Double-Diffusive Sedimentation

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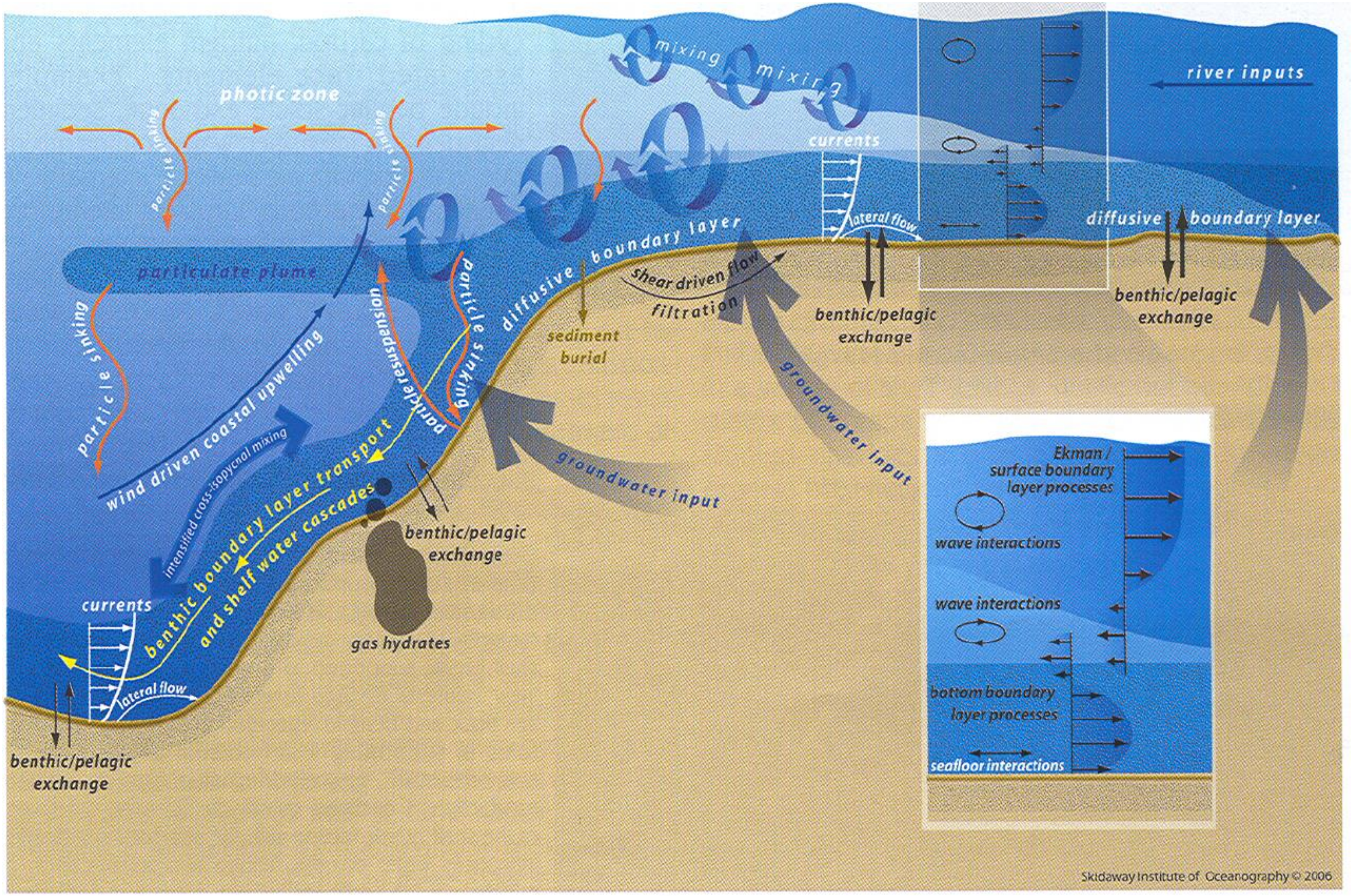
- *Motivation*
- *Governing equations*
- *Results: buoyant river outflows:*
    - *double-diffusive sedimentation*
    - *'fingering' vs. 'leaking' modes*
- *Scaling analysis and physical interpretation*
- *Summary and outlook*

UCSB

Coastal margin processes



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Sedimentation from river plumes: Motivation

- *10^{10} tons of sediment are transported by rivers into the world's oceans every year → important to understand sedimentation in river plumes*

Mississippi river plume
drainage basin size: 3.3×10^6 km^2
annual sediment yield: 1.2×10^2 t/km^2

Santa Clara river plume
drainage basin size: 4.2×10^3 km^2
annual sediment yield: 1.4×10^3 t/km^2

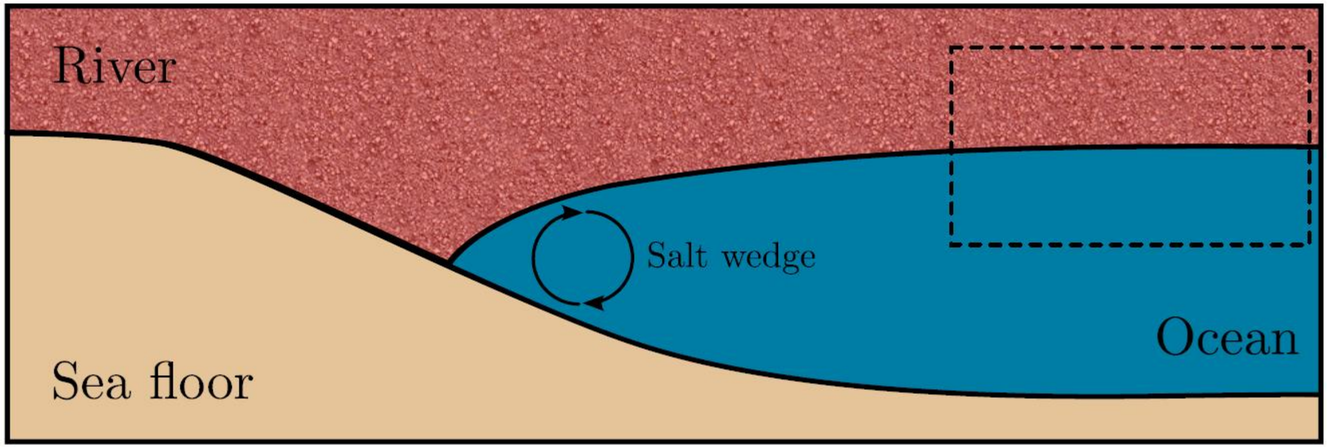
→ a large fraction of the sediment supply into the oceans is due to small, mountainous streams

Sedimentation from river plumes: Configuration

Hypopycnal river plumes:

density of the river (fresh water + sediment) < density of ocean (water + salinity)

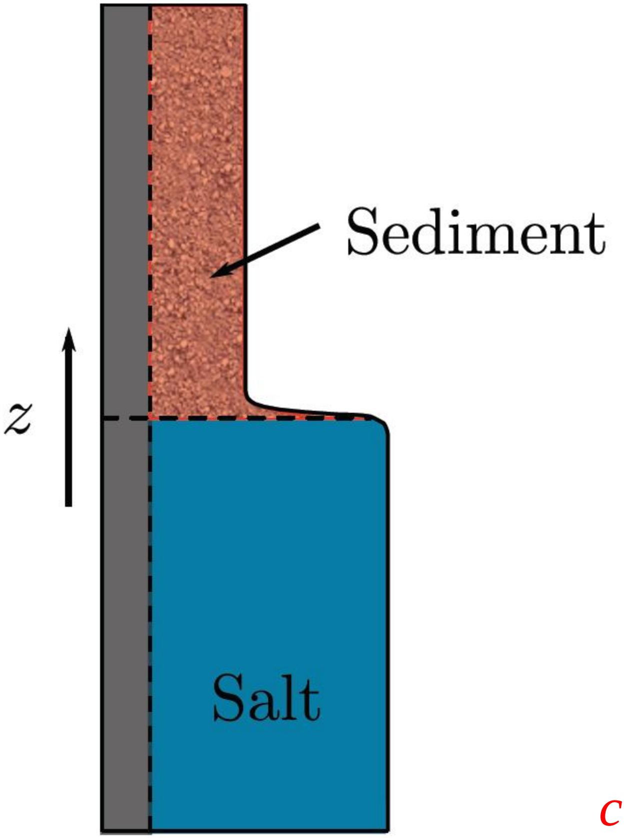
→ river outflow propagates along the ocean surface

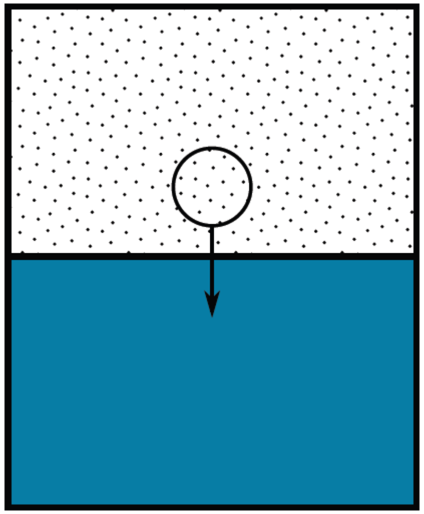


- *focus on the downstream density stratification*

Sedimentation from river plumes: Double-diffusion

Base density profile:

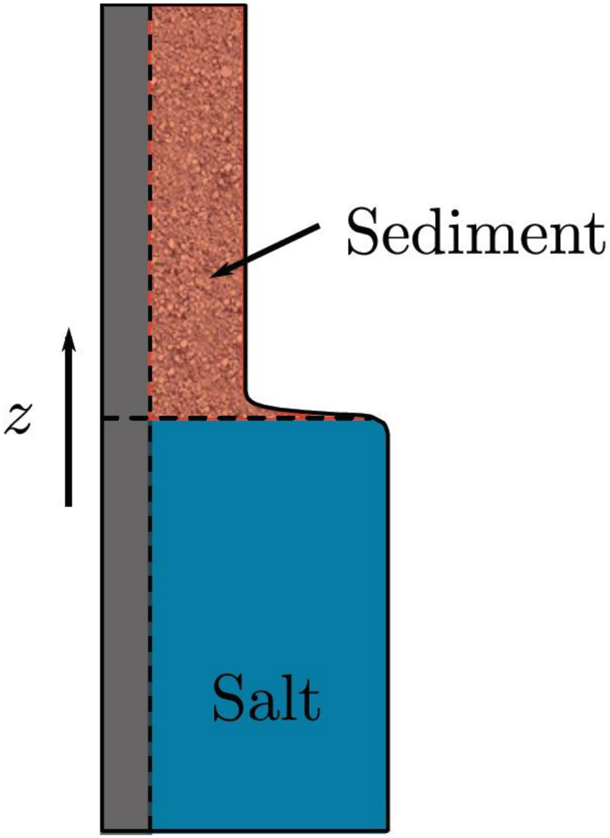


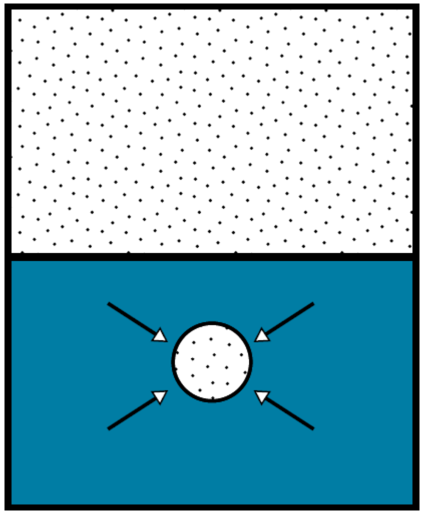


consider local downward perturbation of fluid element across opposing gradients

Sedimentation from river plumes: Double-diffusion

Base density profile:

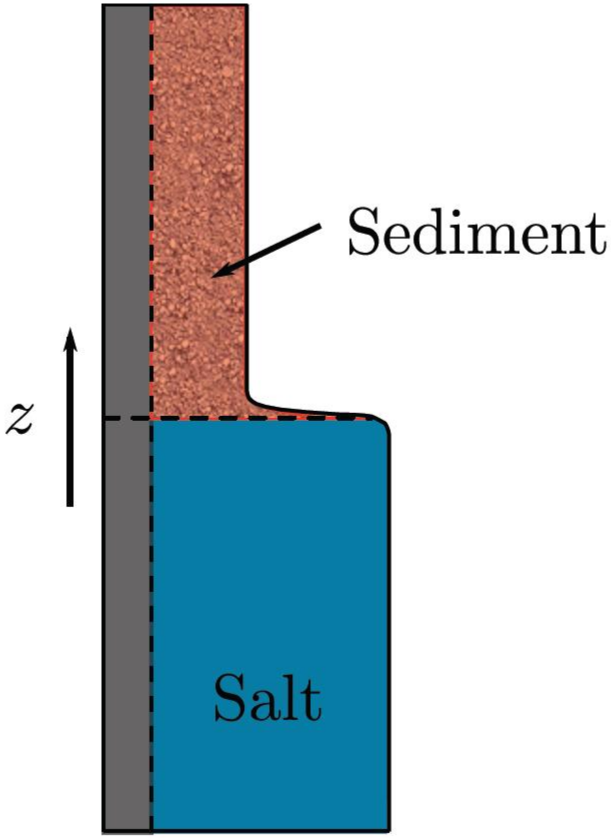


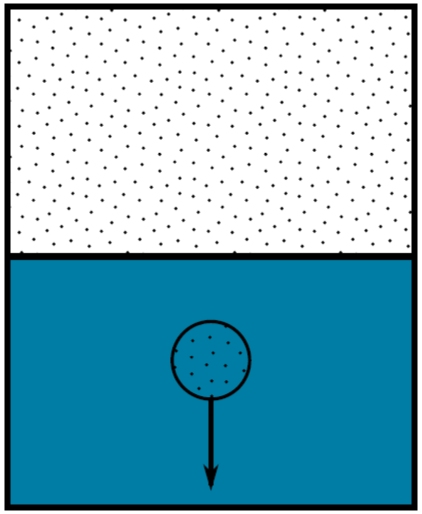


salinity diffuses inward more rapidly
than particles diffuse outward

Sedimentation from river plumes: Double-diffusion

Base density profile:





→ *fluid element will continue to sink*

- *potential for double-diffusive instability*

Traditional case: Salt fingers

- *warm, salty water above cold, fresh water:*

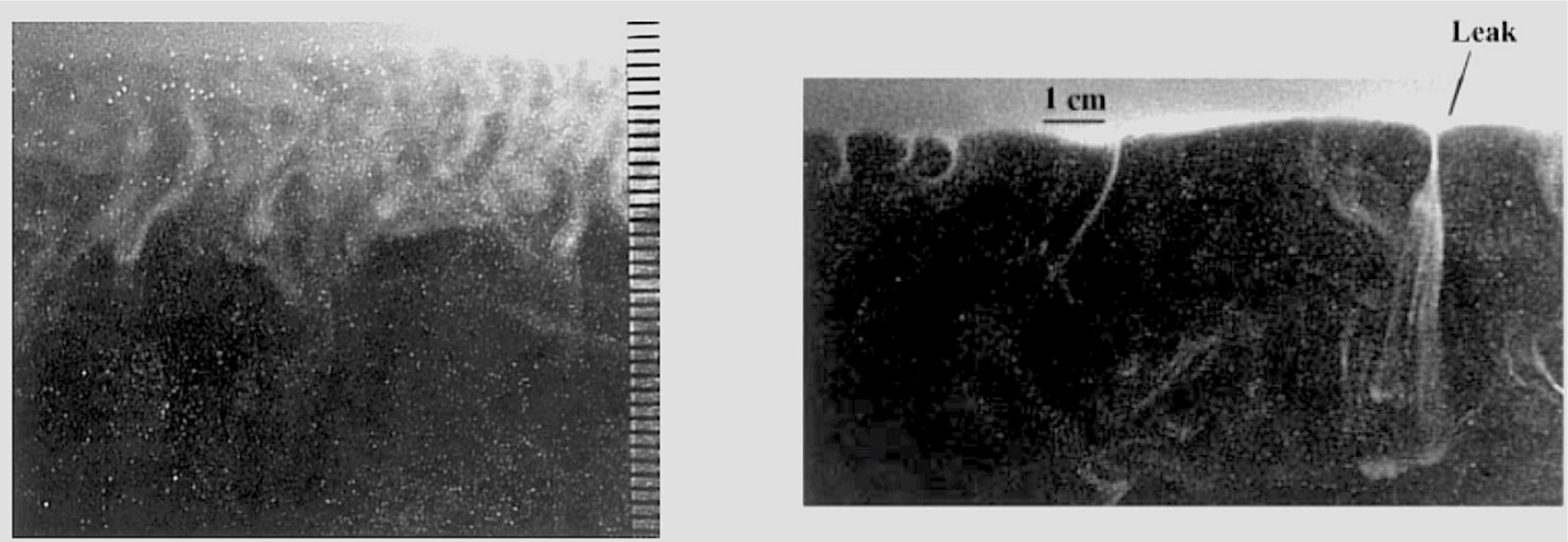
Huppert and Turner (1981)

- *dominant process for the vertical flux of salt in the ocean*
- *robust against shear*
- *believed to be responsible for the formation of the thermohaline staircase*

→ *for salt/sediment system, how does double-diffusion affect sedimentation?*

Sedimentation from river plumes: Experiments

- *previous experimental work by Parsons et al. (2001):*



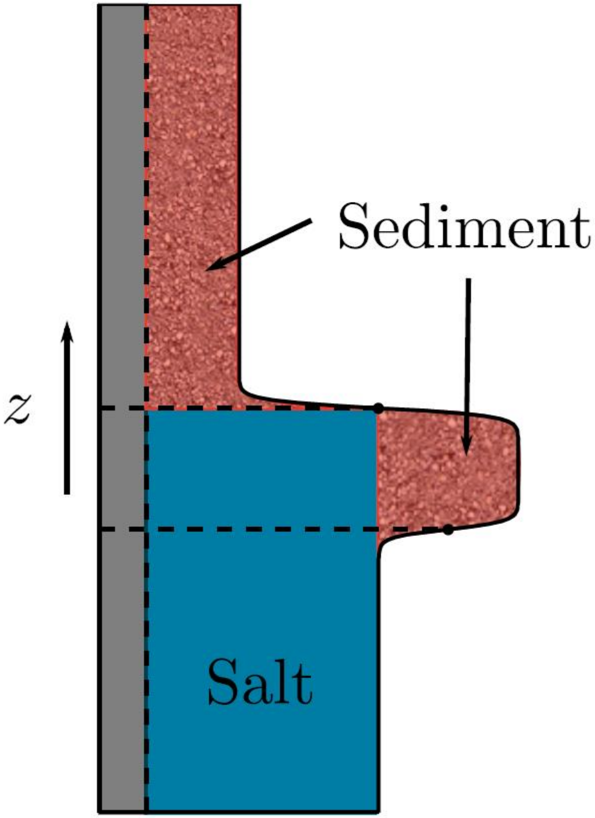
convective 'fingering' mode
space filling

'leaking' mode
localized, structures move along interface

→ *goal: understand mechanisms driving these modes, and their influence on the effective particle settling velocity*

Sedimentation from river plumes

Effect of settling velocity:



nose height H

density profile

- *settling process creates potential for Rayleigh-Taylor instability*

Framework: Dilute flows

Assumptions:

- *volume fraction of particles* $< O(10^{-3})$
- *particle radius « particle separation*
- *small particles with negligible inertia*

Dynamics:

- *effects of particles on fluid continuity equation negligible*
- *coupling of fluid and particle motion primarily through momentum exchange, not through volumetric effects*
- *particle loading modifies effective fluid density*
- *particles follow fluid motion, with superimposed settling velocity*

Moderately dilute flows: Two-way coupling (cont'd)

Governing dimensionless eqns:

$$\rho - 1 = \alpha S + \gamma C$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \nabla^2 \mathbf{u} - \nabla \mathcal{P} + \rho' \frac{\mathbf{g}}{g'}$$

$$\frac{\partial S}{\partial t} + \mathbf{u} \cdot \nabla S = \frac{1}{Sc} \nabla^2 S$$

$$\frac{\partial C}{\partial t} - V_p \frac{\partial C}{\partial z} + \mathbf{u} \cdot \nabla C = \frac{1}{\tau Sc} \nabla^2 C$$

Characteristic quantities:

$$L^c = (\nu^2 / g')^{1/3} \ , \qquad T^c = (L^{c2}/\nu) \ ,$$

$$U^c = (\nu \, g')^{1/3} \ , \qquad g' = \frac{\Delta \rho_c}{\rho_0} g \ ,$$

$$V_{st} = \frac{g \, d_p^2 \, (\rho_p - \rho_f)}{18 \, \mu_f}$$

Dimensionless parameters:

settling velocity $V_p = \frac{V_{st}}{(\nu g')^{1/3}}$

Schmidt number $Sc = \frac{\nu}{\kappa_s}$

stability ratio $R_s = \frac{\alpha}{\gamma}$

diffusivity ratio $\tau = \frac{\kappa_s}{\kappa_c}$

Sedimentation from river plumes: Numerical simulations

- *Two dimensions:*

  - *streamfunction, vorticity-formulation of Navier-Stokes equations*
  - *Boussinesq approximation*
  - *spectral/compact finite differences*

- *Three dimensions:*

  - *IMPACT code (Henniger and Kleiser 2011)*
  - *primitive variable formulation of Navier-Stokes equations*
  - *Boussinesq approximation*
  - *staggered grid*
  - *6th order compact finite differences*
  - *massively parallel*

Sedimentation from river plumes: Numerical simulations

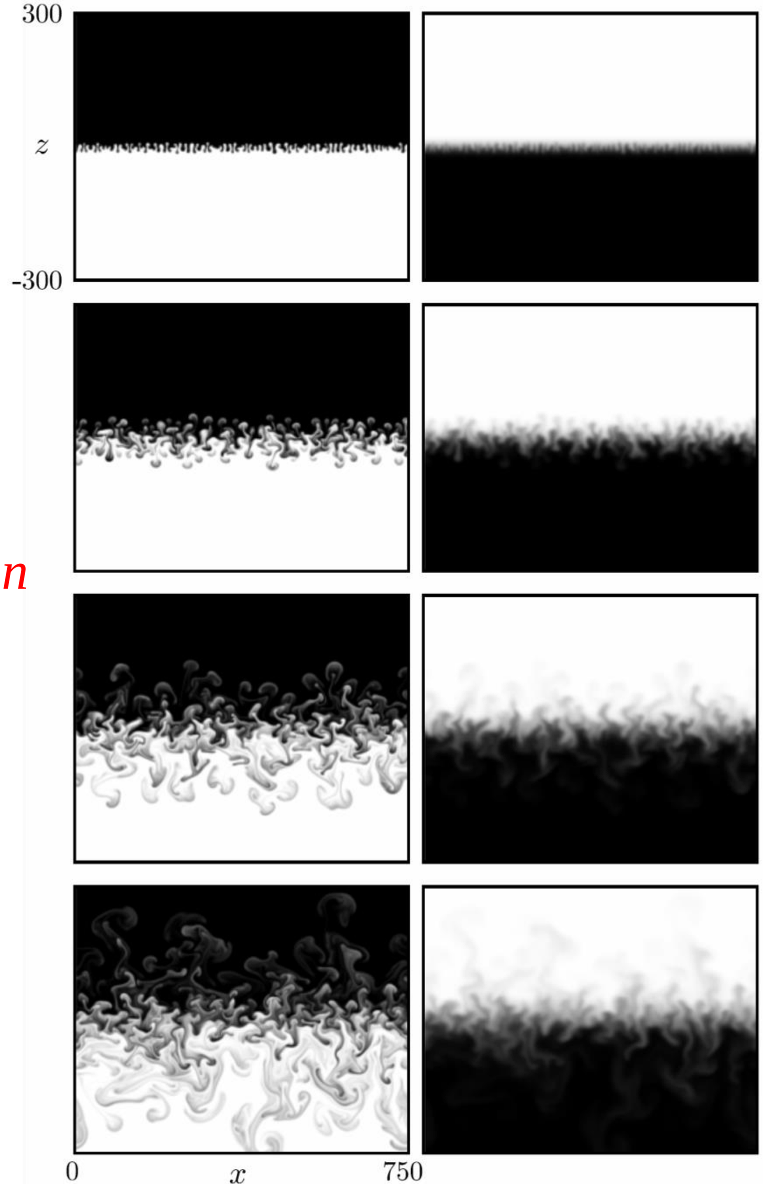
$$V_p = 0.04 \ ,$$

$$Sc = 0.7 \ ,$$

$$R_s = 2 \ ,$$

$$\tau = 25$$

sediment concentration



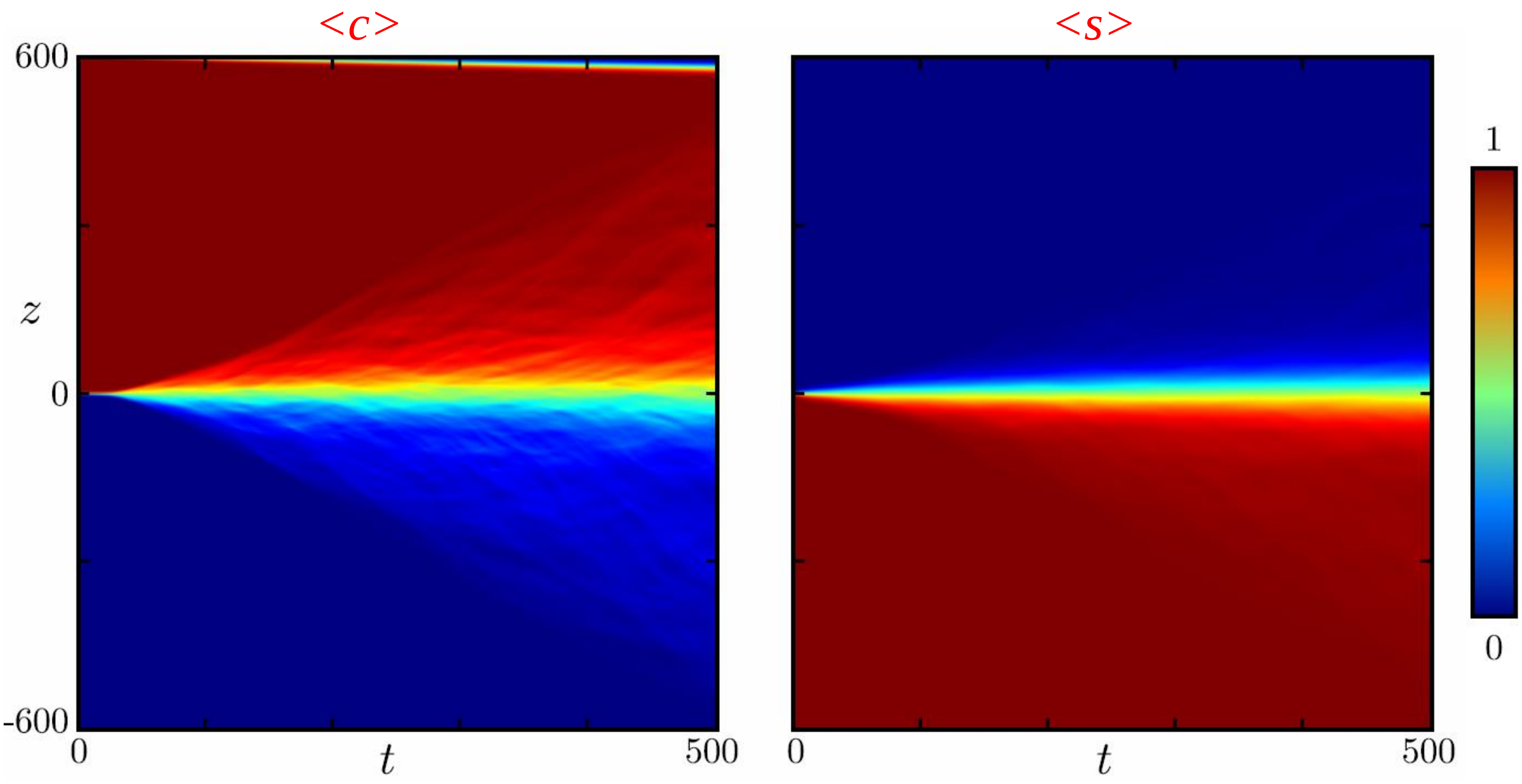
salinity

Sedimentation from river plumes: Numerical simulations

Mammatus clouds

Volcanic ash plume

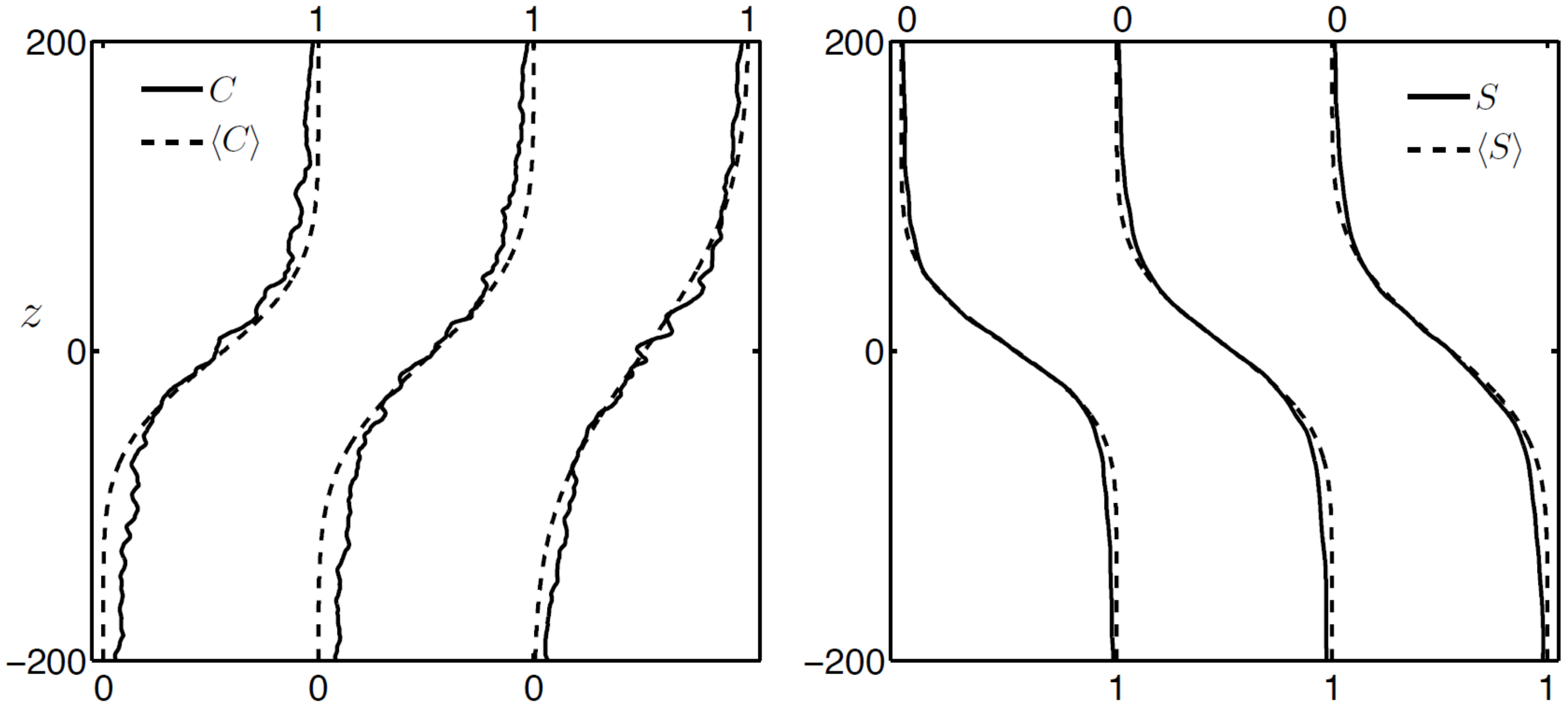
Sedimentation from river plumes: Mean fields



- *thickening of the plume-dominated region ~ time → convectively dominated*
- *vigorous convective motion*
- *'streaks' due to the release of buoyant plumes*

Sedimentation from river plumes: Mean fields

fit concentration profiles with erf → determine interface location, thickness

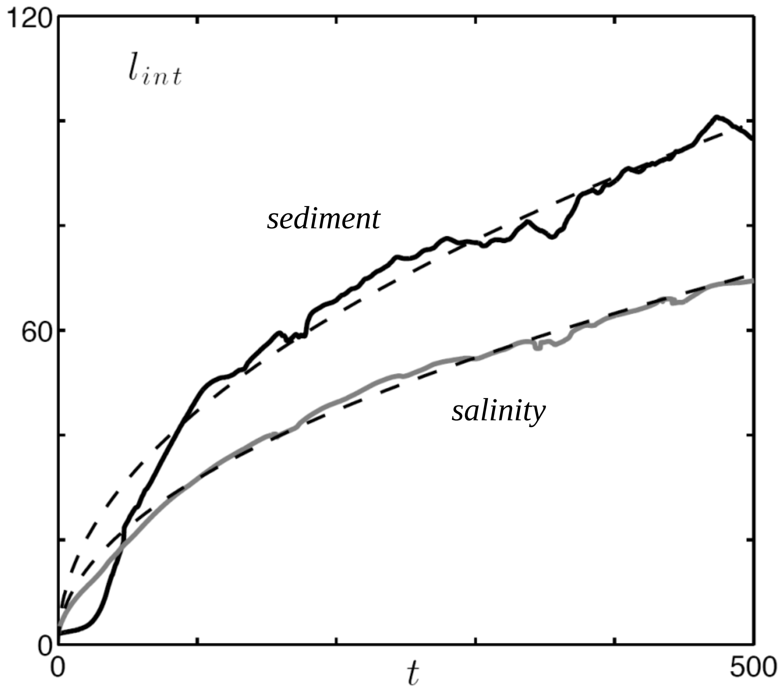


sediment concentration *salinity*

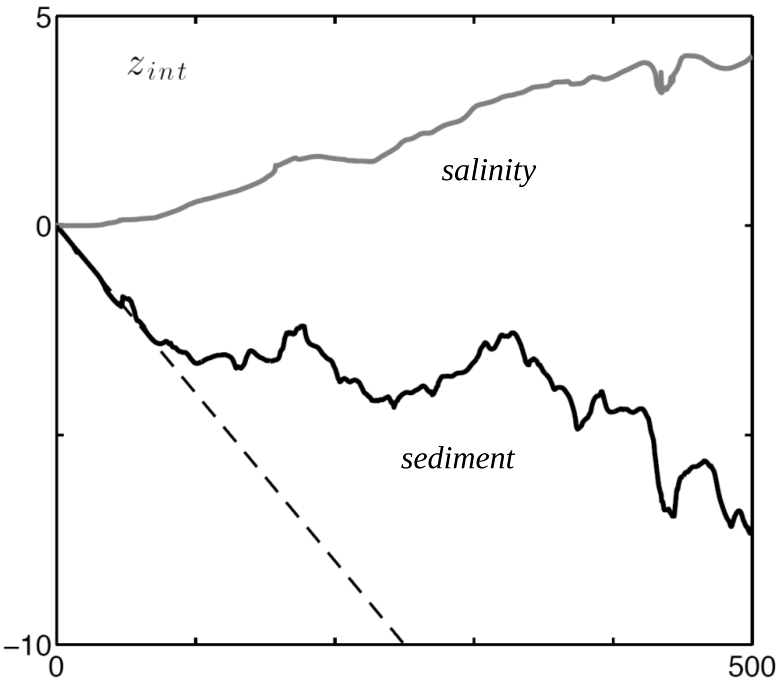
time = 300, 400, 500

Sedimentation from river plumes: Mean fields

interface thickness



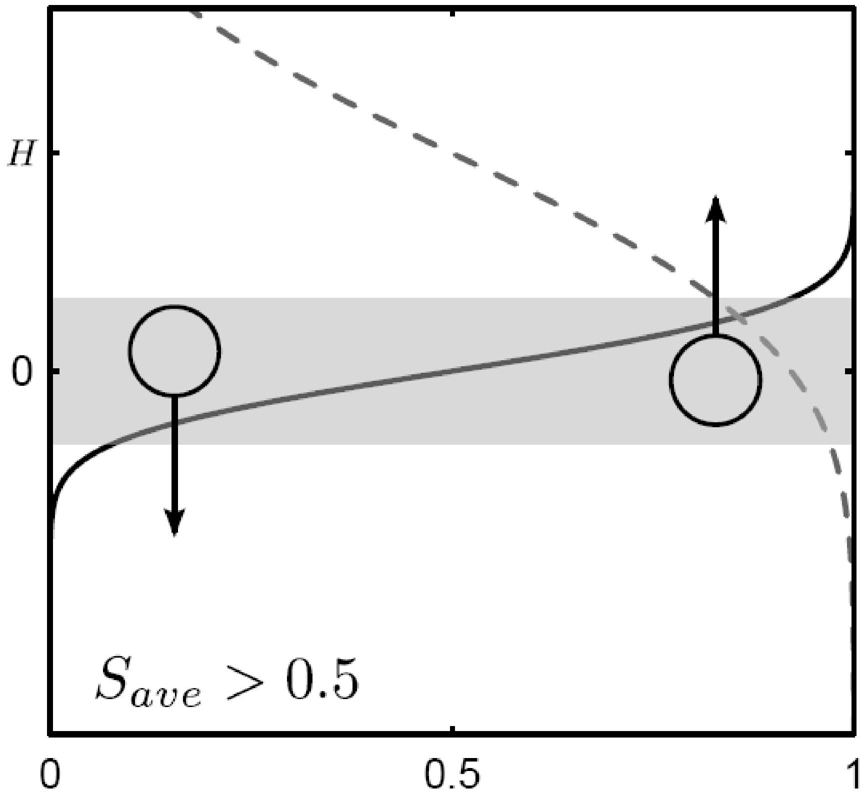
interface location



- *both interface thicknesses grow diffusively*
- *sediment interface thickness grows faster, in spite of smaller molecular diffusivity!*
- *sediment interface moves downward, but more slowly than Stokes settling velocity*
- *salinity interface moves upward*

Sedimentation from river plumes: Mean fields

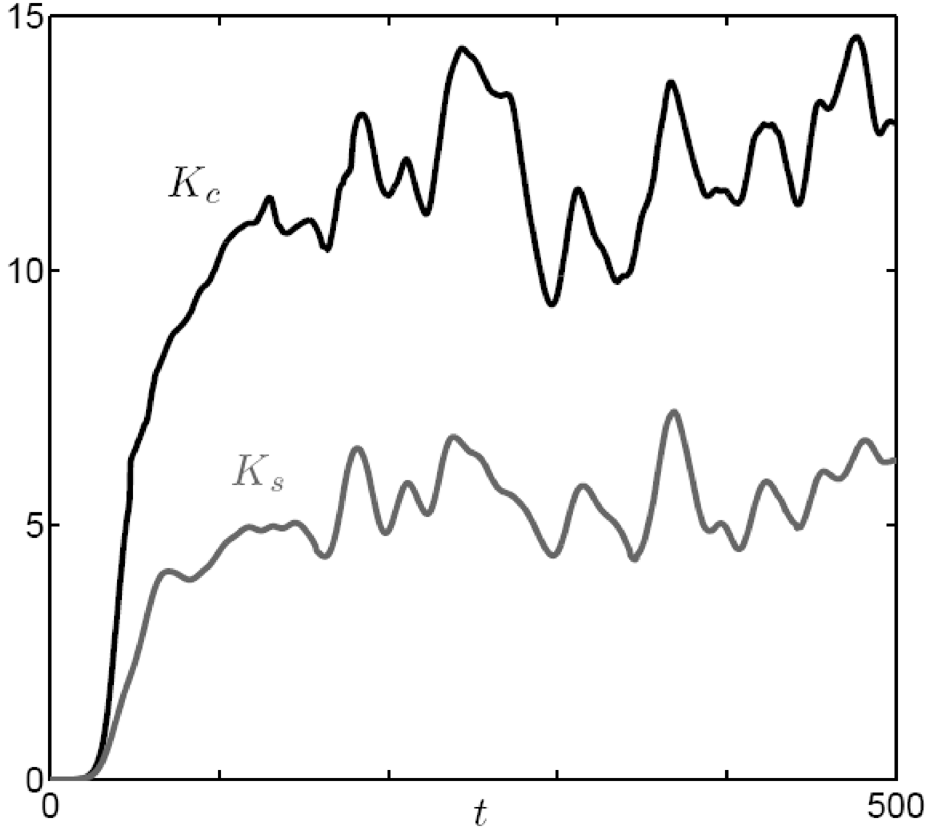
Why does the salinity interface move upward?



- *the instability is centered around the unstable sediment interface, which moves downward into the region of high salinity*
- *the region of high salinity gets mixed more strongly → the s=0.5 contour is displaced upwards*

Sedimentation from river plumes: Mean fields

Turbulent diffusivities:



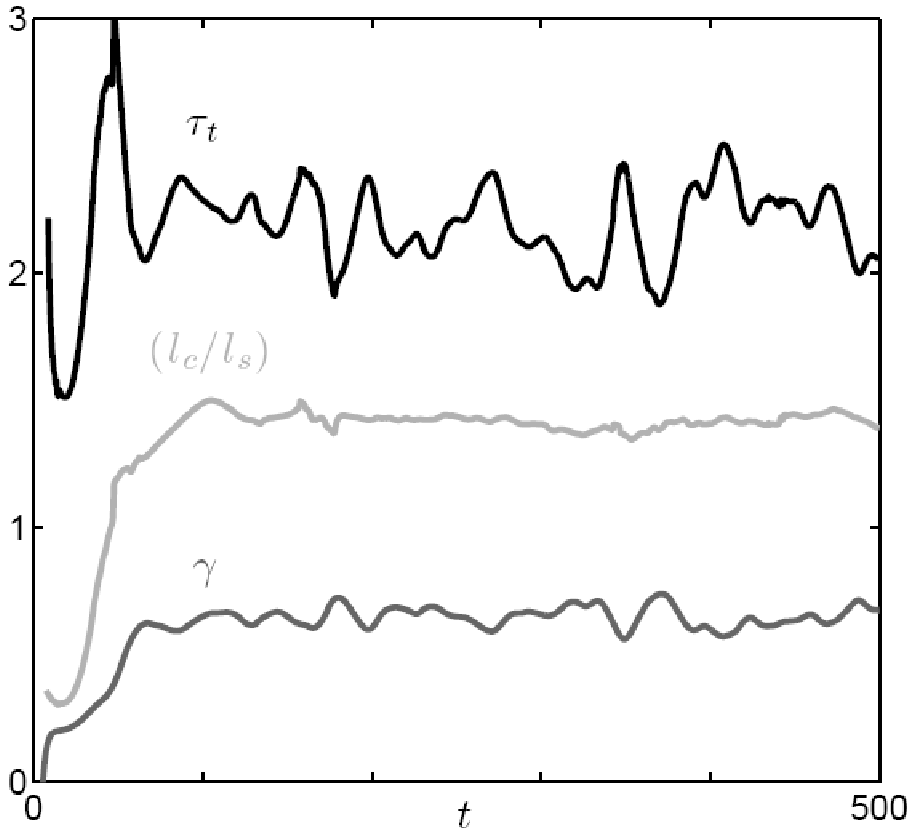
sediment

salinity

- *turbulent sediment diffusivity is about twice as high as turbulent salinity diffusivity, even though the molecular salinity diffusivity is 25 times larger than 'molecular' sediment diffusivity → consistent with numerical observations*

Sedimentation from river plumes: Mean fields

Quasisteady measures of sedimentation dynamics



ratio of turbulent diffusivities:

$$\tau_{turb} = K_c/K_s$$

ratio of interface thicknesses

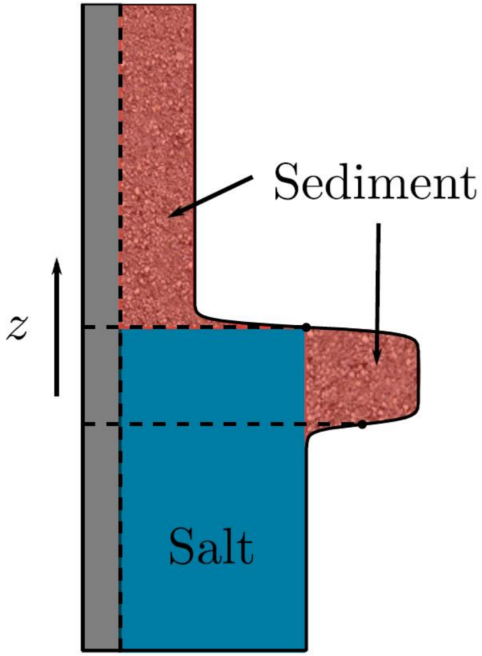
ratio of salinity flux to sediment flux:

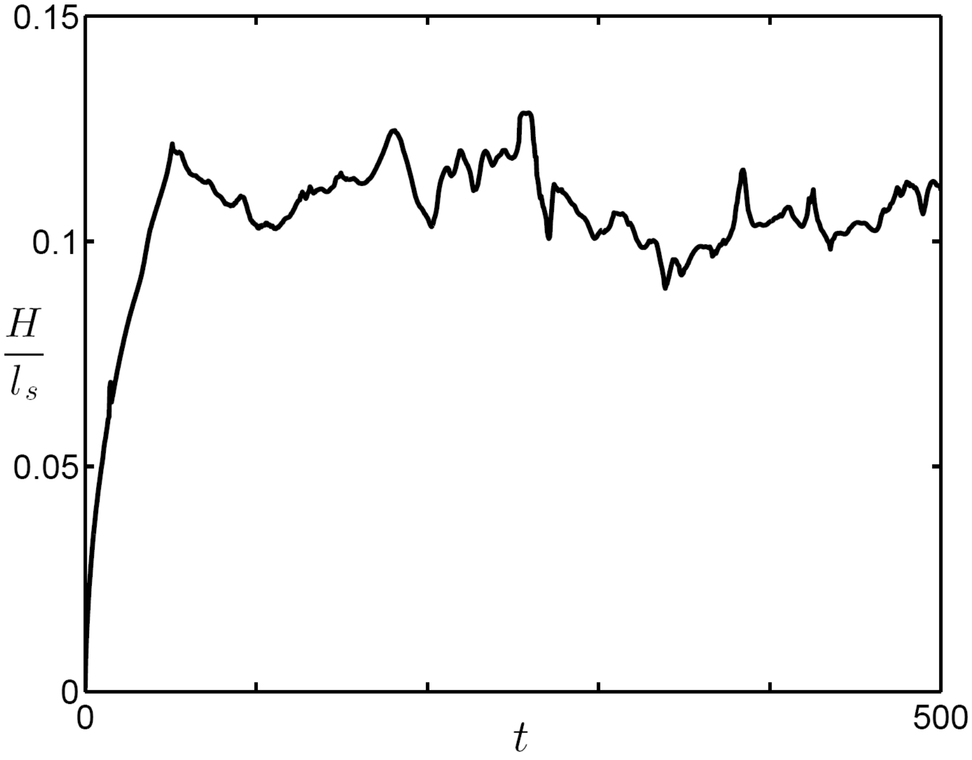
$$\gamma = -\frac{F_s}{F_c}$$

- *ratio of turbulent diffusivities, ratio of interface thicknesses and ratio of turbulent fluxes all approach quasisteady values → will be important for scaling analysis*

Sedimentation from river plumes: Mean fields

Ratio of nose height to salinity interface thickness:





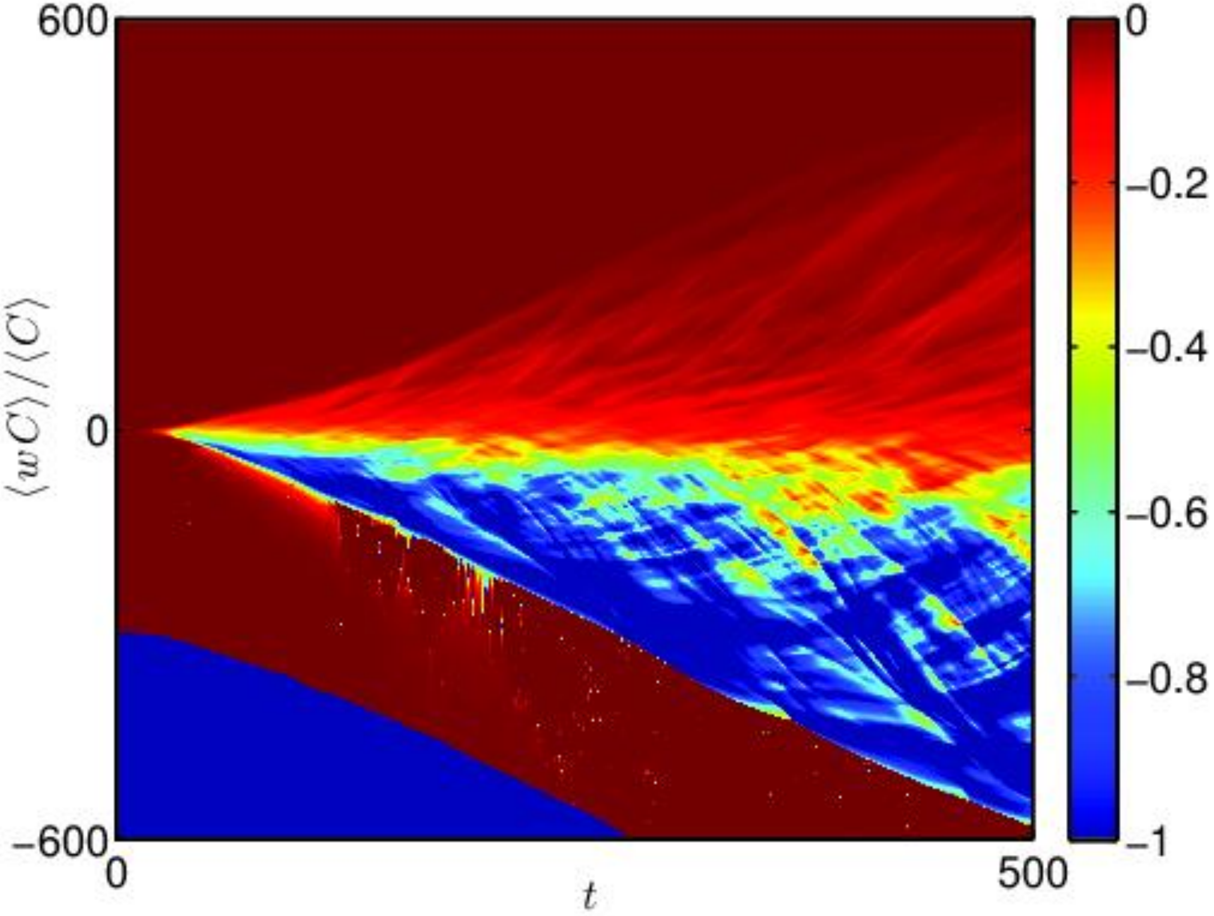
- *ratio of nose height to salinity interface thickness approaches quasisteady state, and remains «1*

→ *sediment interface remains embedded in the region of strong salinity gradient*

→ *double diffusion remains important*

Sedimentation from river plumes: Effective settling velocity

Settling velocity enhancement:

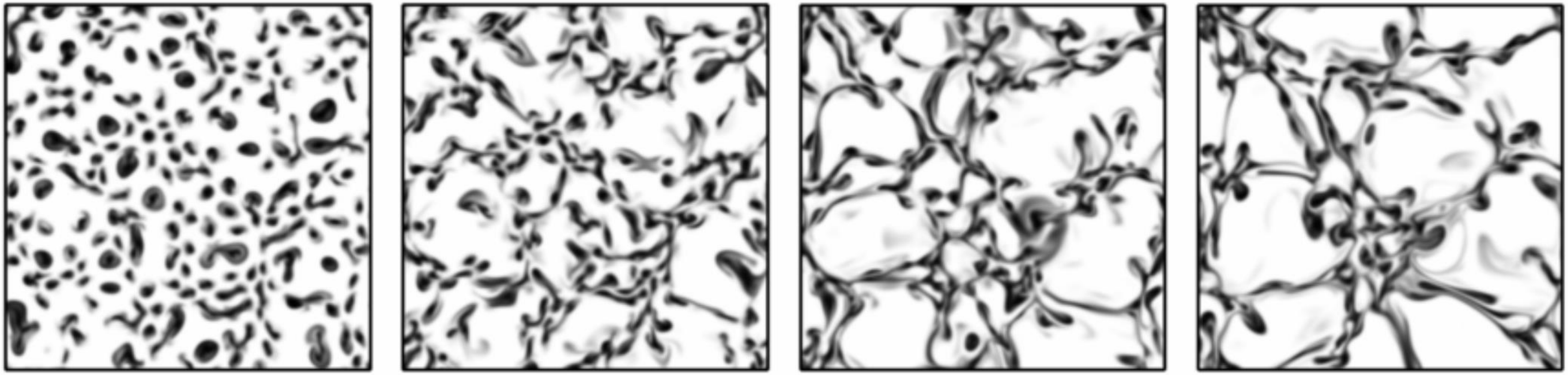


- *in the region $z < 0$, the effective settling velocity is O(1), rather than $V_{st}=0.04$, i.e., it scales with the buoyancy velocity of the system, not the Stokes velocity*

Sedimentation from river plumes: Leaking mode (higher Sc)

Sedimentation from river plumes: Leaking mode

horizontal cross-cuts through sediment concentration field:

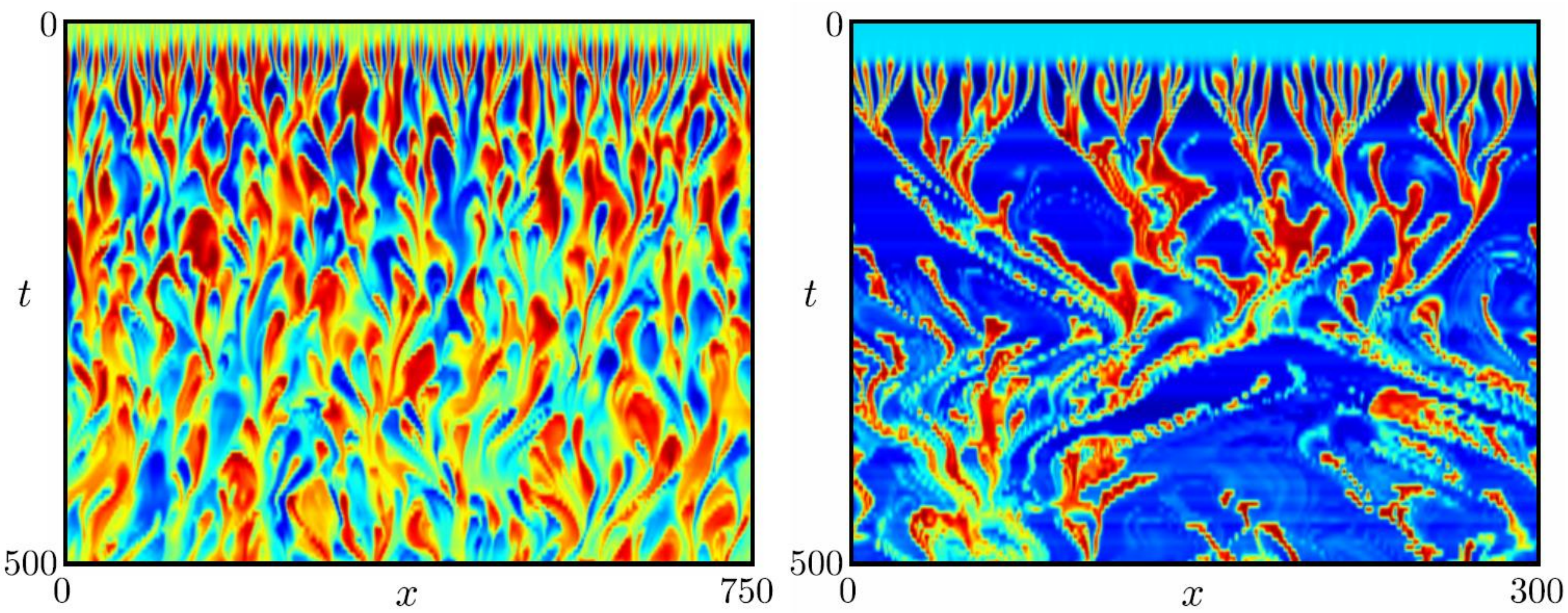


→ *time increases*

- *nonlinear evolution of initial, localized plumes results in web-like structure*
- *characterized by sheets, rather than plumes*

Sedimentation from river plumes: fingering vs. leaking

x,t-diagrams of sediment concentration at fixed vertical location:



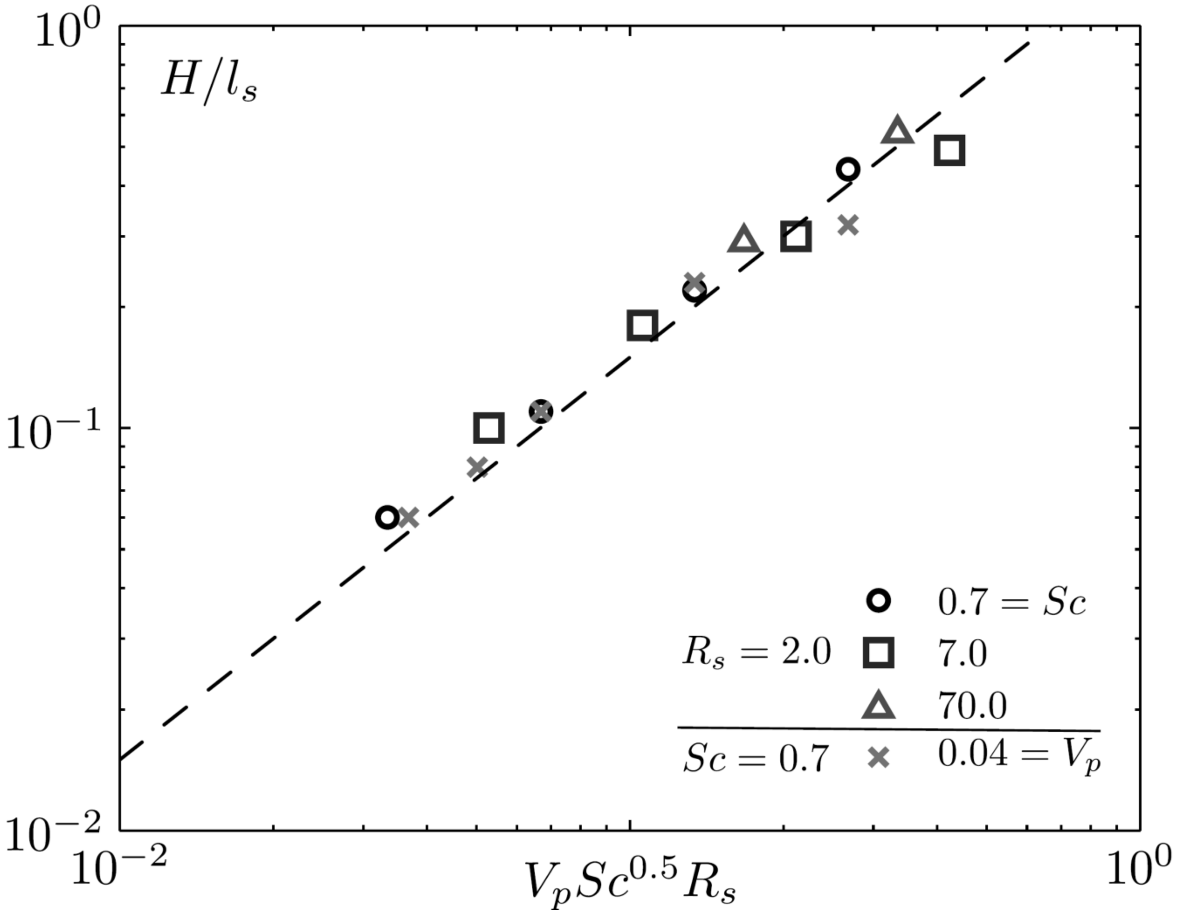
fingering mode
weak horizontal motion

leaking mode
strong horizontal motion and merging

- *explains different modes observed by Parsons et al. (2001)*

Sedimentation from river plumes: Scaling

Scaling of nose height with in-/outflow ratio:



→ *quasisteady ratio of nose height to salinity interface thickness scales with ratio of sediment inflow into nose region to sediment outflow from nose region*

Sedimentation from river plumes: Parametric study

Physical interpretation:

- *for small settling velocity, the rate of sediment inflow from above is low → this low rate of sediment inflow can be balanced by conventional double-diffusive outflow of sediment below → there is little accumulation of sediment in the nose region → height of nose region remains small*

- *for large settling velocity, the rate of sediment inflow from above is high → this high rate of sediment inflow cannot be balanced by traditional double-diffusive sediment outflow below → sediment accumulates in the nose region → height of nose region increases until it is thick enough for Rayleigh-Taylor instability to form, which leads to increased sediment outflow below → new balance between in- and outflow into the nose region is established*

Double-diffusive sedimentation: Open questions

Currently under investigation::

- *linear concentration gradients vs. initial step profiles*

- *influence of shear:*
  - *Kelvin-Helmholtz vs. double-diffusive instabilities*
  - *does Holmboe instability form?*

- *based on recent findings for thermohaline double-diffusive instabilities:*
  - *diffusive vs. convective mode*
  - *do collective instability modes form?*
  - *do horizontal intrusions form?*
  - *do "gamma-instability" and "staircases" form?*

Summary

- *double-diffusive sedimentation in river outflows dramatically enhances the effective settling velocity*
- *settling velocity scales with buoyancy velocity, not with Stokes velocity*
- *two mechanisms drive the process:*
  - *double-diffusive instability of salt vs. sediment*
  - *settling of sediment creates 'nose region,' Rayleigh-Taylor instability*
- *ratio of nose height/salinity interface thickness H/l_s determines regime*
- *for low Schmidt numbers, low stability ratios and small Stokes settling velocities, traditional double-diffusive instability causes convective 'fingering' mode*
- *for high Schmidt numbers, large stability ratios and large Stokes settling velocities, settling of sediment causes 'leaking' mode, via interaction of Rayleigh-Taylor and double-diffusive instability modes through 'phase-locking'*
- *overall dynamics is governed by the in-/outflow of sediment into/from the nose region*