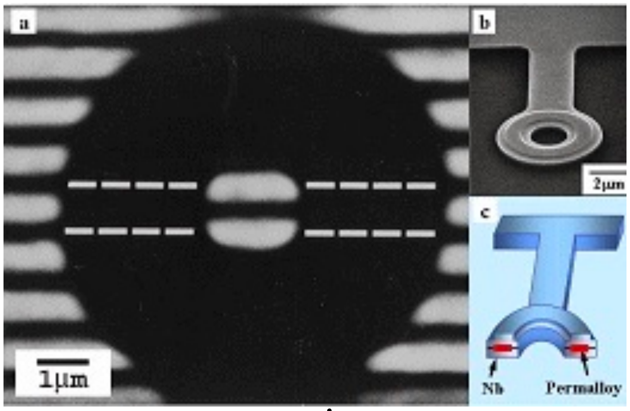
recall : Aharonov-Bohm effect



Tonomura et al., PRL 1986

Berry phase: $\gamma_n(t) = \int_0^t dt' \langle \psi_n(t) | i\partial_t | \psi_n(t) \rangle$

for $H = H(\lambda(t))$, s.t. $H(\lambda(t)) | \psi_n(\lambda(t)) \rangle = E_n(\lambda(t)) | \psi_n(\lambda(t)) \rangle$

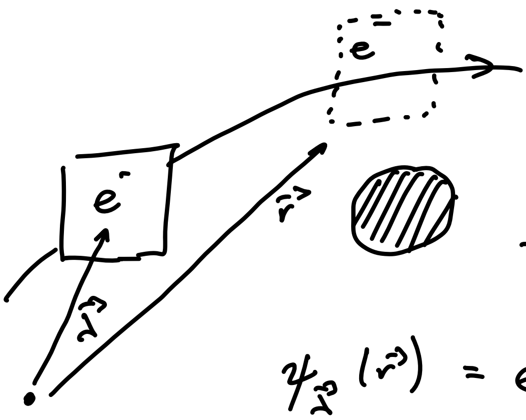
. $i\partial_t = \dot{\lambda}\, i\partial_\lambda$

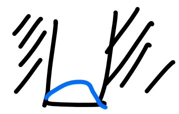
$$\Rightarrow \gamma_n = \int_{\vec{\lambda}(0)}^{\vec{\lambda}(t)} d\lambda \, \langle \psi_n(\lambda) | i\partial_\lambda \psi_n(t) \rangle$$

indep. of protocol $\lambda(t)$

→ only depends on $\lambda(0)$ & $\lambda(t)$

- relation b/w AB effect & Berry's phase?



place e^- in a box: $V(\vec{r}) =$ 

$\Rightarrow \langle \psi_0 | \vec{p} | \psi_0 \rangle = 0$

→ wavefn of e^- in presence of solenoid

$$\psi_{\vec{\lambda}}(\vec{r}) = e^{i g_\lambda(\vec{r})} \psi_0(\vec{r} - \vec{\lambda})$$

$$g_\lambda(\vec{r}) = -q \int_{\vec{\lambda}}^{\vec{r}} \vec{A}(\vec{r}') \cdot d\vec{r}'$$

want Berry's phase: $\gamma = \int_{\lambda_i}^{\lambda_f} d\vec{\lambda} \cdot \langle \psi_{\vec{\lambda}} | i \vec{\nabla}_\lambda | \psi_{\vec{\lambda}} \rangle$

$$i\vec{\nabla}_\lambda \psi_{\vec\lambda}(\vec r) = q\vec A(\vec\lambda)e^{ig_\lambda(\vec r)}\psi_0(\vec r-\vec\lambda) + e^{ig_\lambda(\vec r)}\underbrace{(i\vec\nabla_\lambda)\psi_0(\vec r-\vec\lambda)}_{=-i\vec\nabla_r\psi_0(\vec r-\vec\lambda)}$$

$$\Rightarrow \langle\psi_\lambda|i\vec\nabla_\lambda|\psi_\lambda\rangle = \int d^3r\, \cancel{e^{-ig_\lambda(\vec r)}}\psi_0^*(\vec r-\vec\lambda)\cancel{e^{+ig_\lambda}}[q\vec A(\vec\lambda)\psi_0(\vec r-\vec\lambda)+\vec p\psi_0(\vec r-\vec\lambda)]$$

$$= q\vec A(\vec\lambda)\underbrace{\int d^3r\,|\psi_0(\vec r-\vec\lambda)|^2}_{=1} + \underbrace{\int d^3r\,\psi_0^*(\vec r-\vec\lambda)\,\vec p\,\psi_0(\vec r-\vec\lambda)}_{=\langle\vec p\rangle \underset{\text{box pot.}}{=} 0}$$

$$= q\vec A(\vec\lambda)$$

$$\Rightarrow \langle\psi_\lambda|i\vec\nabla_\lambda|\psi_\lambda\rangle = q\vec A(\vec\lambda)$$

- Berry phase on a closed loop around solenoid:

$$\gamma = \oint q\vec A(\vec\lambda)\cdot d\vec\lambda \underset{\text{Stokes}}{=} q\int \vec\nabla_\lambda\times\vec A(\vec\lambda)\cdot d\vec a = q\,\Phi_0$$

Berry phase coincides w/ AB phase

- analogy:

EM		QM
i) AB phase: $\varphi_{AB} = \oint \vec A\cdot d\vec r$	$\longleftrightarrow$	Berry phase $\gamma = \oint \mathcal{A}(\vec\lambda)\cdot d\vec\lambda$
ii) vector pot. $\vec A(\vec r)$	$\longleftrightarrow$	Berry connection / gauge pot. $\vec{\mathcal{A}}(\vec\lambda) = \langle\psi_0(\vec\lambda)\|i\vec\nabla_\lambda\psi_0(\vec\lambda)\rangle$ $(\vec{\mathcal{A}})_\mu$
iii) magnetic field / EM field tensor $F_{ab} = \partial_a A_b - \partial_b A_a = \varepsilon_{abc}B_c$	$\longleftrightarrow$	Berry curvature $F_{\mu\nu}(\vec\lambda) = \partial_{\lambda_\mu}\mathcal{A}_\nu - \partial_{\lambda_\nu}\mathcal{A}_\mu$

$\hookrightarrow$ quantum geometry

def: gauge pot. (in analogy to EM)

- Hamiltonian: $H(\lambda)$, λ: parameter
- instant. e'system: $H(\lambda)|n(\lambda)\rangle = E(\lambda)|n(\lambda)\rangle$

$$A_\lambda(\cdot) = i\partial_\lambda(\cdot)$$

-> off-diag elements: $n \neq m$

$$\langle n(\lambda)|A_\lambda|m(\lambda)\rangle = i\frac{\langle n(\lambda)|\partial_\lambda H|m(\lambda)\rangle}{E_n(\lambda) - E_m(\lambda)} \qquad \forall n \neq m$$

-> diag. elements:

$$\langle n(\lambda)|A_\lambda|n(\lambda)\rangle = i\langle n(\lambda)|\partial_\lambda n(\lambda)\rangle$$

want: basis-indep. def. of A_λ

1) $$i\partial_\lambda\big(|n(\lambda)\rangle\langle n(\lambda)|\big)(\cdot) = i\partial_\lambda\big(|n(\lambda)\rangle\langle n(\lambda)|(\cdot)\big)$$
$$- |n(\lambda)\rangle\langle n(\lambda)|\, i\partial_\lambda(\cdot)$$
$$= \big(A_\lambda|n(\lambda)\rangle\langle n(\lambda)| - |n(\lambda)\rangle\langle n(\lambda)|A_\lambda\big)(\cdot)$$
$$= [A_\lambda, |n(\lambda)\rangle\langle n(\lambda)|](\cdot) \qquad (*)$$

2) consider: $$H(\lambda) = \sum_n E_n(\lambda)|n(\lambda)\rangle\langle n(\lambda)|$$

$$i\partial_\lambda H(\lambda) = -i\overbrace{\sum_n(-\partial_\lambda E_n(\lambda))|n(\lambda)\rangle\langle n(\lambda)|}^{:=M_\lambda}$$
$$+ \sum_n E_n(\lambda)\Big\{\underbrace{i\partial_\lambda\big(|n(\lambda)\rangle\langle n(\lambda)|\big)}_{\overset{(*)}{=}[A_\lambda, |n(\lambda)\rangle\langle n(\lambda)|]}\Big\}$$
$$= -iM_\lambda + [A_\lambda, H(\lambda)]$$

$$\Rightarrow \quad i\partial_\lambda H(\lambda) = [A_\lambda, H(\lambda)] - i M_\lambda \qquad / \; [H, \cdot\,]$$

$$[H, i\partial_\lambda H] - [H, [A_\lambda, H]] = -i\,[H, M_\lambda] = 0$$

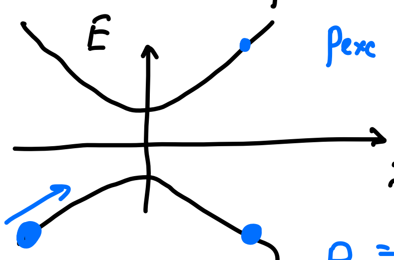
$$\boxed{[H, i\partial_\lambda H - [A_\lambda, H]] = 0}$$

defining eq. for gauge pot. A_λ

Counter-Diabatic Driving

recall LZ problem: $p_{exc} = e^{-\pi \frac{h^2}{2v}}$

$$H(t) = \frac{vt}{2}\sigma^z + \frac{h}{2}\sigma^x$$



$p = 1 - e^{-\pi\frac{h^2}{2v}}$

- excited fraction is exponentially small but it's finite; depends on h & v

<u>issues</u>: 1) p_{exc} increases if speed v increases

2) expression valid in the regime $t \to \infty$
what about finite times?

Q: can we suppress excitations <u>completely</u> & at <u>all times</u> during the ramp?

<u>Yes!</u> $\rightarrow$ <u>transitionless driving</u>:

consider $H = H(\lambda)$

- let $U(\lambda)$ diagonalize $H(\lambda)$ instantaneously, i.e.

$$U^+(\lambda)\, H(\lambda)\, U(\lambda) = \begin{pmatrix} E_1(\lambda) & & 0 \\ & \ddots & \\ 0 & & E_n(\lambda) \end{pmatrix} =: D(\lambda)$$

- for $\lambda = \lambda(t)$, need change of reference frame transf.

$$H_{\text{co-mov}} = \tilde{H}(t) = U^\dagger(\lambda(t))\, H(\lambda(t))\, U(\lambda(t)) - i\, U^\dagger(\lambda(t))\,\partial_t U(\lambda(t))$$

$$= U^\dagger(\lambda)\, H(\lambda)\, U(\lambda) - \dot\lambda\, U^\dagger(\lambda)\, \underbrace{i\partial_\lambda U(\lambda)}_{=A_\lambda}$$

$$= \underset{\text{diagonal matrix}}{D(\lambda)} - \dot\lambda\, \underset{\text{gauge pot. in } U(\lambda) \text{ basis}}{\tilde{A}_\lambda}$$

<u>notice</u>: $D(\lambda)$ is diagonal, <u>therefore</u>:

any excitations during time evolution under $H(\lambda(t))$ must necessarily be caused by $\tilde{A}_\lambda$

$\Rightarrow$ <u>idea</u>: apply a "force" that <u>counteracts</u> excitations

e.g. consider $H(\lambda) \to H(\lambda) + H'(\lambda)$

s.t. $H'(\lambda_i) = 0 = H'(\lambda_f)$ vanishes at beginning & end of protocol, i.e. for $t = 0, T$

— <u>counterdiabatic</u> driving:

$$H_{CD}(t) = H(\lambda(t)) + \underbrace{\dot\lambda A_\lambda}_{\text{cancels } \underline{\text{all}} \text{ excitations}}$$

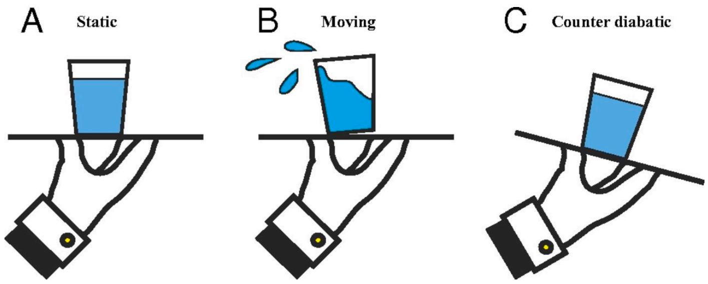
$$\tilde{H}_{CD} = U_\lambda^\dagger (H + \dot\lambda A_\lambda) U - \dot\lambda\, U_\lambda^\dagger\, i\partial_\lambda U_\lambda$$

$$= U_\lambda^\dagger H U_\lambda + \cancel{\dot\lambda\, U_\lambda^\dagger A_\lambda U_\lambda} - \cancel{\dot\lambda\, U_\lambda^\dagger\, i\partial_\lambda U_\lambda}$$

$$= D(\lambda)$$

$\Rightarrow$ for time evo under $H_{CD}(\lambda(t))$, a system starting in an e'state of $H(\lambda(0))$ remains in the inst. e'state of $H(\lambda(t))$ at all times $\to$ transitionless driving

& the state only accumulates a phase.



Sels & Polkovnikov, PNAS 2017

note: we achieve transitionless driving for <u>any</u> protocol $\lambda(t)$!

<u>intuition</u>:

- limiting cases:

a) $\underline{\dot\lambda \to \infty}$ $\Rightarrow$ $H_{CD} \approx \dot\lambda A_\lambda$

Schr. eq: $i\partial_t |\psi(t)\rangle = H_{CD} |\psi(t)\rangle$

$\approx \dot\lambda A_\lambda |\psi(t)\rangle$

$$\Rightarrow i\partial_\lambda |\psi(\lambda)\rangle = A_\lambda |\psi(\lambda)\rangle$$

as $\dot\lambda \to \infty$, A_λ generates time-evo in λ-space

b) adiabatic limit: $\dot\lambda \to 0$

time evo is generated by $H(\lambda)$

$\Rightarrow$ $H_{CD} = H(\lambda) + \dot\lambda A_\lambda$ interpolates b/w infinitely slow adiabatic evo & evo generated by gauge pot.

Examples:

1) two-level system: $H(\lambda) = \Delta \sigma^z + \lambda(t)\, \sigma^x$

i) cheap trick: use $\tilde{A} = U^\dagger i\partial_\lambda U$ & $U^\dagger H U = \text{diag.}$

<u>note</u>: $H(\lambda)$ is real-valued $\Rightarrow$ e'states real

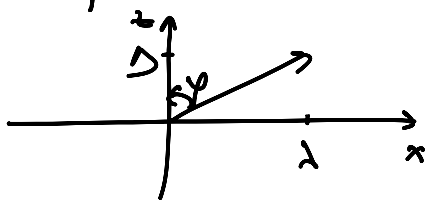
$\Leftrightarrow$ U is orthogonal

most general orthogonal transf. for 2LS

$$U = e^{i f(\lambda)\, \sigma^y}$$

: . σ^y is imaginary-values
. overall factor i

Bloch sphere $\to$ zx-plane



$$U(\lambda) = e^{-i\varphi(\lambda,\Delta)\frac{\sigma^y}{2}}$$

s.t. $\tan\varphi = \dfrac{\lambda}{\Delta}$

ii) need $\tilde{A}_\lambda = U_\lambda^\dagger\, i\partial_\lambda U_\lambda = U^\dagger \partial_\lambda \varphi \dfrac{\sigma^y}{2} U = \dfrac{1}{2}\partial_\lambda \varphi\, \sigma^y$

$$\partial_\lambda \varphi = \partial_\lambda \arctan\left(\frac{\lambda}{\Delta}\right) = \frac{1}{\Delta}\,\frac{1}{1+(\lambda/\Delta)^2} = \frac{\Delta}{\Delta^2+\lambda^2}$$

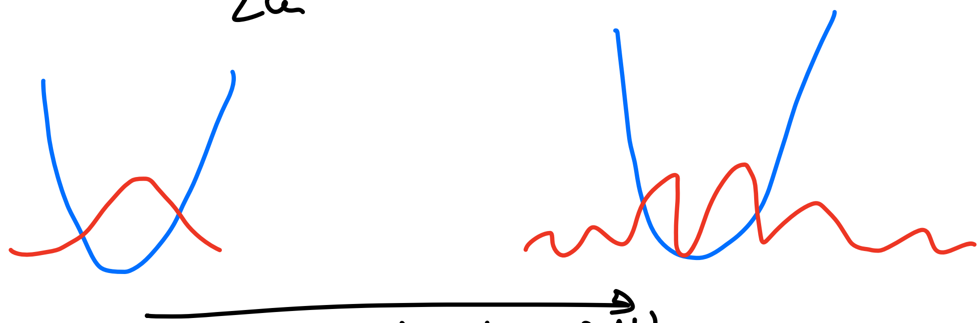
$$\Rightarrow \tilde{A}_\lambda = \frac{1}{2}\,\frac{\Delta}{\Delta^2+\lambda^2}\,\sigma^y \underset{[A_\lambda,\, U_\lambda]=0}{=} A_\lambda$$

iii) construct H_{CD}:

$$H_{CD}(\lambda(t)) = \Delta\sigma^z + \lambda(t)\,\sigma^x + \frac{\dot{\lambda}(t)}{2}\,\frac{\Delta}{\Delta^2+\lambda^2(t)}\,\sigma^y$$

2) particle in a moving harmonic potential

$$H(\lambda) = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 \left(x - \lambda(t)\right)^2$$



i) find a frame to eliminate drive $\lambda(t)$ from harmonic pot.

=> co-moving frame

$U_\lambda = e^{-i\lambda p}$ (momentum op. p generates translations)

ii) $$H_{\text{co-mov}} = \tilde{H} = U_\lambda^\dagger \left(\frac{p^2}{2m} + \frac{1}{2} m\omega^2 (x-\lambda)^2 \right) U_\lambda - \dot{\lambda} U_\lambda^\dagger \, i\partial_\lambda U_\lambda$$

$$= \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 - \dot{\lambda} p$$

$$= \underbrace{\omega\left(\hat{n} + \frac{1}{2}\right)}_{\text{diag. ✓}} - \underbrace{\dot{\lambda} p}$$

$\Rightarrow \tilde{A}_\lambda = p = A_\lambda$

gauge pot. is the generator of translations (translating harmonic pot.)

iii) $$H_{CD} = \frac{p^2}{2m} + \frac{m\omega^2}{2}\left(x - \lambda(t)\right)^2 + \dot{\lambda} p$$

→ potential issue w.r.t. experimental implementation:

· difficult to couple to p

→ apply gauge transf. : $p \to p' = p + m\dot{\lambda}$

$x \to x' = x$

$$\Rightarrow H'_{CD} = \frac{p'^2}{2m} + \frac{m\omega^2}{2}\left(x' - \lambda(t)\right)^2 - m\ddot{\lambda}x' - \frac{m}{2}\dot{\lambda}^2$$

all drives couple to position op.

$\Rightarrow$ can be realized in exp.

requirement: want
$$\left.\begin{aligned} p(0) &= p'(0) \\ p(T) &= p'(T) \end{aligned}\right\} \overset{CD}{\underset{\text{gauge transf.}}{\Rightarrow}} \begin{aligned} \dot{\lambda}(0) &= 0 = \dot{\lambda}(T) \\ \ddot{\lambda}(0) &= 0 = \ddot{\lambda}(T) \end{aligned}$$