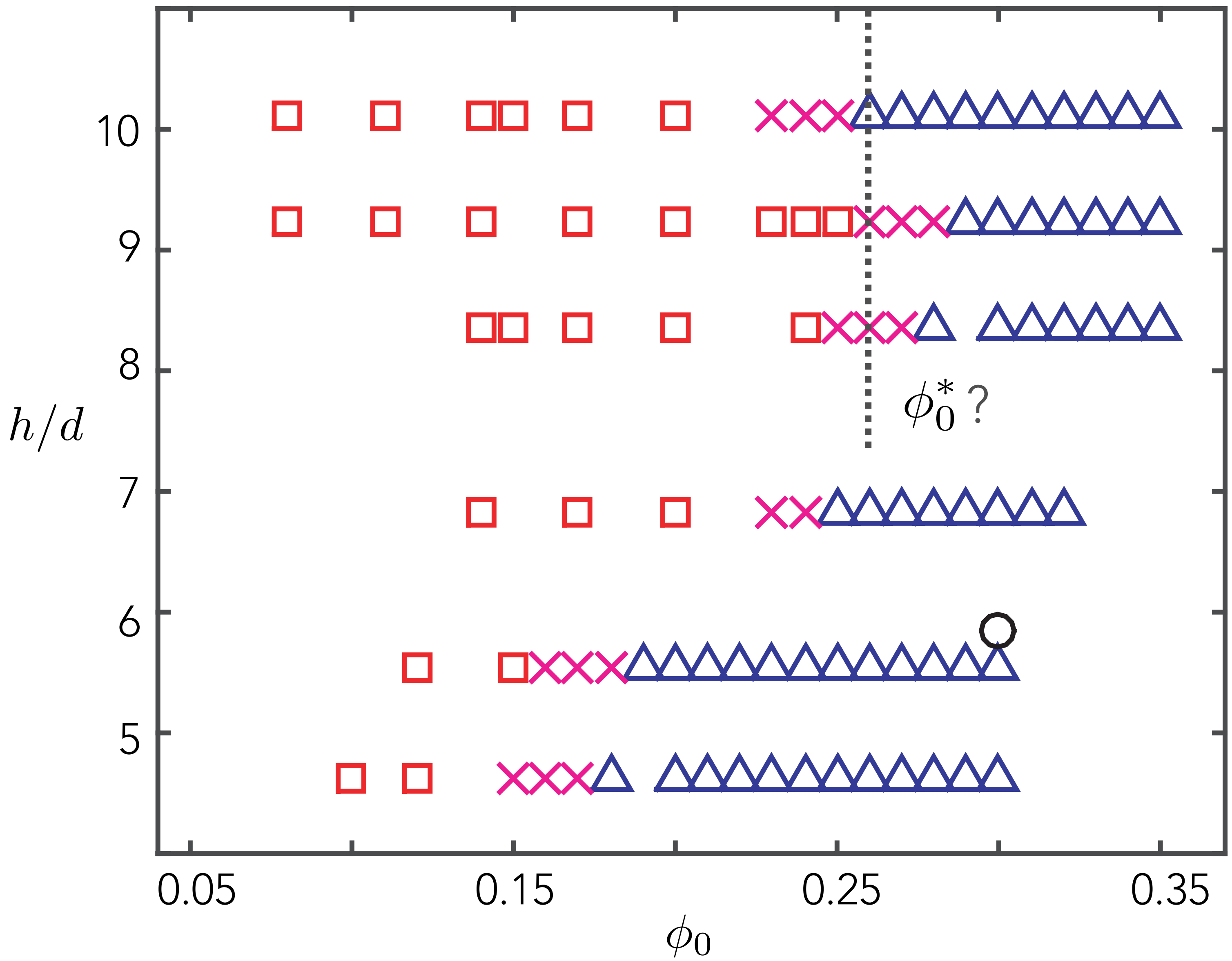
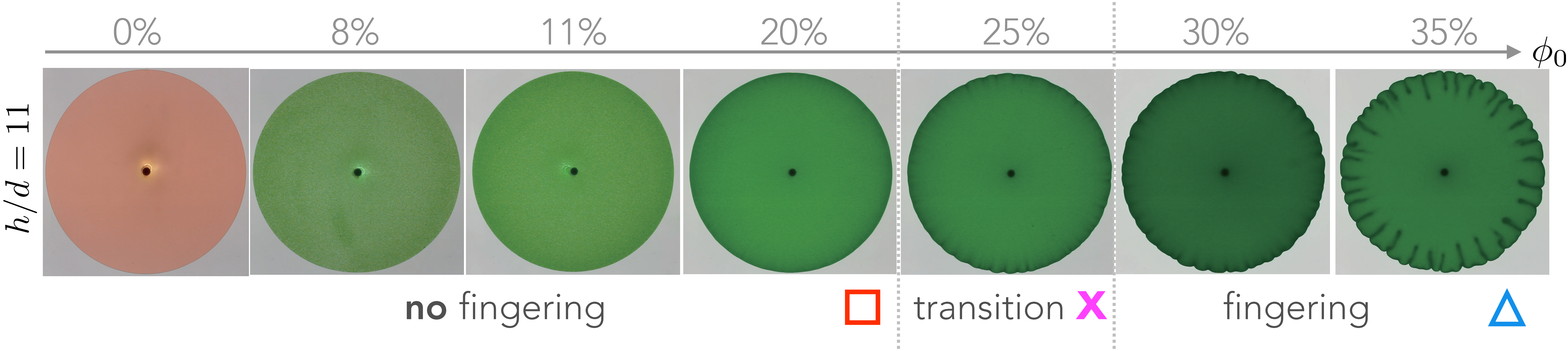


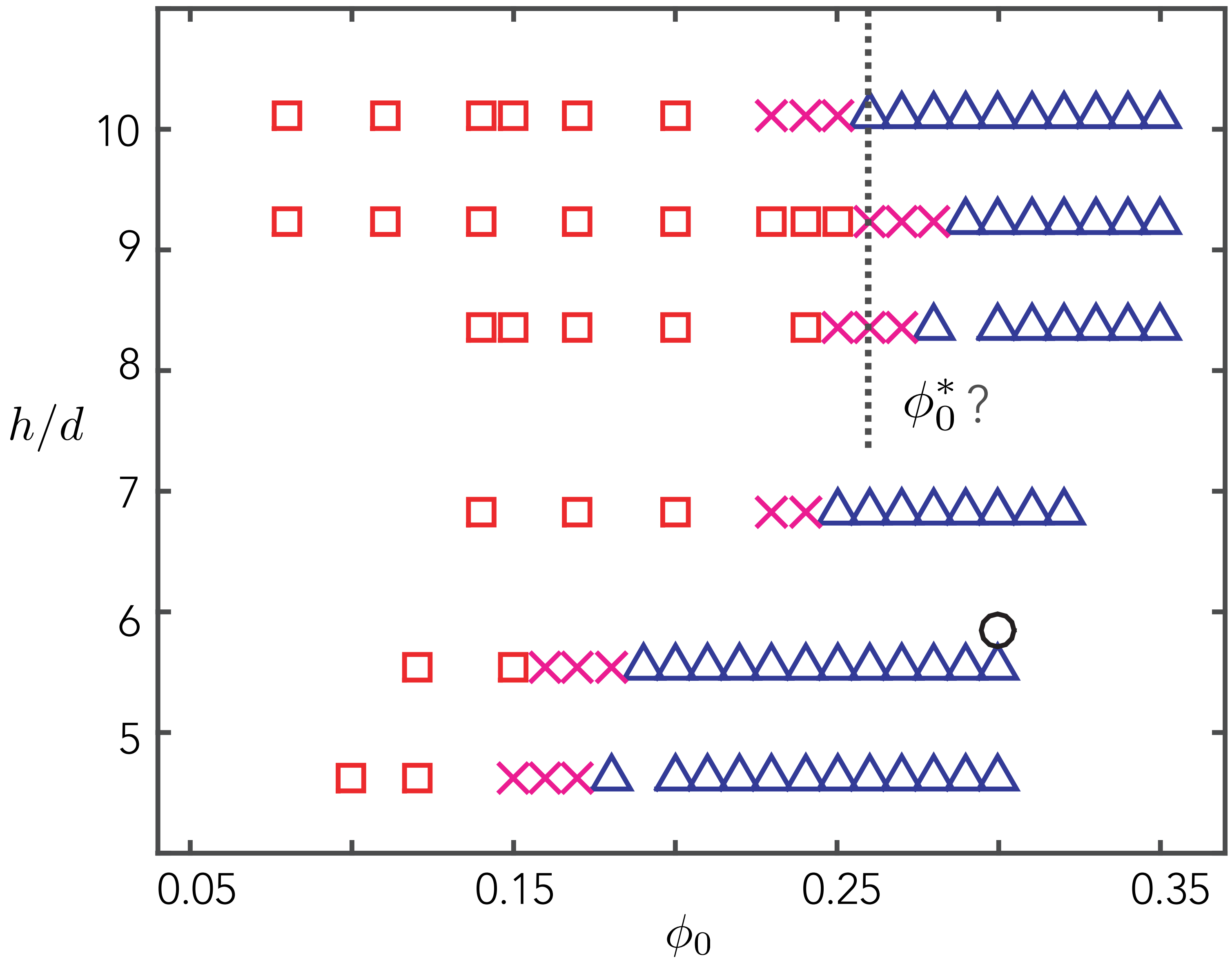


Eventually:

What is the **critical particle concentration** in the continuum limit?

PARTICLE-INDUCED FINGERING





In this talk, we will focus on:

1. What is the **mechanism of fingering**?

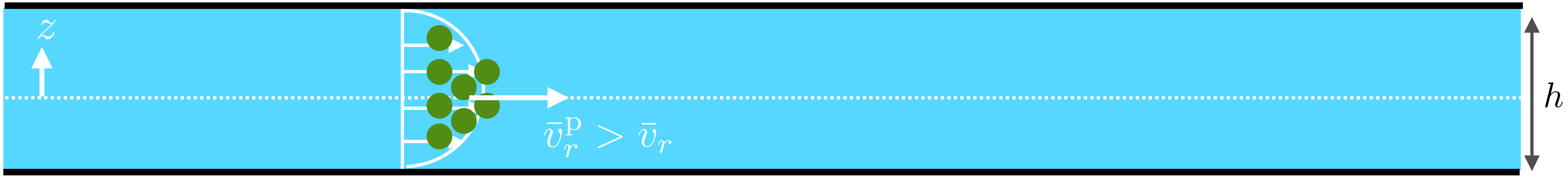
2. Can we model the **particle-laden flow** upstream of the interface?

MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

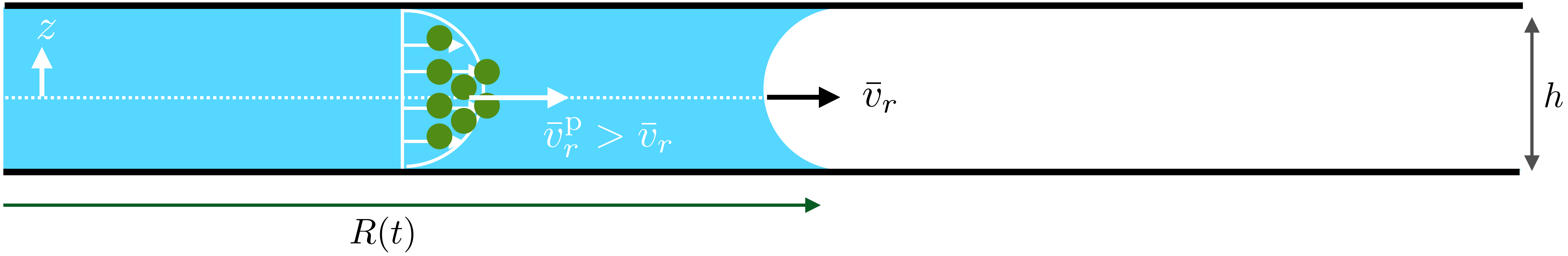
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.



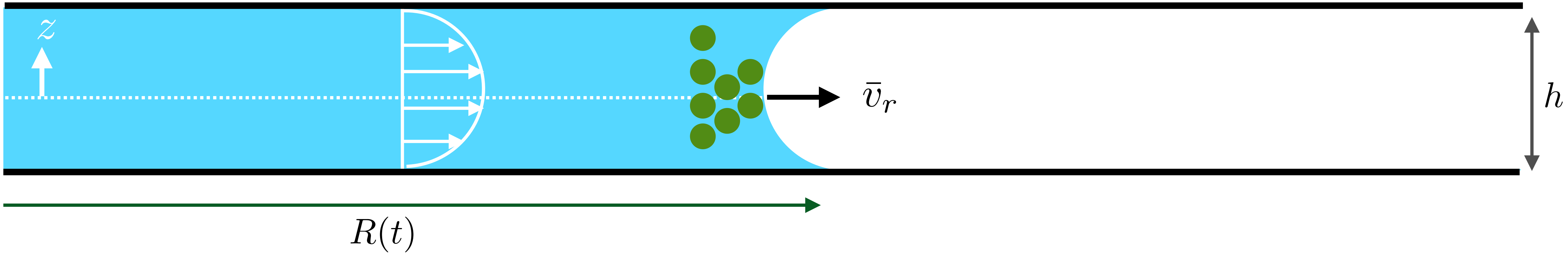
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.



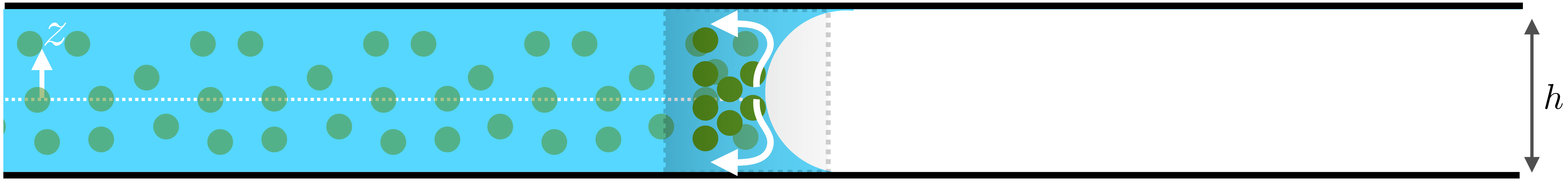
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.



MECHANISM OF FINGERING

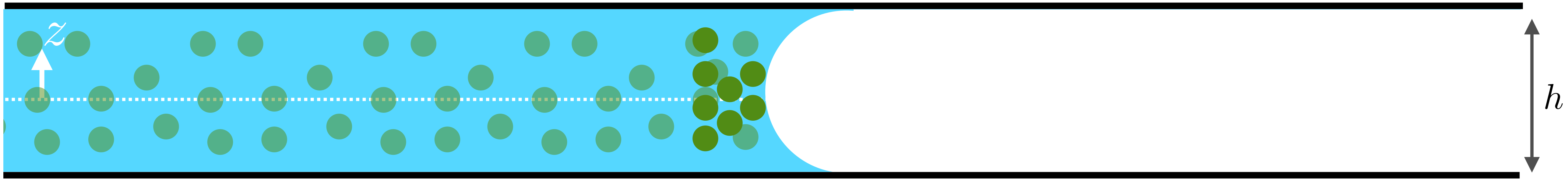
particle accumulation on the fluid-fluid interface.

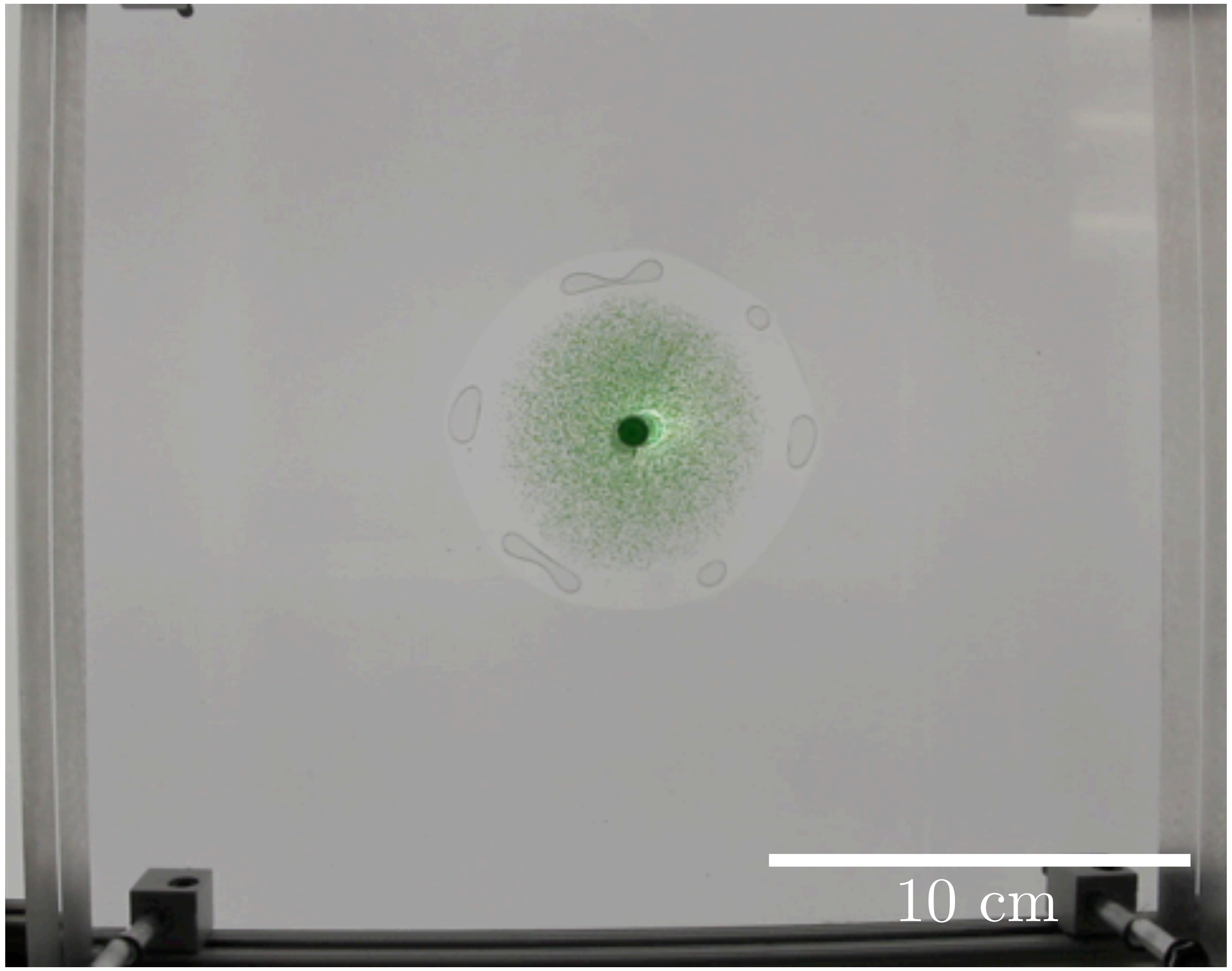


"fountain flow" Coyle et al 1987; Karnis & Mason 1967

MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

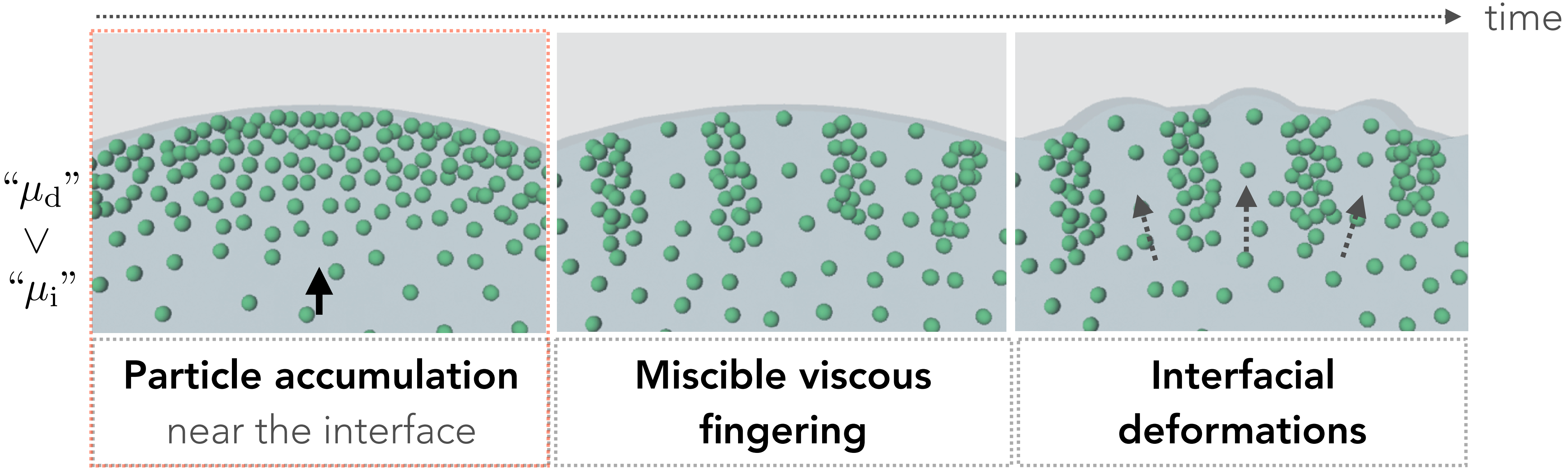




Tang et al 2000; Ramachandran & Leighton 2010, 2007 (particle accumulation on the meniscus in a tube)

MECHANISM OF FINGERING

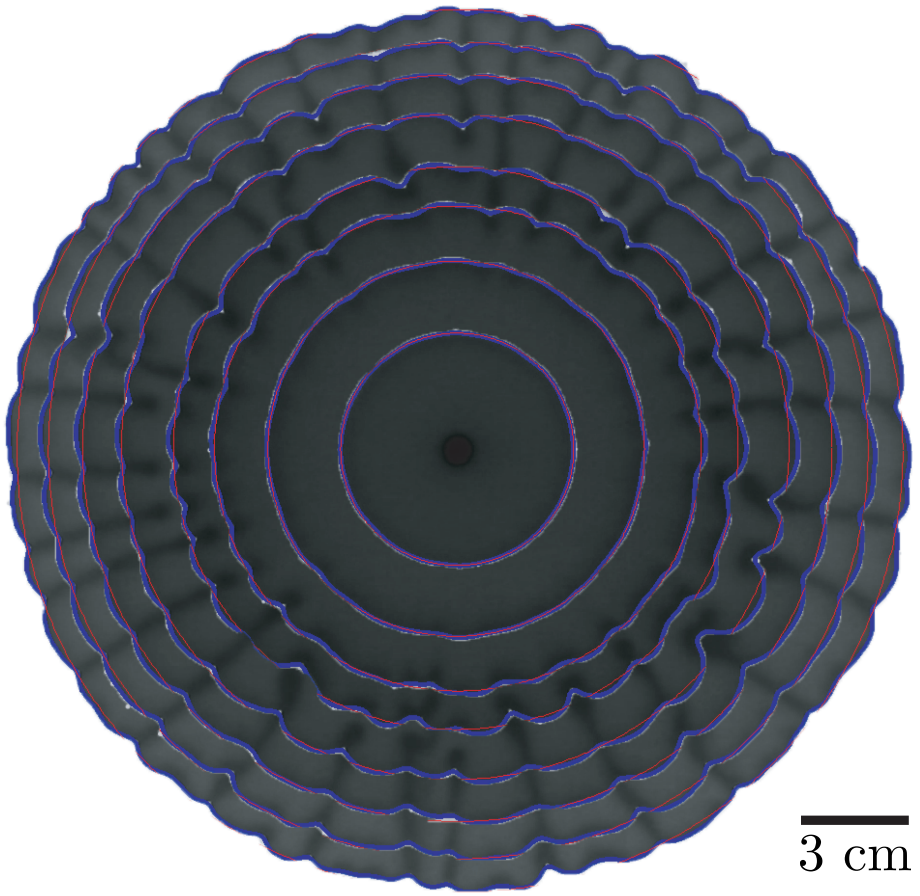
particle accumulation on the fluid-fluid interface.



Particle accumulation near the interface

Miscible viscous fingering

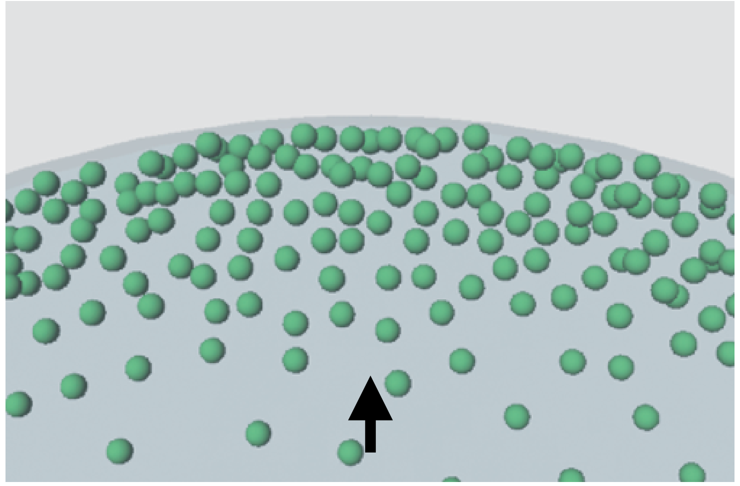
Interfacial deformations



$\phi_0 = 35\%$

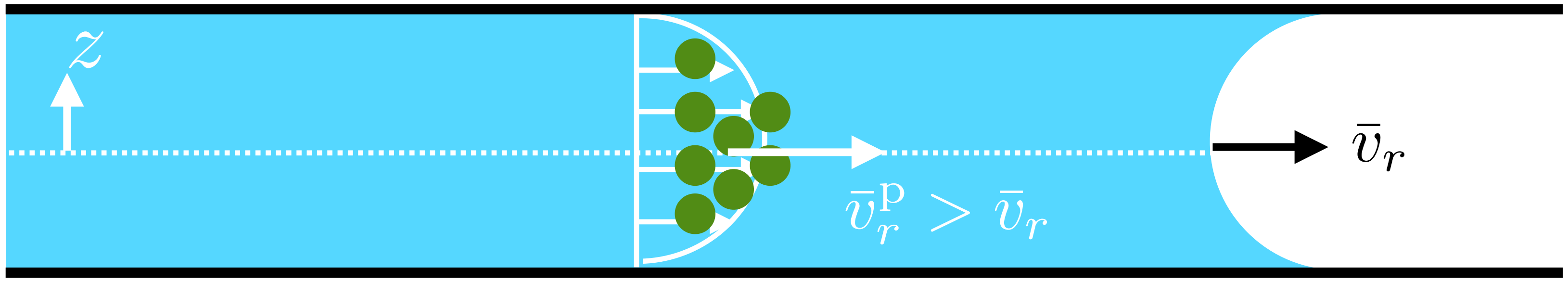
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.



Particle accumulation

due to shear-induced migration

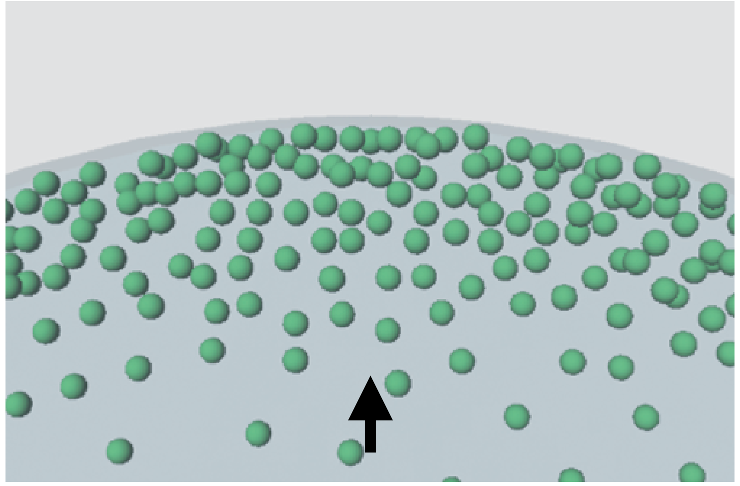


velocity ratio $\beta \equiv \frac{\bar{v}_r^{\mathrm{p}}(r)}{\bar{v}_{\mathrm{r}}(r)}$ >1 necessary condition for **accumulation**

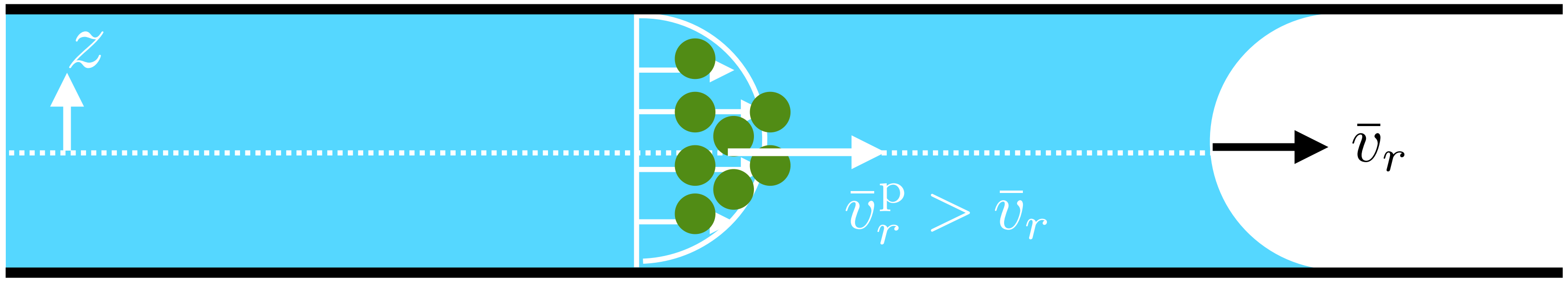
where $\bar{v}_r^{\mathrm{p}} \equiv \frac{\int_{-h/2}^{h/2} v_r^{\mathrm{p}}(r,z)\phi(r,z)dz}{\bar{\phi}(r)h}$ & $\bar{v}_r = \frac{Q}{2\pi r h}$

MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.



Particle accumulation
due to shear-induced migration

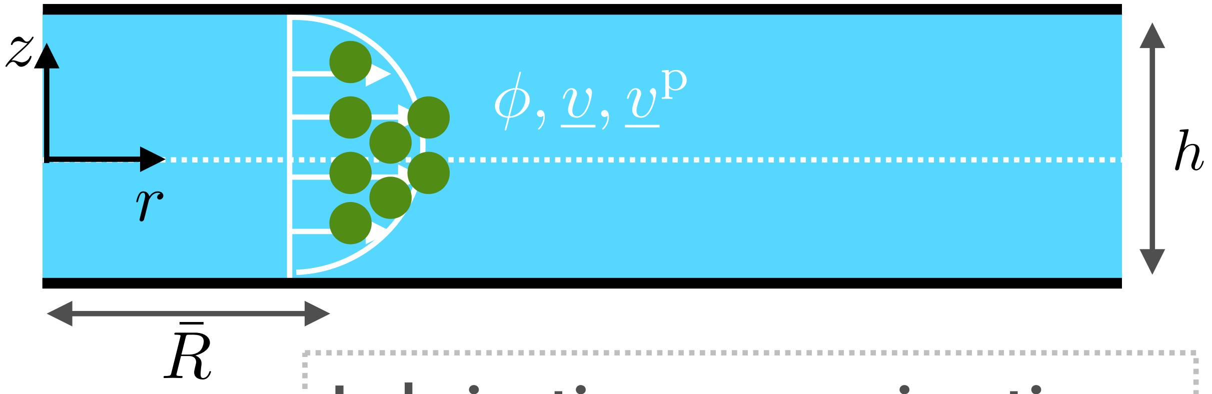


velocity ratio $\beta \equiv \dfrac{\bar{v}_r^{\mathrm{p}}(r)}{\bar{v}_{\mathrm{r}}(r)}$ >1 necessary condition for **fingering**

Calculate $\beta(\phi_0, h/d)$
by resolving the upstream flow field!

CONTINUUM MODEL

suspension balance approach Nott & Brady 1994



mixture mass conservation

$$\nabla \cdot \underline{v} = 0$$

mixture momentum

$$\nabla \cdot \underline{\underline{\Pi}} = 0$$

particulate mass conservation

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \underline{v}^{\mathrm{p}}) = 0$$

particulate momentum

$$\nabla \cdot \underline{\underline{\Pi}}^{\mathrm{p}} - \frac{18\mu_{\mathrm{l}}}{d^2} \frac{\phi}{f(\phi)} (\underline{v}^{\mathrm{p}} - \underline{v}) = 0$$

Lubrication approximations:

$$h/\bar{R} \ll 1$$

$$\underline{v} = \frac{F(z)}{r} \underline{e}_r, \quad \frac{\partial}{\partial \theta} = 0$$

(axisymmetric)

---

kinematic relationship

$$\underline{v} = \phi \underline{v}^{\mathrm{p}} + (1 - \phi) \underline{v}^{\mathrm{f}}$$

constitutive relationships

$$\underline{\underline{\Pi}} = -p\underline{\underline{I}} + \mu_{\mathrm{l}} \left(\nabla \underline{v} + \nabla \underline{v}^{\intercal} \right) + \underline{\underline{\Pi}}^{\mathrm{p}}$$

$$\underline{\underline{\Pi}}^{\mathrm{p}} = \boxed{\underline{\underline{\Pi}}^{\mathrm{p}}_{\mathrm{n}}} (\dot{\gamma}, \mu_{\mathrm{n}}(\phi)) + (\mu_{\mathrm{s}}(\phi) - \mu_{\mathrm{l}}) \left(\nabla \underline{v} + \nabla \underline{v}^{\intercal} \right)$$

particulate normal stress

$$\sim \mu_{\mathrm{l}} \nabla \dot{\gamma}$$

empirical relationships

$$\mu_{\mathrm{s}} \approx \mu_{\mathrm{l}} \frac{e^{-2.34\phi}}{(1 - \phi/\phi_{\mathrm{m}})^3}$$

Zarraga et al 2000

CONTINUUM MODEL

suspension balance approach + **lubrication** approximations

mixture momentum

$$r^*\frac{dp^*}{dr^*}(z^*-1/2)=\frac{\mu_{\mathrm{s}}(\phi)}{\mu_{\mathrm{l}}}\frac{dF^*}{dz^*}$$

particulate mass + momentum

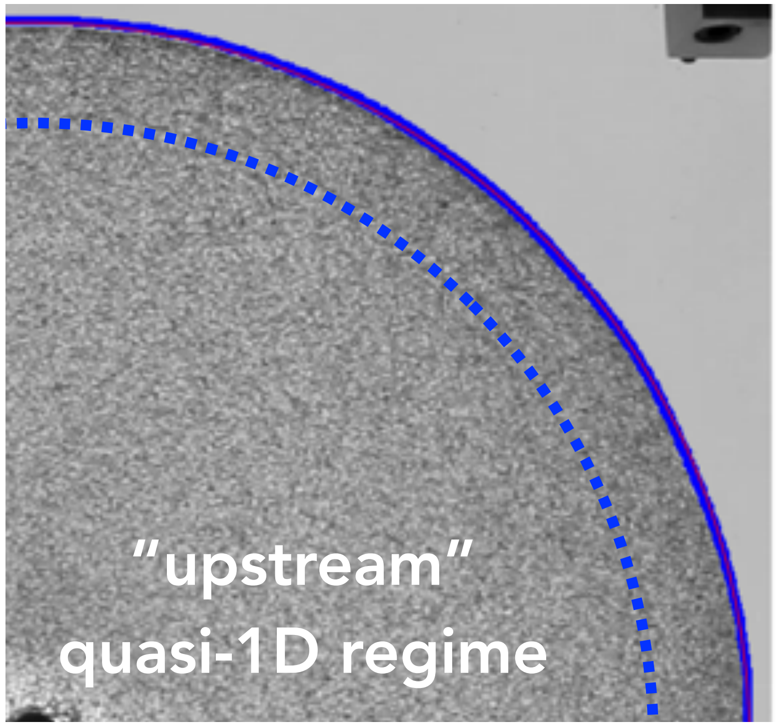
$$\mathrm{const}=\frac{\mu_{\mathrm{n}}(\phi)}{\mu_{\mathrm{l}}}\frac{dF^*}{dz^*}$$

constant mixture flux

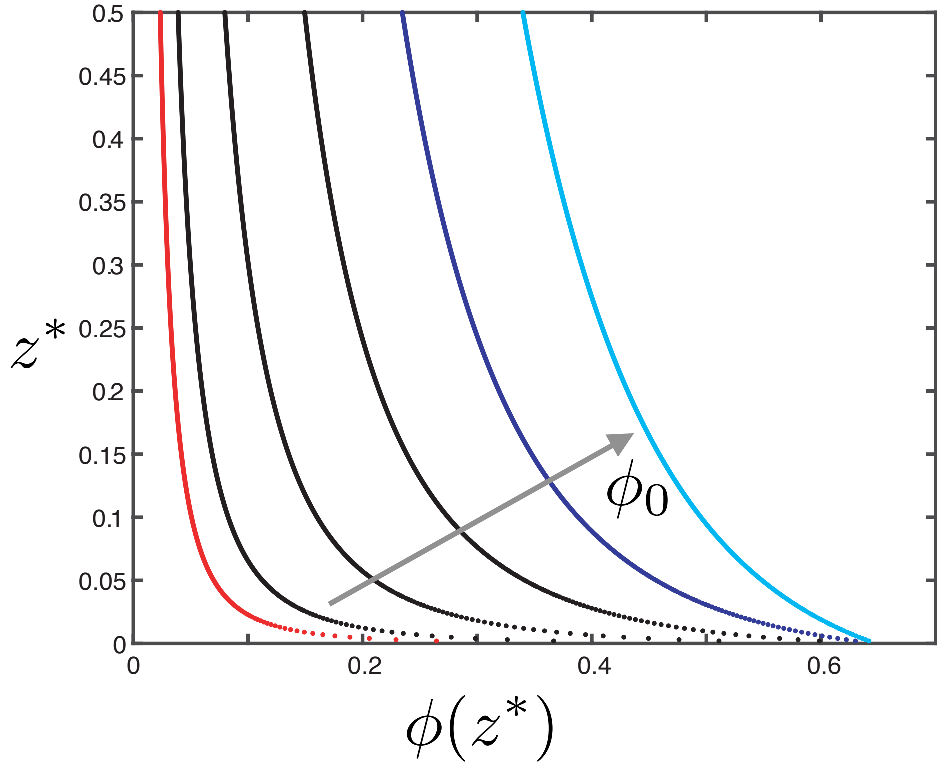
$$\frac{1}{2}=\int_0^{1/2}F^*(z^*)dz^*$$

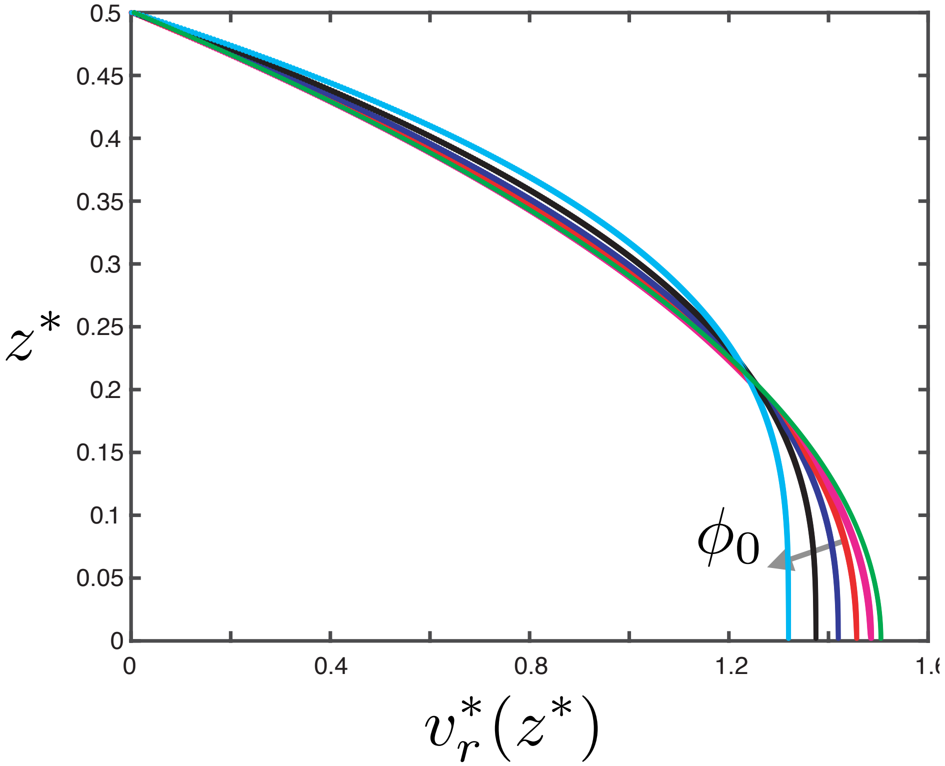
constant particulate flux

$$\frac{\phi_0}{2}=r^*\int_0^{1/2}\phi(z)v_r^{\mathrm{p}*}dz^*$$


“upstream” quasi-1D regime

$\phi=\phi(z)$ only





CONTINUUM MODEL

suspension balance approach + **lubrication** approximations

mixture momentum

$$r^*\frac{dp^*}{dr^*}(z^*-1/2)=\frac{\mu_s(\phi)}{\mu_l}\frac{dF^*}{dz^*}$$

particulate mass + momentum

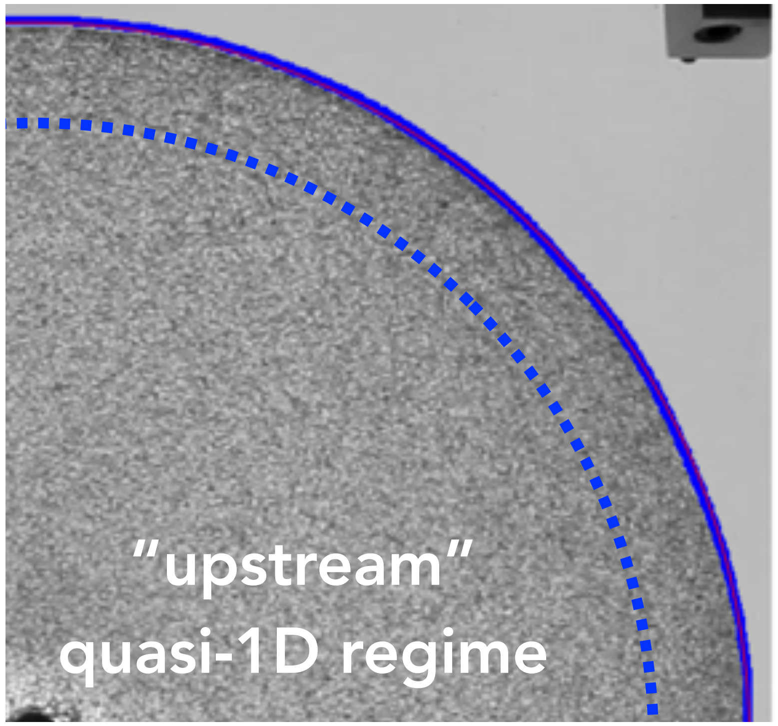
$$\text{const}=\frac{\mu_n(\phi)}{\mu_l}\frac{dF^*}{dz^*}$$

constant mixture flux

$$\frac{1}{2}=\int_0^{1/2}F^*(z^*)dz^*$$

constant particulate flux

$$\frac{\phi_0}{2}=r^*\int_0^{1/2}\phi(z)v_r^{p*}dz^*$$



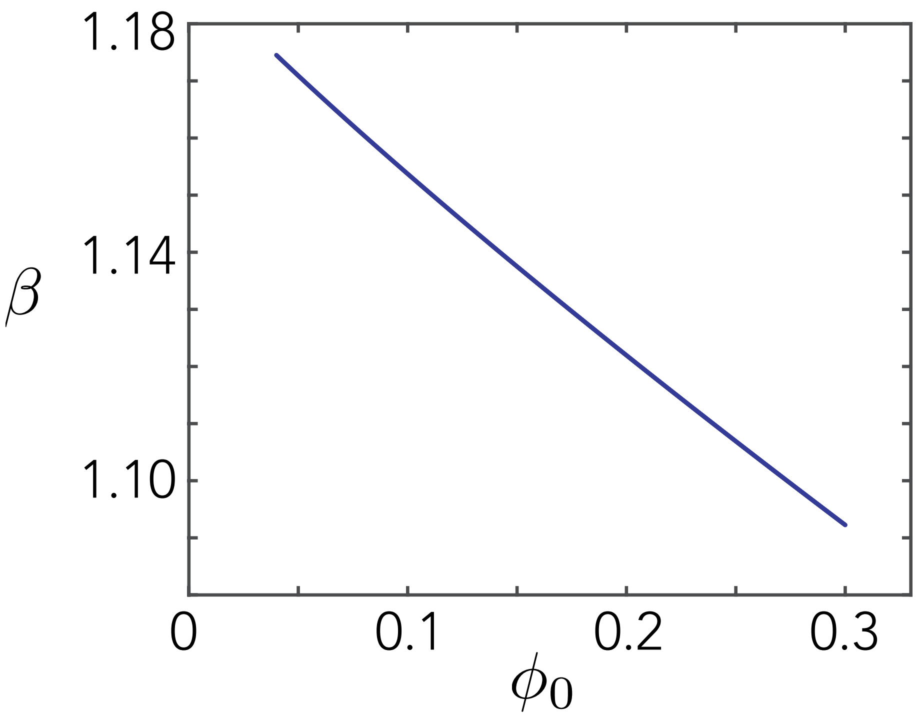
$\phi=\phi(z)$ only

---

velocity ratio

$$\beta=\frac{\int_{-h/2}^{h/2}v_r^p(r,z)\phi(r,z)dz}{\bar{\phi}(r)\bar{v}_r(r)h}\quad >1$$

necessary condition for **fingering**



CONTINUUM MODEL

suspension balance approach + **lubrication** approximations

mixture momentum

$$r^* \frac{dp^*}{dr^*}(z^* - 1/2) = \frac{\mu_\mathrm{s}(\phi)}{\mu_\mathrm{l}} \frac{dF^*}{dz^*}$$

particulate mass + momentum

$$\mathrm{const} = \frac{\mu_\mathrm{n}(\phi)}{\mu_\mathrm{l}} \frac{dF^*}{dz^*}$$

constant mixture flux

$$\frac{1}{2} = \int_0^{1/2} F^*(z^*) dz^*$$

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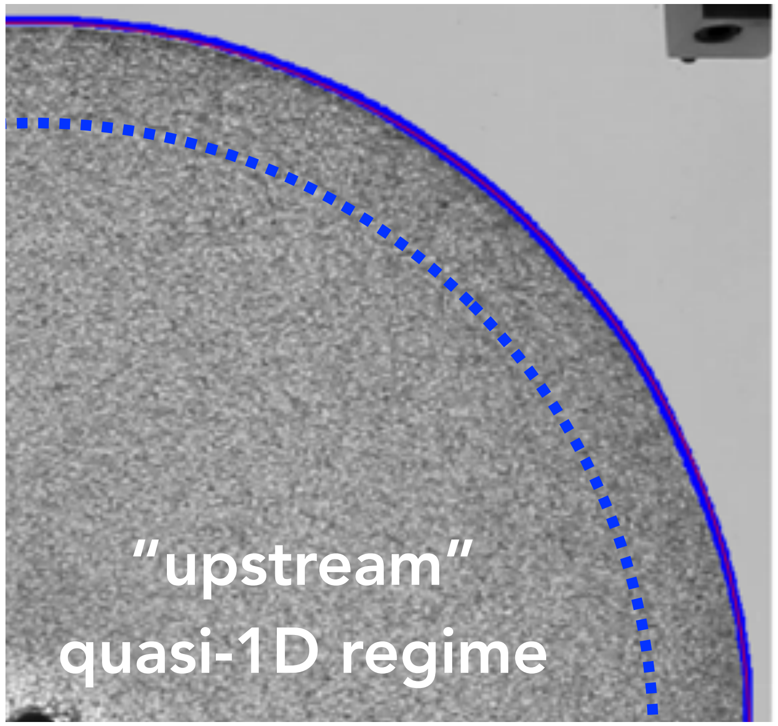
$$\frac{\phi_0}{2} = r^* \int_0^{1/2} \phi(z) v_r^{\mathrm{p}*} dz^*$$



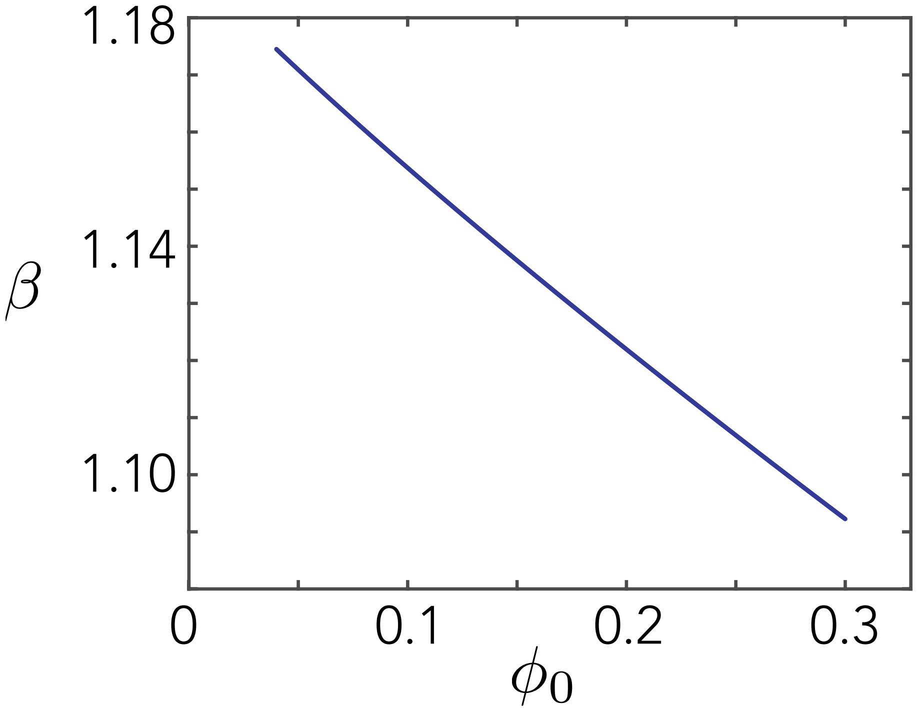
$\phi = \phi(z)$ only

---

velocity ratio

$$\beta = \frac{\int_{-h/2}^{h/2} v_r^\mathrm{p}(r,z)\phi(r,z)dz}{\bar{\phi}(r)\bar{v}_\mathrm{r}(r)h} > 1$$

Can we validate the velocity ratio experimentally?



MODEL VALIDATION:

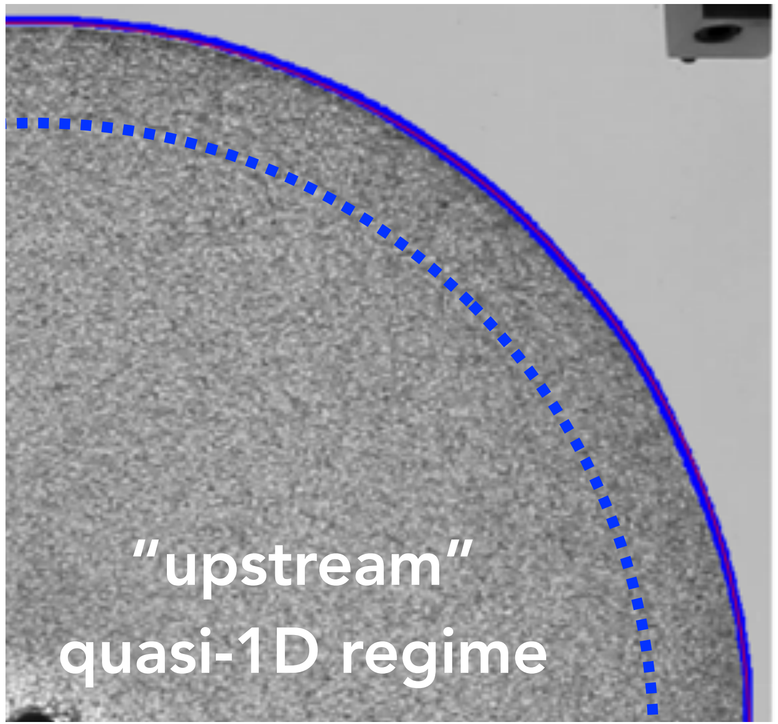
from **accumulation** to **fingering**

mass conservation in upstream regime

$$\phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v_r^{\mathrm{p}}(z)\phi(z)dz$$

$$\beta = \phi_0/\bar{\phi}_{\mathrm{up}}$$

measure experimentally



$$\bar{\phi}_{\mathrm{up}} = \frac{1}{h}\int_{-h/2}^{h/2} \phi(z)dz$$

MODEL VALIDATION:

from **accumulation** to **fingering**

mass conservation
in upstream
regime

$$\phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v_r^{\mathrm{p}}(z)\phi(z)dz$$

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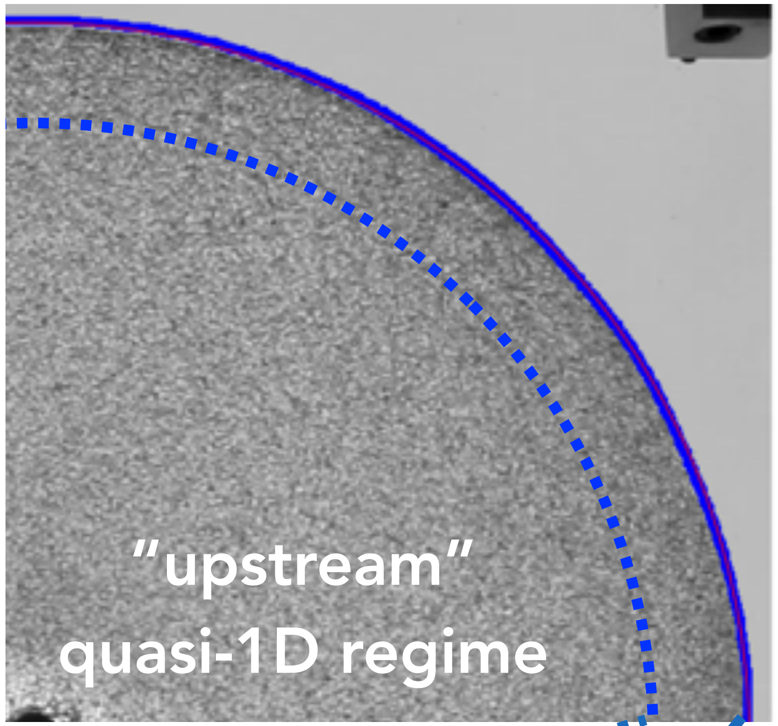
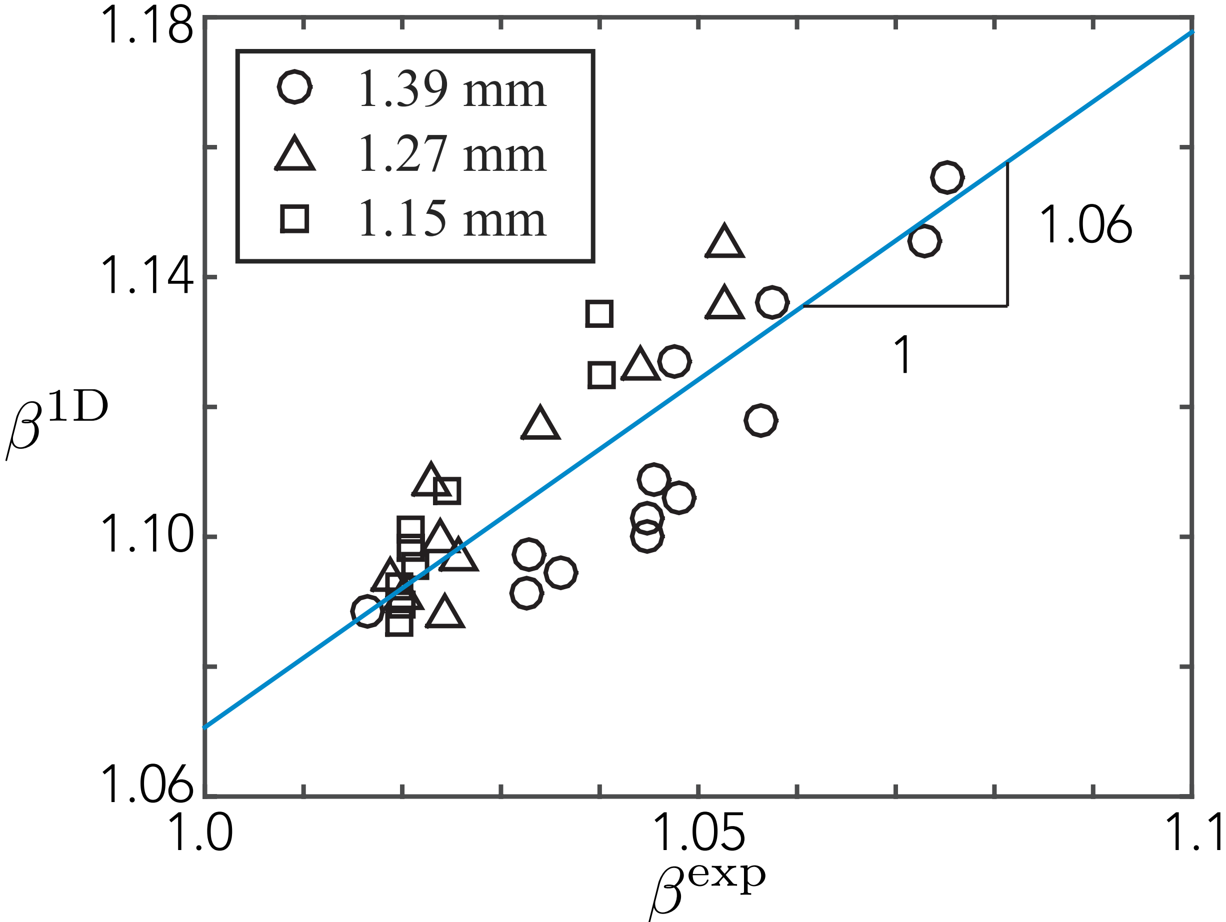
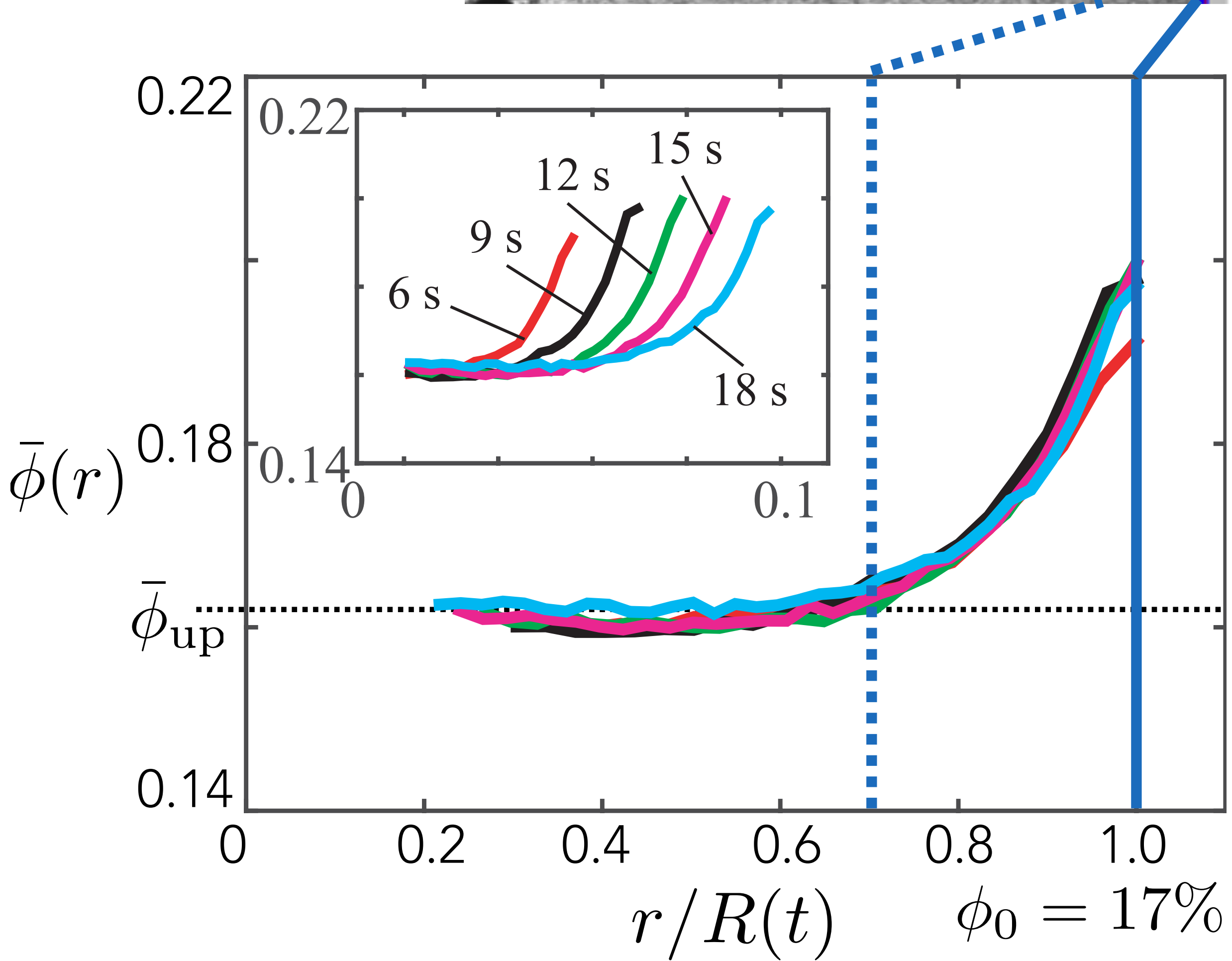
measure experimentally







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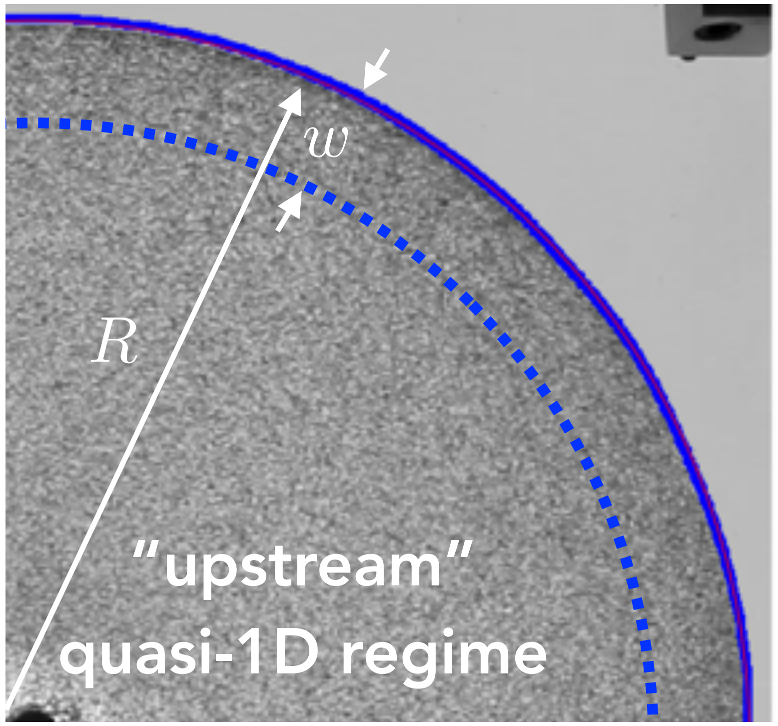
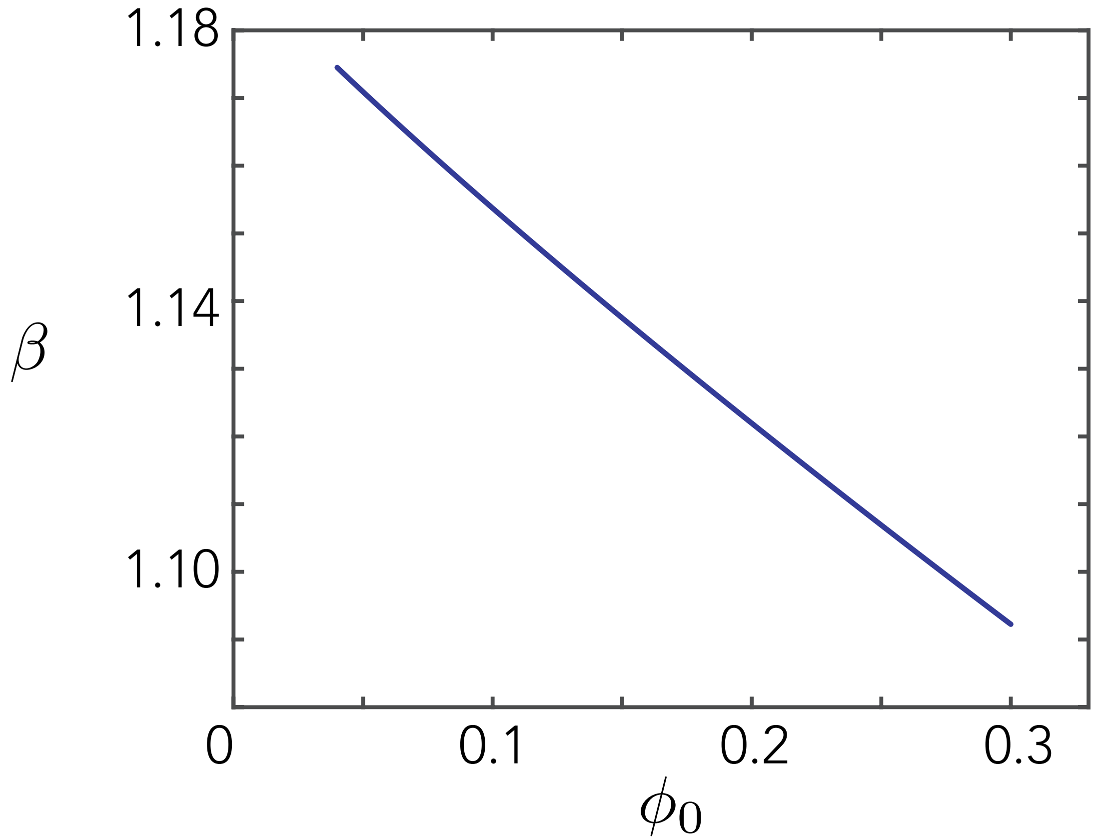




Rate of particle accumulation
decreases with increasing ϕ_0.

↓

Rate of particle accumulation
does not directly determine
likelihood of fingering.

MODEL VALIDATION:

from **accumulation** to **fingering**

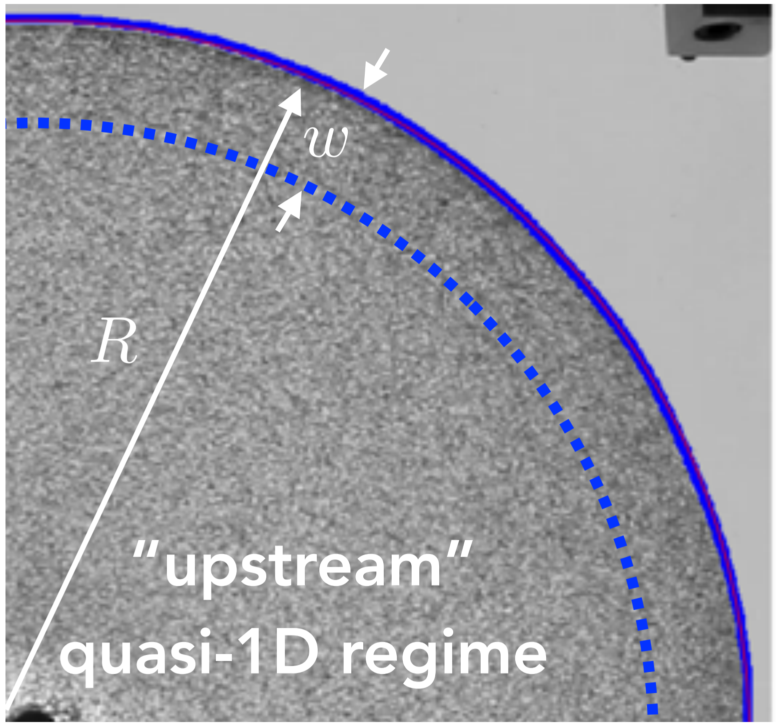
mass conservation in upstream regime

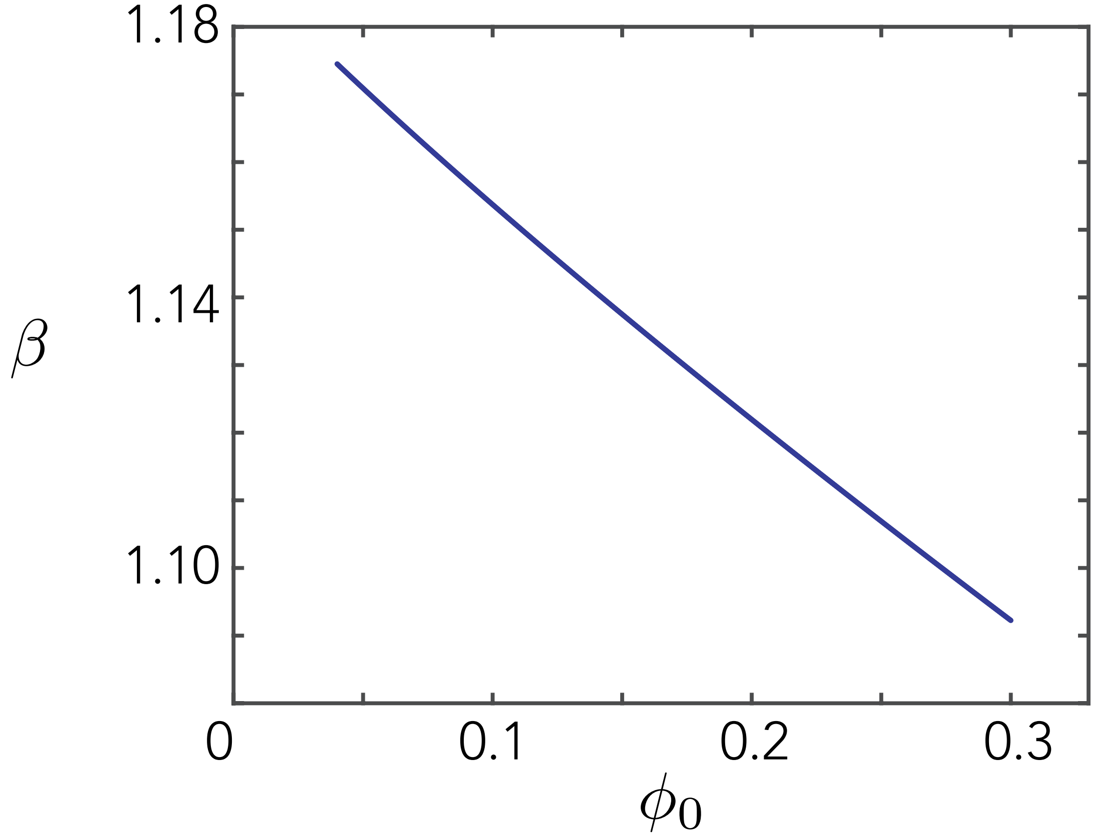
$$\phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v_r^{\mathrm{p}}(z)\phi(z)dz$$

$$\beta = \phi_0/\bar{\phi}_{\mathrm{up}}$$

$$V^{*\mathrm{p}}_{\mathrm{accum}} = \frac{V^{\mathrm{p}} - V^{\mathrm{p}}_{\mathrm{up}}}{V^{\mathrm{p}}} \sim \phi_0\left(1 - \frac{1}{\beta}\right)$$

where $V^{\mathrm{p}} = \phi_0 \pi R^2 h$ & $V^{\mathrm{p}}_{\mathrm{up}} = \frac{\phi_0}{\beta}\pi(R-w)^2 h$





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from **accumulation** to **fingering**

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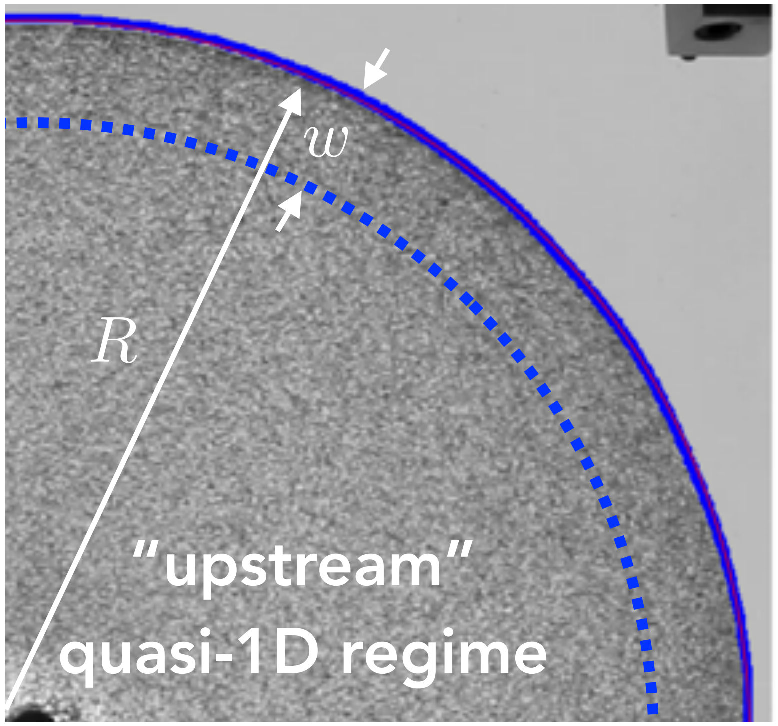
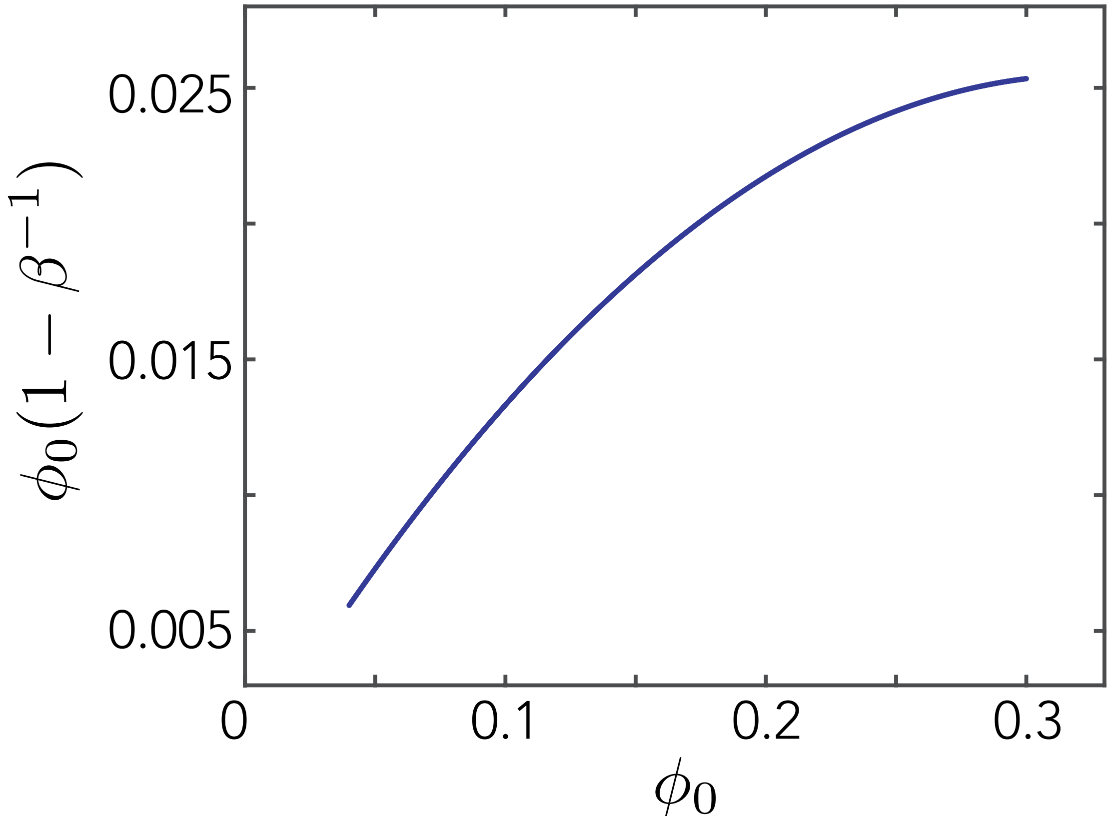




More particles collect near the interface with increasing ϕ_0.

↓

More likely to finger with increasing ϕ_0.

Xu, Kim & Lee [in review]

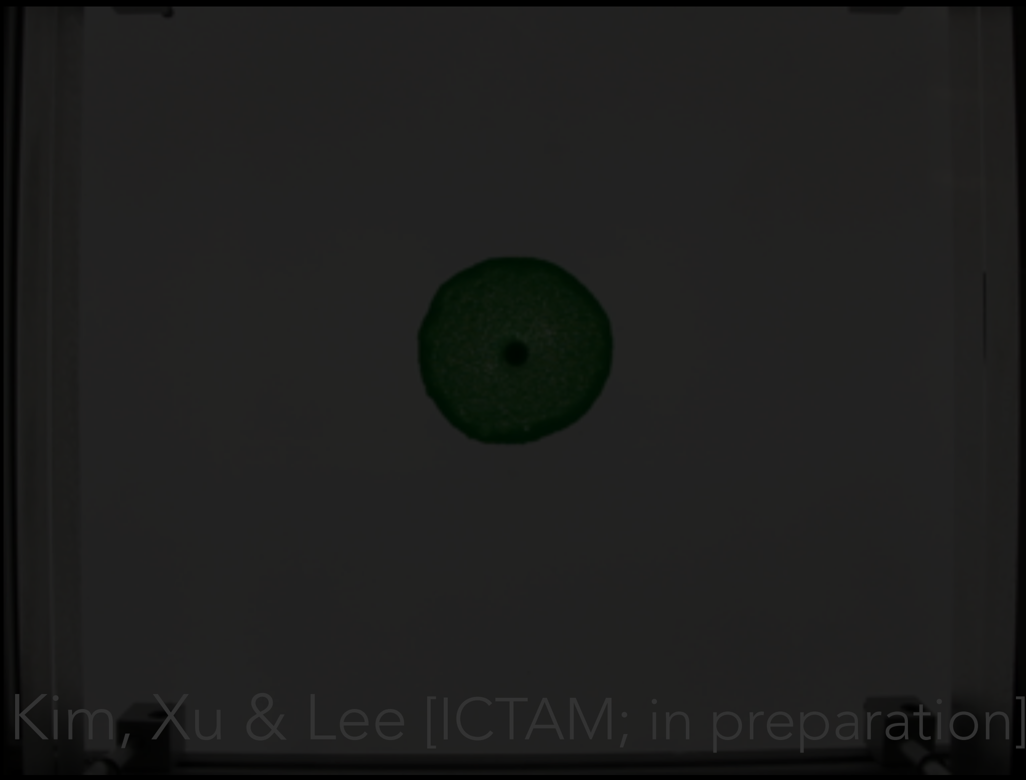
particle-induced fingering

Summary: characterization of particle-induced fingering; continuum model formulation

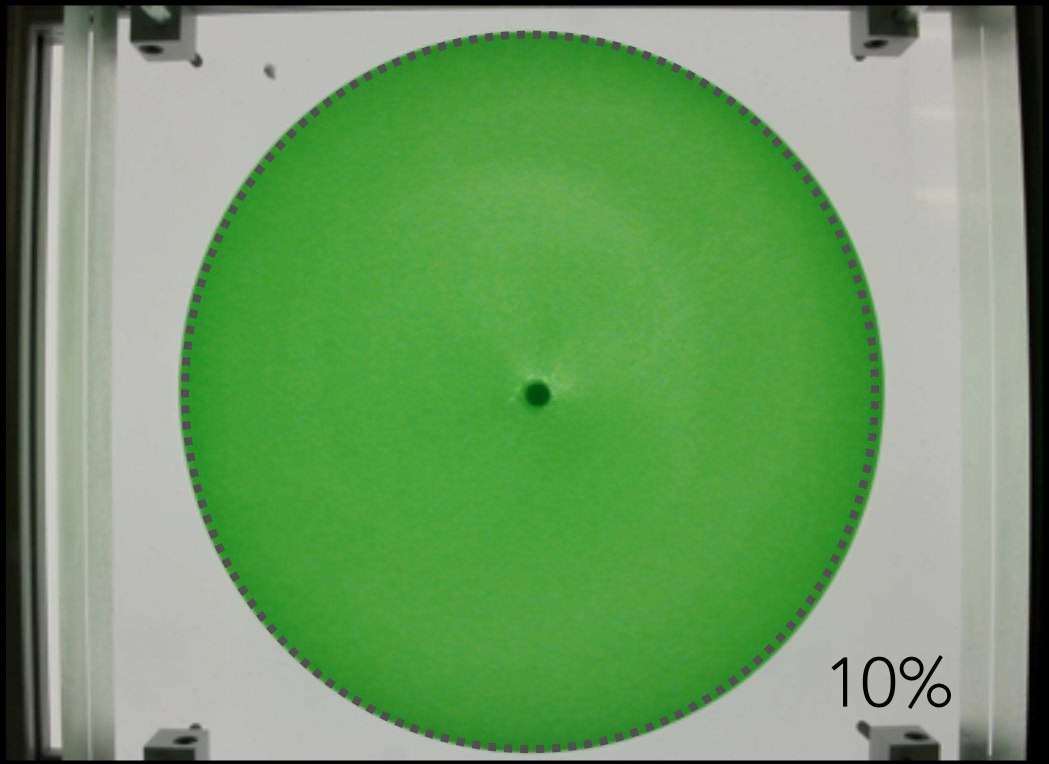
On-going: prediction of the onset of fingering based on stability analysis

Big picture: coupled dynamics of particles & interface dynamics

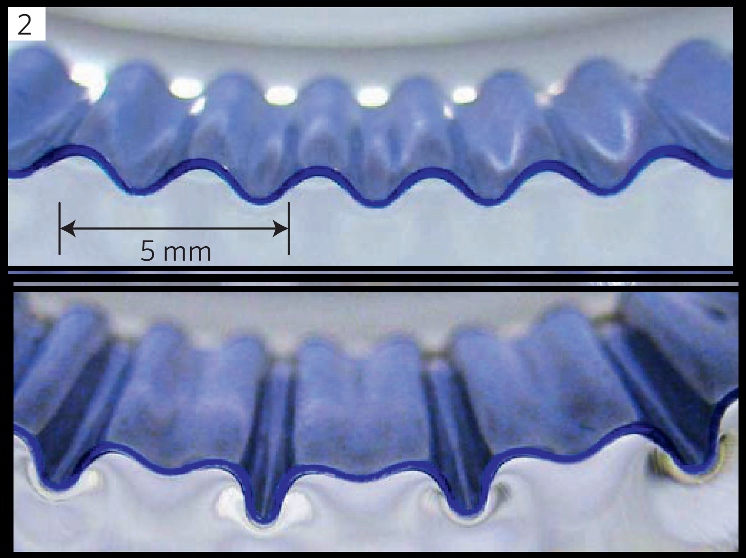
formation & breakage of **particle band**


Kim, Xu & Lee [ICTAM; in preparation]

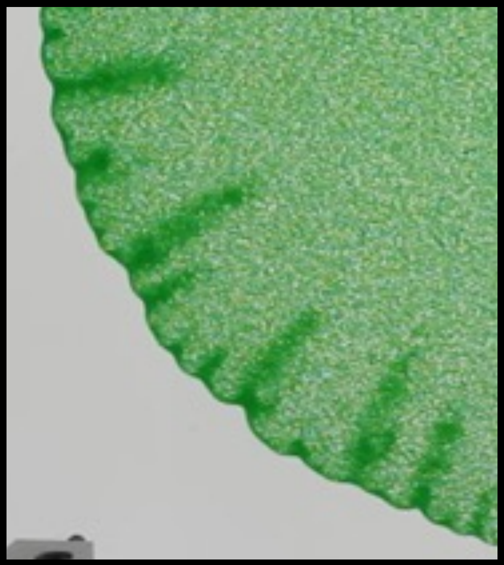
effect of particles on **draining**



pattern formation with analogy to elastic instability



Brau et al 2011



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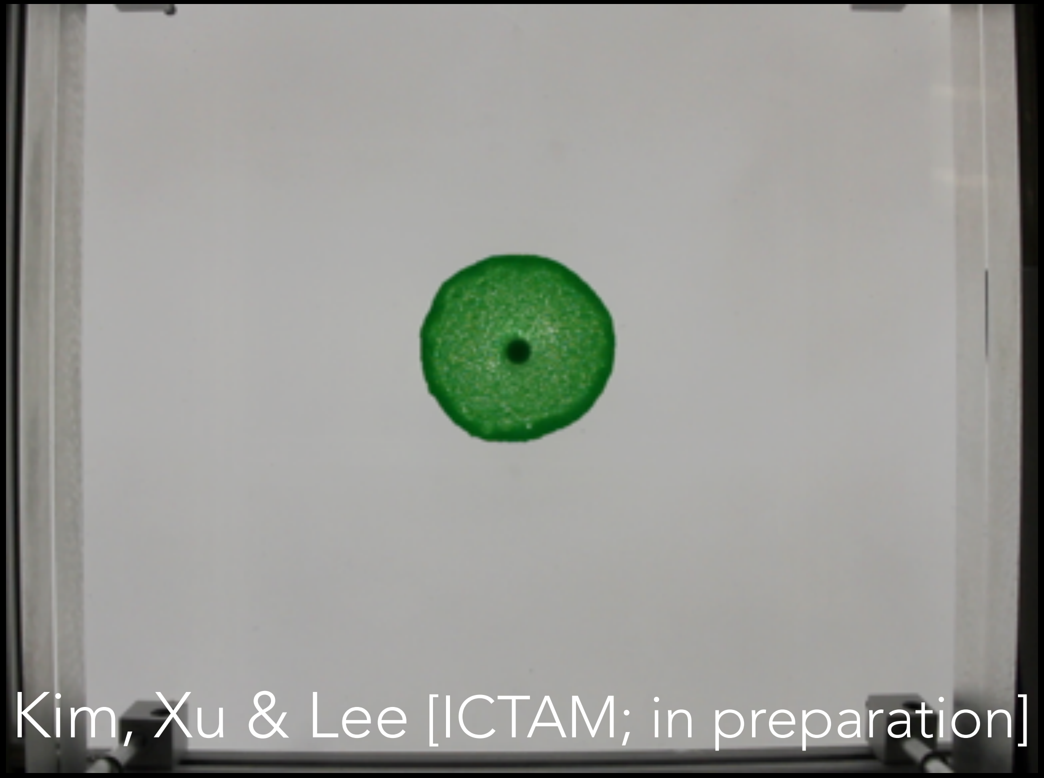

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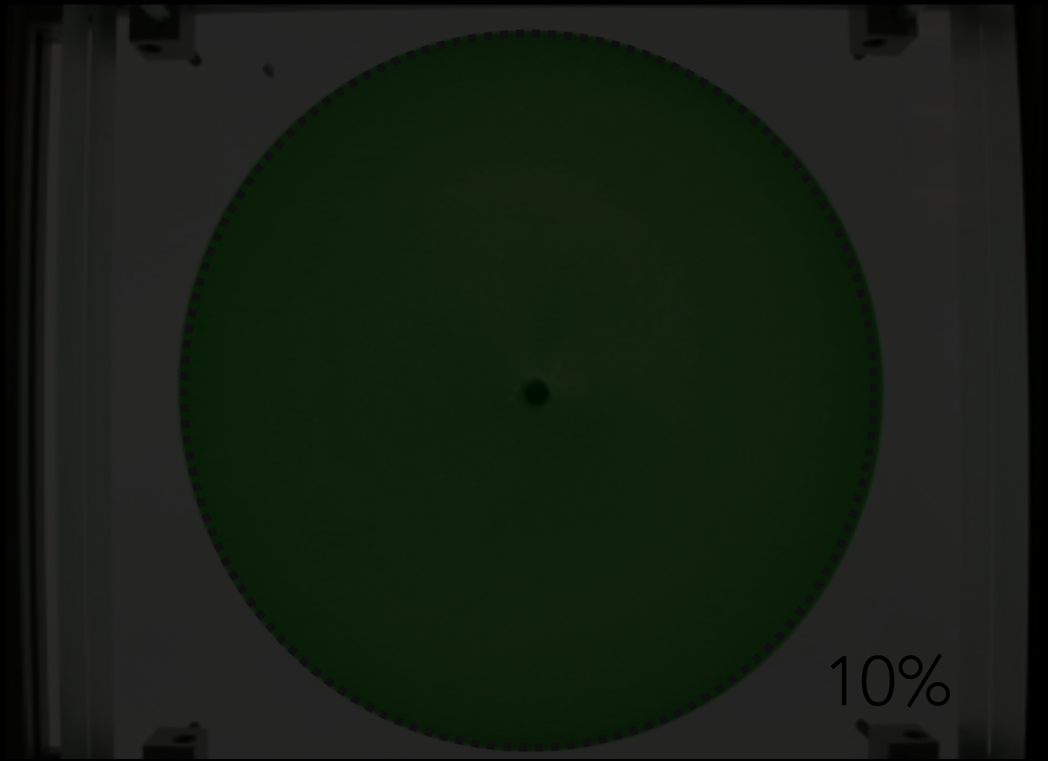

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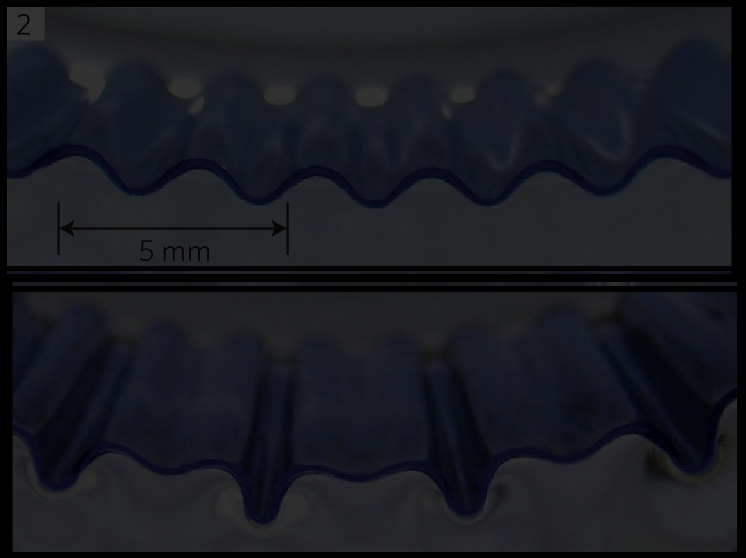
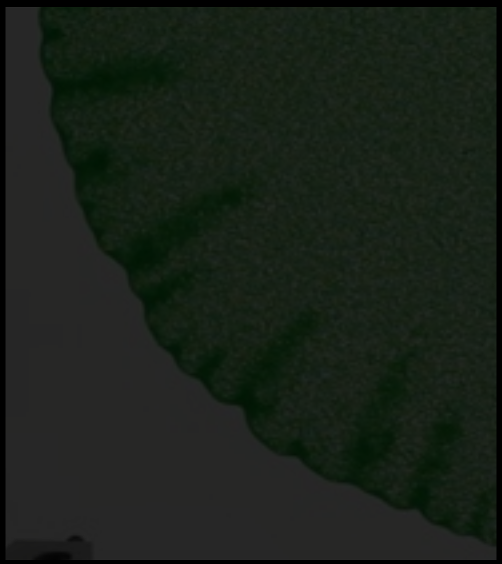


Brau et al 2011



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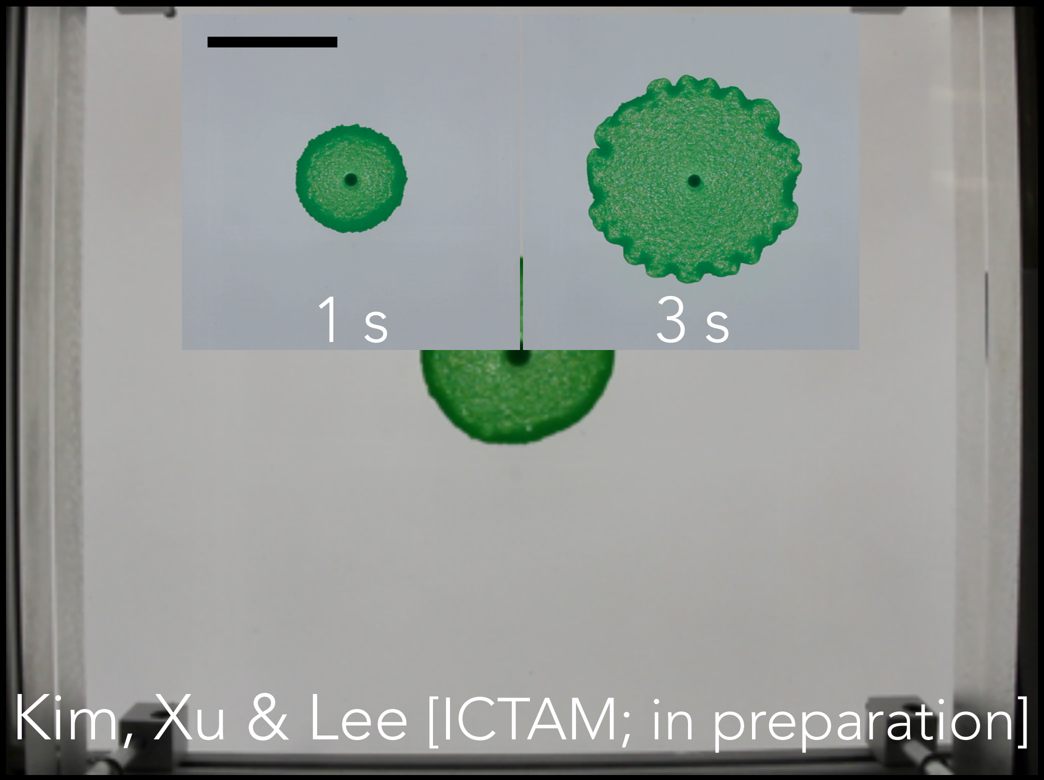

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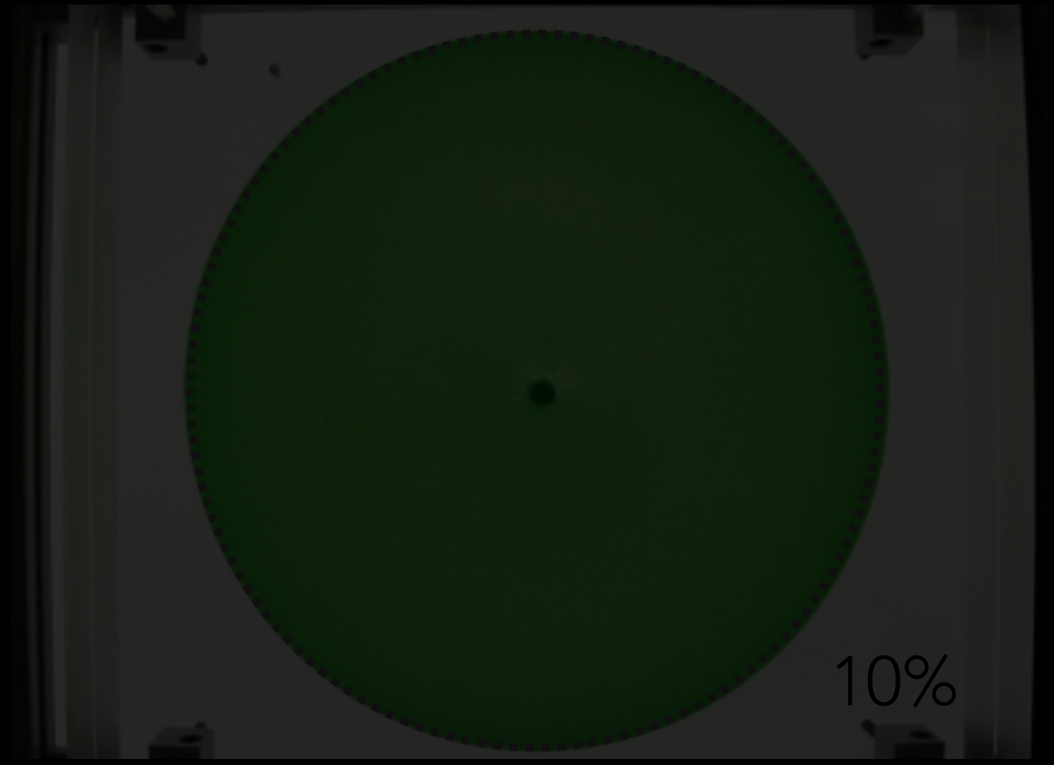

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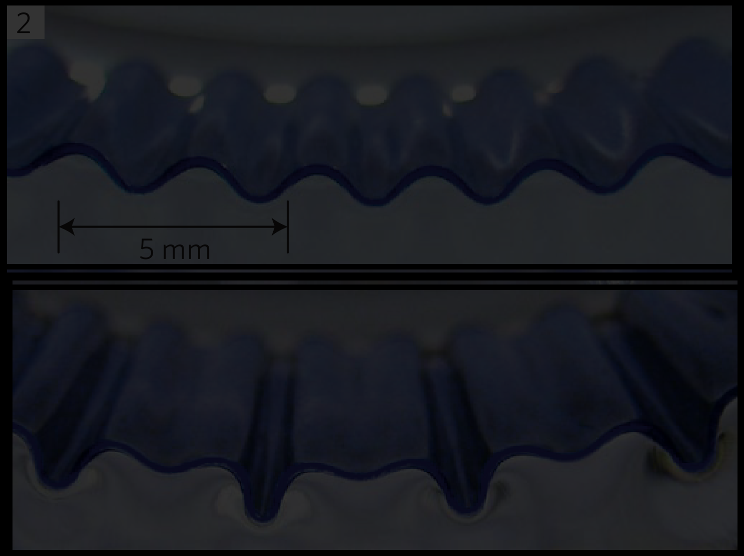
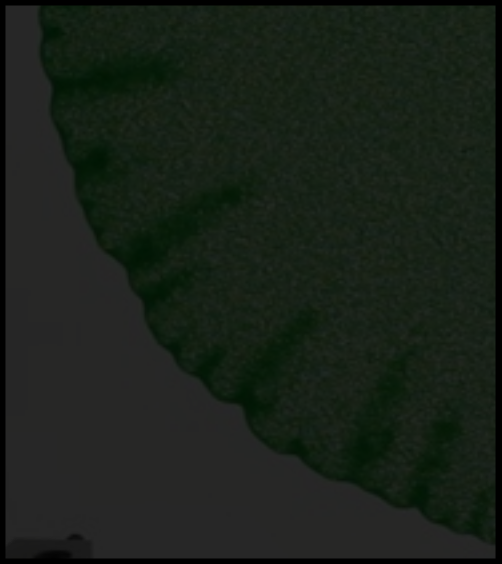
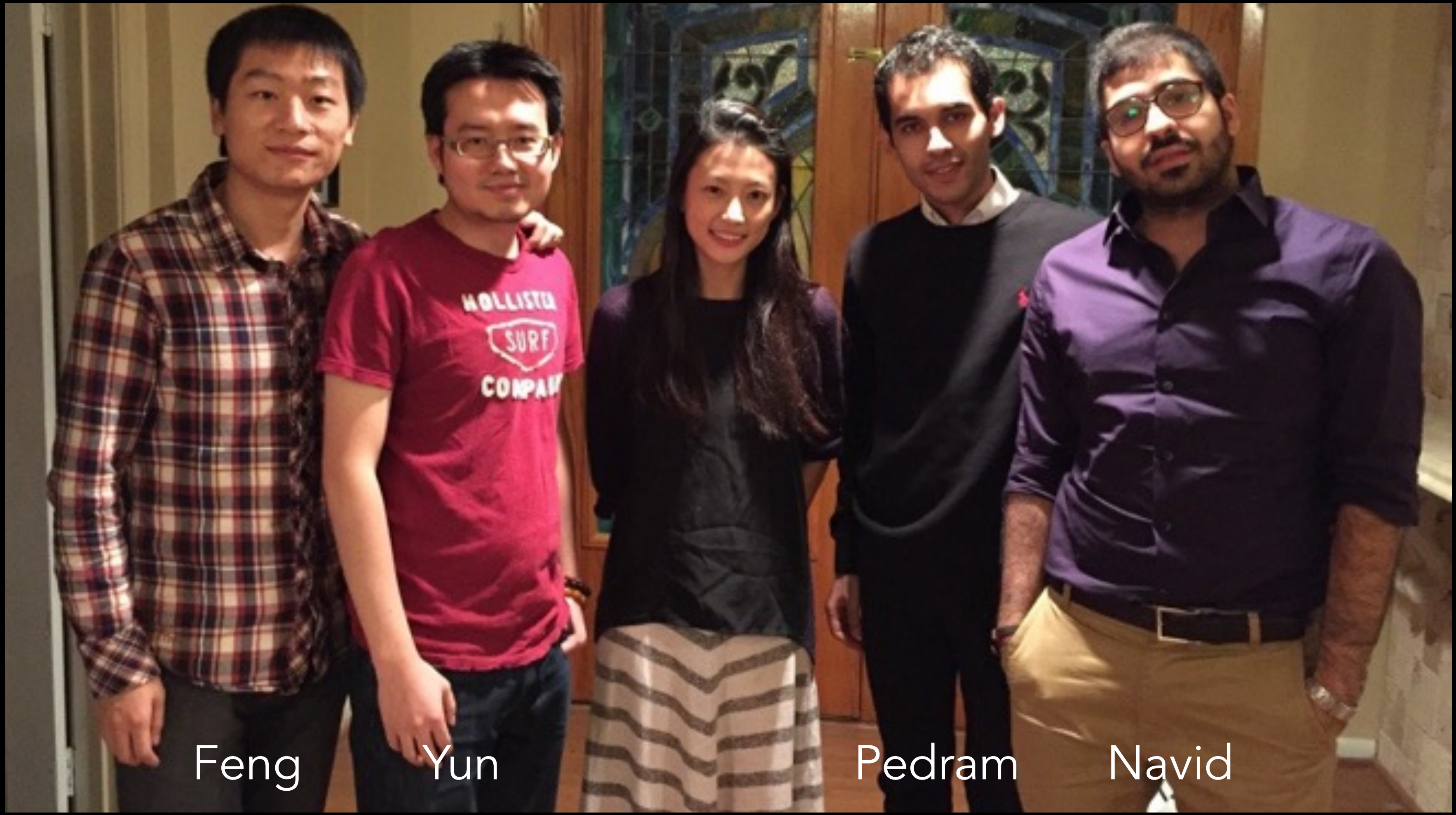


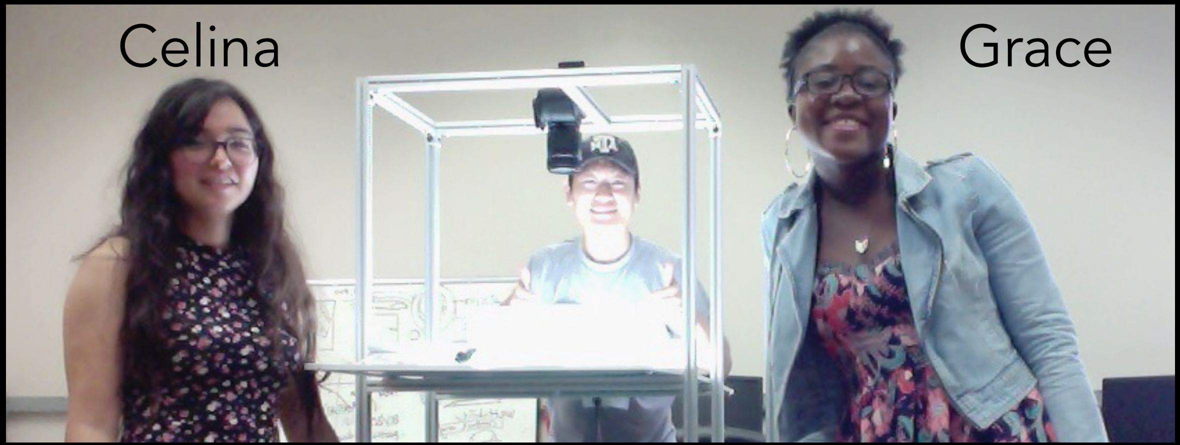


Brau et al 2011

DrIPs

DROPS, **INTERFACES** & **P**ARTICULATE **S**YSTEMS





Thank you.