Particulate Gravity Currents with Resuspension

Jim McElwaine

Professor of Geohazards
Department of
Earth Sciences
Durham University

Acknowledgments

THE ROYAL SOCIETY

VEÐURSTOFA ÍSLANDS

NORGES GEOTEKNISKE INSTITUTT
NGI
NORWEGIAN GEOTECHNICAL INSTITUTE

EPSRC
Engineering and Physical Sciences Research Council

Betty Sovilla
Kouichi Nishimura
Dieter Issler
Eckart Meiburg
Nathalie Vriend
Liz Bowman

Barbara Turnbull
Christophe Ancey
Takahiro Ogura
Shane Byrne
Jan-Thomas Fischer

Plan of Talk

- Transition Experiments
- Direct Numerical Simulations
- Dynamic Models

Powder Avalanche on K2

Pierre Beghin

Head of Powder Snow Avalanche

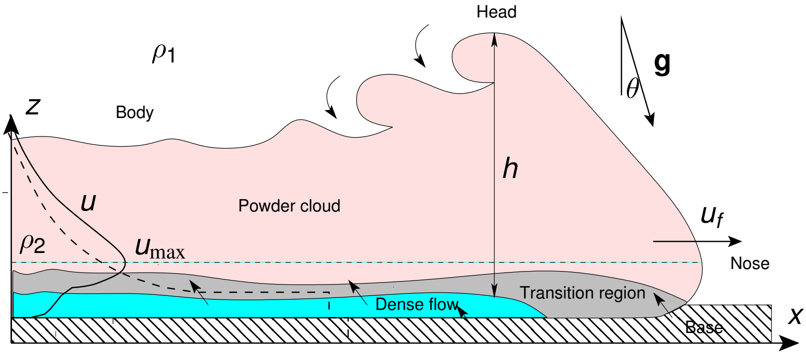
Cemagref

Slab Avalanche Fracture Line

Test Chute in Davos

film

Schematic of Avalanche, Turbidity Current, Pyroclastic Flow



Aim

Understand formation of Suspension Currents

Use steep slopes to give a low Richardson number for large density difference

- Transition to suspension
- Limited entrainment
- Steady flows
- Understand air interaction
- Comparison of field observations with experiments

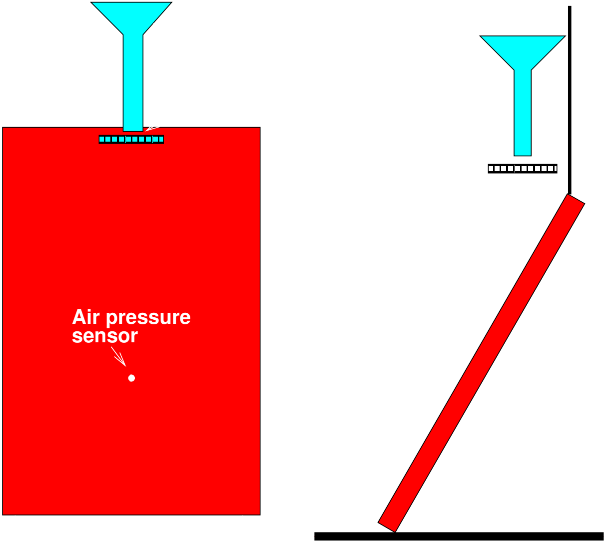
Similarity Criteria

Experiment	Materials	Re_p	Ri	St	$\frac{\Delta\rho}{\rho_a}$	Re
Powder snow avalanches	**Snow-air**	**3000**	**1**	**0.02**	**10**	**10^9**
Ancey (2004)	sawdust water	50	1.7	0.006	0.05	10^4
Bozhinskiy (1998)	aluminium air	0.1	20	0.03	1	10^3
Beghin (1981) Beghin (1983) Beghin (1983)	Brine-water/ Sand-water suspension	–	5	–	0.02	10^4
Nishimura (1998) McElwaine (2001)	Ping-pong balls air	2×10^4	2	10	50	10^7
Hampton (1972)	Kaolinite and water slurry	–	< 0.5	–	0.1	–
Hermann (1987)	Polystyrene powder water	1.5	0.1	10^{-4}	0.002	10^4
Hopfinger (1977) Tochon-Danguy (1974)	Brine water	–	2	–	0.01	10^3
Present Study	**Snow-air**	**150**	**1**	**10**	**10**	**10^5**
Present Study	**Polystyrene-air**	**150**	**2**	**1**	**5**	**10^4**

Similarity Summary

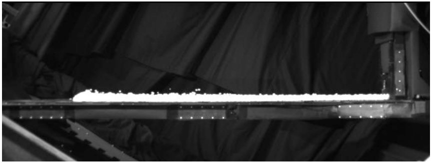
- Exact similarity of Ri and $\Delta\rho/\rho_a$
- Re and Re_p not matched but qualitatively similar
- St is different $\Rightarrow$ sedimentation is important
- Slope angles are different.
  Appears unimportant at high Re

Experiments

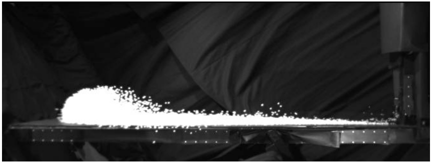


front view *side view*

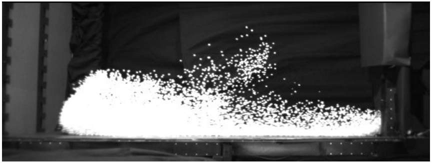
Side View 8 Litre Avalanches



31.5° slope



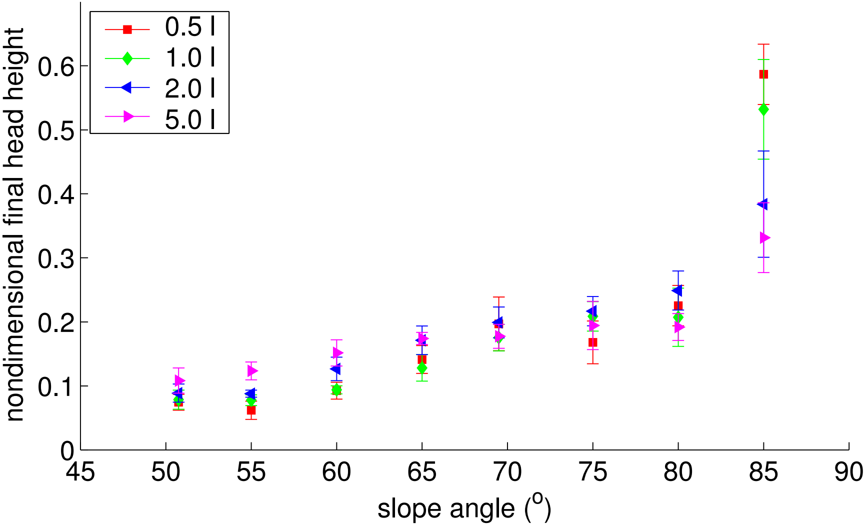
58.5° slope



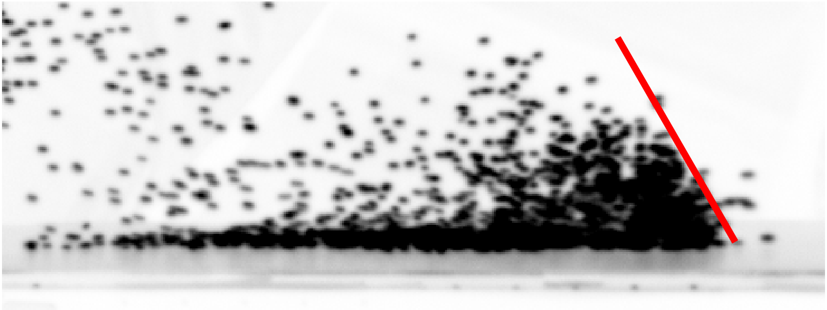
91.0° slope

100 ml side 100 ml front
8000 ml side 8000 ml front

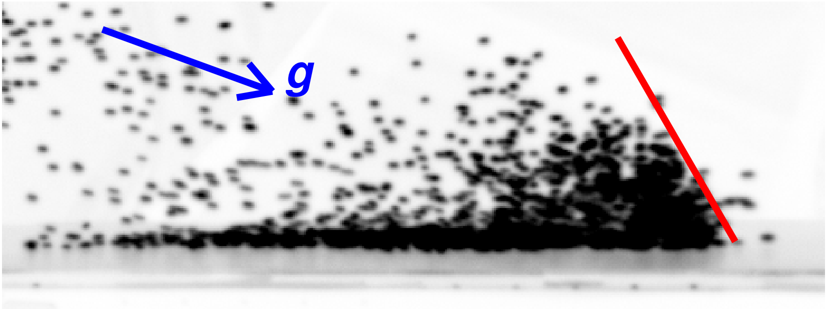
Non-dimensional Height — $\tilde{h} = \frac{h}{V^{1/3}}$



Polystyrene balls

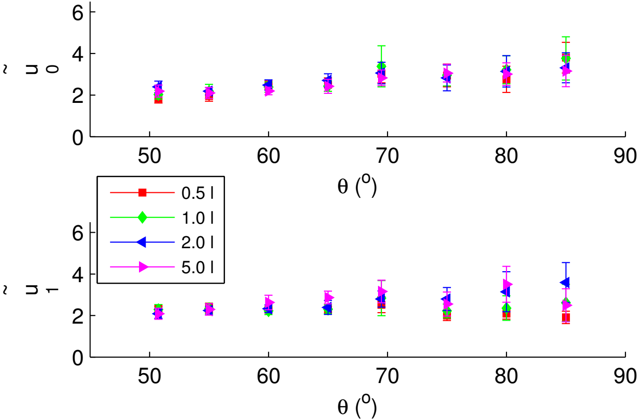


Polystyrene balls on 70° surface

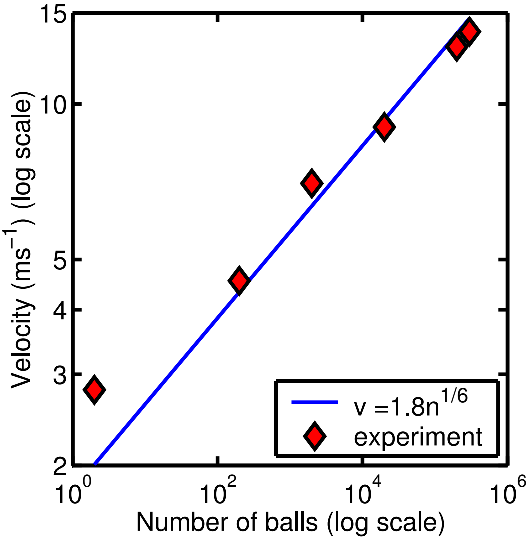


Ping-Pong Avalanches

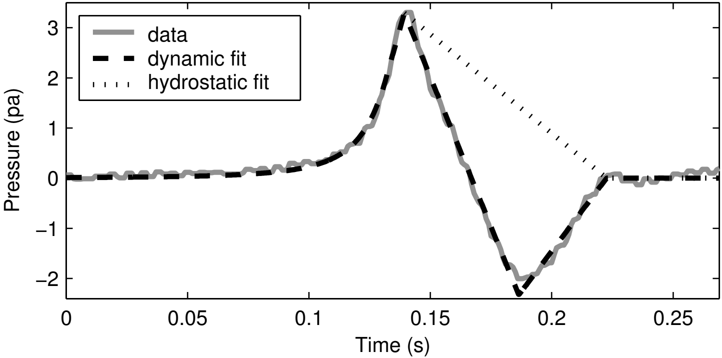
Non-Dimensional Velocity — $\tilde{u} = \frac{u}{V^{\frac{1}{6}} g^{\frac{1}{2}}}$



Front Velocities at the K-Point

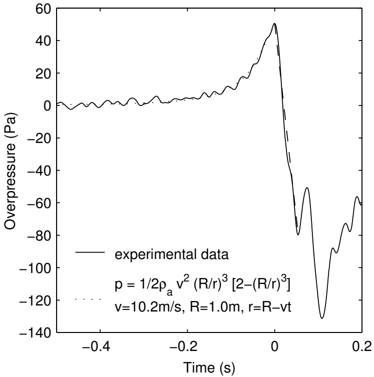


Pressure Theory Comparison

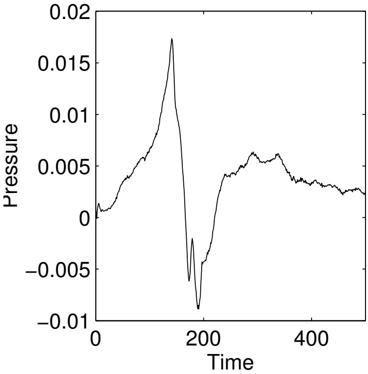


Air pressure in a small powder cloud

DNS simulations and Ping-Pong experiments



Air pressure data from a ping-pong ball avalanches and comparison with theory



Air pressure data from a direct numerical simulation of a gravity current

Front Instability, 51 polystyrene, 51° slope

Conclusions

- Transition to suspension can be achieved in the laboratory
- Can deduce avalanche length, height, speed, and front angle from pressure data
- Good agreement between theory, experiments and field observations
- Pressure measurements can distinguish suspended from dense flows
- Coherent internal velocities can be twice front velocity
  Take care estimating forces !

Direct Numerical Simulations

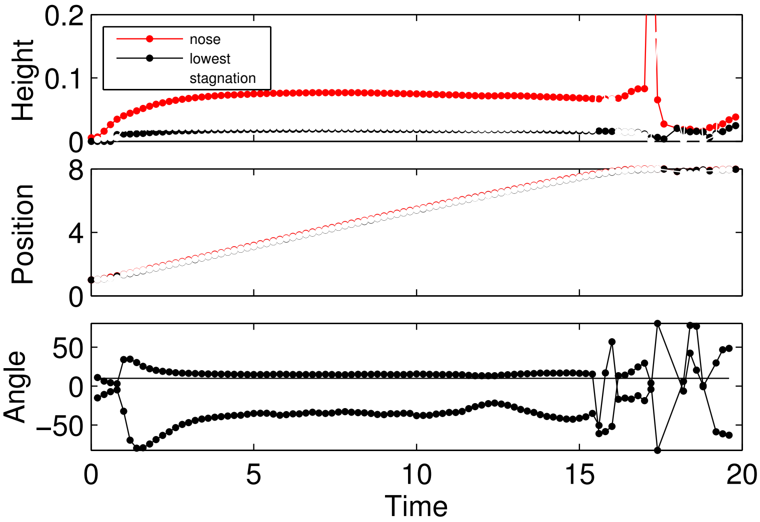
- Meiburg Code
  2d spectral with compact finite differences
- Diablo from John taylor
  3d spectral with low order finite differences
- Simulation region 8×1
- Release area 2×0.5
- Slope angles 0–90°
- Boussinesq and non-Boussinesq

Test hypothesis:

stagnation point is lowest point as Re$\to \infty$

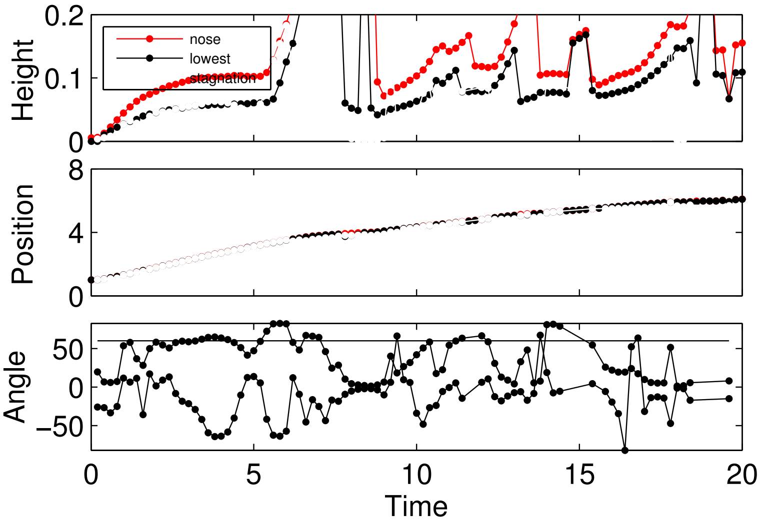
Time evolution, Re=32,000, Slope=10°

film front

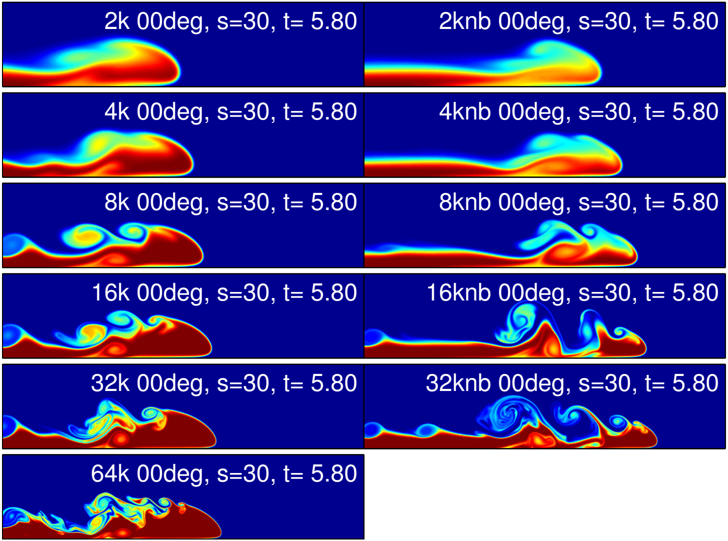


Time evolution, Re=32,000, Slope=60°

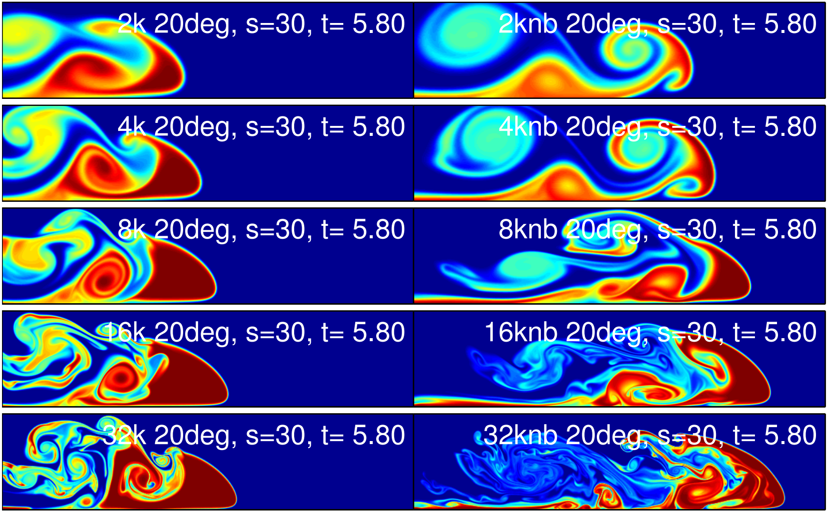
film front



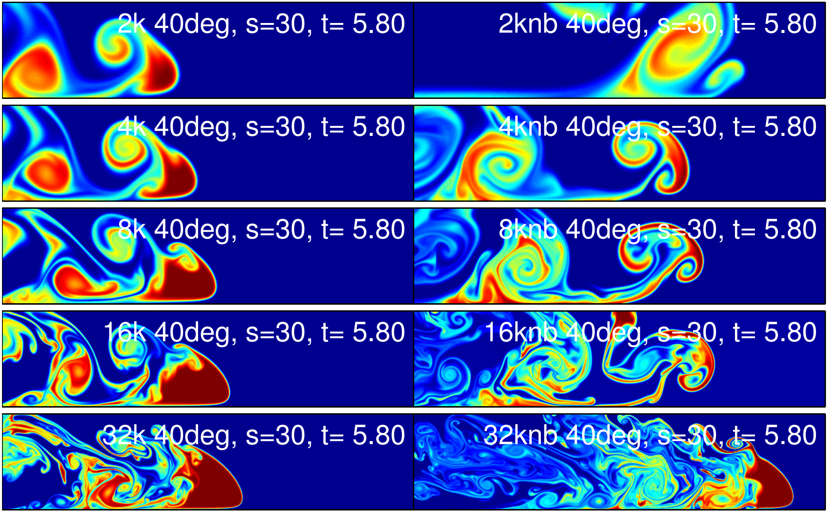
Re Comparison at slope 0°



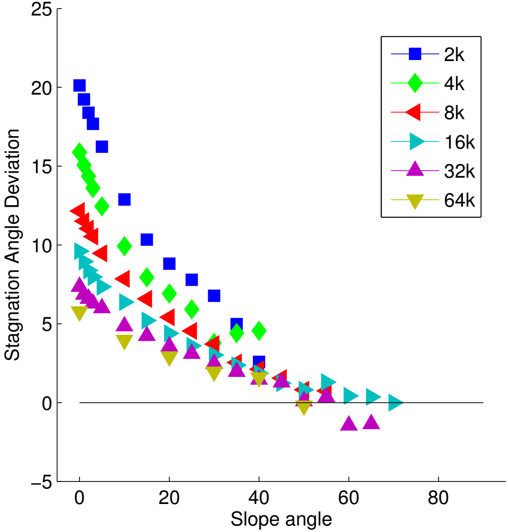
Re Comparison at slope 20°



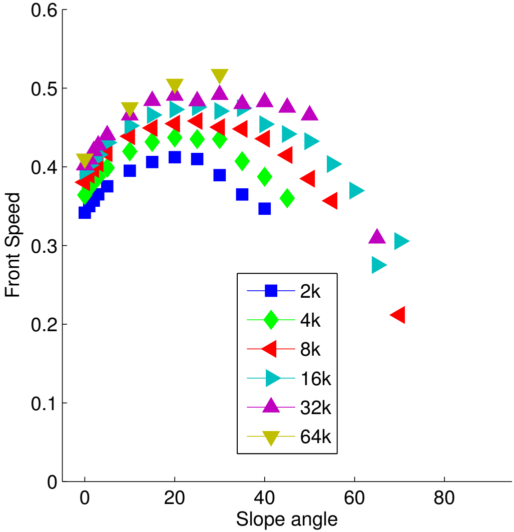
Re Comparison at slope 40°



Stagnation Point Angle



Front Speed — High angles evaporate



Hindered Sedimentation, with Particle Pressure

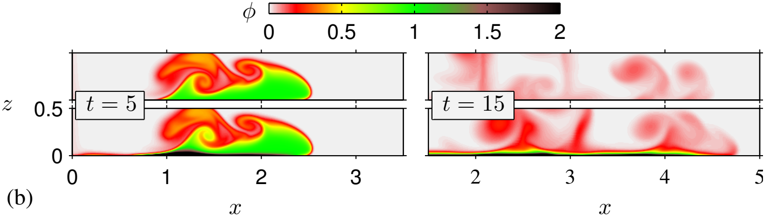
$$\frac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{q} = 0,$$

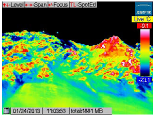
where particle flux

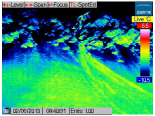
$$\mathbf{q} = \mathbf{u}\phi + \mathbf{u_s}\phi(1 - \alpha\phi) - \frac{D\nabla\phi}{1 - \beta\phi}$$

Sedimenting Boundary condition $\mathbf{n} \cdot \nabla(\mathbf{n} \cdot \mathbf{q}) = 0$ or $\nabla\phi = 0$
Resuspending boundary condition $\mathbf{n} \cdot \mathbf{q} = 0$
Use mixed compact finite differences and finite volume schemes to exactly conserve mass.

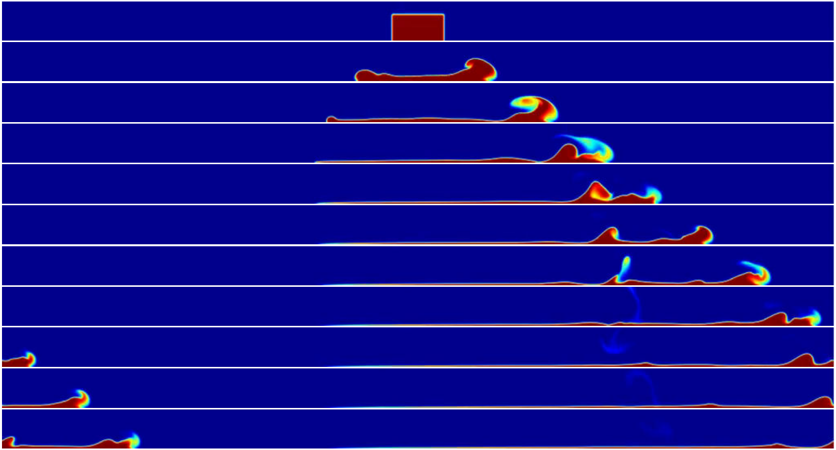
Sedimentation vs Resuspension



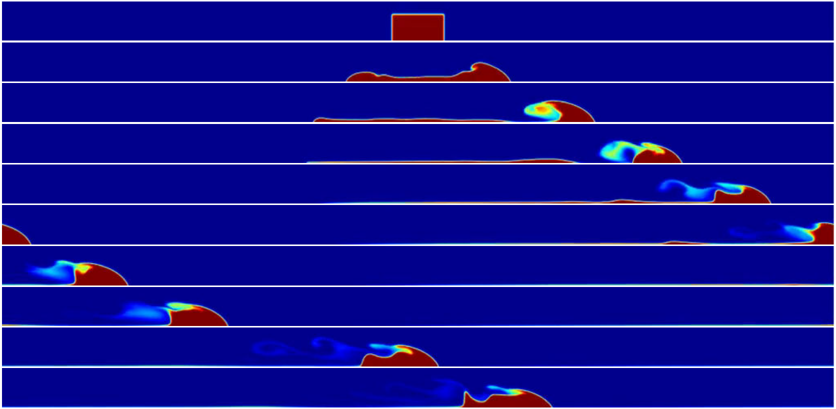




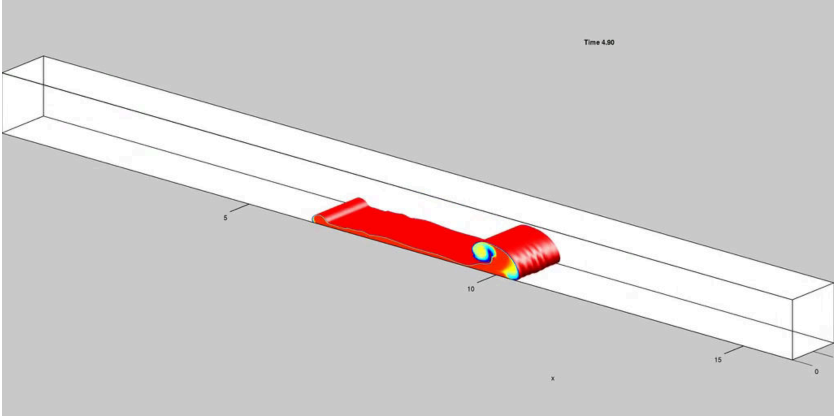
2D with no-slip and hindered sedimentation, $\mathrm{Re} = 2\,000$



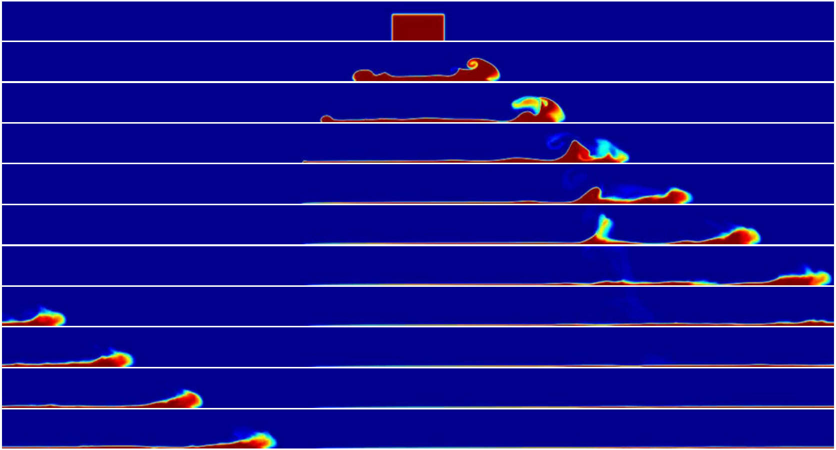
2D with slip and hindered sedimentation, $\mathrm{Re} = 2\,000$



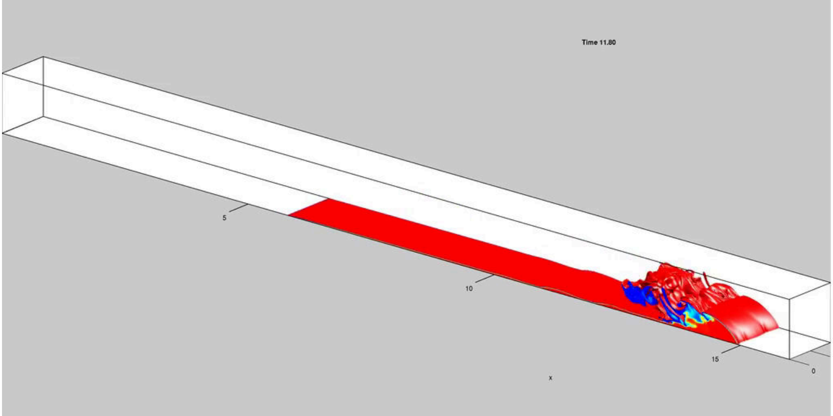
3D with no-slip and hindered sedimentation, $\mathrm{Re} = 4\,000$



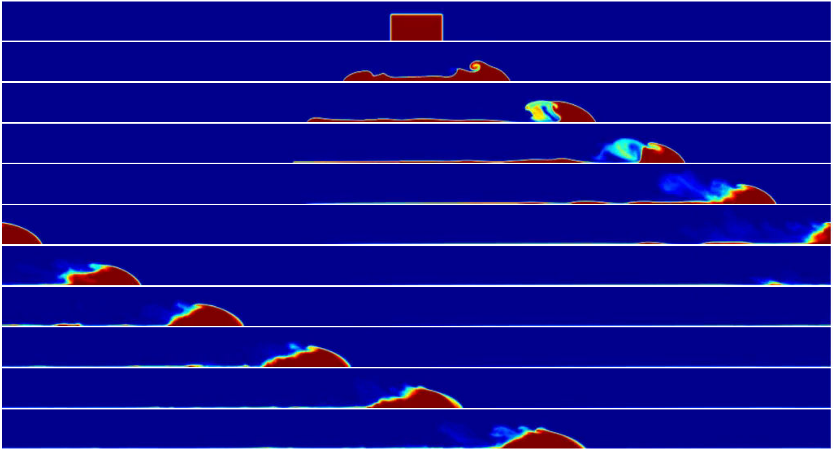
3D with no-slip and hindered sedimentation, $\mathrm{Re} = 4\,000$



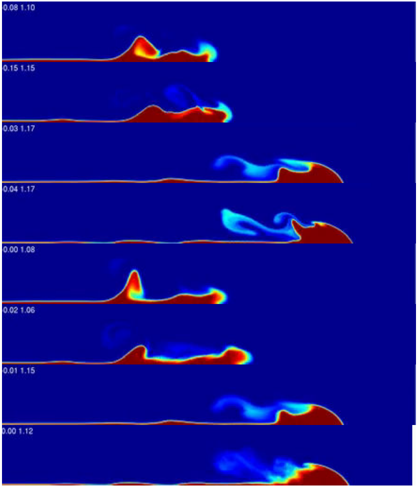
3D with slip and hindered sedimentation, $\mathrm{Re} = 4\,000$



3D with slip and hindered sedimentation, $\mathrm{Re} = 4\,000$



Comparison



2D	Re=2 000	no-slip
2D	Re=4 000	no-slip
2D	Re=2 000	slip
2D	Re=4 000	slip
3D	Re=2 000	no-slip
3D	Re=4 000	no-slip
3D	Re=2 000	slip
3D	Re=4 000	slip

Kulikovskiy–Sveshnikova–Beghin (KSB)

Three conservation equations

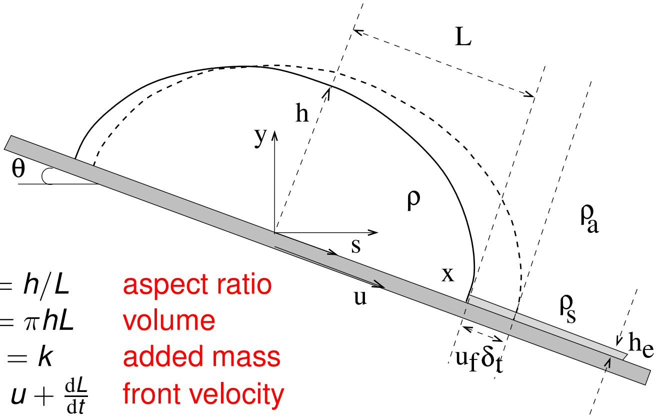
volume $$\frac{dV}{dt} = q_s + q_a$$

buoyancy $$\frac{dB}{dt} = (\rho_s - \rho_a) q_s$$

momentum $$\frac{d}{dt}\left\{\left[B + (1+\chi)\,V\rho_a\right]u\right\} = Bg\sin\theta$$

ρ_s	snow density	ρ_a	air density	χ	added mass
g	gravity	θ	slope angle		
q_s	snow flux	q_a	air flux		

Geometric Closure



$k = h/L$	aspect ratio
$V = \pi hL$	volume
$\chi = k$	added mass
$u_f = u + \frac{dL}{dt}$	front velocity

Flux Closures

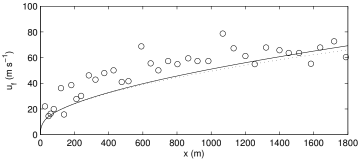
$q_s = u_f h_e - \beta u \sqrt{V}$	snow entrainment/detrainment
$q_a = (\alpha u - u_s)\sqrt{V}$	air entrainment/detrainment

$$\alpha(\mathrm{Ri}) = \begin{cases} \mathrm{e}^{-\lambda \mathrm{Ri}^2} & \mathrm{Ri} \le 1 \\ \frac{\mathrm{e}^{-\lambda}}{\mathrm{Ri}} & \mathrm{Ri} > 1 \end{cases}$$

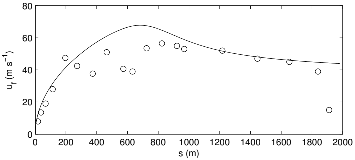
$$\mathrm{Ri} = \frac{\rho - \rho_a}{\rho_a} \frac{g h \cos\theta}{u^2}$$

h_e	erodible snow depth	β	mass loss coefficient
u_s	sedimentation velocity		

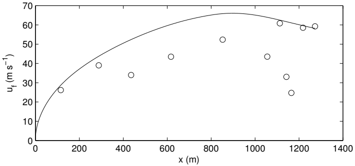
Comparison with Velocity Data



Avalanche no. 200

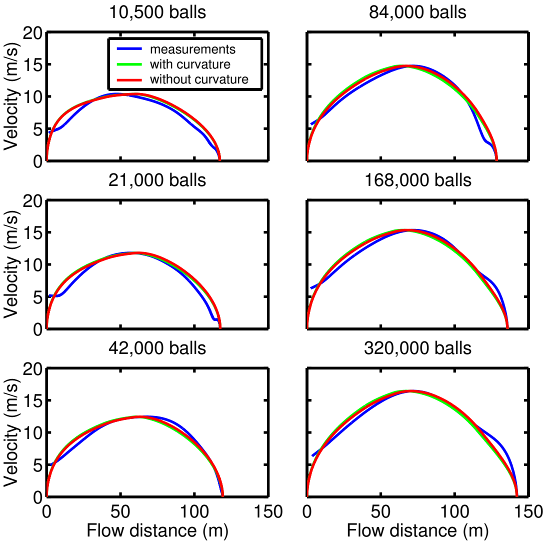


Avalanche no. 509

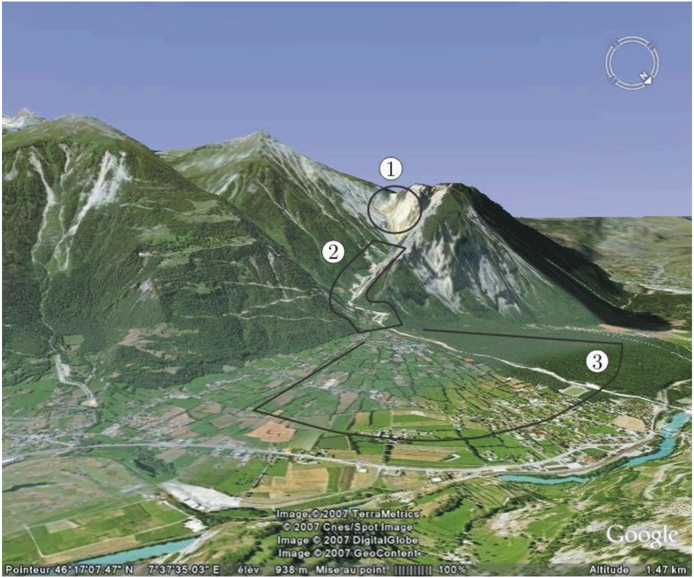


Avalanche no. 628

Comparison with Ping-Pong Avalanches

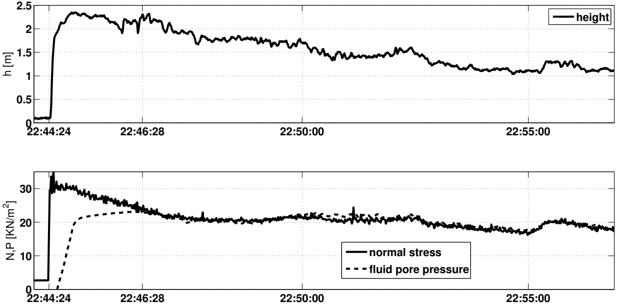


Illgraben Situation

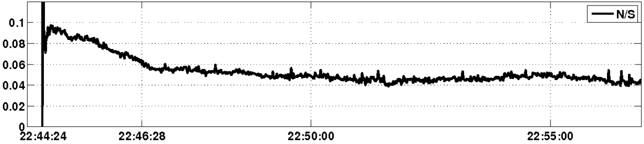


Illgraben Bridge

Height and Stress Data



Ratio of shear force to normal force

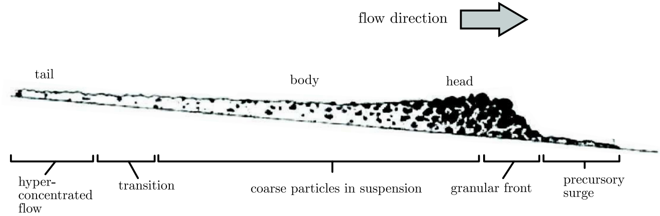


Motivation – Zeroth order shallow water

$$\frac{\partial u}{\partial t} = g \sin\theta - \mu g \cos\theta$$

- Chezy $\mu(\mathrm{Fr}, h/d, \theta) \propto \mathrm{Fr}^2$
- Viscous $\mu(\mathrm{Fr}, h/d, \theta) \propto \mathrm{Fr}$
- μ should also depend on solids concentration at the bottom

$$\mathrm{Fr} = \frac{u}{\sqrt{gh\cos\theta}}$$



Two phase debris flow model

After much algebra . . .
vertical equation of mass conservation is

$$\partial_t\alpha = \partial_y\left[V\alpha(1-\alpha) + D\partial_y\alpha\right] = \partial_y D\left[\mathrm{Pe}\,\alpha(1-\alpha) + \partial_y\alpha\right]$$

α density of solids relative to max
V sedimentation velocity
D diffusion
$\mathrm{Pe} = V/D$ Peclet number

This is a diffusion equation with hindered settling and the same as Gray's segregation theory

Reduction to slow manifold

Define the vertical centre of mass for the solids fraction

$$h_p(t) = \int_0^H y\alpha(t, y)\,\mathrm{d}y.$$

Then use previous result and more algebra to get

$$\frac{\mathrm{d}h_p(t)}{\mathrm{d}t} = \frac{h_p^* - h_p(t)}{T}\left[1 + \epsilon(h_p^* - h_p(t) + \cdots\right].$$

For resuspension dominated regime $h_p^* = \frac{H}{2}$ and mixture becomes well mixed

For sedimenation dominated regime $h_p^* = \frac{H_p}{2}$ where H_p is total height of particles at maximum packing fraction

Excess pore pressure is

$$\frac{p}{\rho_f g_y} = \frac{c_1(2h_p - H_p) + c_2(h_p - h_p^*)}{H - H_p}$$

Complete Set of Depth Averaged Equations

$$\begin{aligned}
\partial_t M + \partial_x (MU) &= 0 \\
\partial_t (cM) + \partial_x [cMU(1 + a_1 m_1)] &= 0 \\
\partial_t (UM) + \partial_x \left[MU^2(a_2 + a_3 m_1^2) + g_y hM \right] &= Mg_x - \mu M g_y \\
\partial_t m_1 + \partial_x [m_1 U + (a_4 + a_5 m_1^2) U] &= \frac{m_1^* - m_1}{T_m} [1 + \cdots]
\end{aligned}$$

m_1 $= 2h_p/H - 1$ dimensionless deviation from mixed

M total mass hold up

c relative concentration of particles

h centre of mass

Conclusions

- Resuspension very important in many geophysical flows
- DNS expensive but can reproduce two layer structure
- Integral point mass models work well for some cases
- Depth integrated equations with vertical resdistribution a good compromise
- Easy to construct empirical models
- Also works for *n* component mixtures and segregation

Publications

- N.A. Konopliv and S.G. Llewellyn-Smith and **J.N. McElwaine** and E. Meiburg. Modeling gravity currents without an energy closure. *J. Fluid Mech.*, **789**:806–829, 2016.
- B. Sovilla, **J.N. McElwaine**, and M.Y. Louge. The structure of powder snow avalanches. *Comptes Rendus Physique*, **16**(1):97–104, 2015.
- B. Turnbull, E.T. Bowman, and **J.N. McElwaine**. Debris flows: Experiments and modelling. *Comptes Rendus Physique*, **16**(1):86–96, 2015.
- M. Ash, P.V. Brennan, C.J. Keylock, N.M. Vriend, **J.N. McElwaine**, and B. Sovilla. Two-dimensional radar imaging of flowing avalanches. *Cold Regions Science and Technology*, **102**:41–51, 2014.
- A.J. Hafiz, **J.N. McElwaine**, and C.P. Caulfield. The instantaneous froude number and depth of unsteady gravity currents. *J. Hydraulic Res.*, 51:432–445, 2013.
- J. Kowalski and **J.N. McElwaine**. Shallow two-component gravity-driven flows with vertical variation. *J. Fluid Mech.*, 714:434–462, 2013.
- N. M. Vriend, **J.N. McElwaine**, B. Sovilla, C. J. Keylock, M. Ash, and P. V. Brennan. High resolution radar measurements of snow avalanches. *Geo. Res. Let.*, 40(4):727–731, 2013.
- B. Turnbull and **J.N. McElwaine**, 2008. Experiments on the non-Boussineq Flow of Self-Igniting Suspension Currents on a Steep Open Slope., *J. Geophys. Res.*, **113**(F01003).
- B. Turnbull, **J.N. McElwaine** and C. Ancey, 2007. The Kulikovskiy–Sveshnikova–Beghin Model of Powder Snow Avalanches: Development and Application, *J. Geophys. Res.*, **112**(F01004).
- Turnbull, B., and **J.N. McElwaine**, 2007. A Comparison of Powder Snow Avalanches at Vallée de la Sionne with Plume Theories, *J. Glaciol.*, **53**(30)
- **J.N. McElwaine**, and B. Turnbull, 2006. Plume Theories Versus Compact Models for Powder Snow Avalanches, *Sixth International Symposium on Stratified Flows, Perth, December 11-14*,
- **J.N. McElwaine**, 2005. Rotational flow in gravity current heads, *Phil. Trans. R. Soc. Lond.*, **363**, 1603–1623, 10.1098/rsta.2005.1597.
- **J.N. McElwaine** and B. Turnbull, 2005. Air Pressure Data from the Vallée de la Sionne Avalanches of 2004, *J. Geophys. Res.*, **110**(F03010).
- **J.N. McElwaine** and K. Nishimura, 2001. Particulate Gravity Currents, Blackwell Science, chap. Ping-pong Ball Avalanche Experiments, no. 31 in Special Publication of the International Association of Sedimentologists, 135–148.