

Femtosecond photoionization of atoms under noise

Kamal P. Singh and Jan M. Rost

Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, 01187 Dresden, Germany

(Received 3 September 2007; published 6 December 2007)

We investigate the effect of incoherent perturbations on atomic photoionization due to a femtosecond midinfrared laser pulse by solving the time-dependent stochastic Schrödinger equation. For a weak laser pulse which causes almost no ionization, an addition of a Gaussian white noise to the pulse leads to a significantly enhanced ionization probability. Tuning the noise level, a stochastic resonancelike curve is observed showing the existence of an optimum noise for a given laser pulse. Besides studying the sensitivity of the obtained enhancement curve on the pulse parameters, such as the pulse duration and peak amplitude, we suggest that experimentally realizable broadband chaotic light can also be used instead of the white noise to observe similar features. The underlying enhancement mechanism is analyzed in the frequency domain by computing a frequency-resolved atomic gain profile, as well as in the time domain by controlling the relative delay between the action of the laser pulse and noise.

DOI: [10.1103/PhysRevA.76.063403](https://doi.org/10.1103/PhysRevA.76.063403)

PACS number(s): 32.80.Rm, 02.50.Ey, 42.50.Hz

I. INTRODUCTION

The role of noise on driven quantum systems is a subject of considerable interest and importance. Several studies exist in the literature on this broad topic. For instance, the noise-induced effects in the nanoscale quantum devices such as Josephson junctions [1,2], the macroscopic phase transitions due to quantum fluctuations [3], driven multilevel quantum systems under incoherent environment [4], and stochastic ionization of Rydberg atoms by microwave noise [5], to mention just a few. In these examples, the action of noise on a system can be broadly classified as being of two types: either of destructive nature, i.e., noise must be avoided; or of constructive nature necessitating its presence.

It is this nontrivial latter aspect of noise-induced effects that has been the subject of intense investigation [1,2,6,7]. In particular, the stochastic resonance phenomenon (SR) provides a paradigm for the constructive role of noise in nonlinear classical as well as quantum systems [7]. The essence of quantum SR is the existence of an optimum amount of noise in a nonlinear system that enhances its response to a weak coherent input forcing [8–11]. Despite the diversity of nonlinear dynamics exploiting classical SR, most of the quantum mechanical studies of the effect have focused on the so-called spin-Boson model, which provides an analog of a classical double-well potential [1,2,12,13]. However, many other physical systems exist, particularly atoms or molecules exposed to strong laser pulses, where the quantum dynamics can be nonlinear and therefore added noise could play an important role.

The presence of noise is also worth studying from the point of view of steering quantum dynamics of atomic or molecular systems [14–16]. Traditionally this is achieved with strong, tailored laser pulses by exploiting their nonperturbative and nonlinear interaction with the atomic systems. In this context, it has been shown that weak noise of various origin in multilevel ladder systems plays a crucial role [14]. Many scenarios have been discussed, such as the need to either cooperate or fight with decoherence in the closed-loop control [17], and to engineer the environment to achieve the

steering of quantum systems toward a desired state [18]. In the same spirit, white shot noise has been used to dissociate diatomic molecules [19]. This has implications for the field of quantum control. Motivated by the concept of exploiting noise in nonlinear systems, one can ask the question if noise can serve as an extra tool for quantum control. Indeed, the concept of the quantum SR effect has not been exploited for additional insight in controlling the quantum phenomenon.

In this paper, we provide a detailed study of the influence of noise in a generic quantum situation, namely, the photoionization of a single-electron atom interacting with an ultrashort laser pulse. We will demonstrate the conditions under which a resonancelike behavior emerges in the stochastic photoionization process. This noise-induced phenomenon is studied for a variety of laser pulses, from a few optical cycles duration to very long ones, and of varying intensities. Furthermore, we suggest the experimental observability of the effect by employing a broadband chaotic light instead of white Gaussian noise. Last, we characterize the underlying gain mechanism in the frequency domain in order to identify the crucial frequency bands in the broad noise spectrum.

The paper is organized as follows. Section II introduces our model of the simplest atom interacting with a femtosecond laser pulse and white noise, and describes our method to solve its stochastic Schrödinger equation. In Sec. III, we show the results of the ionization probability (IP) for various combinations of the laser pulse and noise. The existence of a stochastic resonancelike behavior is quantified using an enhancement factor, which is computed from IP. The sensitivity of this noise-induced effect is tested with laser pulses of varying duration and strength, and other types of noise such as a realizable broadband chaotic light. To characterize the enhancement mechanism, we compute the frequency-resolved gain profile of the driven atom, and study the role of relative time delay between noise and the laser pulse. Finally, Sec. IV provides a summary of results with our conclusions.

II. DESCRIPTION OF THE MODEL

A. Hydrogen atom interacting with a laser pulse and noise

We consider as an example the simplest single-electron atom, i.e., the hydrogen atom. Due to the application of an

intense linearly polarized laser field $F(t)$, the electron dynamics is effectively confined in one dimension along the laser polarization axis [20]. The Hamiltonian for such a simplified description of the hydrogen atom, which is here also perturbed by a stochastic force $\xi(t)$ [21], reads as (atomic units $\hbar=m=e=1$ are used unless stated otherwise)

$$H(x,t) = \frac{\hat{p}^2}{2} + V(x) + x\{F(t) + \xi(t)\}, \quad (1)$$

where x is the position of the electron and $\hat{p}=-i\partial/\partial x$ is the momentum operator. The external perturbations $F(t)$ and $\xi(t)$ are dipole coupled to the atom. The potential is approximated by a nonsingular Coulomb-type form,

$$V(x) = -\frac{1}{\sqrt{x^2 + a^2}}. \quad (2)$$

Such a soft-core potential with parameter a has been routinely employed to study atomic dynamics in strong laser fields [22]. It successfully describes many experimental features of multiphoton or tunnel ionization [20], and the observation of the plateau in higher harmonic generation spectra [22].

The laser field is a nonresonant midinfrared (MIR) femtosecond pulse described as

$$F(t) = f(t)F_0 \sin(\omega t + \delta). \quad (3)$$

Here F_0 defines the peak amplitude of the pulse, ω denotes the angular frequency, and δ is the carrier-envelope phase. We choose a smooth pulse envelope $f(t)$ of the form

$$f(t) = \begin{cases} \sin^2[\pi t/(2\tau)], & t < \tau \\ 1, & \tau \leq t \leq T_p - \tau \\ \cos^2[\pi(t + \tau - T_p)/(2\tau)], & T_p - \tau < t \leq T_p, \end{cases}$$

where T_p is the pulse duration and τ is the time for turning the field on and off.

The noise term $\xi(t)$ is a zero-mean Gaussian white noise having the following properties:

$$\langle \xi(t) \rangle = 0, \quad (4)$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'), \quad (5)$$

and noise intensity D [23].

B. Stochastic quantum dynamics

The presence of the stochastic forcing term in the Hamiltonian as described above makes the quantum evolution non-deterministic. Thus an averaging over a large number of realizations of the stochastic force is required in order to produce a statistically meaningful solution of the following time-dependent stochastic Schrödinger equation:

$$i\frac{\partial\Psi(x,t)}{\partial t} = H(x,t)\Psi(x,t). \quad (6)$$

For a given realization, the numerical solution of the Schrödinger equation amounts to propagating the initial

wave function $|\Psi_0\rangle$ using the infinitesimal short-time stochastic propagator,

$$U_\xi(\Delta t) = \exp\left(-i\int_t^{t+\Delta t} H(x,t)dt\right). \quad (7)$$

One can compute $U_\xi(\Delta t)$ using the split-operator fast Fourier algorithm [24]. Details of the method employed are described in the Appendix. Successive applications of the stochastic propagator $U_\xi(\Delta t)$ advance $|\Psi_0\rangle$ forward in time.

Note that the initial state $|\Psi_0\rangle$ is always chosen to be the *ground state* of the system having an energy of $I_b = -0.5$ a.u. This is obtained by the imaginary-time relaxation method for $a^2=2$ [20]. To avoid parasitic reflections of the wave function from the grid boundary, we employ an absorbing boundary [24].

The ionization flux leaking in the continuum on one side is defined as [25]

$$J_R(x_R,t) = \text{Re}[\Psi^*\hat{p}\Psi]_{x_R}, \quad (8)$$

where x_R is a distant point (typically 500 a.u.) near the absorbing boundary. The ionization rate is integrated over a sufficiently long time interval to obtain the corresponding total ionization probability,

$$P = \int_0^\infty J_R(x_R,t)dt. \quad (9)$$

In the following section, we shall use both the ionization flux and the photoionization yield to study the interplay between the laser pulse and noise. From the point of view of stochastic resonancelike phenomena, we aim at establishing the constructive role of noise in atomic photoionization due to a femtosecond laser pulse.

III. RESULTS

A. Optimal stochastic enhancement of photoionization

1. Photoionization as a nonlinear effect

Let us first consider the response of the atom interacting with a short but strong laser pulse only. Figure 1 shows the ionization probability P_I versus the peak pulse amplitude F_0 for a 20-cycle-long MIR laser pulse ($\omega=0.057$). This figure shows that with increasing values of F_0 the ionization probability first increases nonlinearly, and then saturates to the maximum value of unity, for $F_0 > 0.05$. The behavior of $P_I(F_0)$ is a characteristic signature for many atomic and molecular systems interacting with nonresonant intense laser pulses [22].

The laser pulse produces (nonlinear) ionization of the atom, which is most easily understood, especially in the time domain, with the picture of a periodically changing tunneling barrier. Ionization flux is produced close to those times when the effective potential $U(x,t) = V(x) + xF(t)$ is maximally bent down by the dipole-coupled laser field. This is illustrated in the inset of Fig. 1 with the temporal evolution of the ionization flux for laser pulses (shown in the top parts of the inset) with two different peak amplitudes $F_0=0.05$ and $F_0=0.02$

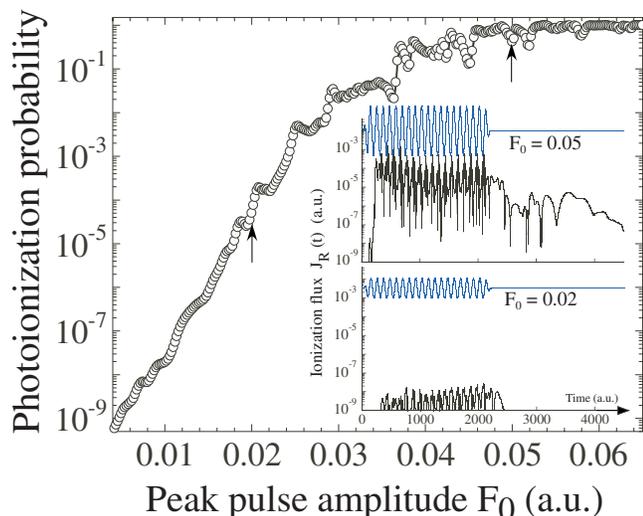


FIG. 1. (Color online) The ionization probability P_I as a function of the laser peak amplitude F_0 . The insets show the ionization flux versus time for two different pulses (top parts of the curves) of amplitudes, $F_0=0.05$ and $F_0=0.02$, marked by arrows on the IP curve. Here $\omega=0.057$, $\delta=0.0$, $T_p=20\pi/\omega$, and $\tau=2\pi/\omega$.

(see the arrows in Fig. 1). Time-resolved ionization peaks separated by the optical period ($2\pi/\omega$) are clearly visible for both peak field amplitudes. In addition, $J_R(t)$ shows a complex interference pattern (the inset of Fig. 1) due to the modulated Coulomb barrier for $F_0=0.05$. However, quite strikingly, if F_0 is reduced to 0.02 a.u., the ionization flux collapses by around five orders of magnitude as shown in Fig. 1. One can therefore conclude that the photoionization dynamics is highly nonlinear, and in particular, it exhibits a form of “threshold” dynamics where the threshold is created by the condition for over-the-barrier ionization.

2. Ionization induced by noise alone

Here we look into the possibility of efficiently ionizing the atom, when it is subjected to white Gaussian noise only. The interaction time of the atom with the noise is kept identical to the laser pulse duration T_p (see the inset of Fig. 3). Figure 2 shows the evolution of the ionization flux $\langle J_R(t) \rangle$, which is averaged over 50 different realizations of the noise. One can see that for small noise amplitudes the ionization flux exhibits a featureless curve, producing the ionization flux around 10^{-9} . As the noise level is increased, the featureless ionization curve rises monotonically as shown in Figs. 2(a)–2(c). By integrating the stochastic ionization flux, one can compute the corresponding ionization probability P_n , which is simply equal to the area under the curve $\langle J_R(t) \rangle$. The resulting noise-induced ionization probability P_n versus the noise amplitude is shown in Fig. 3. As can be seen the stochastic ionization probability rises monotonically with the noise level. For ultraintense noise, such that its strength becomes comparable to the atomic binding field, obviously full ionization can be achieved. We should mention that similar effects have been observed in other systems, for example, the purely noise-induced molecular dissociation [19], and the

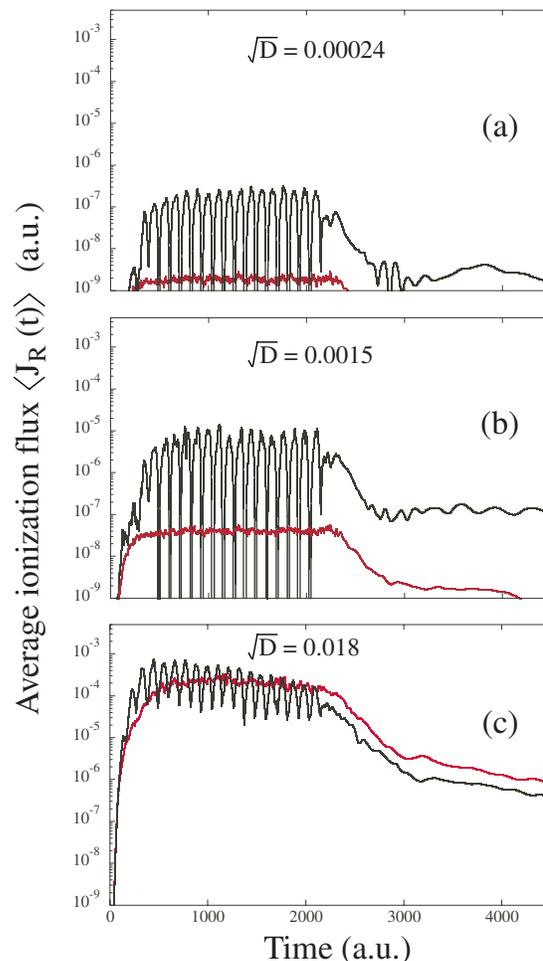


FIG. 2. (Color online) Ionization flux for a weak laser pulse $F_0=0.02$, with three values of noise amplitude: (a) $\sqrt{D}=0.00024$, (b) 0.0015, and (c) 0.018. Background featureless curves (red) show the corresponding purely noise-driven ($F_0=0$) flux. The flux is averaged over 50 realizations.

ionization induced by weak noise of the highly excited Rydberg atoms [5]. However, in our case we consider the atom to be initially in its ground state.

Although the application of noise alone, or the laser pulse alone, can lead to the atomic ionization, we aim to study whether a combination of both the laser pulse and noise makes the ionization process more efficient as compared to the individual cases.

3. Simultaneous application of the laser pulse and noise

We have seen that the atomic photoionization due to an intense femtosecond laser pulse is a highly nonlinear quantum phenomenon, and in particular, the ionization response collapses for a “weak” laser pulse (see the inset of Fig. 1). Motivated by the quantum SR effect, we wish to explore if the noise can recover the strong periodic ionization flux for the weak laser pulse. To answer this question, in Fig. 2(a) we show the average ionization flux when a small noise of amplitude, $\sqrt{D}=0.00024$, is added to the previously weak laser pulse ($F_0=0.02$). Note that the atomic excitation time by the

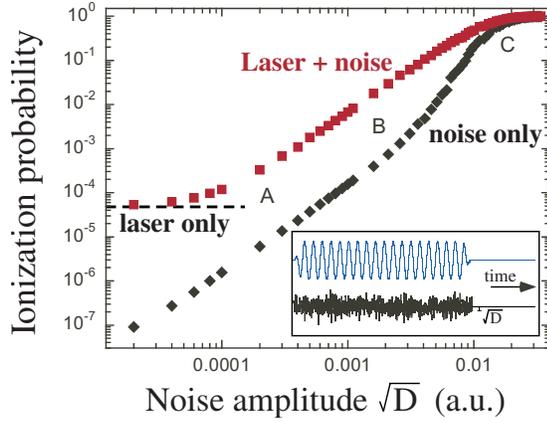


FIG. 3. (Color online) The ionization probability versus the noise amplitude \sqrt{D} for the noise alone P_n (diamonds) and for the laser pulse ($F_0=0.02$) with noise P_{l+n} (squares). The points A–C correspond to the three cases considered in Fig. 2. The dashed line shows the limit of vanishing noise, i.e., the probability due to the laser pulse alone P_l . Inset: An example of laser pulse and noise.

laser and the noise here are identical. One can see that for such a feeble noise amplitude, the periodic structure in atomic ionization gets enhanced by more than one order of magnitude, as compared to the case of the noise alone, which is shown as the background featureless curve. Hence, the observed net enhancement can be attributed to a nonlinear quantum interaction between the coherent pulse and noise.

As the noise level is further increased, we observe an enhancement of the periodic ionization profile by around three orders of magnitude as shown in Fig. 2(b). However, the increase in noise level also causes the background structureless stochastic ionization curve to rise monotonically. For strong noise [Fig. 2(c)], these periodic structures tend to wash out and the process is effectively controlled by the noise. Hence one expects the existence of an intermediate noise level where the nonlinear ionization is optimally enhanced.

The net enhancement of the atomic ionization due to interplay between the laser pulse and the noise can be characterized by the enhancement factor [21],

$$\eta = \frac{P_{l+n} - P_0}{P_0}, \quad (10)$$

with $P_0 = P_l + P_n$. Although this is different compared to the quantifiers commonly used [1,2], η is more suitable for our case. One can verify that a zero value of η corresponds to the case when either the laser pulse ($P_l \gg P_n$) or the noise ($P_l \ll P_n$) dominates. In Fig. 4, we have plotted the enhancement factor η versus the noise amplitude \sqrt{D} . It exhibits a sharp rise, followed by a maximum at a certain value of the noise (point B), and then a gradual falloff. It is worth mentioning that only a modest noise-to-laser ratio ($\sqrt{D}_{opt}/F_0 = 0.075$) is required to reach the optimum enhancement (here $\eta_{max} = 36$).

Before investigating other properties of the enhancement effect, it is worth making three remarks. First, although the

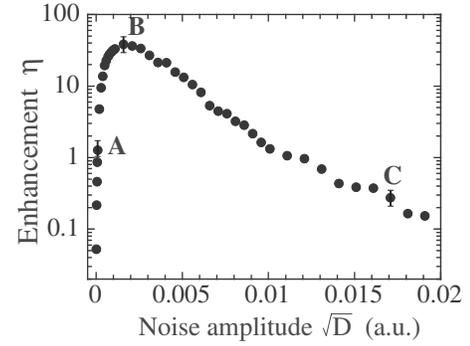


FIG. 4. The enhancement in photoionization due to quantum SR. The points marked A–C correspond to the noise amplitudes of Figs. 2(a)–2(c), respectively. At point B the ratio $\sqrt{D}_{opt}/F_0 = 0.075$. The error bars indicate the standard deviation of η calculated using more than 1000 different noise realizations.

enhancement curve bears a striking resemblance to the typical SR curve, it is not the SR effect where the matching of the time scales between coherent and incoherent driving exists. The underlying gain mechanism here is completely different, as we shall see later. One can perhaps call this as a generalized quantum SR for such atomic systems, in the sense of the existence of an optimum noise level. Second, the location of the optimum enhancement is governed by an empirical condition, when the strengths of the laser pulse and noise are comparable, in terms of the ionization flux produced by their individual action $P_l \sim P_n$. This can be verified in Fig. 3 for the enhancement curve shown in Fig. 4. Third, due to the presence of the random noise term in the Hamiltonian, the optimal solution is only statistically unique. We have computed the standard deviation of the enhancement factor η using 1000 realizations, $\sigma = \sqrt{\langle \eta^2 \rangle - \langle \eta \rangle^2}$. The corresponding σ values are shown by error bars on the η curve in Fig. 4.

4. Enhancement curves for a variety of laser pulses

In this section, we study the sensitivity of the stochastic enhancement curves on the laser pulse. In particular, we study the role of two parameters: (i) the pulse duration T_p , and (ii) the peak pulse amplitude F_0 . In Fig. 5, we have plotted enhancement curves versus the noise amplitude for pulses of fixed amplitude ($F_0 = 0.02$) but of varying durations from 5 to 30 optical cycles. One can clearly see that the enhancement features (particularly the location and strength of the optima) are robust for pulses ranging from ultrashort few cycles duration to quite long ones.

Although we do not show, we have also verified that variation in the carrier envelope phase δ of the laser pulse $F(t)$ [see Eq. (3)] does not modify the enhancement effect. This can be expected due to the presence of the noise term, by which any effect of δ is averaged out. Furthermore, we have also observed similar enhancement curves for other forms of the pulse envelope, such as $f(t) = \sin^2(\pi t/T_p)$.

To investigate the dependence of η on the laser pulse amplitude F_0 , we choose some moderate noise amplitude value, for example, $\sqrt{D} = 0.0015$. For this fixed \sqrt{D} , we now

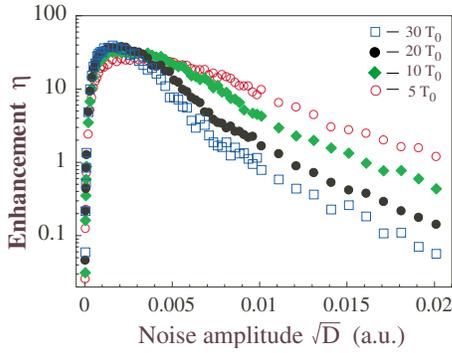


FIG. 5. (Color online) The enhancement factor versus the noise amplitude for four different values of pulse durations T_p : 5, 10, 20, and 30 optical period T_0 . Here $F_0=0.02$, and other parameters are the same as in previous figures.

increase the peak pulse amplitude F_0 of the 20 cycles pulse from zero to a large value, and calculate the IP for each value of F_0 . The obtained probabilities for different cases are plotted in Fig. 6(a), which are then used to compute the enhancement factor η in Fig. 6(b). Here, η also exhibits a nonmonotonic feature versus the laser peak amplitude, thus suggesting a range of F_0 , where the addition of noise can be useful. This dependency is intuitively explained. Since for weak laser pulse the process is dominated by the noise and the η collapses. On the other hand, if the laser peak amplitude is too strong (comparable to over-the-barrier ionization threshold), the pulse can ionize the atom by itself, and the noise has no role to play. Thus, it is for the intermediate values of the noise and laser pulse amplitudes where this nonlinear enhancement mechanism can be significant. From Figs. 4 and 6, one can conclude that in order to maximize the net ion-

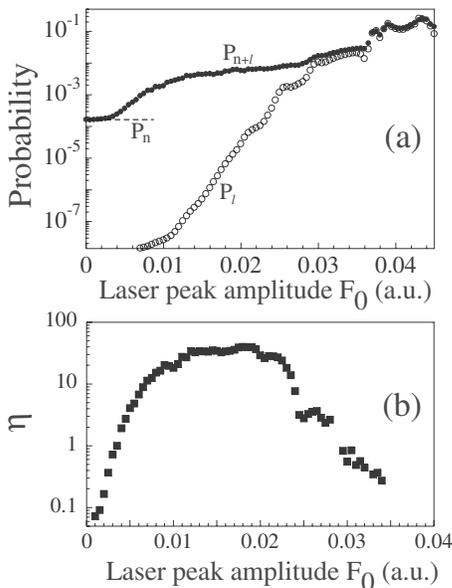


FIG. 6. The dependence of enhancement on F_0 for a fixed noise amplitude $\sqrt{D}=0.0015$. (a) Ionization probabilities versus F_0 for laser only P_n , and for laser with noise P_{n+n} . The dashed line shows the limiting value of P_n . (b) The enhancement versus the F_0 curve obtained from the probability curves shown in (a).

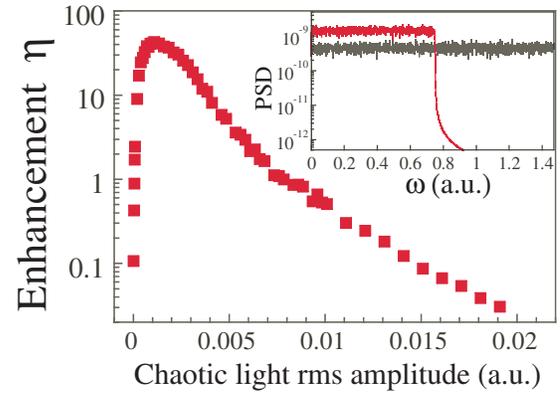


FIG. 7. (Color online) Enhancement induced by a broadband chaotic light. The peak amplitude and frequency of the 20 cycle laser pulse are $F_0=0.02$ and $\omega=0.057$, respectively. The bandwidth of chaotic light is $\Delta\omega=0.75$ with central frequency $\omega_0=0.375$. Inset: power spectral density (PSD) of the chaotic light compared to the one for the white noise.

ization yield, a particular pair of F_0 and \sqrt{D} is required.

B. Employing chaotic light instead of the white noise

1. Generation and characterization of chaotic light

To experimentally observe this effect, the most challenging task is the generation of intense white noise. For instance, if one considers the thermal radiation from a blackbody (such as the sun) as a possible source of the white noise, its noise intensity falls short by many orders of magnitude [26], compared to the one required for the optimum of Fig. 4. We thus look into alternatives for generating a noise-like wave form. One possibility is to employ modern pulse shaping techniques, whereby one can design wave forms of almost arbitrary shapes [27,28]. To realize such a chaotic light, we choose a large number of frequency modes N in a finite but broad bandwidth $\Delta\omega$. These modes can be, for example, different Fourier components of an ultrashort laser pulse. The total electric field $Z(t)$ is a sum of these N individual modes as [29]

$$Z(t) = \sqrt{\frac{2}{N}} \sum_{n=1}^N F_{rms} \sin(\omega_n t + \phi_n), \quad (11)$$

where ω_n, ϕ_n denote the angular frequency and phase of the n th mode, respectively, and F_{rms} is the root-mean-square amplitude of $Z(t)$. Note that here we consider these frequency modes to oscillate independently with their phases ϕ_n assuming random values relative to each other. In this particular case of phase-randomized coherent modes, the total field $Z(t)$ at any point will be noise-like, fluctuating in intensity due to the interference between modes. The inset in Fig. 7 shows an example of such a chaotic light spectrum for $N=1024$ in a chosen bandwidth (BW) of 0.75 (corresponding to a 32 attosecond pulse) [30]. Such a construct tends to the white noise, in the limit of $\Delta\omega, N \rightarrow \infty$. In the following section, we consider a simultaneous application of the weak laser pulse

and chaotic light (instead of the white noise) and see if the enhancement effect can be preserved.

2. Photoionization by the chaotic light

When we replace white noise by the chaotic light ($BW = 0.75$ and $N=1024$), one can see in Fig. 7 that most of the features of the enhancement phenomenon remain intact. In particular, the intensity and location of the optimum is very close to the one obtained for the white noise case in Fig. 4. We also recover other features of the enhancement mechanism with the chaotic light such as its dependence on the pulse duration T_p and pulse amplitude F_0 .

This observation not only suggests the possibility of observing the effect using a finite but broadband chaotic light, but also raises the question concerning the relevant frequency components in the chaotic light spectrum, to be discussed next.

C. Spectral and temporal analysis of the gain mechanism

1. Frequency-resolved gain profile

In this section, we will analyze the mechanism of stochastic enhancement in both the frequency domain and the time domain. In particular, we aim to identify frequency components in the broad spectrum of noise (or chaotic light), which are the crucial ones to provide the gain. For this purpose, we compute a frequency-resolved atomic gain (FRAG) profile using a pump-probe type of setting as described below.

It is well known that when an atom interacts linearly with a weak external field, the energy absorption takes place at its resonant frequencies. However, due to its interaction with a strong laser pulse, the unperturbed atomic states are significantly modified leading to a completely different frequency response of the driven atom. Indeed, such a modified spectral response is the relevant quantity when the atom is also subjected to noise. So, how can one measure precisely the frequency-resolved gain $G(\nu)$ offered by the atom? One possible way to observe FRAG is to consider the previously employed laser pulse ($\omega=0.057$, $F_0=0.02$) as a pump pulse and replace noise by a tunable monochromatic probe pulse, $F_p(t)=f(t)F_p \sin(\omega_p t)$. The probe amplitude F_p is much weaker than the driving laser pulse [$F_p/F_0=0.01$], such that it does not significantly alter the quasienergy levels, but can drive transitions between them. For an atom prepared in its ground state, a FRAG profile is obtained by measuring either the depletion of the ground state population or the net energy absorption, as a function of the probe frequency ω_p .

Such a gain profile $G(\omega_p)$ is shown in Fig. 8 for the atom driven by a ten-cycle-long laser pulse of amplitude $F_0=0.02$. Although the FRAG shows lots of structure, the dominant peaks appear around the first atomic transition frequency. A closer inspection of Fig. 8 reveals that there is a dip at the unperturbed resonance frequency $\omega_{01}=0.267$. The main peak is indeed shifted beyond the linear Stark shift for our model. These shifts and broadening of the atomic resonances are caused by the strong laser pulse, since the field can drive the electron significantly away from the nucleus leading to strong nonlinear perturbation to its bound states.

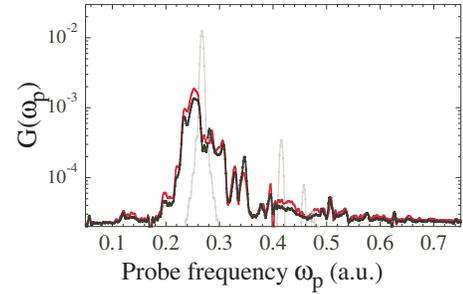


FIG. 8. (Color online) Frequency-resolved gain $G(\omega_p)$ of the atom, which is driven by a ten cycle laser pulse of $F_0=0.02$ at $\omega=0.057$. The atomic gain is probed by a simultaneous application of a weak probe beam $F_p=0.0002$ of tunable frequency ω_p . Both the depletion of the ground state population (thick red line) and the net absorption of energy (black) due to the probe are plotted. The gain of the driven atom is also compared with the bare atomic gain (thin gray line).

Such a FRAG curve provides a fingerprint of the atomic gain under the strong laser pulse. The frequency bands that correspond to the peaks in the obtained gain curve are indeed the most useful ones to obtain the enhancement. In the following, we will test the validity of this statement by designing a chaotic light where significant resonance frequencies are filtered from its spectrum.

2. Chaotic light with missing resonant frequencies

To show that the useful frequencies are *not* simply the resonant frequencies of the atom, we have designed a chaotic light spectrum perforated by digging holes in its spectral density around the first few atomic transition frequencies. As can be seen from Fig. 9, almost no enhancement is lost if the hole width w is below 0.013 a.u., which already includes the linear ac Stark shift of the atomic states. However, by increasing the hole width such that no frequency component exists in the noise spectrum where the FRAG has dominant

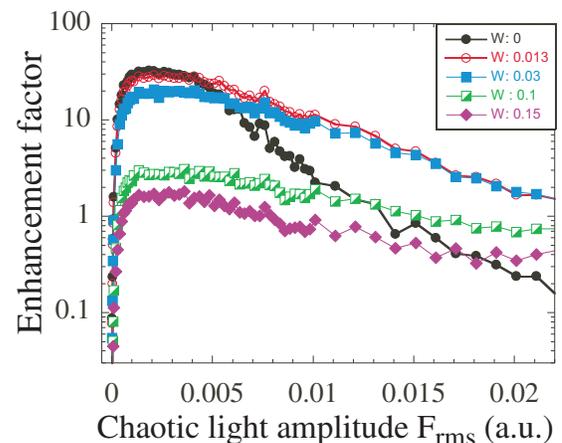


FIG. 9. (Color online) Enhancement curves due to chaotic light spectrum perforated by holes at the first three resonance frequencies. Difference curves correspond to the increasing hole widths of $w=0.0$, 0.013, 0.03, 0.1, and 0.15.

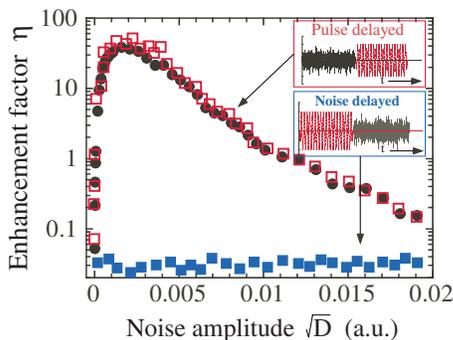


FIG. 10. (Color online) Enhancement factor η versus the noise amplitude \sqrt{D} for various cases, application of noise first and then the laser pulse (empty squares), application of laser pulse first and then noise (filled squares), simultaneous application of laser and noise (circles). The insets depict the schematically temporal signal applied to the atom for corresponding cases.

peaks leads to a collapse of the enhancement mechanism, as shown in Fig. 9. This observation validates the importance of a FRAG structure in identifying the useful spectral bands in noise. It suggests that for the simultaneous presence of the laser pulse and noise, nonresonant frequency components are the dominant ones.

It is worth making a few remarks here. First, the FRAG is a property of the atom interacting with a particular strong laser pulse. Thus, the detailed features of the gain curve depend on both the atomic system and on the laser pulse parameters. But this does not affect the general conclusion drawn from such a curve. Second, it is also possible to obtain the enhancement using a monochromatic beam instead of the chaotic broadband light, if its frequency is properly tuned to the new “resonances.” But, the advantage of using a broadband source is that the enhancement becomes independent of both, the particular atom and the FRAG structure for different pulse parameters.

3. Role of relative delay between signal and noise

Up to now we have considered the case of a perfect synchronization, i.e., a simultaneous application of the laser pulse and noise to observe the enhancement. We now wish to relax this synchronization constraint between the laser pulse and noise to test if the enhancement still exists. Such a scenario would not only help the experimental search of similar effects, but also provides an alternative aspect of the enhancement mechanism as compared to the one mentioned above.

There are two possible ways to expose the atom to a laser pulse and noise sequentially: (i) the atom first interacts with noise only and then we apply the laser pulse, or conversely, (ii) the laser pulse is applied first and then the noise is applied. Note that for both cases there is no direct interplay between laser and noise. The enhancement factor η , which is defined as before, is shown in Fig. 10 for both cases. One can clearly see that for the case (i) the η curve looks very similar to the one for simultaneous action of laser and noise. But for case (ii), the enhancement curve collapses.

Although we obtain almost identical enhancement curves for cases of first noise then laser pulse, and simultaneous action of noise with laser pulse, the frequency components providing the gain are fundamentally different in each case. The gain curves for both cases are given in Fig. 8. For the sequential application of noise and laser, the atomic gain is basically at the resonant frequencies of the unperturbed atom. Thus the resonant frequencies (particularly the first few) are the most fertile ones in the noise spectrum. In this case, a two-step picture of the enhancement mechanism applies, where the atom first absorbs energy from the noise leading to an exponential population distribution, and the laser causes ionization from a “noise-heated” atom in a second step.

IV. SUMMARY AND CONCLUSION

We have investigated photoionization of a hydrogen atom which is subjected to both a MIR femtosecond laser pulse and white Gaussian noise. Due to the inherent nonlinearity of the ionization process, a form of quantum stochastic resonancelike behavior has been observed. This quantum SR leads to a dramatic enhancement (by several orders of magnitude) in the nonlinear ionization when a specific but small amount of white noise is added to the weak few cycle laser pulse. We have further shown the signatures of the enhancement effect for different types of the laser pulses from a few cycles to a few tens of cycles duration. Moreover, if the noise amplitude is kept fixed to some level, and the peak pulse amplitude is varied, again a curve with a specific maximum is obtained for the enhancement parameter. These results suggest the existence of an optimum combination of the laser pulse and noise, if one is interested in optimizing the relative ionization enhancement. The same effect is also achieved if one uses realizable broadband chaotic light instead of white noise. We emphasize that the effect is robust with respect to a range of experimentally accessible parameters such as the pulse duration.

The enhancement mechanism is analyzed in the frequency domain by measuring the frequency-resolved gain profile of the atom under a strong laser pulse, employing a pump-probe type of setting. The frequencies providing the gain are significantly modified from the unperturbed atomic resonances, suggesting the nonresonant nature of the noise absorption. However, if we introduce a relative time delay between the laser pulse and noise the enhancement is still present, provided the noise acts first on the atom. In this case, the useful frequencies in the noise spectrum are at the atomic resonances.

Finally, analogous effects are also expected in other systems provided the following three conditions are fulfilled: (i) The system has a single-well finite binding potential with multiple energy levels, (ii) it can be subjected to a (nonresonant) coherent optical driving, and (iii) it can be subjected to an incoherent perturbation. Since these conditions are sufficiently general, other systems [superconduction quantum interference devices (SQUIDS), molecules, etc.] might also display similar features [31,32].

ACKNOWLEDGMENTS

We thank A. Kenfack, W. Peijie, N. Singh, A. Buchleitner, and P. Hänggi for fruitful discussions.

APPENDIX

In this appendix we briefly present an algorithm for the numerical simulation of the time-dependent stochastic Schrödinger equation [Eq. (6) in the text], where the Hamiltonian is given by Eq. (1). The properties of the white Gaussian noise $\xi(t)$ are defined in Eqs. (4) and (5). Our basic approach is to use the split-operator fast-Fourier transform (SOFFT) method due to Feit and Fleck (which is well known for the deterministic case) [25], and adapt it to the case when the Hamiltonian contains an additional stochastic term $\xi(t)$.

Recalling briefly that if there were no random term, the solution of Eq. (6) can be obtained by defining the standard propagator $U(t_0, t)$, which when applied on initial state wave function $|\Psi_0\rangle$ propagates it forward in time. The usual approach to compute the solution of the Schrödinger equation is to discretize the total propagation time into N small steps of equal intervals Δt . The resulting exact short-time propagator can be written as

$$U_\xi(\Delta t) = \exp\left(-i \int_t^{t+\Delta t} [H_{\text{det}}(x, t) + x\xi(t)] dt\right). \quad (\text{A1})$$

Here, $H_{\text{det}}(x, t) = \hat{p}^2/2 + V(x) + xF(t)$, is the deterministic atomic Hamiltonian including the laser-atom interaction term. In order to incorporate the white noise term within the framework of the SOFFT method, one can rewrite the propagator as

$$U_\xi(\Delta t) = U(\Delta t) \exp\left(-i \int_t^{t+\Delta t} x\xi(t) dt\right), \quad (\text{A2})$$

where $U(\Delta t) = \exp[-iH_{\text{det}}(x, t)\Delta t]$ denotes the deterministic part of the propagator. The stochastic integral in the exponential can be interpreted in the Stratonovitch sense [23] us-

ing the properties of the white Gaussian noise as follows:

$$\int_t^{t+\Delta t} \xi(t) dt = \sqrt{2D\Delta t} X_t, \quad (\text{A3})$$

where X_t is a random number having Gaussian distribution and of unit variance. This makes the propagator a stochastic operator. Note that even in the presence of the noise term the operator is unitary, i.e., it preserves the norm of the wave function.

One can approximate the exact propagator given by Eq. (A2) following a three-step splitting leading to the following expression:

$$U_\xi(\Delta t) = \exp\left(-i \frac{p^2}{2} \Delta t\right) \times \exp[-iV_\xi(x, t)\Delta t] \exp\left(-i \frac{p^2}{2} \Delta t\right), \quad (\text{A4})$$

with $V_\xi(x, t) = V(x) + xF(t) + \sqrt{2D/\Delta t} X_t$. Note that the effect of noise is simulated by inserting at every time step a random number X_t whose statistical properties are described above. The right-hand side of Eq. (A4) is thus equivalent to the free propagation over a half time increment $\Delta t/2$, a random phase change from the action of potential $V_\xi(x, t)$ over the whole time Δt , and an additional free particle propagation over $\Delta t/2$.

This operator splitting is correct up to second order in the time step Δt for the noise-free part, but due to the stochastic integration it is accurate up to only first order for the stochastic part. In the actual calculation Δt is chosen sufficiently small, such that a further reduction in its value does not alter the accuracy of the physical results. For a given realization of the random number sequence entering in the propagator via X_t , one generates a quantum ‘‘trajectory’’ for the wave function. To extract the physical observable, an ensemble average of the desired quantity over a large number of noise realizations is needed. Other simulation parameters such as the grid size and the grid resolutions should be taken as described in the literature [24].

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