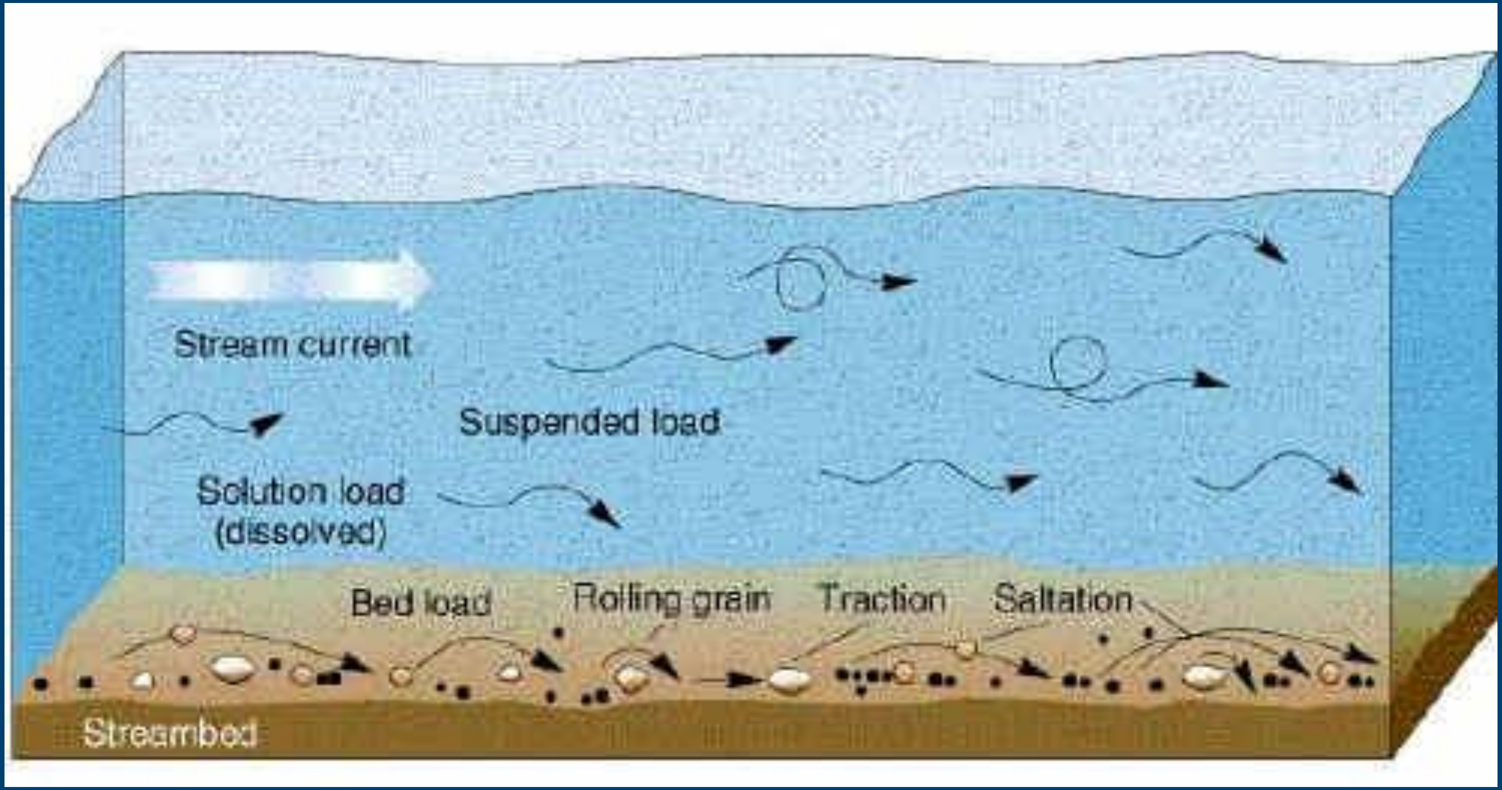
POLITECNICO DI MILANO

From collisional to turbulent-collisional suspensions

Diego Berzi

Outline

- Classification of steady sediment transport
- Pressure-driven collisional suspensions
- Transition to turbulent-collisional suspensions
- Gravity-driven suspensions
- Effective fluid shear viscosity in collisional suspensions



Too many ingredients:

motion regimes, polydispersity, grain shape, type of bed…

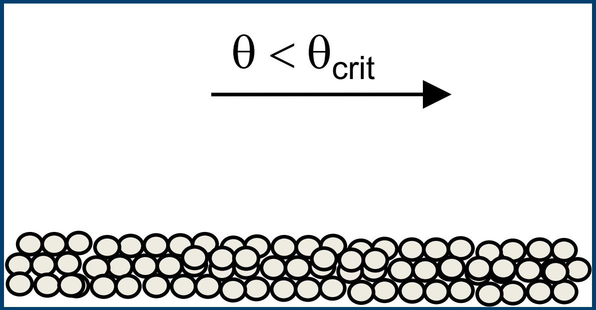
Simplification

Transport of monosized spheres over an horizontal bed of flowing-like particles by a steady, shearing, turbulent fluid

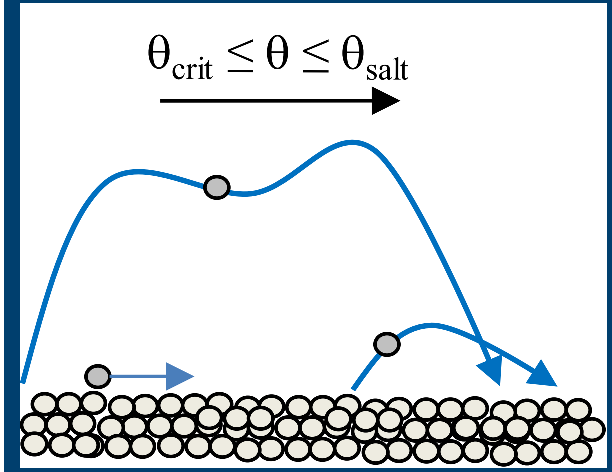
Dimensionless units using particle diameter **d**, particle density ρ_p and reduced gravity $g(\sigma-1)/\sigma$, with σ ratio of particle to fluid density

In dimensionless units, the fluid shear stress is the Shields parameter θ

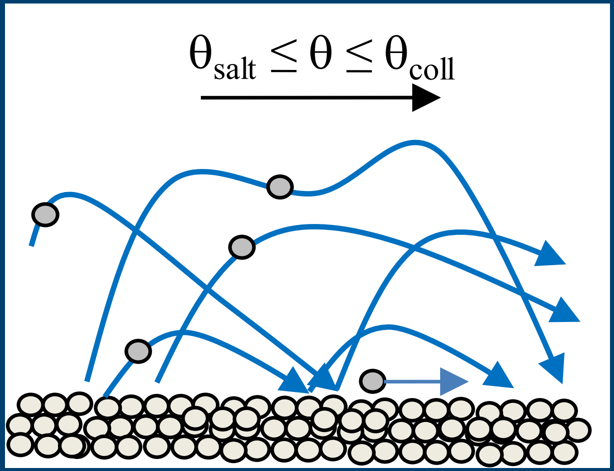
no motion



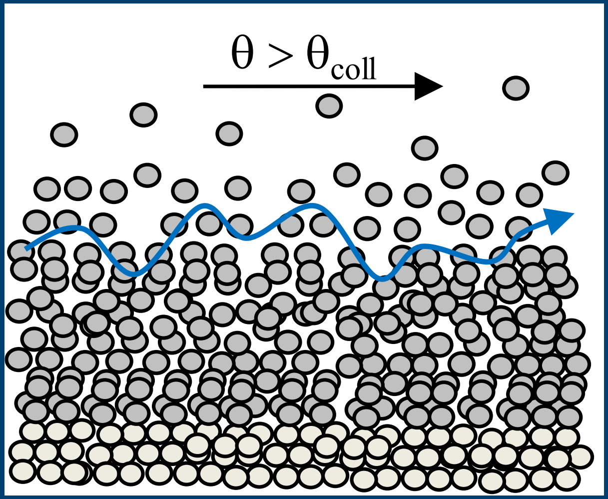
intermittent motion



continuing saltation

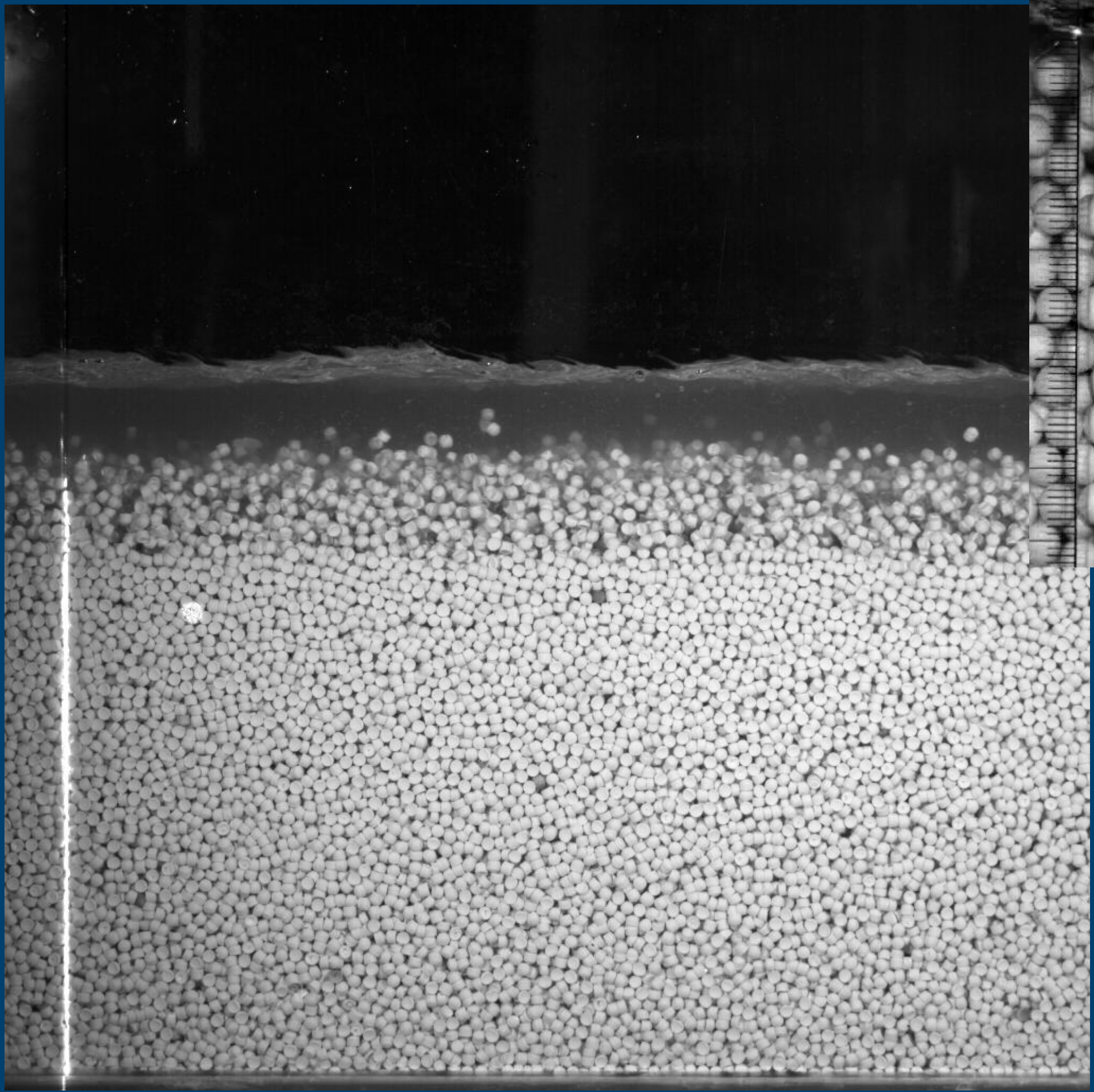


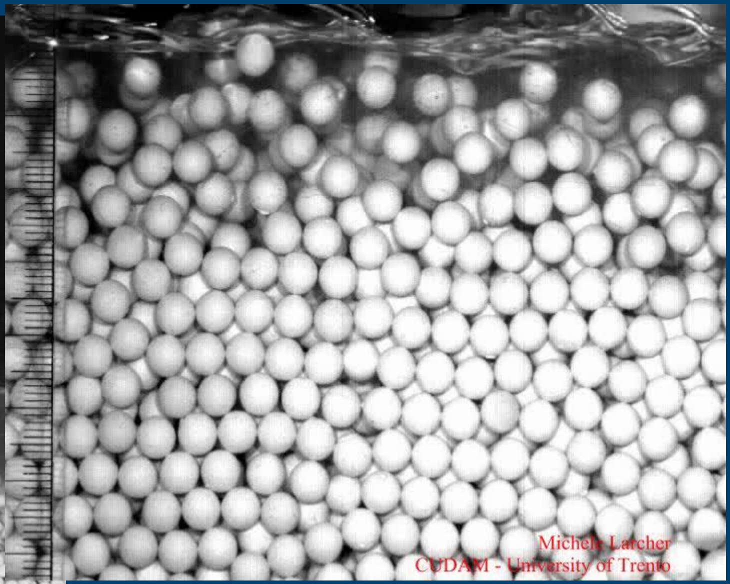
suspension



(ordinary) Bed Load

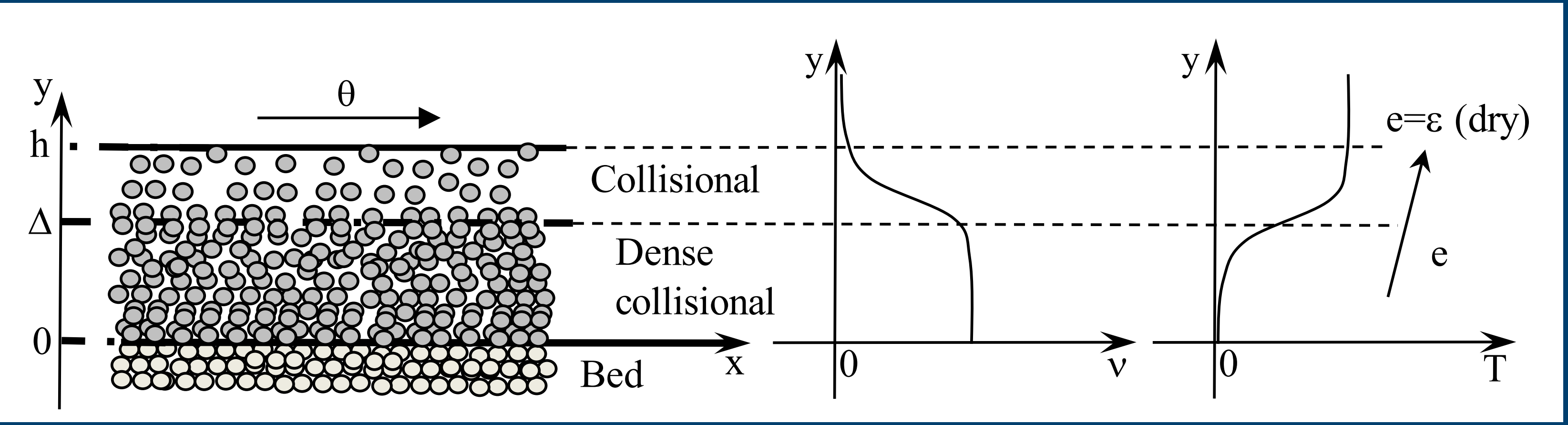
Collisional suspensions: experiments





Plastic cylinders and water (Capart and Fraccarollo, GRL 2011)

Steady, horizontal, collisional sediment transport over an erodible bed in a turbulent fluid (Berzi, JHE 2011, 2013)



T is granular temperature;

ν is solid volume fraction;

e is coefficient of collisional restitution;

u is particle x-velocity

Governing Equations

$$p' = -v$$ Particle y-momentum balance

$$s + S = \theta$$ Boundary layer

$$su' = \Gamma$$ Algebraic energy balance

$$p = f_1(v,e)T$$ Particle pressure (kinetic theory)

$$s = f_2(v,e)T^{1/2}u'$$ Particle shear stress (kinetic theory)

$$\Gamma = \frac{f_3(v,e)}{L}T^{3/2}$$ Dissipation rate (kinetic theory)

Lubrication forces damp collisions:

$$e = \varepsilon - 6.9\frac{1+\varepsilon}{\mathrm{St}}$$

Coefficient of restitution decreases with the Stokes number (Yang and Hunt, PHF 2006)

$$\mathrm{St} \equiv \sigma \mathrm{R} T^{1/2}$$

$$\mathrm{R} \equiv \frac{\rho_f \sqrt{g\frac{\sigma-1}{\sigma}d}\,d}{\eta}$$

$$S = \theta$$

$y = h$ — $s = p = v = 0$

Collisional layer: classic kinetic theory

$$L = 1$$

$y = \Delta$ — $v = 0.49$

Dense collisional layer: extended kinetic theory

$$L \geq 1$$

$$s \approx \theta$$

$y = 0$ — $s / p = \alpha, u = 0,$ $v = v_c$

Erodible bed

Key assumptions

- $s+S \approx \theta$ (boundary layer)
- concentration and velocity linearly distributed in the layers
- turbulence suppressed in the dense layer ($S \approx 0$)
- algebraic balance between production and dissipation of fluctuation energy
- yielding at the bed (s/p has a characteristic value α)

Analytical solution

INPUT

Particle properties: ε, α

Fluid properties: σ, R

Flux strength: $S^* = \theta$ *(Shields number)*

OUTPUT

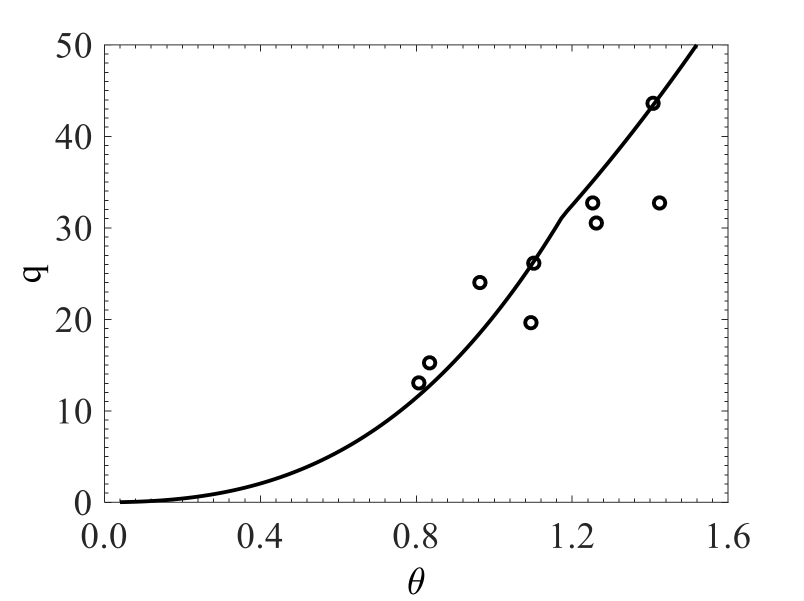
Layer depths

Velocity

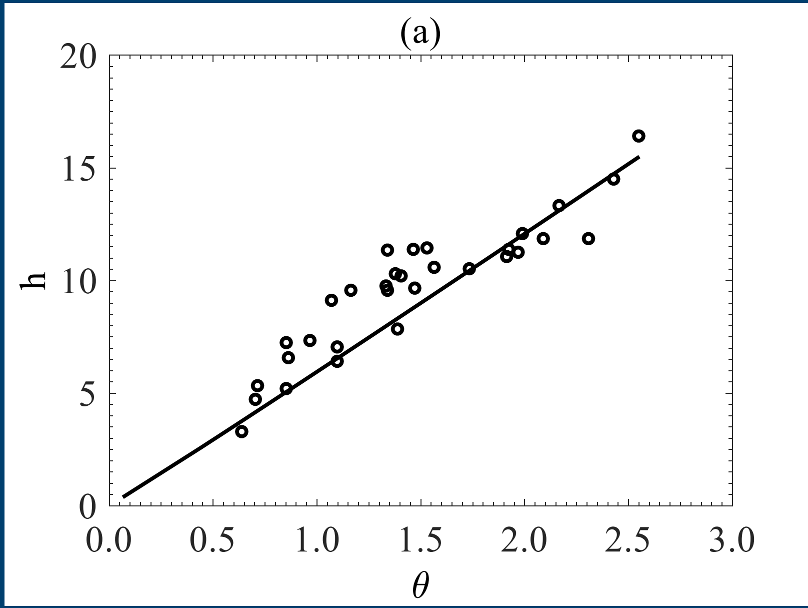
Flow rate

BLACK(GREEN)BOARD PLEASE

Comparisons with experiments



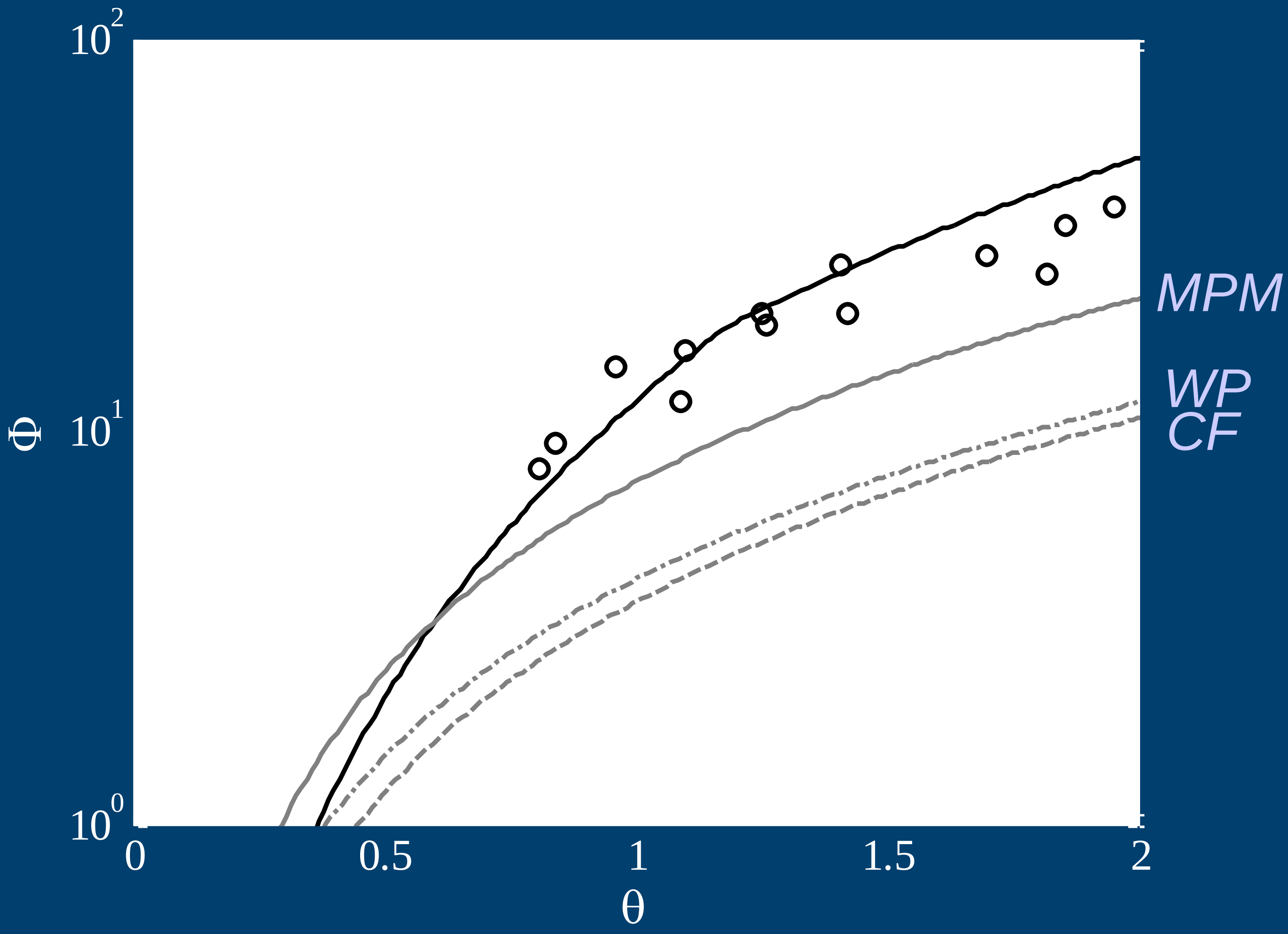
0.7 mm sand in water
(Nnadi and Wilson, JHE 1992)



3 mm plastic cylinders in water (Sumer et al., JHE 1996)

Comparisons with experiments

0.7 mm sand in water (Nnadi and Wilson, JHE 1992)

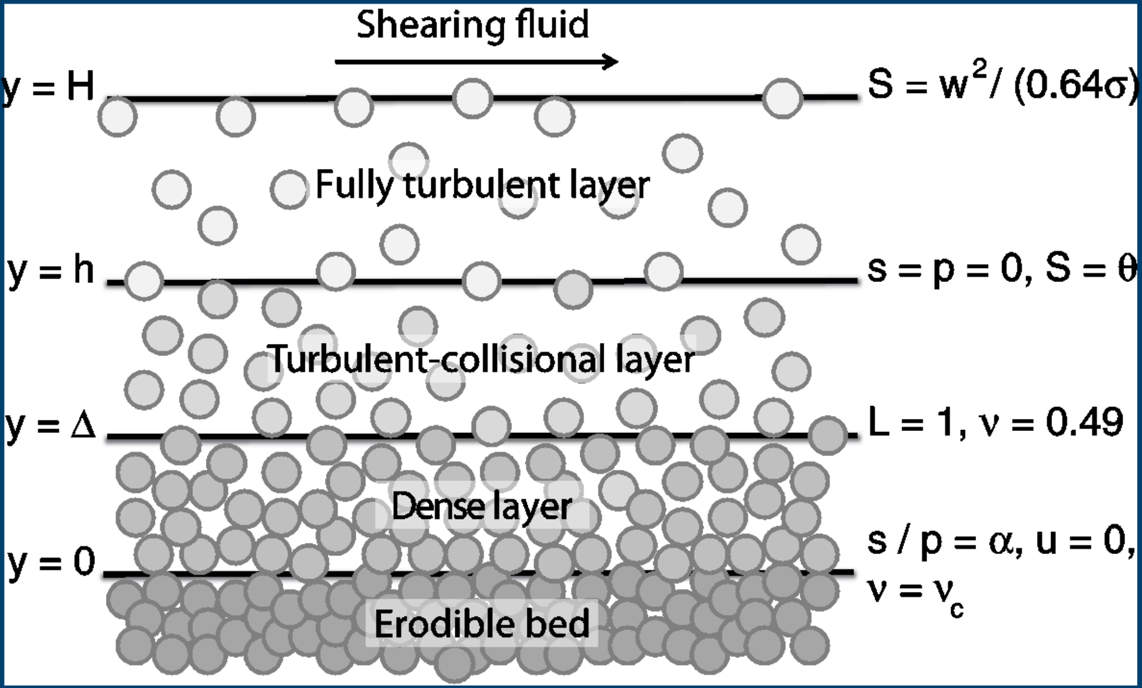


Limits

Particle depth (h) must be at least 1 diameter (otherwise no suspension: ordinary bedload) ⟹ *Minimum value for the Shields number (around 0.2 for sand, i.e. 4 times the critical Shields number)*

Absence of turbulent suspension, i.e., ratio of fluid shear velocity at the top and single particle settling velocity less than 1 ⟹ *Maximum value for the Shields number (around 1.3 for 0.7 mm sand)*

Steady, horizontal, turbulent-collisional sediment transport over an erodible bed in a turbulent fluid (Berzi and Fraccarollo, PHF 2016)



$$p' = -v - C\left(\sigma S\right)^{1/2} l_m v'$$

Particle y-momentum balance (McTigue, JHD 1981)

$$s + S = \theta$$

Boundary layer (not in the FT layer)

$$s u' = \Gamma$$

Algebraic energy balance

$$p = f_1\left(v, e\right) T$$

Particle pressure (kinetic theory)

$$s + S = f_2\left(v, e\right) T^{1/2} u' + \frac{1-v}{\sigma} l_m^{\,2} u'^2$$

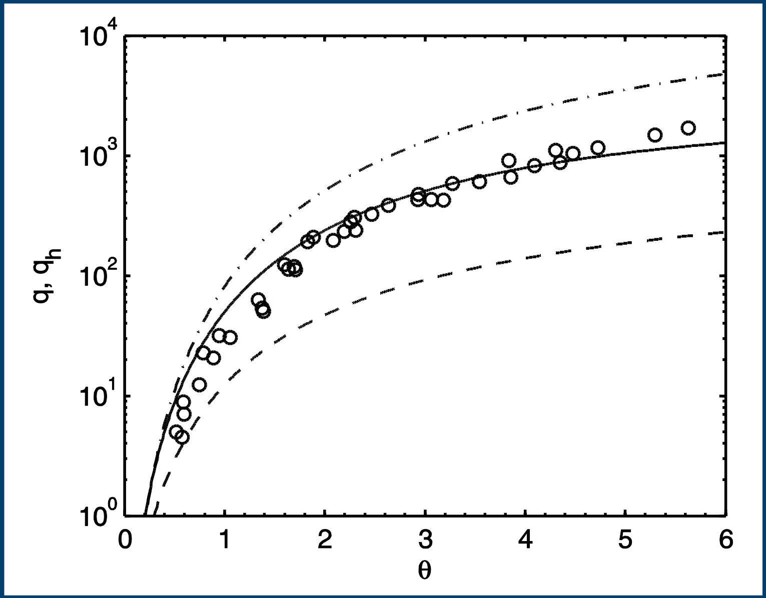
Shear stress (kinetic theory+turbulence)

Nonlocal mixing length

$$\Gamma = \frac{f_3\left(v, e\right)}{L} T^{3/2}$$

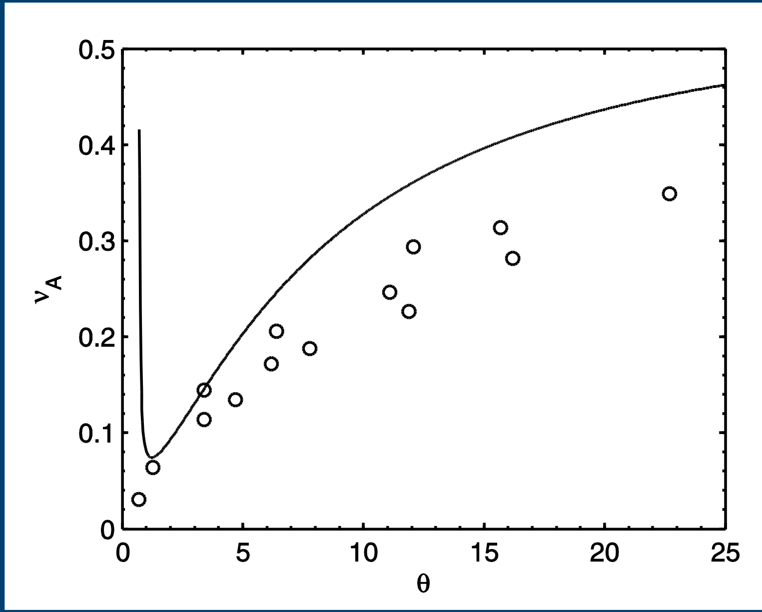
Dissipation rate (kinetic theory)

Comparisons with experiments

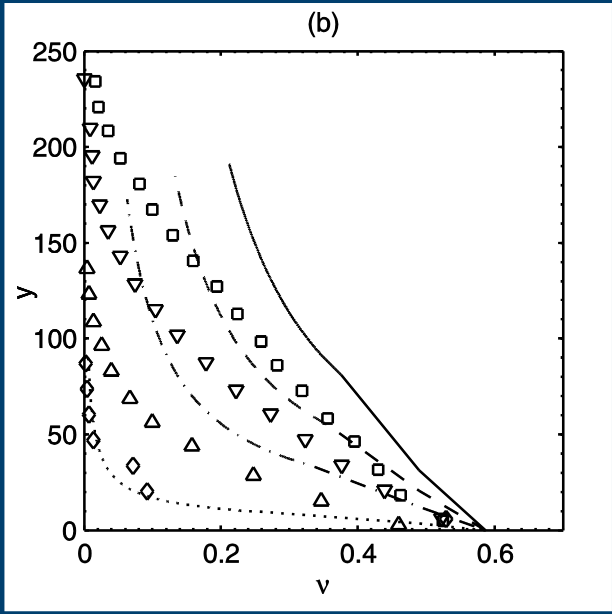


0.18 mm glass spheres in water
(Matousek et al., JHH 2013)

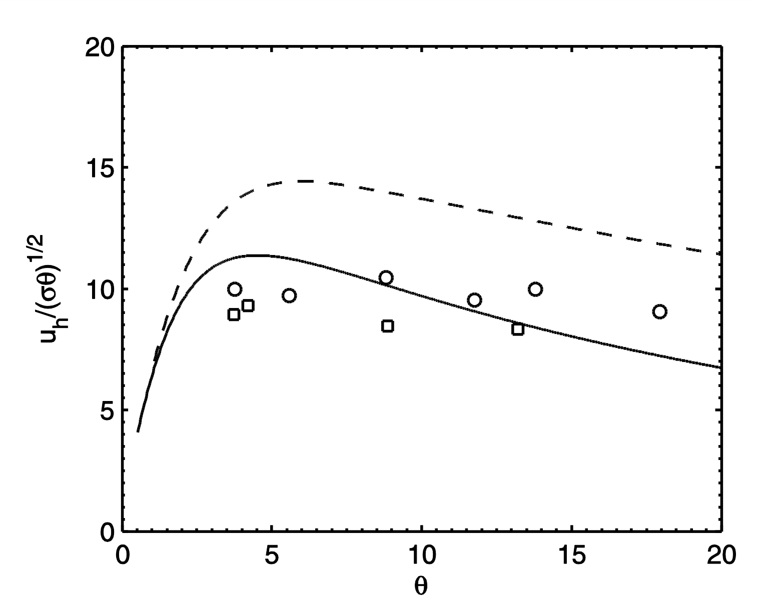
0.37 mm sand in water
(Matousek, JHE 2009)



Comparisons with experiments



0.37 mm sand in water
(Matousek, JHE 2009)



0.3 and 0.56 mm sand in water
(Pugh and Wilson, JHE 1999)

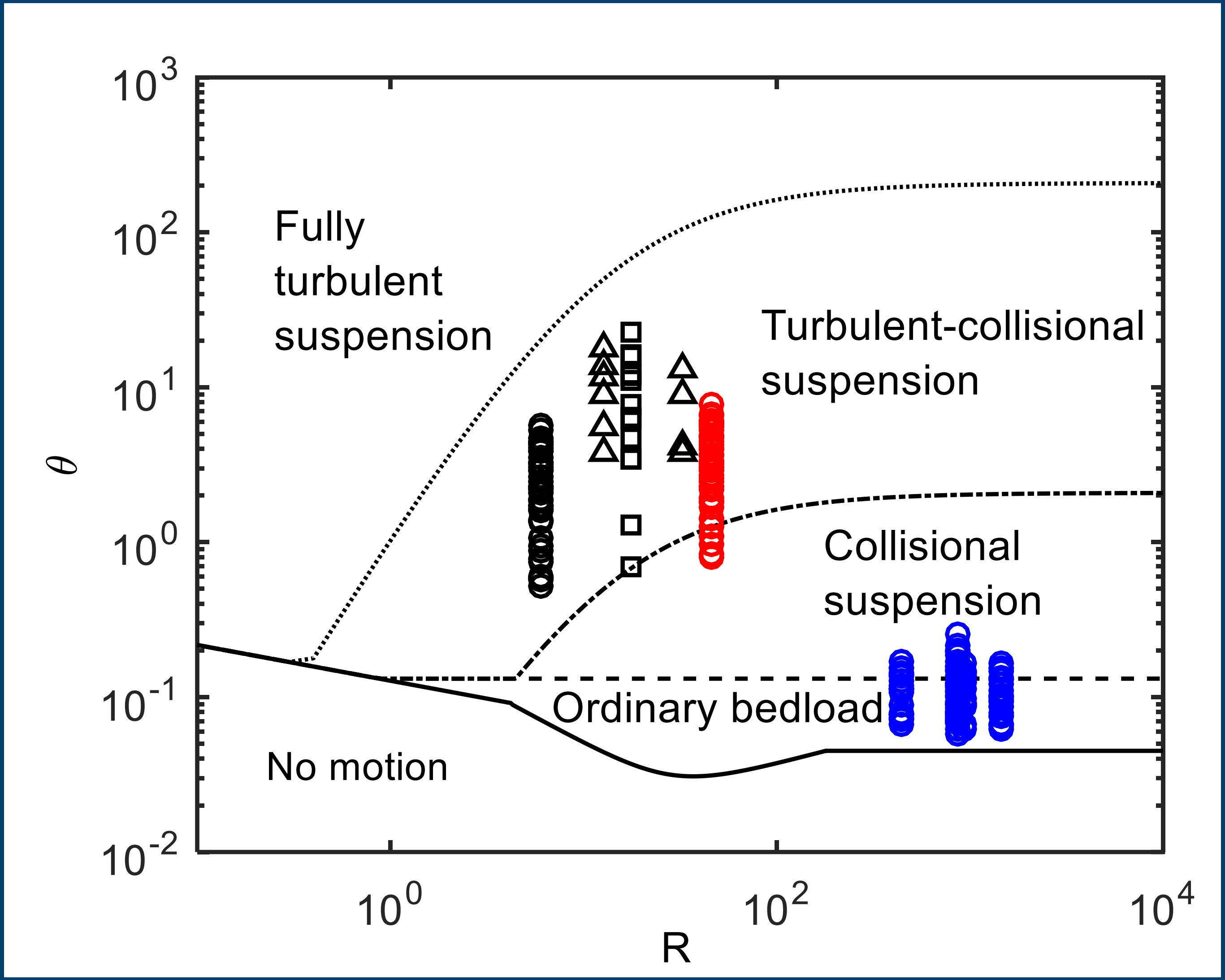
Limits

$h=1$ → Boundary between bedload and collisional suspension

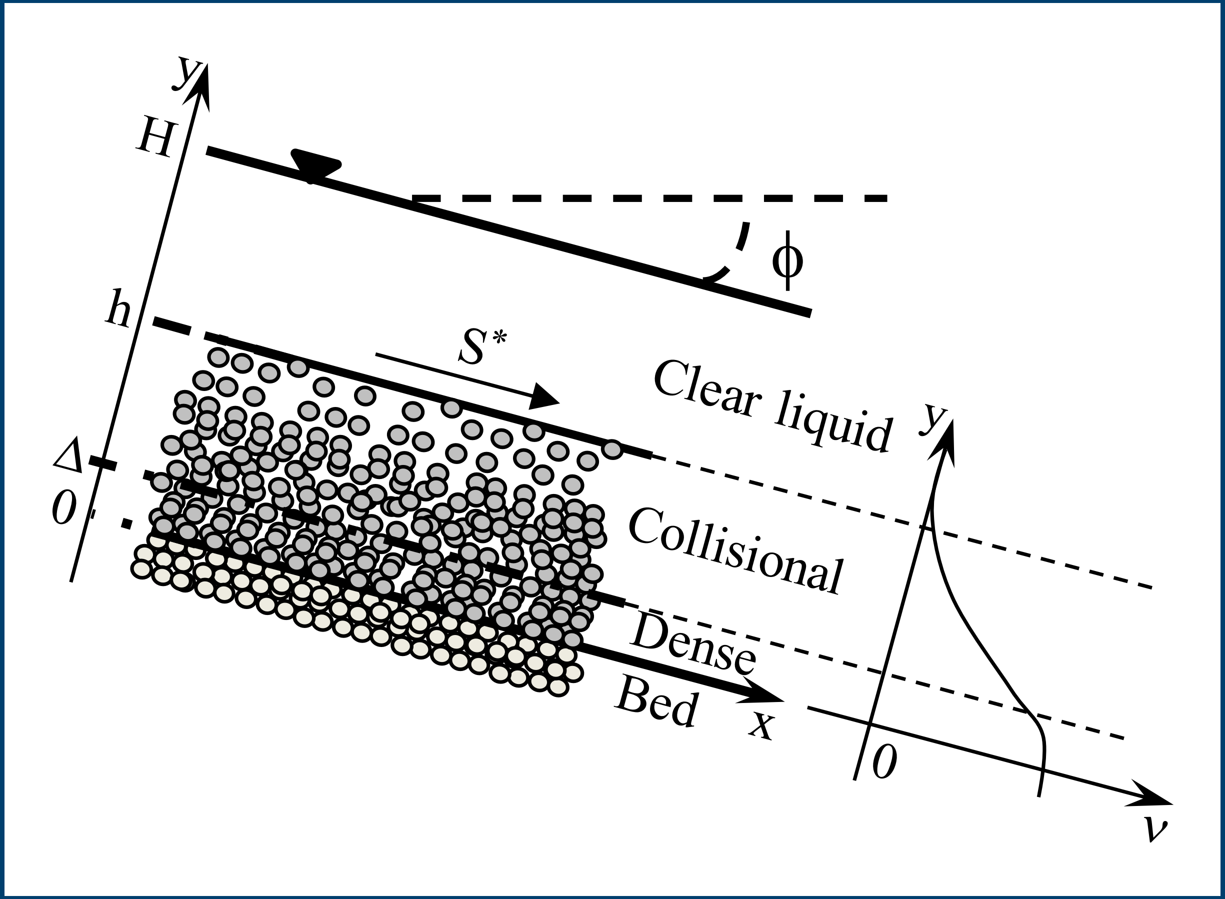
$(\sigma\theta)^{1/2}/w=0.8$ → Boundary between collisional and turbulent-collisional suspensions

$h=\Delta$ → Boundary between turbulent-collisional and fully turbulent suspensions

Regime map 1


10^3
10^2
10^1
10^0
10^-1
10^-2
θ
Fully
turbulent
suspension
Turbulent-collisional
suspension
Collisional
suspension
Ordinary bedload
No motion
10^0
10^2
10^4
R

Steady, inclined, collisional sediment transport over an erodible bed in a turbulent fluid (Berzi and Fraccarollo, PHF 2013)



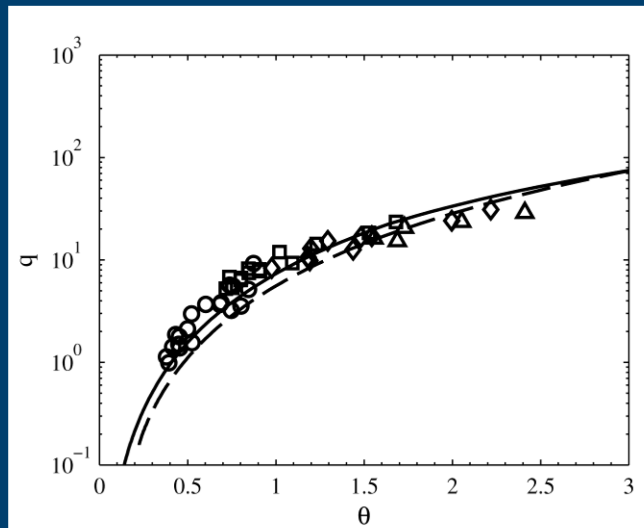
Governing Equations

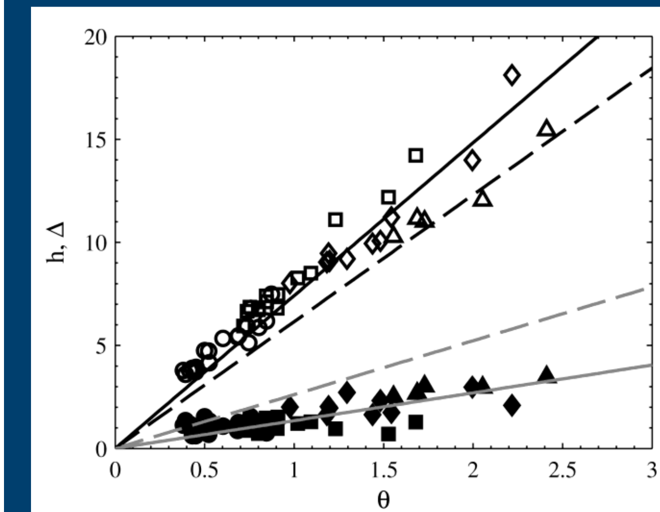
$p' = -\nu \cos\phi$	Particle y-momentum balance
$s' + S' = -\dfrac{1-\nu+\sigma\nu}{\sigma-1}\sin\phi$	x-momentum balance
$su' = \Gamma$	Algebraic energy balance
$p = f_1(\nu, e)T$	Particle pressure (kinetic theory)
$s = f_2(\nu, e)T^{1/2}u'$	Particle shear stress (kinetic theory)
$\Gamma = \dfrac{f_3(\nu, e)}{L}T^{3/2}$	Dissipation rate (kinetic theory)

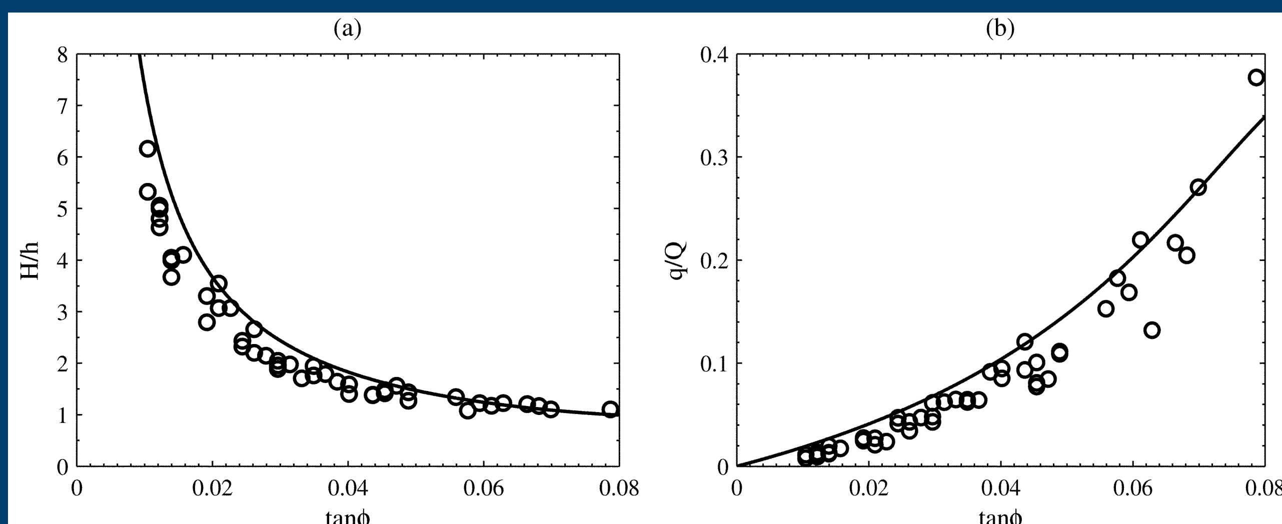
Comparisons with experiments

3.35 mm plastic beads in water, ϕ=0.5÷4.5°

(Capart and Fraccarollo, GRL 2011)







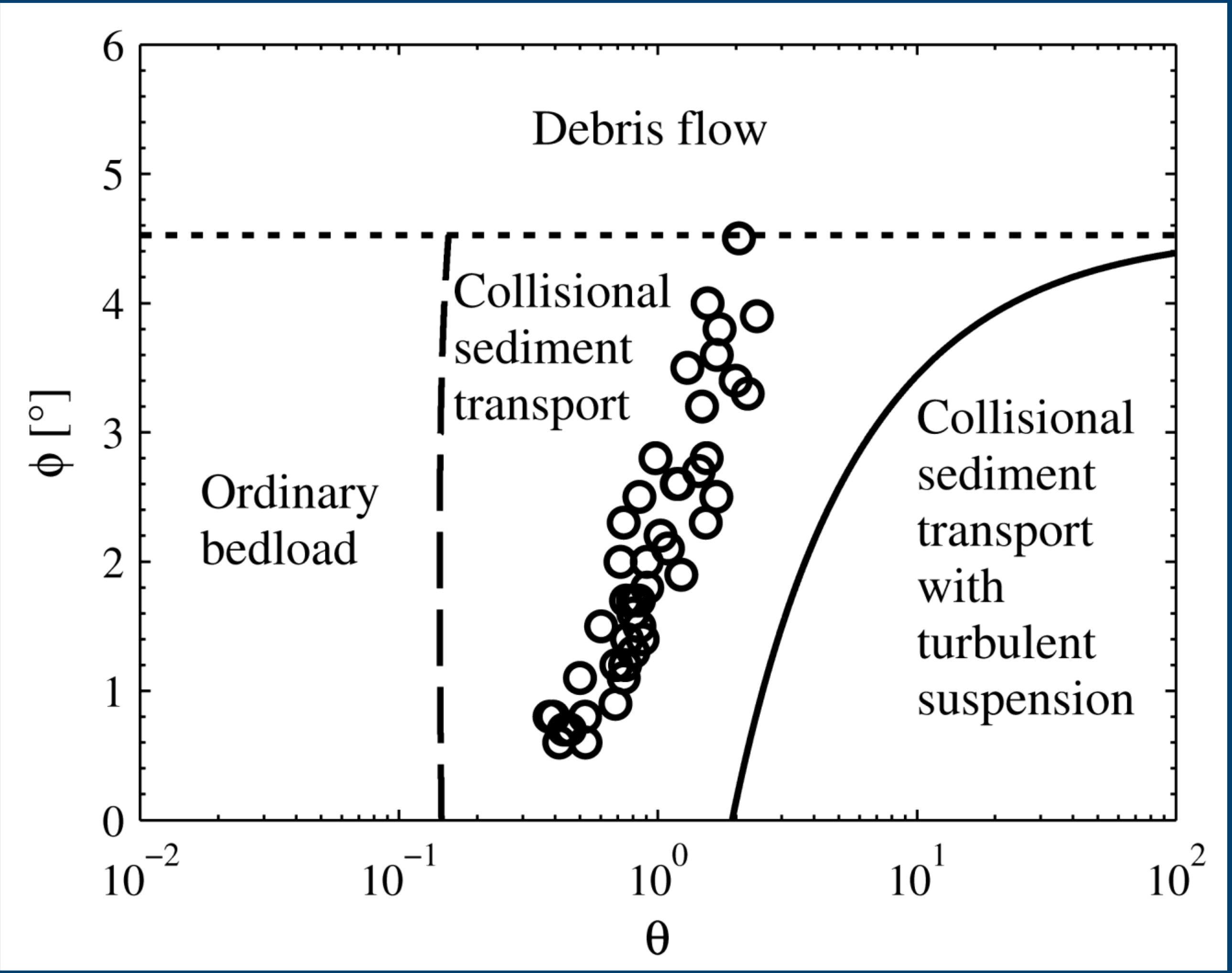
Limits

$h=1$ ⟹ *Boundary between bedload and collisional suspension*

$(\sigma S^*)^{1/2}/w=0.8$ ⟹ *Boundary between collisional and turbulent-collisional suspensions*

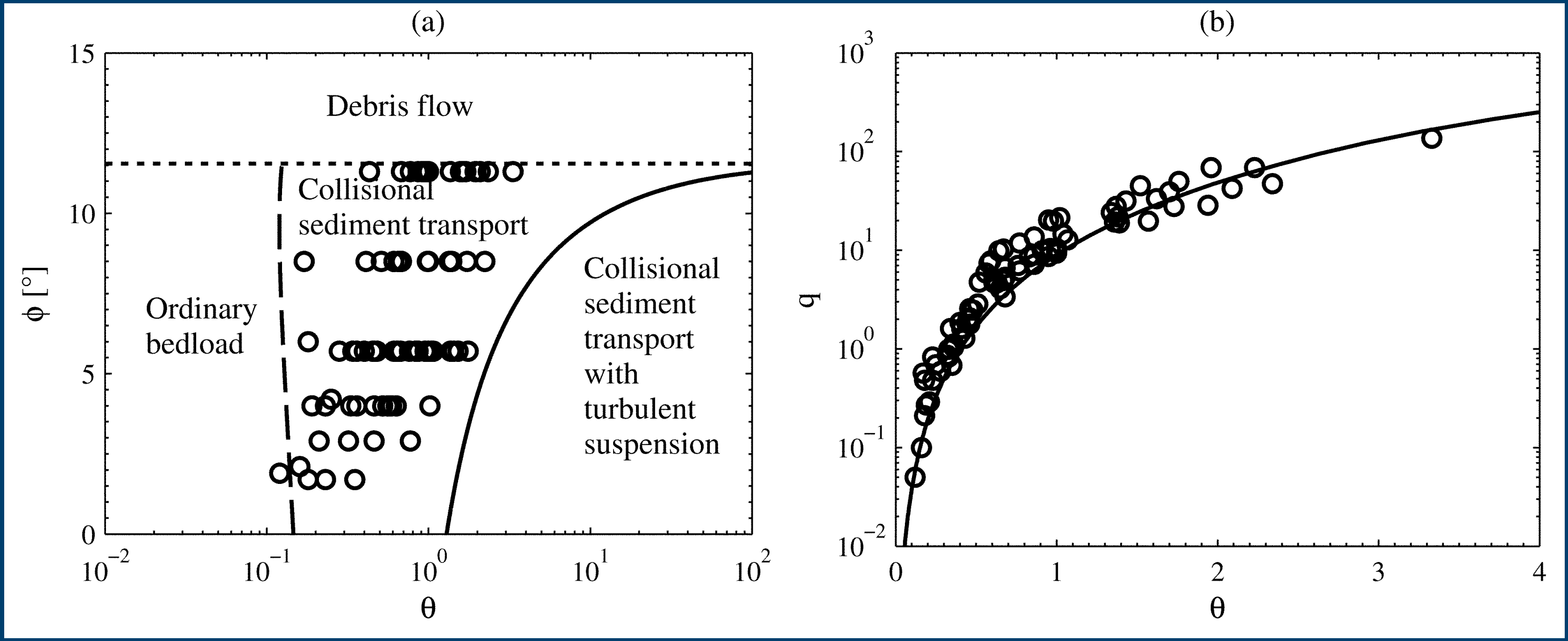
$h=H$ ⟹ *Boundary with debris flows*

Regime map 2



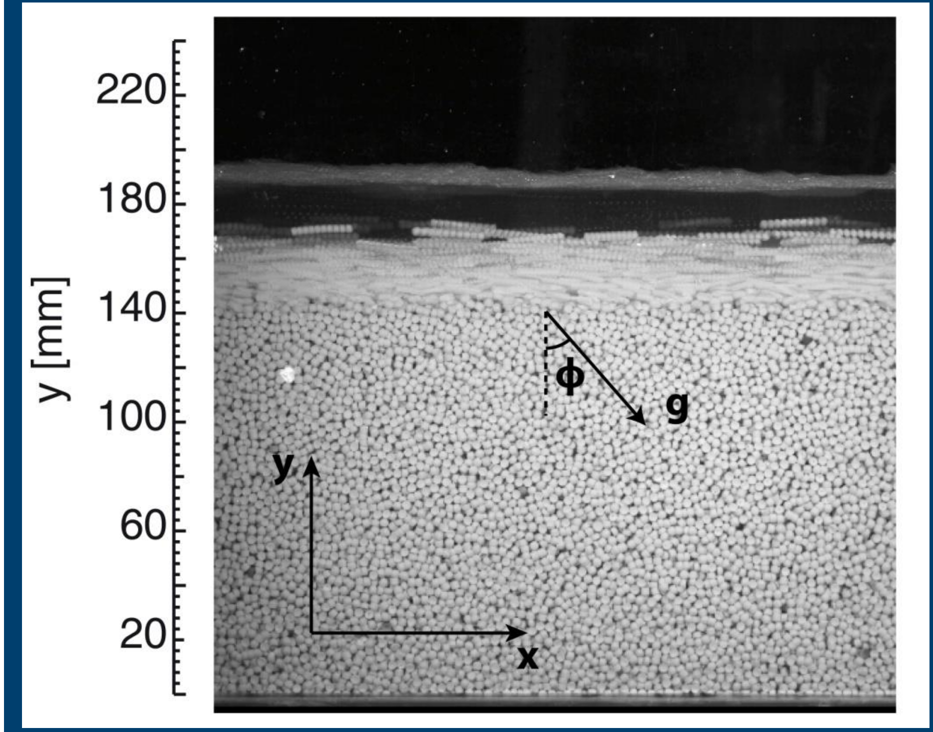
6
5
4
3
2
1
0
φ [°]
Debris flow
Collisional
sediment
transport
Ordinary
bedload
Collisional
sediment
transport
with
turbulent
suspension
10^−2
10^−1
10^0
10^1
10^2
θ

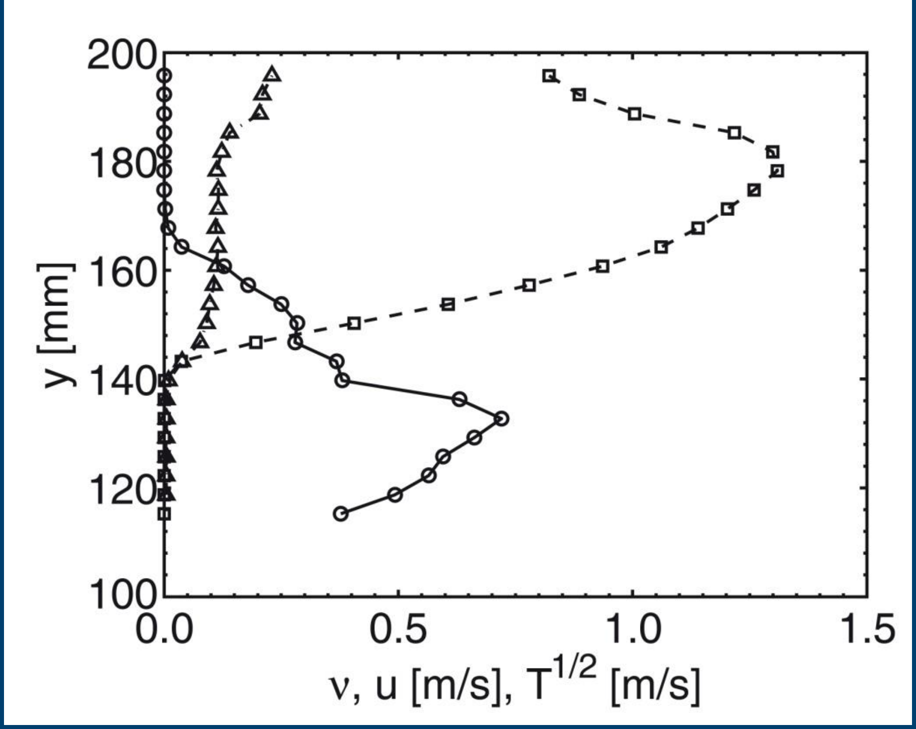
Non-uniform 2 to 10 mm gravel in water (Smart, JHE 1984)



Berzi and Fraccarollo, PRL 2015

Profiles of solid volume fraction ν, mean particle velocity **u** and granular temperature **T** (intensity of velocity fluctuations)



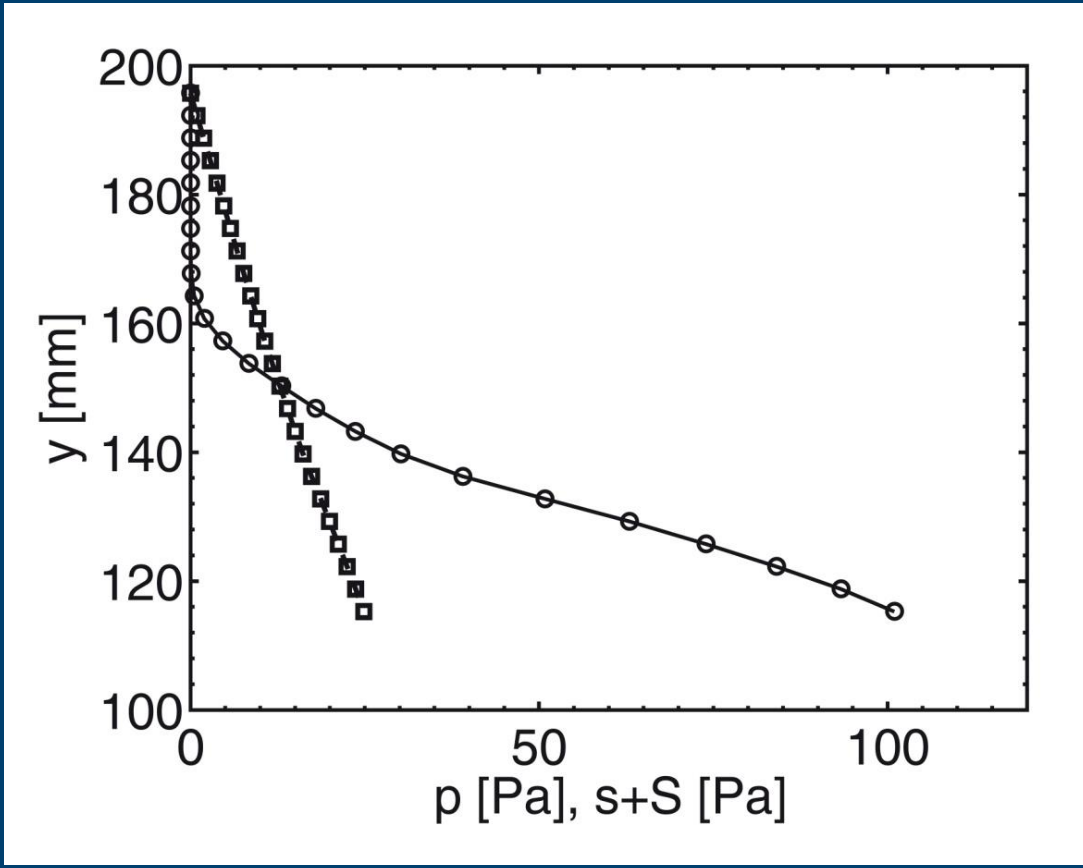


Plastic cylinders and water (Capart and Fraccarollo, GRL 2011)

Integrating the momentum balances

$$p' = -\left(\rho_p - \rho_f\right)\nu g \cos\phi$$

$$\left(s + S\right)' = -\left[\rho_p \nu - \rho_f \left(1 - \nu\right)\right] g \sin\phi$$



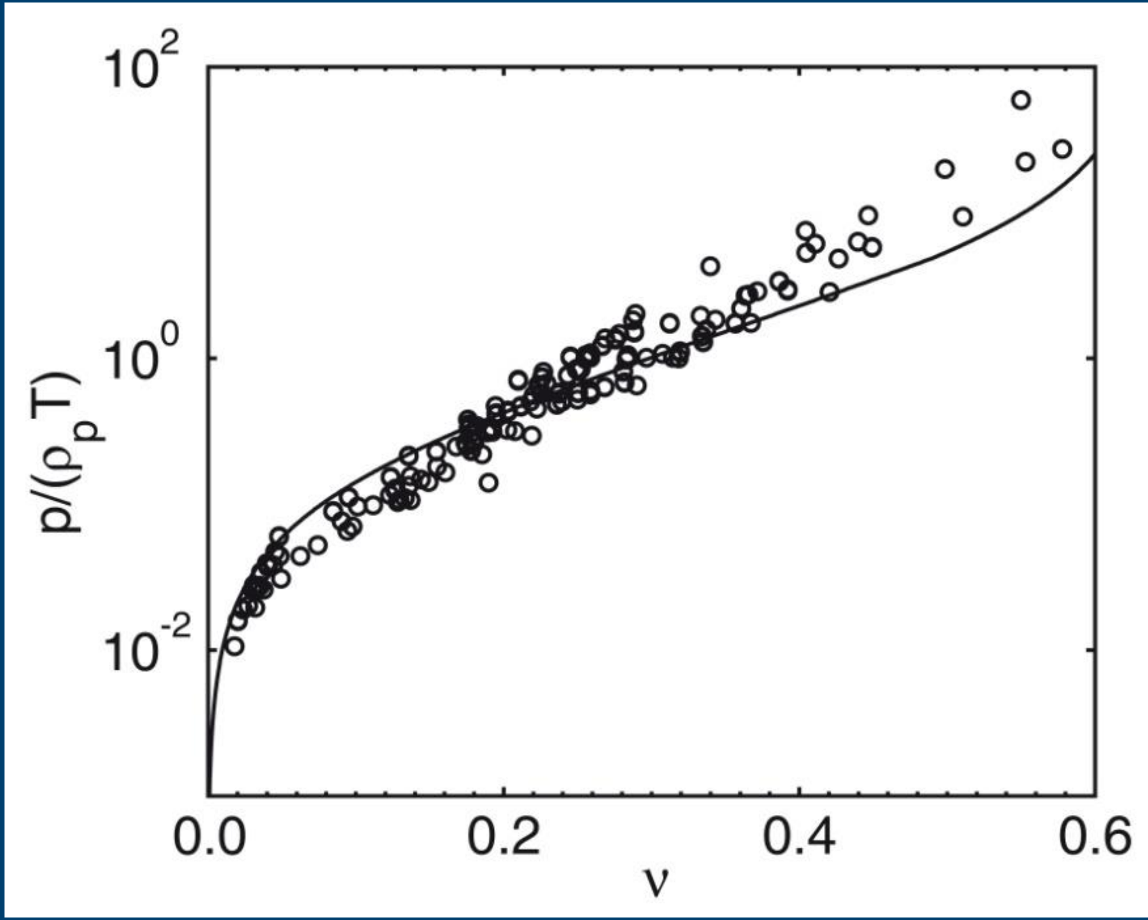
e.g., Garzo and Dufty, PRE 1999

$$\frac{p}{\rho_p T} = 4\nu^2 g_0 \left(\frac{1}{4\nu g_0} + \frac{1+e}{2} \right)$$

$$s = \rho_p \frac{8 J \nu^2 g_0}{5\pi^{1/2}} d T^{1/2} u'$$

g_0 radial distribution function at contact; **e** coefficient of collisional restitution

Still valid for suspensions?



Particle pressure scales with granular temperature!

(solid line when g_0 is given by Torquato, PRE 1995)

Hence, we can assume that also the particle shear stress is given by Kinetic Theory

$$\frac{s}{u'} = \rho_p \frac{8 J \nu^2 g_0}{5\pi^{1/2}} d T^{1/2}$$

⇓

s+S (from integrating momentum balance) –

s (from constitutive relation of KT) =

S (fluid shear stress)

⇓

$$\eta = \frac{S}{u'}$$ Effective fluid shear viscosity

Three components to the fluid viscosity

$$\eta = \eta_{visc} + \eta_{turb} + \eta_{gran}$$

$$\eta_{visc} = \left[1 + \frac{5}{2}\nu\left(1 - \frac{\nu}{\nu_m}\right)^{-1}\right]\eta_{mol}$$

viscous hydrodynamic component (Boyer et al , PRl 2011): negligible here

turbulent hydrodynamic component: mixing length approach

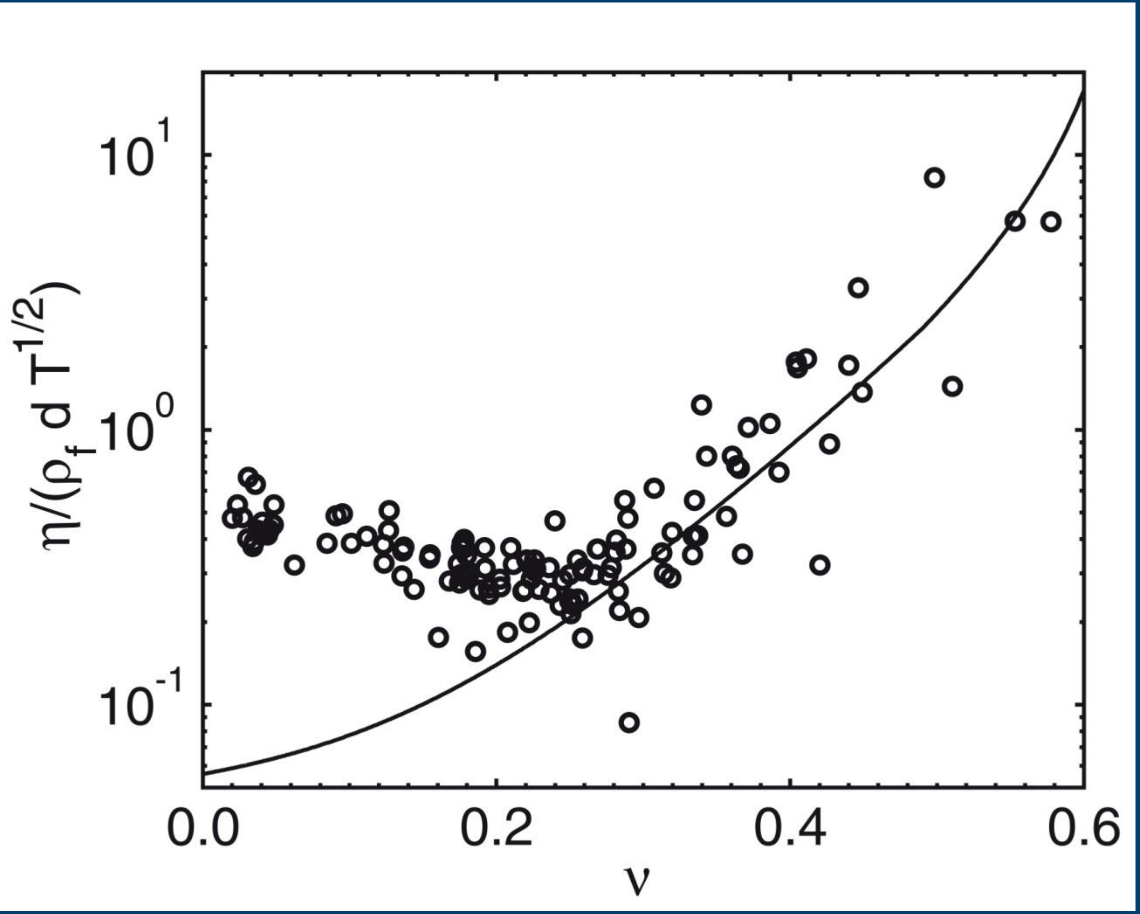
$$\eta_{turb} = \rho_f \left(1 - \nu\right) l_m^2 u'$$

$$\eta_{gran} = \rho_f \frac{1 + 2\nu}{2\left(1 - \nu\right)} \frac{8 J \nu^2 g_0}{5\pi^{1/2}} d T^{1/2}$$

granularlike component: portion of fluid stuck with the particle and fluctuates with it (added mass, Lamb 1932)

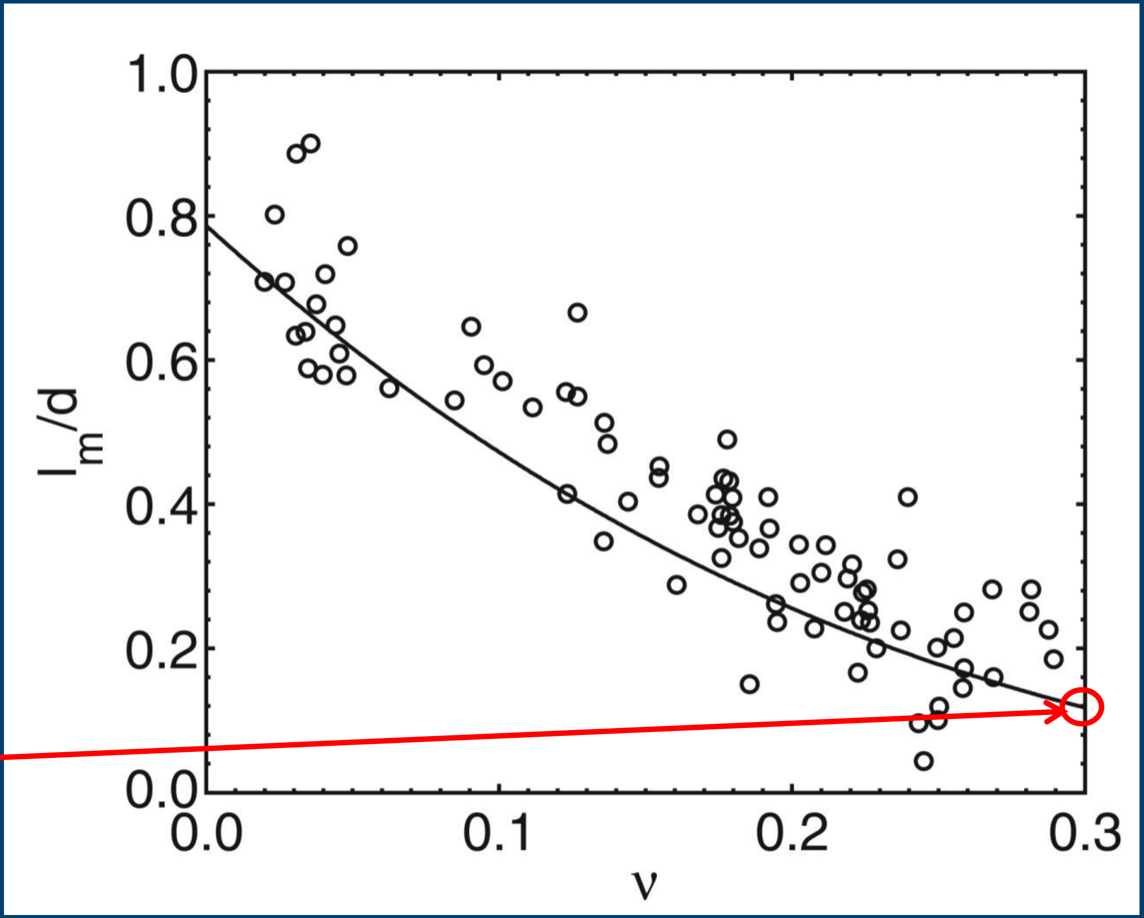
Granularlike viscosity

$$\eta_{gran} = \rho_f \frac{1+2\nu}{2(1-\nu)} \frac{8J\nu^2 g_0}{5\pi^{1/2}} dT^{1/2}$$



Turbulent mixing length

$$\frac{l_m}{d} = \sqrt{\frac{\eta - \eta_{gran}}{\rho_f \left(1-\nu\right) d^2 u'}}$$



Agreement with numerical simulations (Verberg and Koch, PHF 2006)

Turbulence is local: it depends on the local value of the volume fraction; the interparticle distance limits the size of turbulent eddies

Asymptotic cases

GRANULAR LIMIT

High volume fraction

⇒ Fluctuation energy production due to particle shear stress balanced by collisional dissipation (Jenkins and Berzi, Granul. Matt. 2010)

$$\frac{T}{d^2 u'^2} = \frac{2J}{15\left(1-e^2\right)}$$

TURBULENT LIMIT

Low volume fraction

⇒ Fluctuation energy production due to turbulent eddies balanced by dissipation due to drag (Hsu et al., Proc. R. Soc. A 2004)

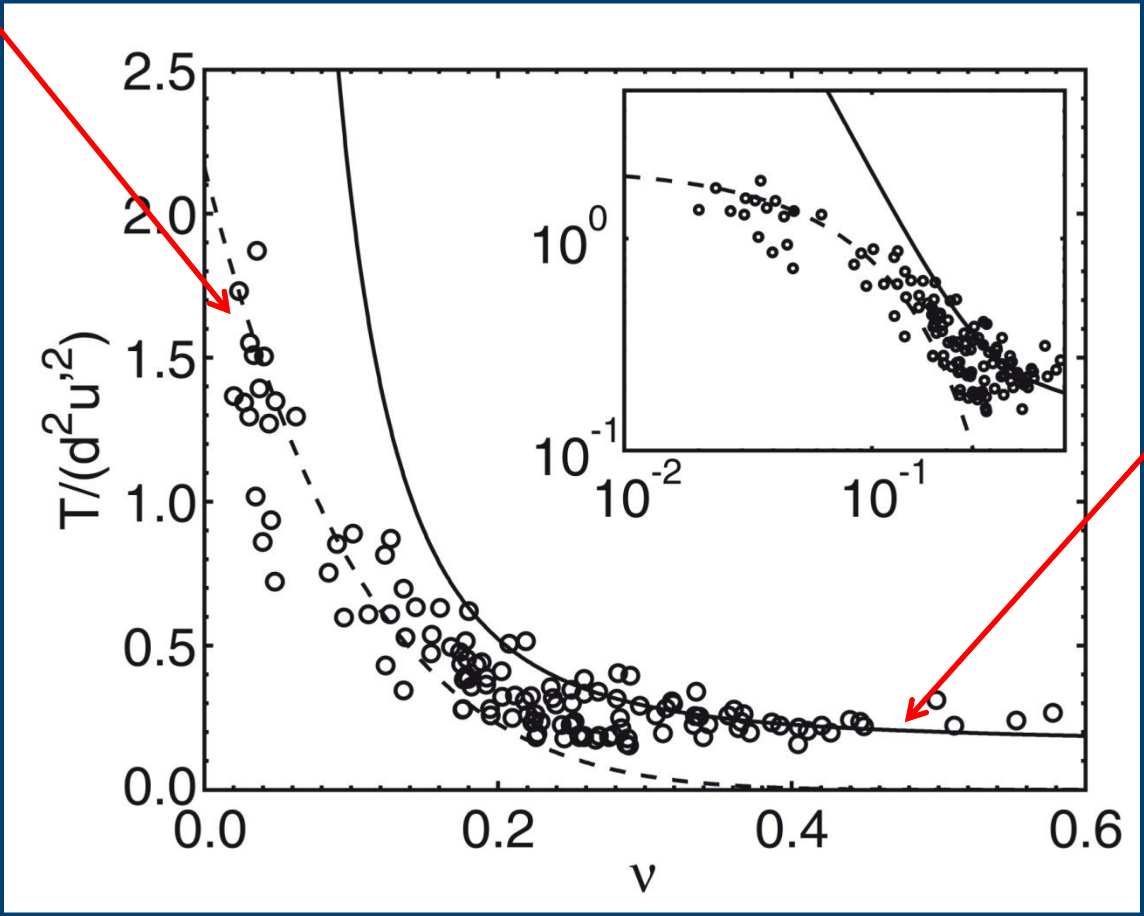
Granular temperature proportional to the square of the fluid shear velocity

$$T \propto S \Big/ \left[\rho_f \left(1-\nu\right) \right]$$

$$\frac{T}{d^2 u'^2} = 3.5 \left(\frac{l_m}{d} \right)^2$$

Granular temperature

TURBULENT LIMIT

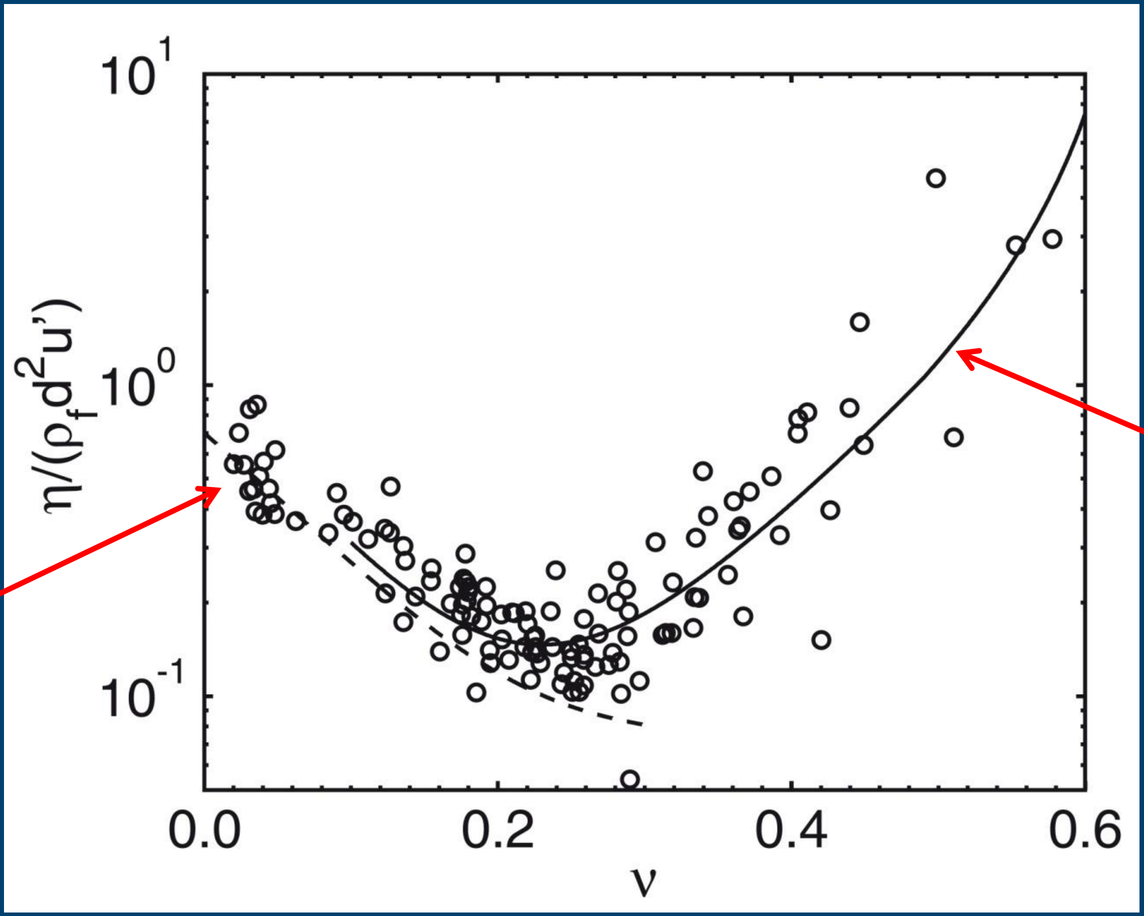


GRANULAR LIMIT

Fluid shear viscosity

$$\frac{\eta}{\rho_f d^2 u'} = (1-\nu)\left(\frac{l_m}{d}\right)^2 + \frac{1+2\nu}{2(1-\nu)}\frac{8J\nu^2 g_0}{5\pi^{1/2}}\left(\frac{T}{d^2 u'^2}\right)^{1/2}$$

TURBULENT LIMIT



GRANULAR LIMIT

Minimum in the viscosity (also in Revil-Baudard et al., JFM 2015)

Conclusions

- Two components of effective fluid viscosity in colllisional suspensions
- Turbulence originates at the surface of the particles
- Local mixing length: bounded by interparticle distance and decreasing with volume fraction
- Momentum tranfer due to added mass in conjugate motion with the fluctuating particles
- Granularlike viscosity given by kinetic theory using density of added mass
- Scaling of the granular temperature in the granular and turbulent limits
- Transition to non-local turbulence in turbulent-collisional suspensions?