Marco Colombini – Università di Genova – Italy

Stability of Alternate Bars and Oblique Dunes

Alternate bars in the Tokachi River, Japan
Image courtesy of V. Langlois

Marco Colombini – Università di Genova – Italy

Stability of River Bed Forms Part III

- Part I – linear stability of roll-waves, dunes & antidunes;
- Part II – linear stability of ripples and dunes;

Stability of River Bed Forms
Part III

- Part I – linear stability of roll-waves, dunes & antidunes;
- Part II – linear stability of ripples and dunes;
- Part III – linear stability of alternate bars & oblique dunes;

Marco Colombini – Università di Genova – Italy

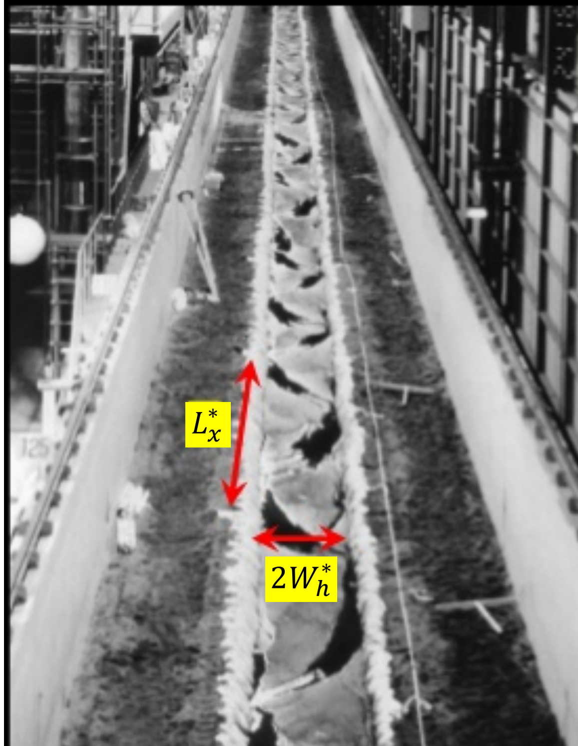
Stability of River Bed Forms
Part III

- Part I – linear stability of roll-waves, dunes & antidunes;
- Part II – linear stability of ripples and dunes;
- Part III – linear stability of alternate bars & oblique dunes;

- Part IV – weakly nonlinear analysis

Marco Colombini – Università di Genova – Italy

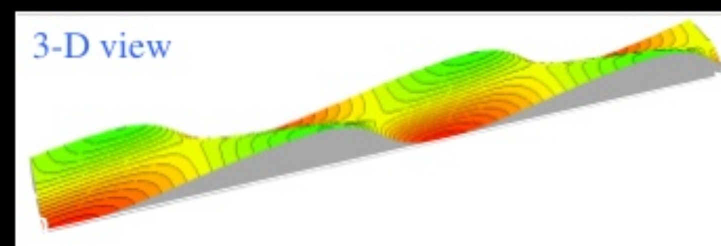
Alternate bars in straight channels



Lisle et al. (1997)

Naka River, Parker's e-book

$$\beta = \frac{W_h^*}{D^*}$$



Federici and Colombini (2003)

Ingeniería de ríos, Jorge Abad 2011

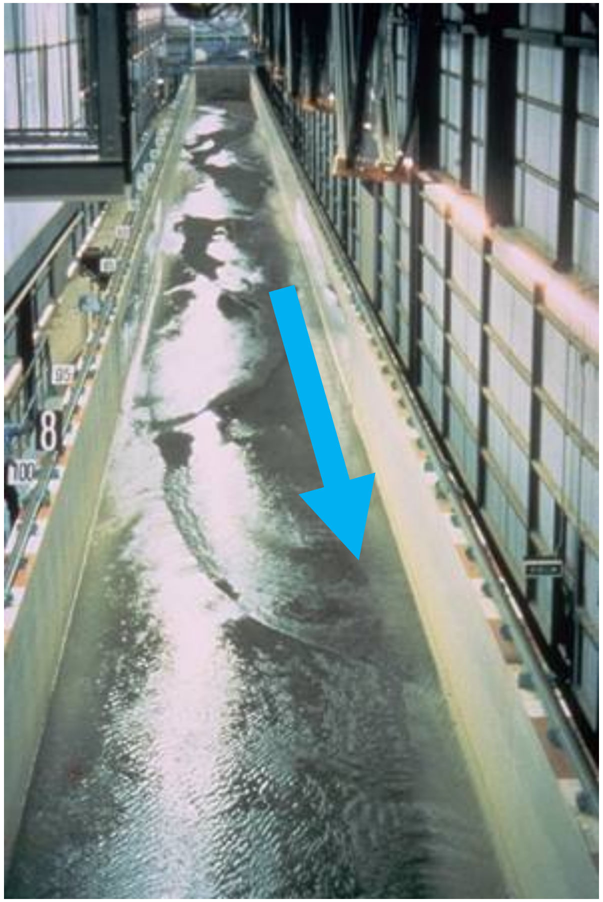
The name game: alternate bars

Super fun group game!

The name game: alternate bars

Alternate bars in a flume.

Image courtesy of S. Ikeda.



Super fun group game!

The name game: alternate bars

Super fun group game!

The name game: alternate bars

Super fun group game!

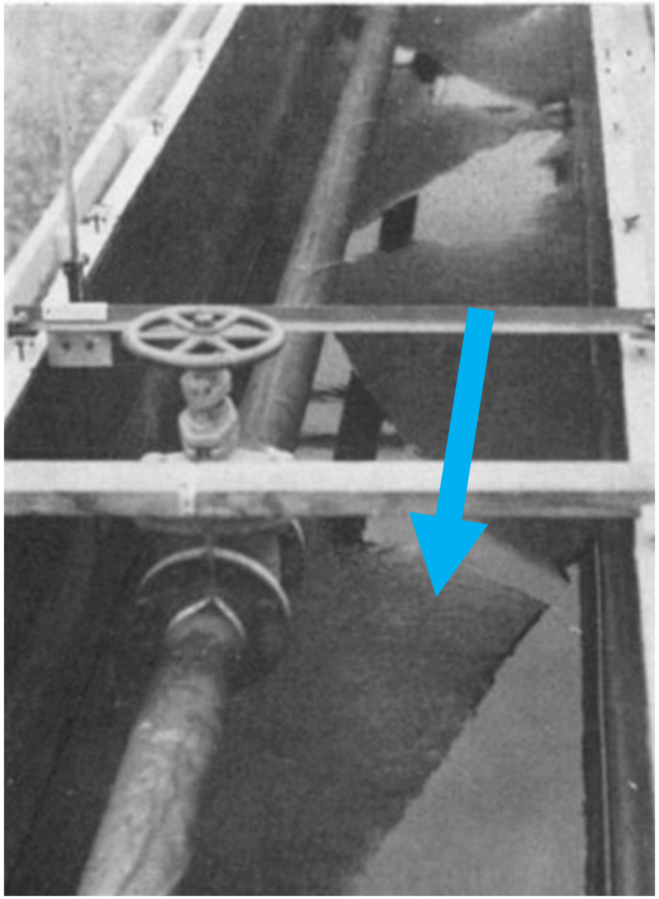
"A small dune sings only the few days in which there is no wind and no clouds”

(Andreotti, B. - LiveScience, 2005)

The name game: alternate bars

Diagonal bars in a flume.

From Einstein & Shen, JGR 1964

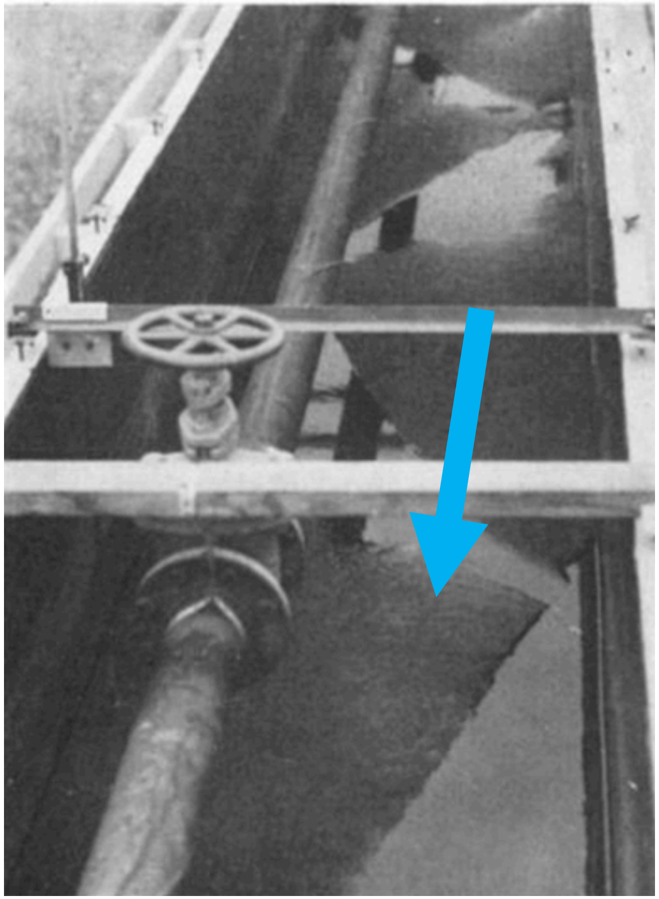


Super fun group game!

The name game: alternate vs. diagonal bars

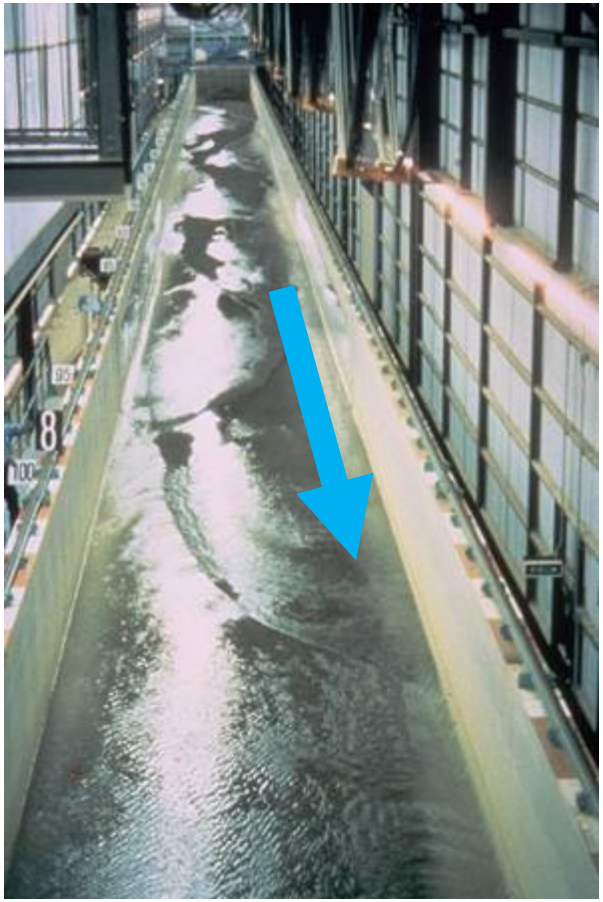
Diagonal bars in a flume.

From Einstein & Shen, JGR 1964



Alternate bars in a flume.

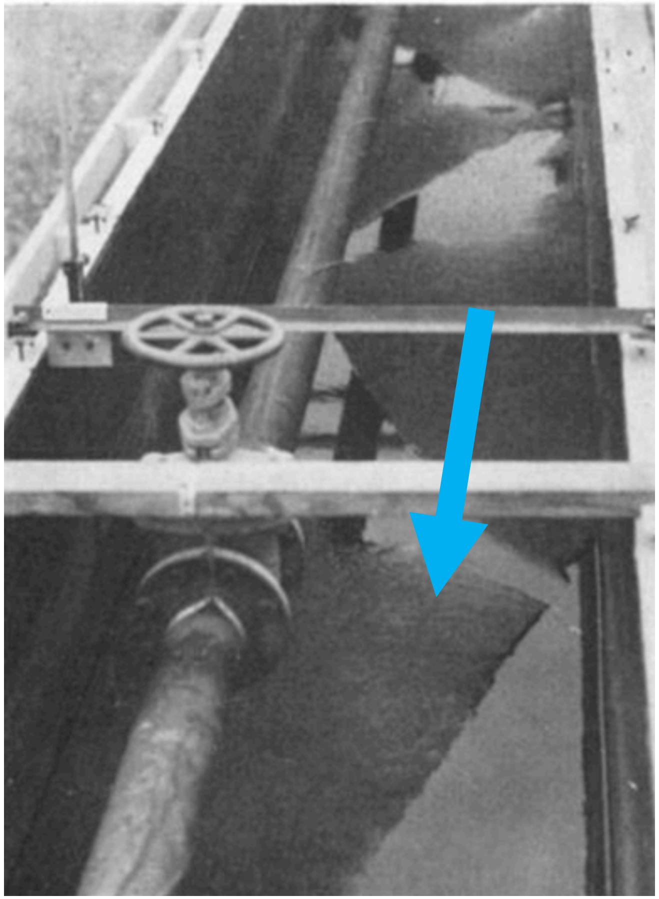
Image courtesy of S. Ikeda.



The name game: alternate vs. diagonal bars

Diagonal bars in a flume.

From Einstein & Shen, JGR 1964



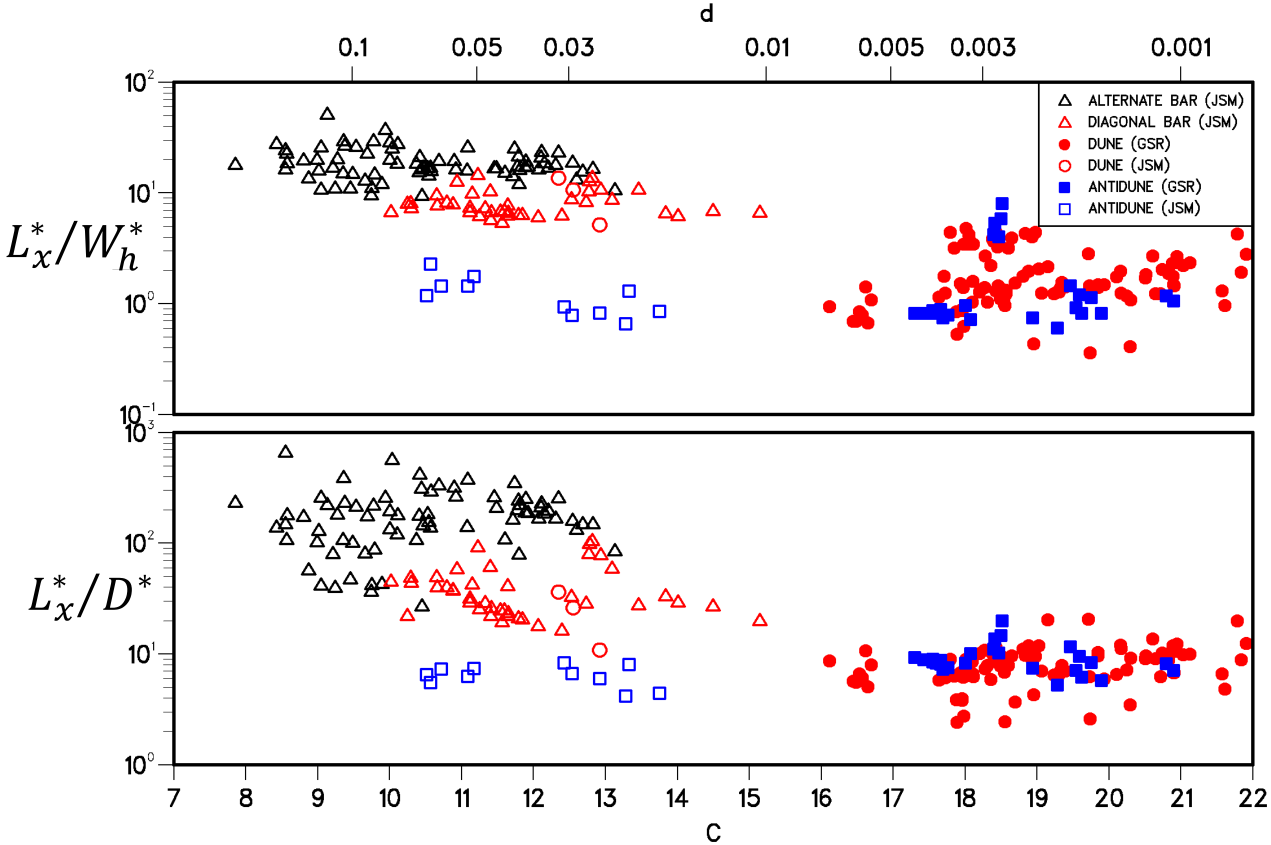
- “ ... it is probable that diagonal bars have sometimes been classified as alternate bars by some authors.”
- “ ... experiments seem to indicate that a grouping of three-dimensional mesoforms [i.e. scaling with flow depth], in which the fronts of the mesoforms were diagonally alligned over the channel width, was responsible for these features.”

From Jaeggi, JHE 1984

Longitudinal scaling: bars & dunes

JSM: Jaeggi (1984), Sukegawa (1971), Muramoto & Fujita (1978)

GSR: Guy, Simons & Richardson (1966)



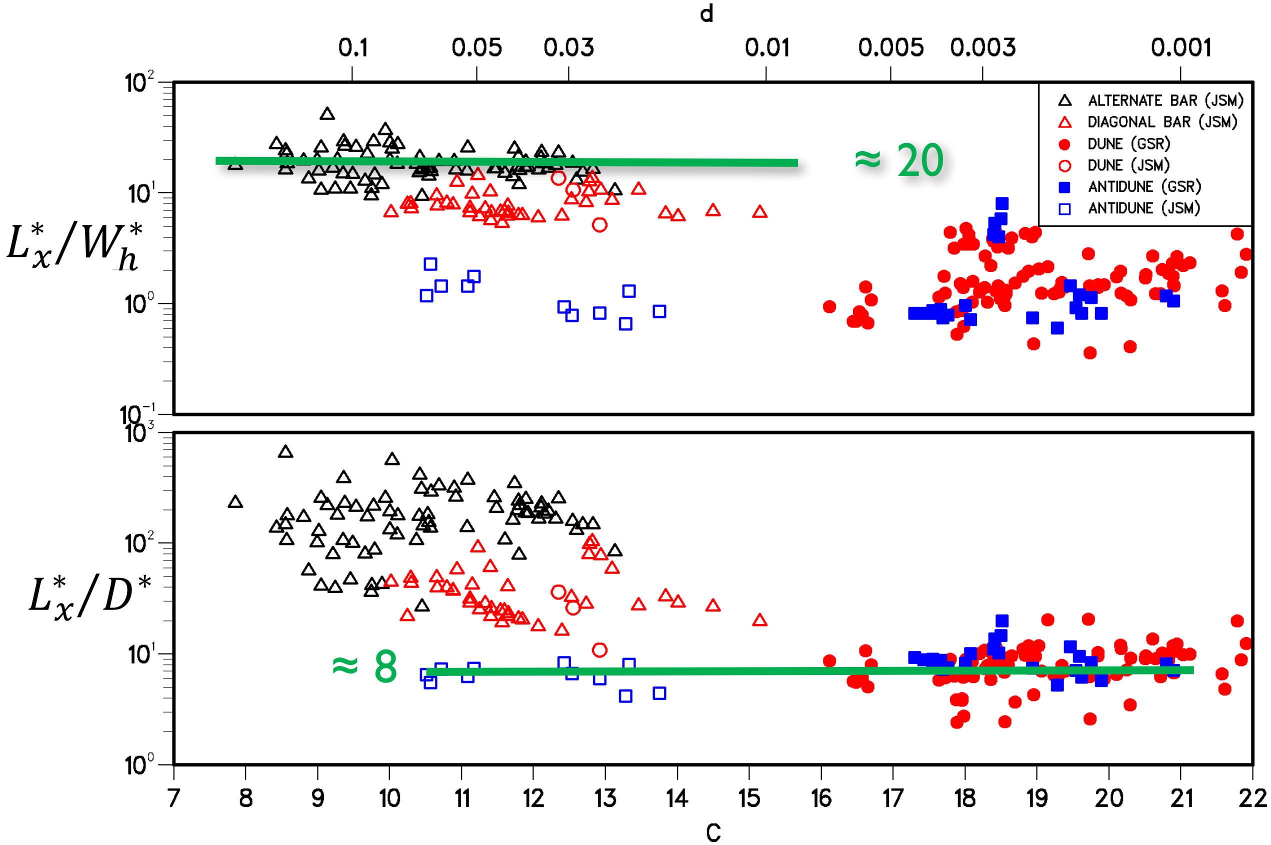
$$C = \frac{U^*}{u_f^*} = \sqrt{\frac{8}{f}} = \frac{1}{\kappa}\ln\left(\frac{11.09}{2.5d}\right)$$

(Rough regime)

Longitudinal scaling: bars & dunes

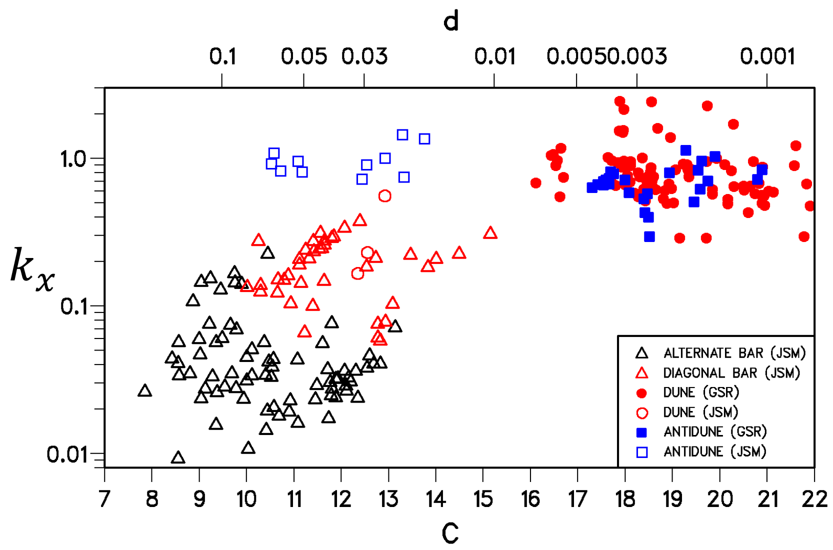
JSM: Jaeggi (1984), Sukegawa (1971), Muramoto & Fujita (1978)

GSR: Guy, Simons & Richardson (1966)



$$C = \frac{U^*}{u_f^*} = \sqrt{\frac{8}{f}} = \frac{1}{\kappa}\ln\left(\frac{11.09}{2.5d}\right)$$

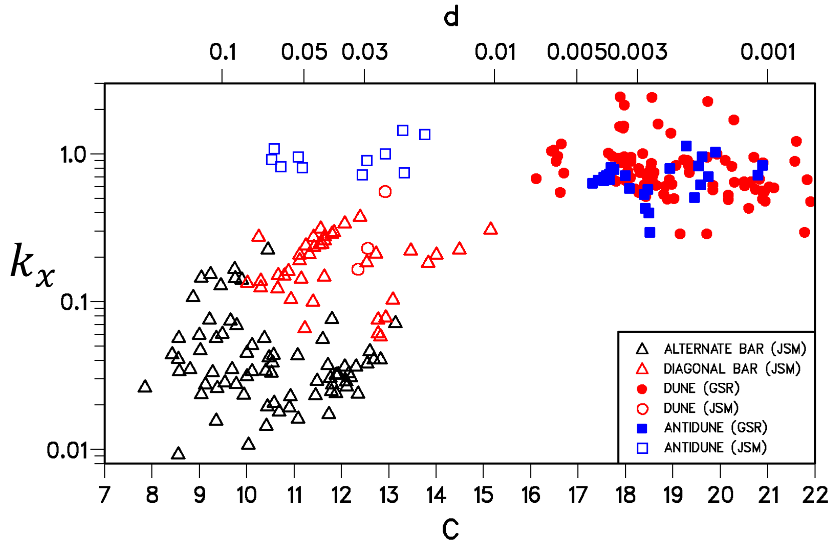
(Rough regime)



- Longitudinal wavenumbers for Diagonal Bars are larger than for Alternate Bars

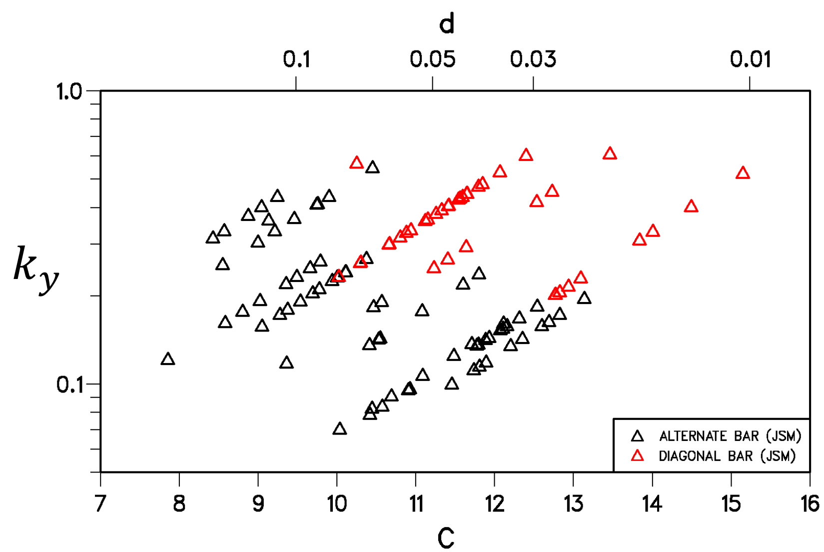
$$k_x = \frac{2\pi D^*}{L_x}$$

Longitudinal & transverse wavenumbers



- Longitudinal wavenumbers for Diagonal Bars are larger than for Alternate Bars

$$k_x = \frac{2\pi D^*}{L_x}$$



- Transverse wavenumbers for Diagonal Bars are larger than for Alternate Bars

$$k_y = \frac{2\pi D^*}{L_y} = \frac{2\pi D^*}{4W_h^*} = \frac{\pi}{2\beta}$$

Stability of alternate bars

Instability and river channels
R. A. Callander
Journal of Fluid Mechanics, Volume 36, Issue 03, May 1969, pp 465-480
doi: 10.1017/S0022112069001765, Published online by Cambridge University Press 29 Mar 2006

- Linear
- 2D Flow model
- Free Bars

A unified bar–bend theory of river meanders
P. Blondeaux and G. Seminara
Journal of Fluid Mechanics, Volume 157, August 1985, pp 449-470
doi: 10.1017/S0022112085002440, Published online by Cambridge University Press 20 Apr 2006

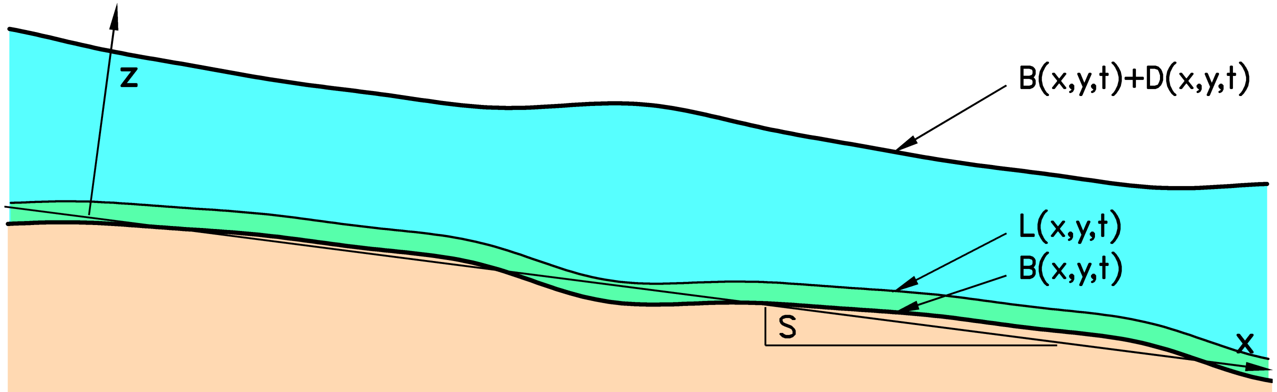
- Linear
- 2D Flow model
- Free and Forced Bars

Finite-amplitude alternate bars
M. Colombini and G. Seminara and M. Tubino
Journal of Fluid Mechanics, Volume 181, September 1987, pp 213-232
doi: 10.1017/S0022112087002064, Published online by Cambridge University Press 21 Apr 2006

- Weakly Non Linear
- 2D Flow model
- Free Bars

Three-dimensional river bed forms
M. Colombini and A. Stocchino
Journal of Fluid Mechanics, Volume 695, March 2012, pp 63-80
doi: 10.1017/jfm.2011.556, Published online by Cambridge University Press 07 Feb 2012

- Linear
- 3D Flow model
- Free Bars



FLOW MODEL

- 3D ROTATIONAL FLOW MODEL (infinitely wide channel)
- BOUSSINESQ'S CLOSURE (algebraic mixing length)
- COORDINATE TRANSFORMATION (rectangular domain)

SEDIMENT TRANSPORT MODEL

- EQUILIBRIUM MODEL (Exner)
- BEDLOAD ONLY (MPM bedload function)
- CORRECTIONS FOR SEDIMENT WEIGHT (x – Fredsøe, y – Engelund)
- CORRECTION FOR BEDLOAD LAYER THICKNESS

$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

$$\Omega = \Omega\left(k_x, k_y; Fr, C\right)$$

DUNE FLAVOUR

$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

$$\Omega = \Omega\left(k_x, k_y; Fr, C\right) \qquad \Omega = \Omega(\lambda, \beta; \vartheta, d)$$

DUNE FLAVOUR

BAR FLAVOUR

Alternate Bars: 3D linear stability

$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

$$\Omega = \Omega\left(k_x, k_y; Fr, C\right) \qquad \Omega = \Omega(\lambda, \beta; \vartheta, d)$$

DUNE FLAVOUR

BAR FLAVOUR

$$\lambda = k_x\,\beta \qquad \beta = \frac{\pi}{2k_y}$$

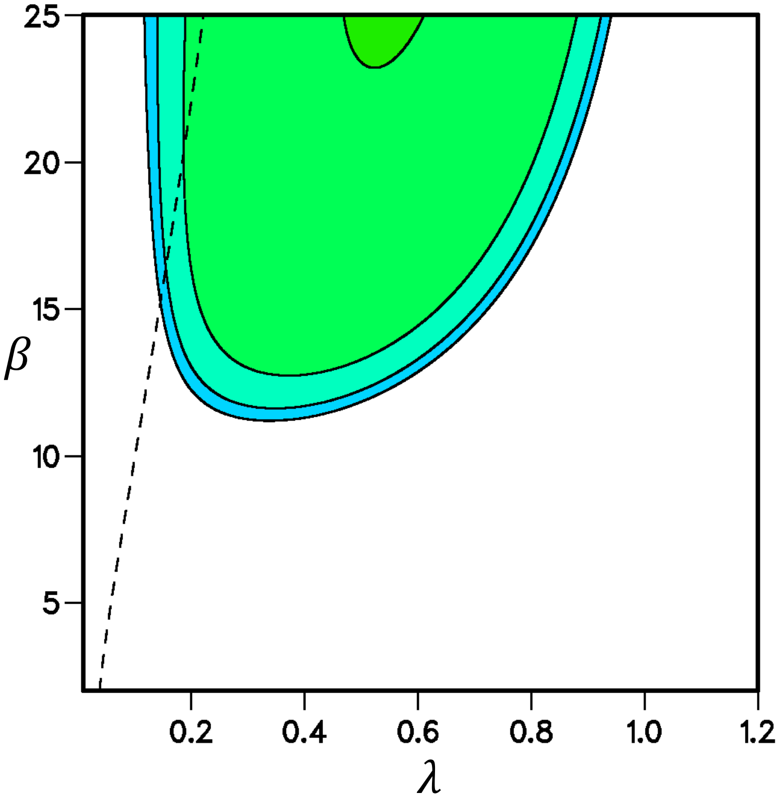
$$\vartheta \cong 0.14\,\frac{Fr^2 e^{\kappa C}}{C^2} \qquad C = \frac{1}{\kappa}\ln\left(\frac{11.09}{2.5d}\right)$$

Alternate Bars: 3D linear stability

$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

$$\Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x\,\beta \quad \beta = \frac{\pi}{2k_y}$$



$$\frac{\vartheta}{\vartheta_C} = 3 \quad d = 0.025$$

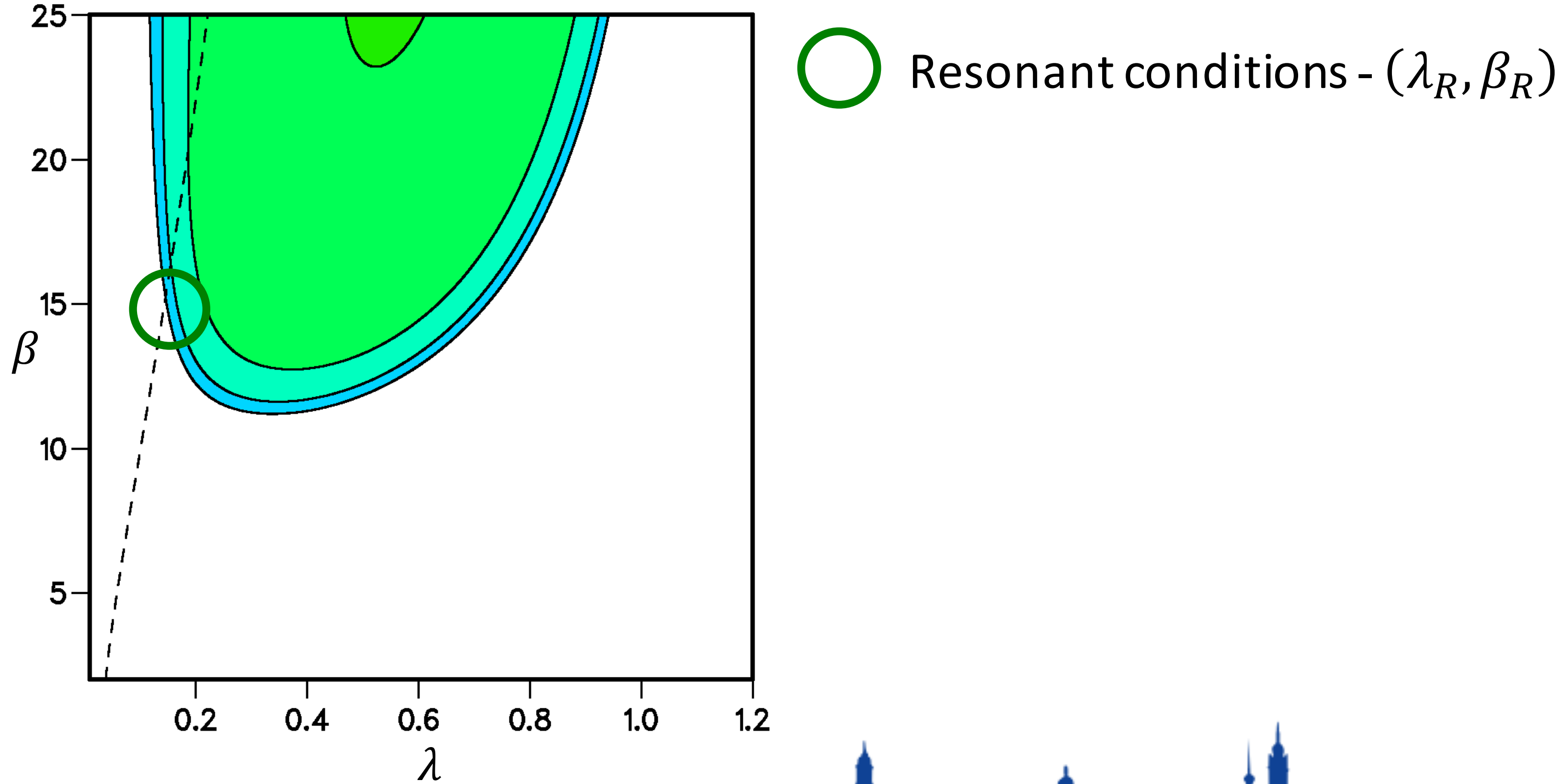
$$Fr = 1 \quad C = 13$$

Alternate Bars: 3D linear stability

$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

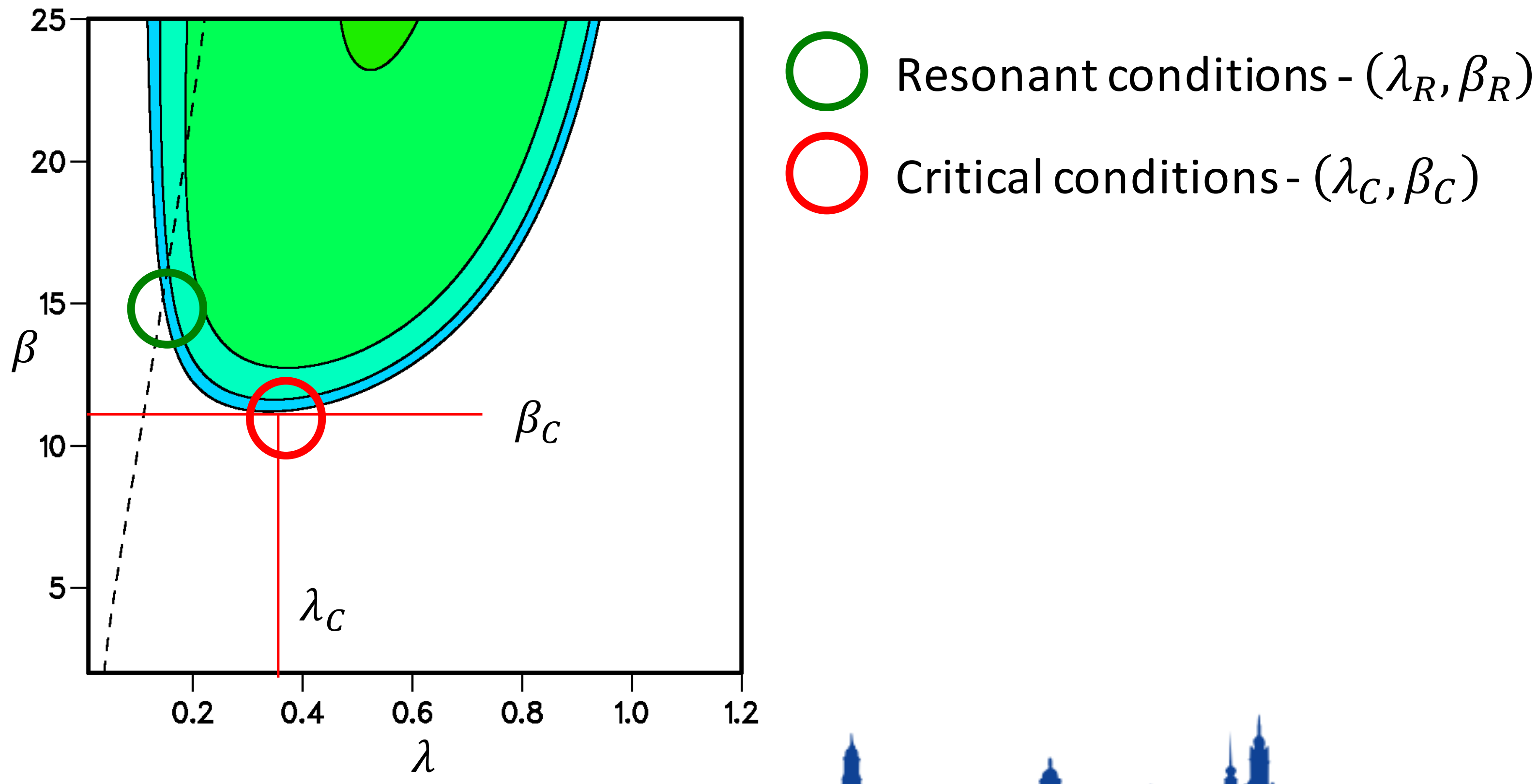
$$\Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x \beta \quad \beta = \frac{\pi}{2k_y}$$



$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

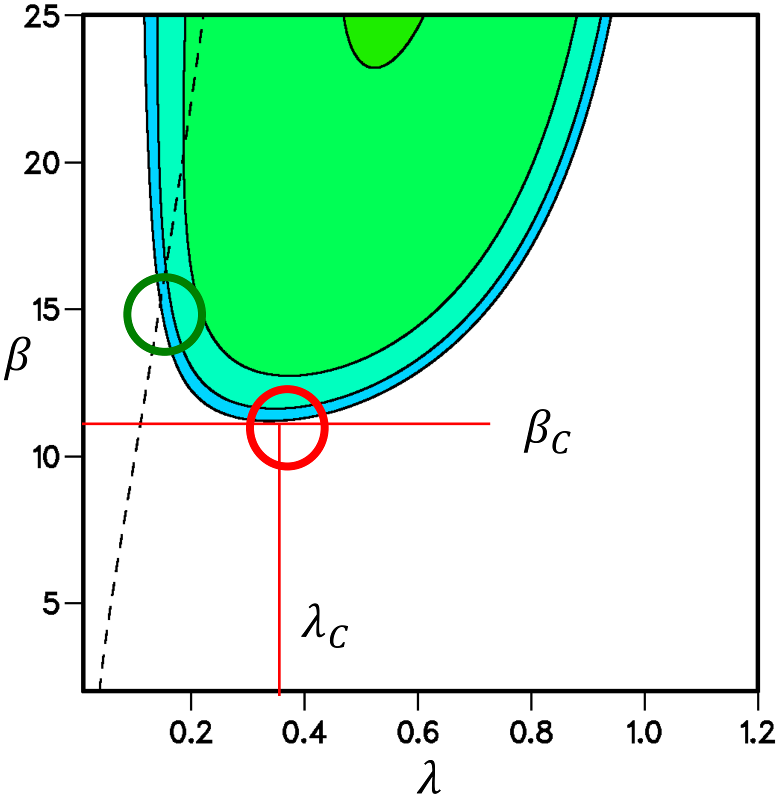
$$\Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x\, \beta \quad \beta = \frac{\pi}{2k_y}$$



$$G(x,y,z,t) = G_0(z) + \varepsilon g_1(z)\exp\left[ik_x(x-\omega t) + ik_y y + \Omega t\right] + c.c.$$

- **LINEAR LEVEL: differential eigenvalue problem**

$$\Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x\,\beta \quad \beta = \frac{\pi}{2k_y}$$



Resonant conditions - (λ_R, β_R)

Critical conditions - (λ_C, β_C)

$$\beta > \beta_C(\vartheta, d)$$

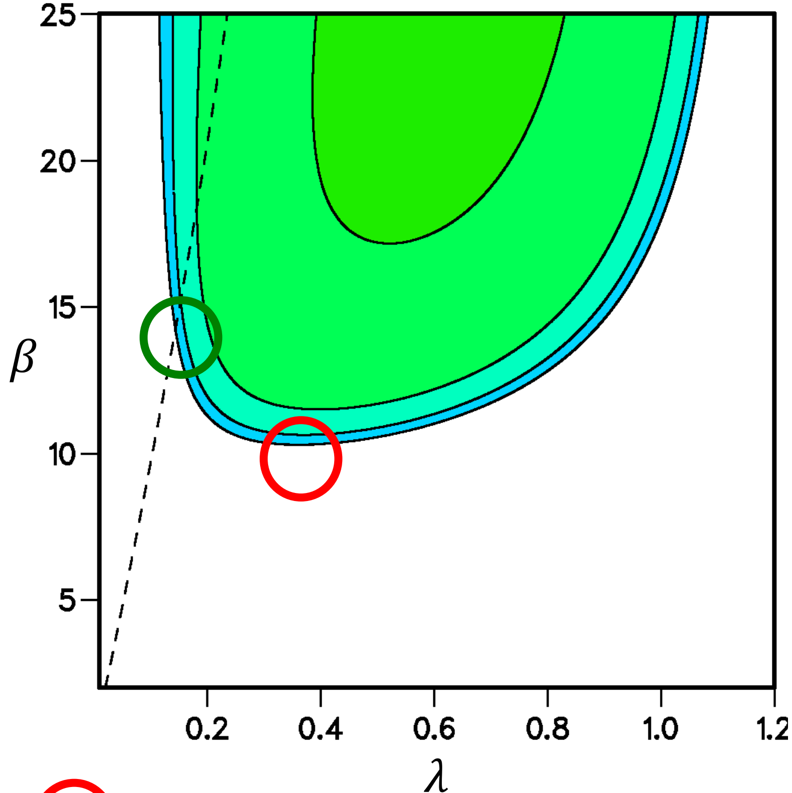
Alternate Bars do not form in a narrow channel

Alternate Bars: 3D linear stability

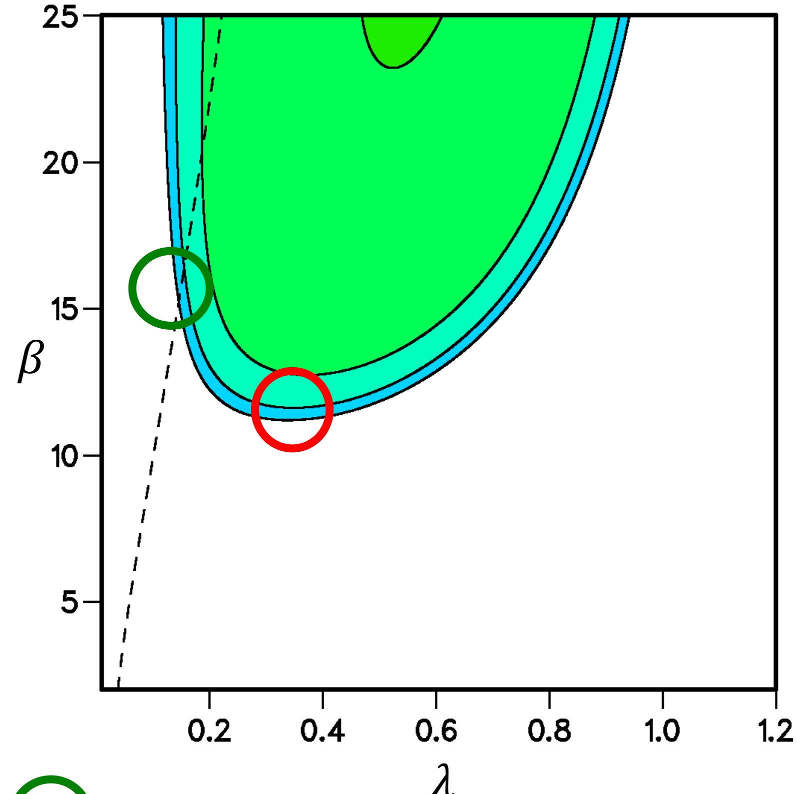
Alternate Bars: linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

SW MODEL



3D MODEL



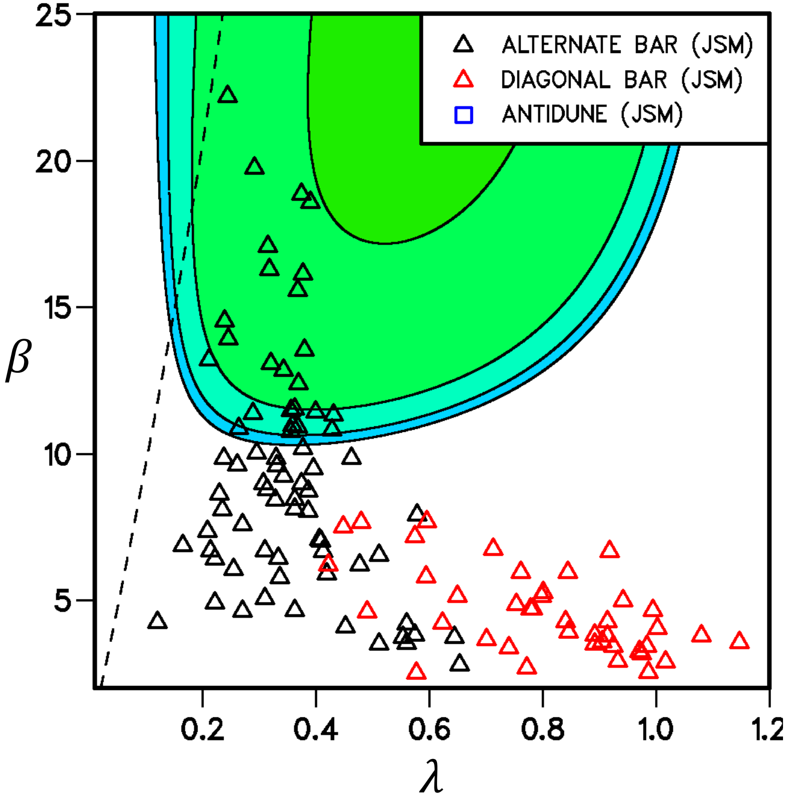
Red circle: Critical conditions - (λ_C, β_C)

Green circle: Resonant conditions - (λ_R, β_R)

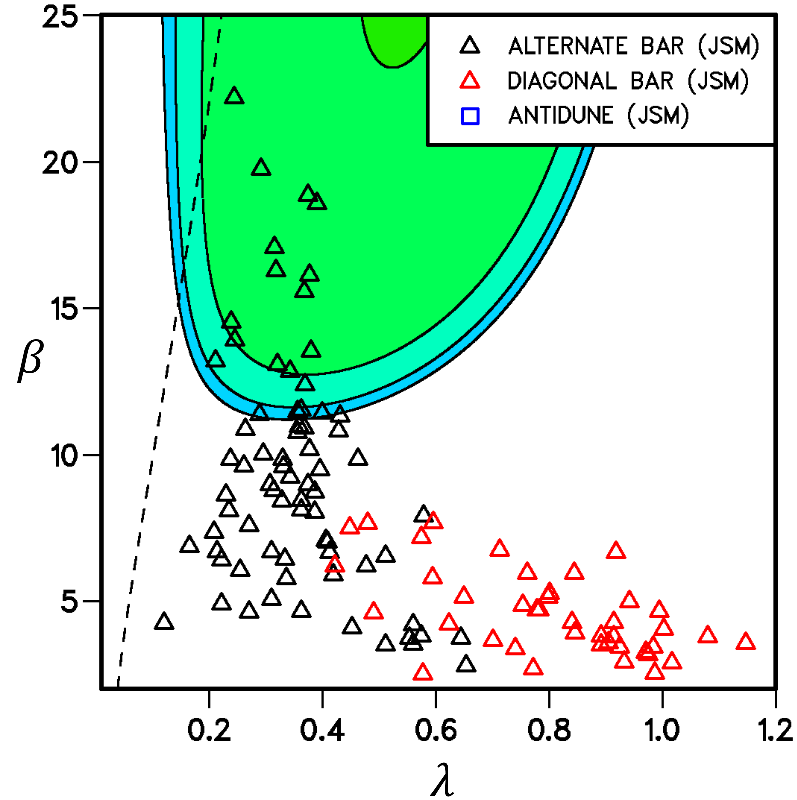
Alternate Bars: linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

SW MODEL

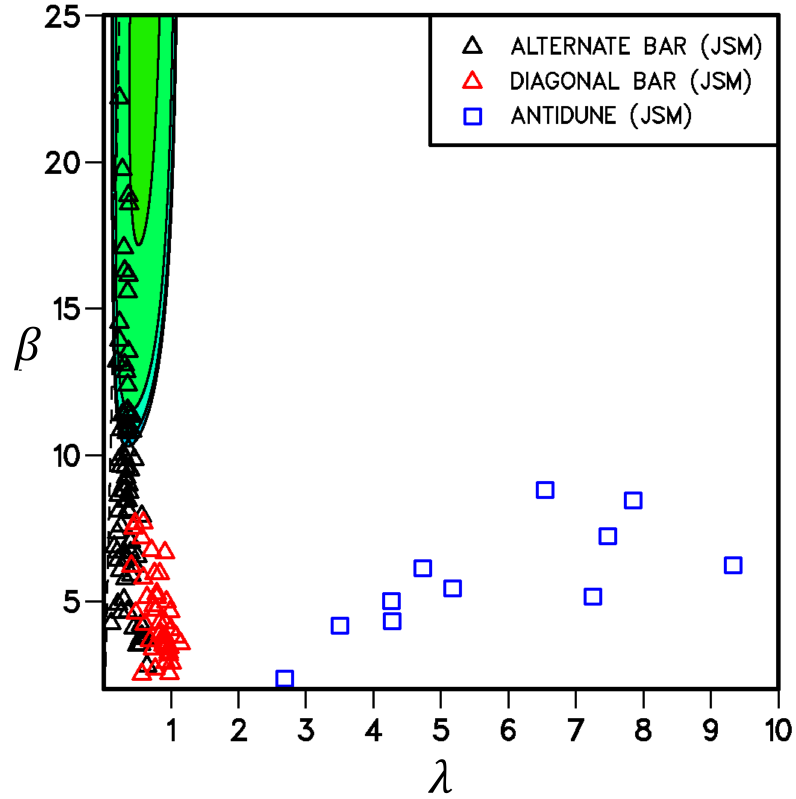


3D MODEL

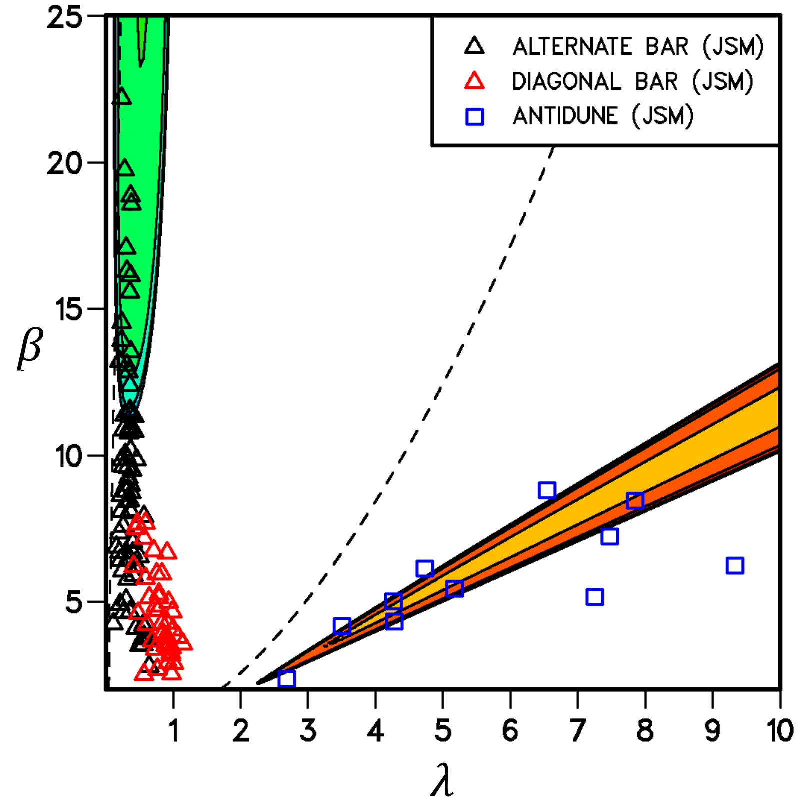


$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Leftrightarrow \quad Fr = 1 \quad d = 0.025$$

SW MODEL



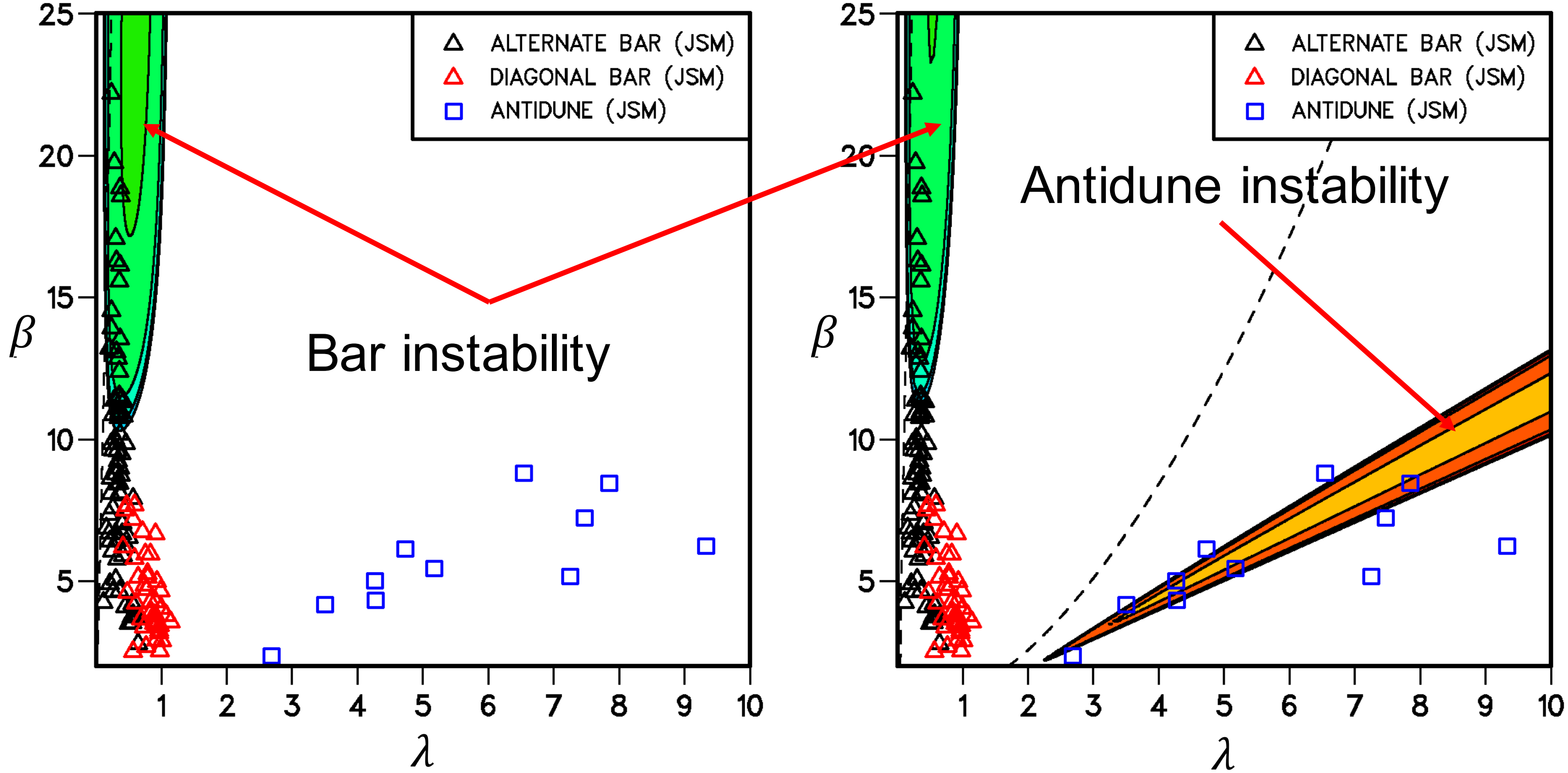
3D MODEL



Alternate Bars: linear stability

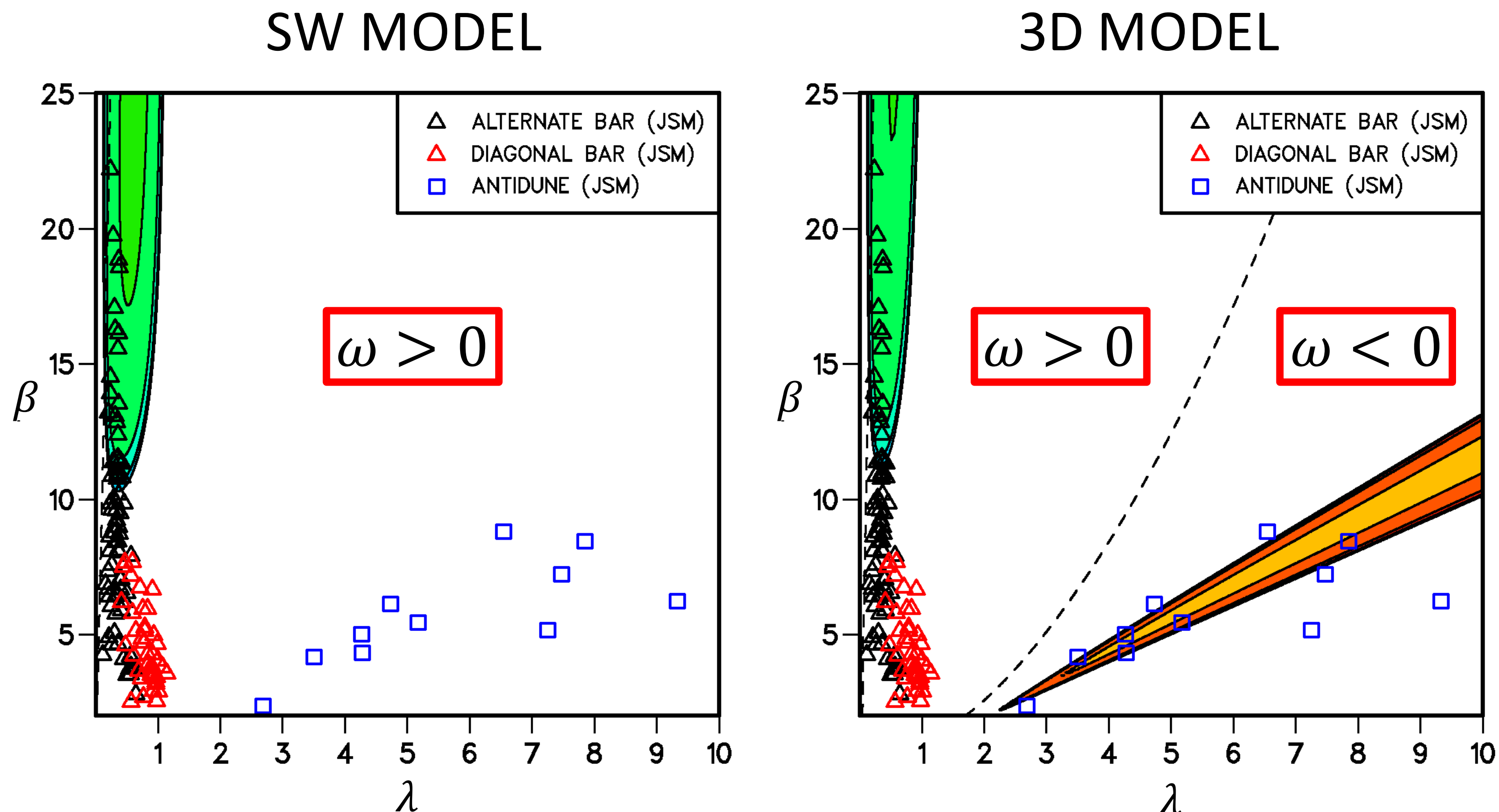
$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Leftrightarrow \quad Fr = 1 \quad d = 0.025$$

SW MODEL | 3D MODEL



Alternate Bars: linear stability

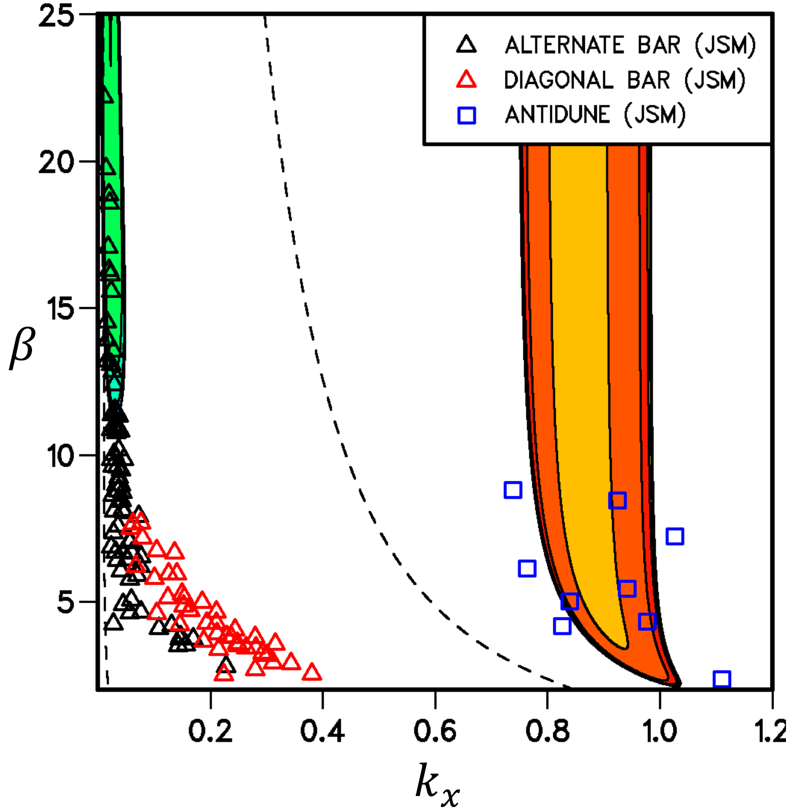
$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

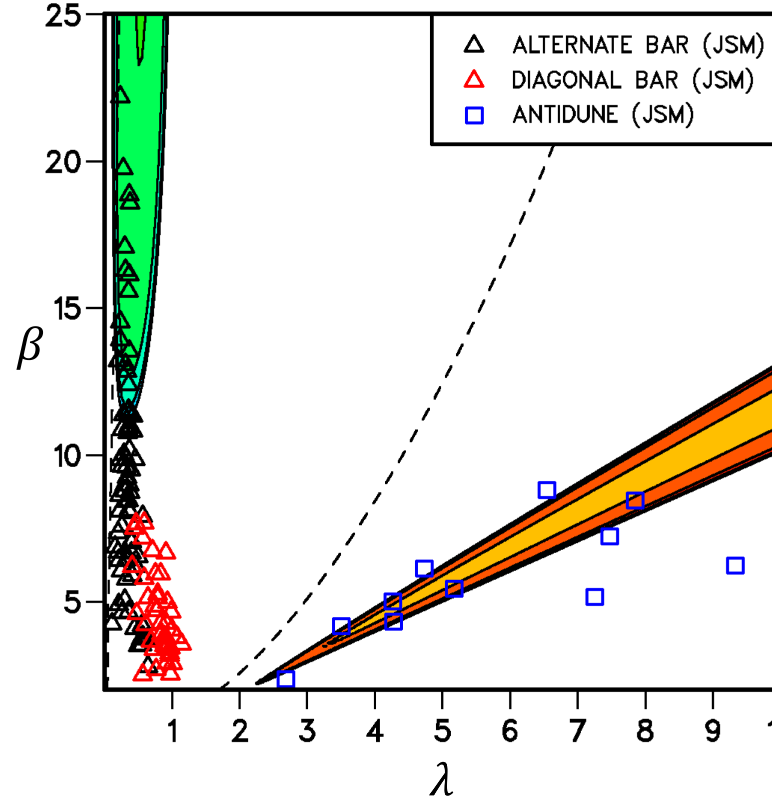


$$\beta = \frac{\lambda}{k_x}$$

Alternate Bars: 3D linear stability

$$\frac{\vartheta}{\vartheta_c} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

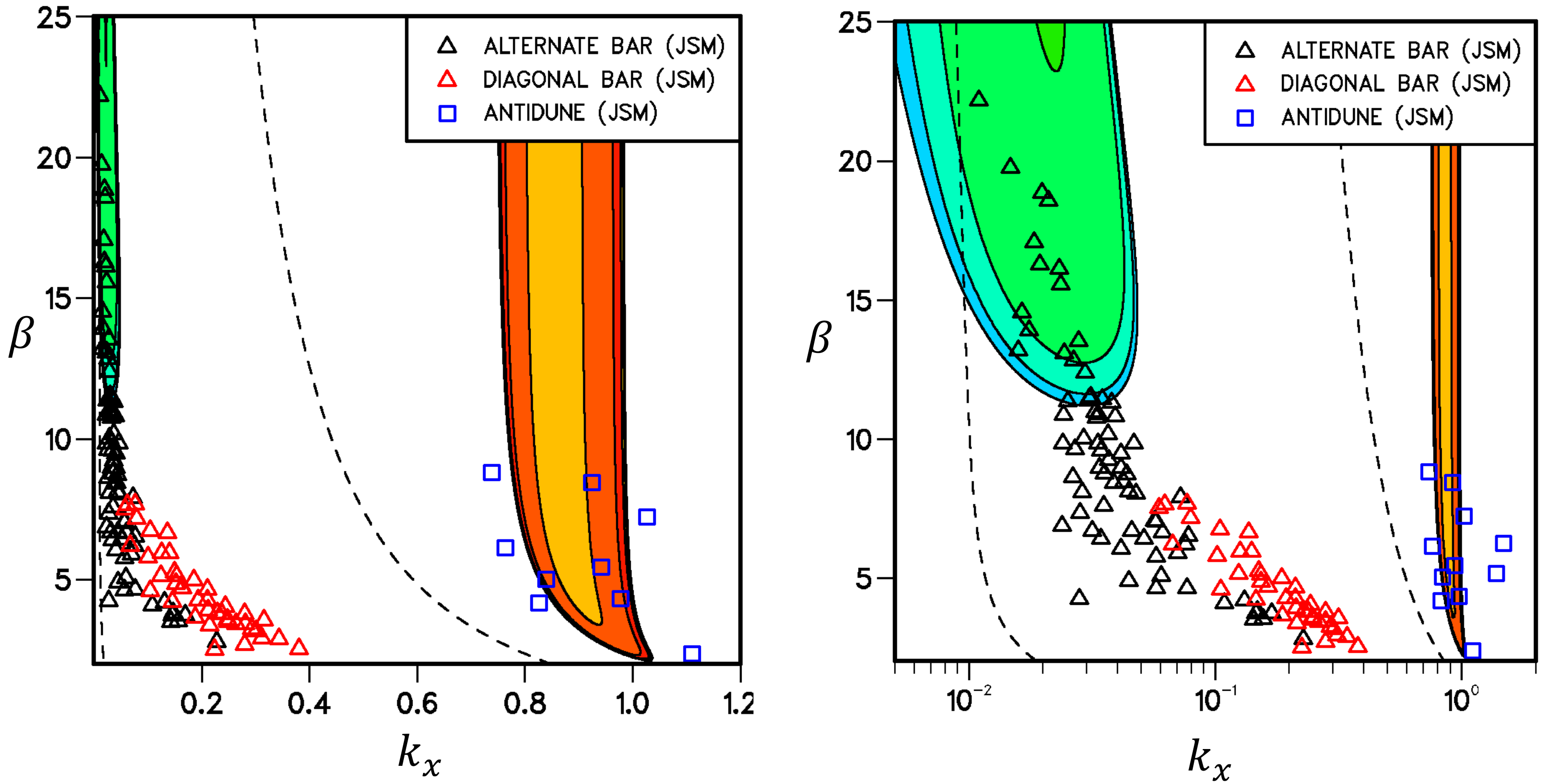




$$\beta = \frac{\lambda}{k_x}$$

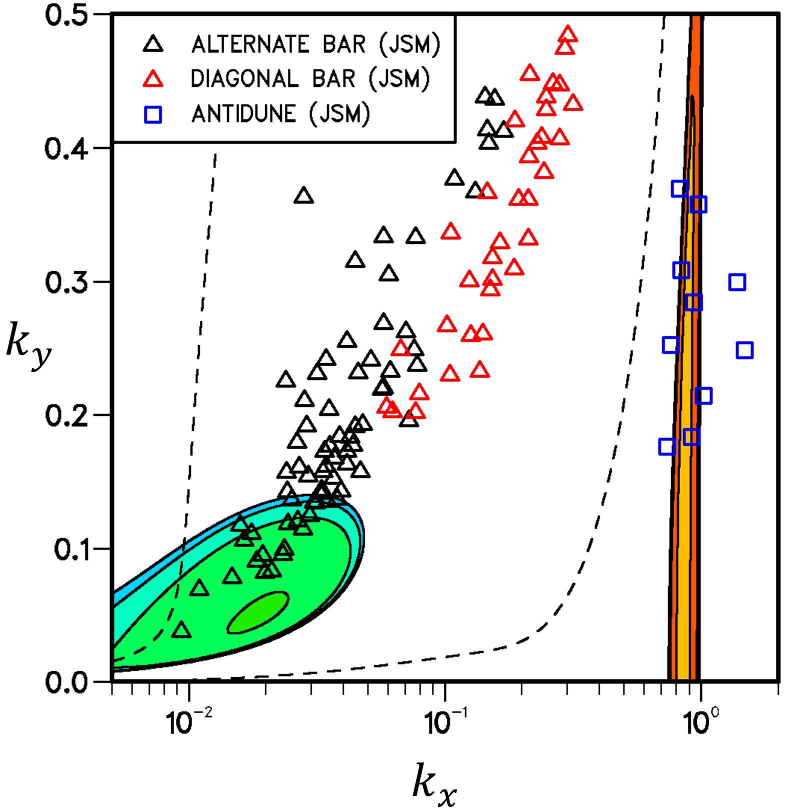
Alternate Bars: 3D linear stability

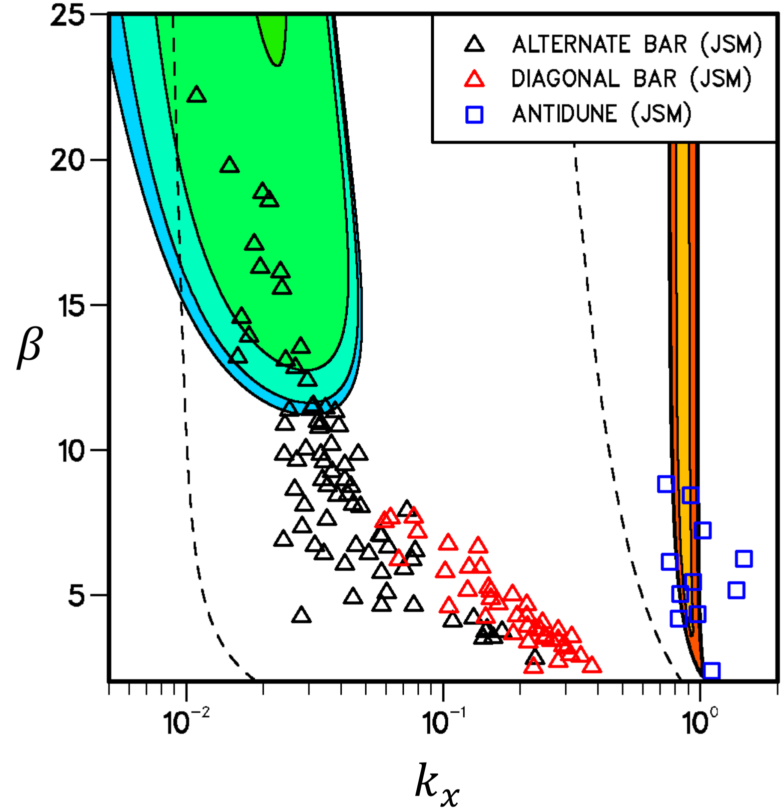
$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$



Alternate Bars: 3D linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

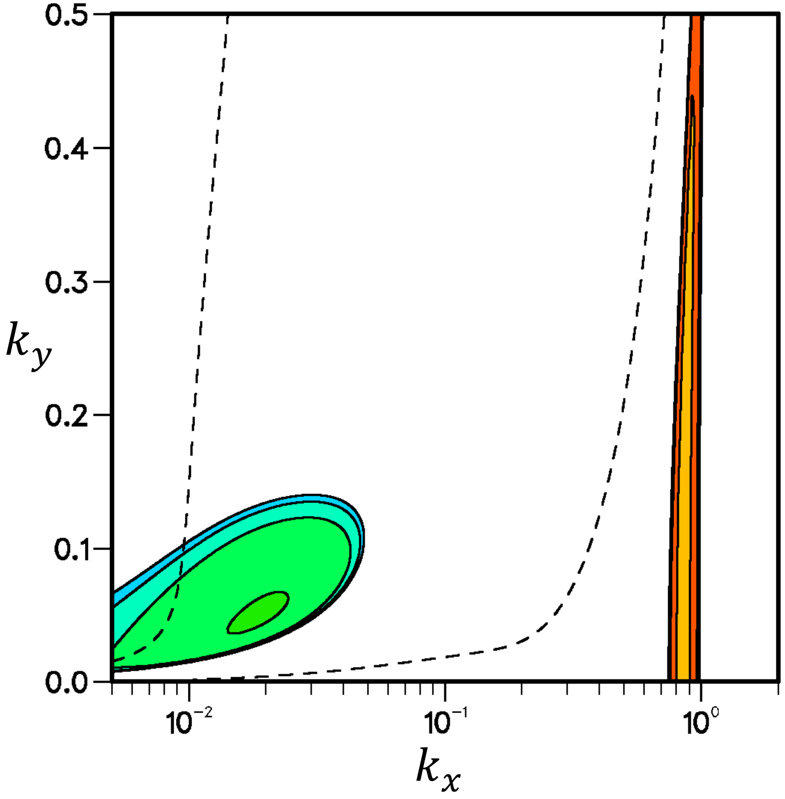




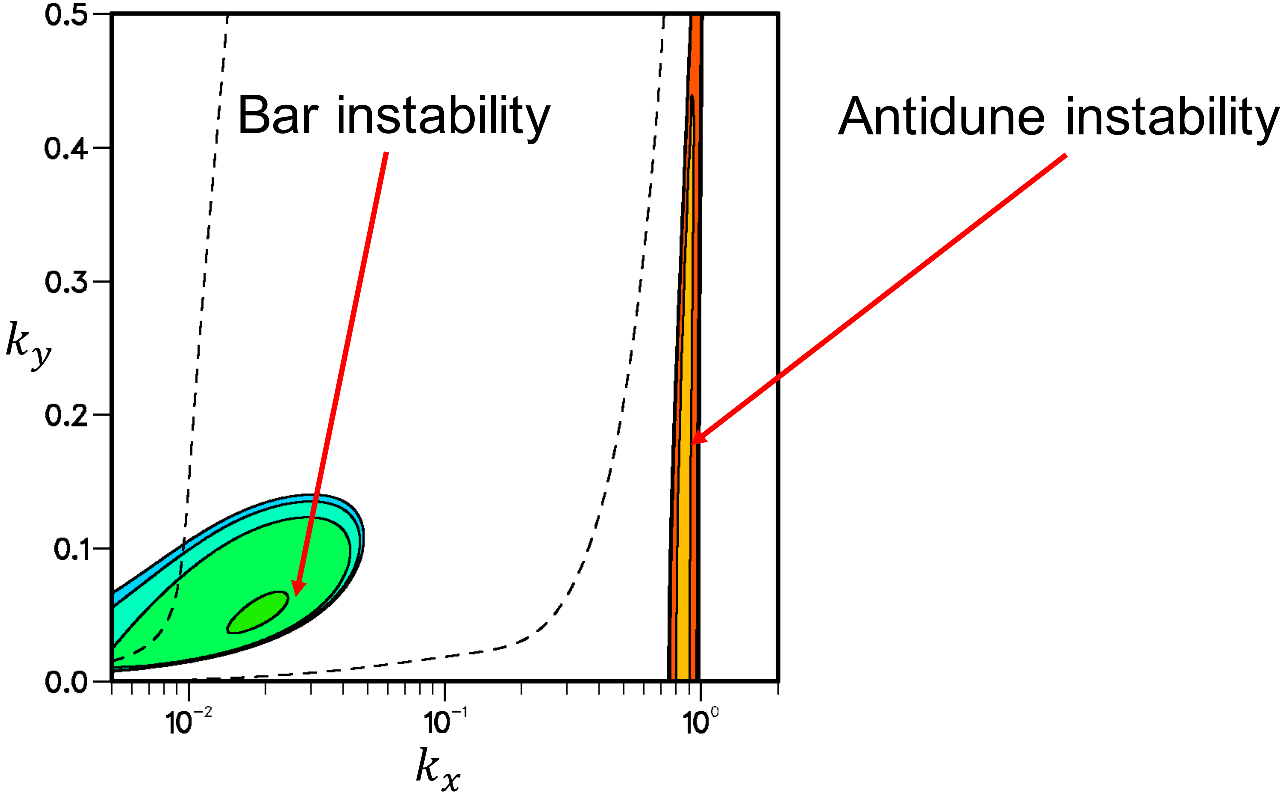
$$k_y = \frac{\pi}{2\beta}$$

Alternate Bars: 3D linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Leftrightarrow \quad Fr = 1 \quad d = 0.025$$

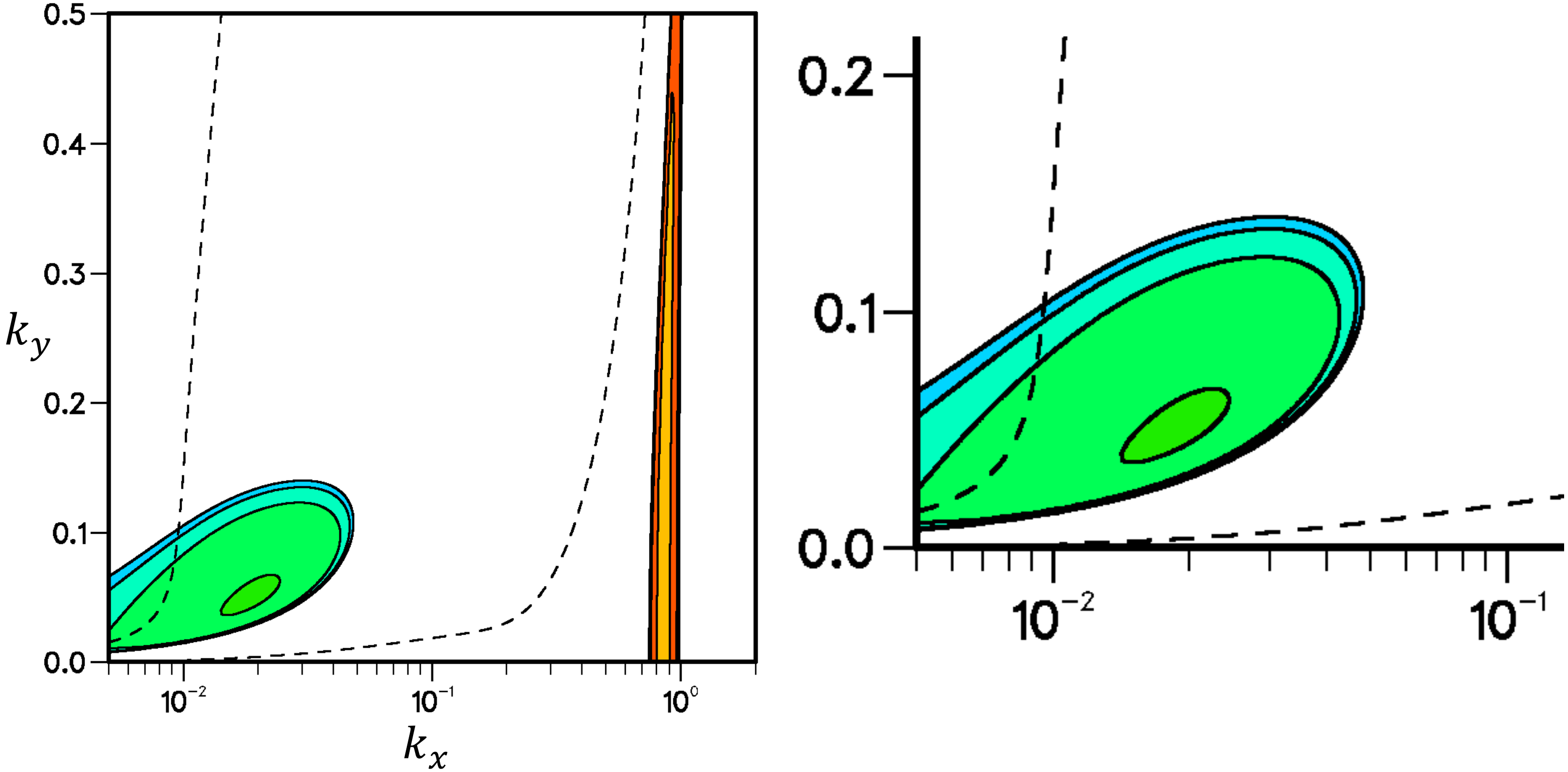


$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$



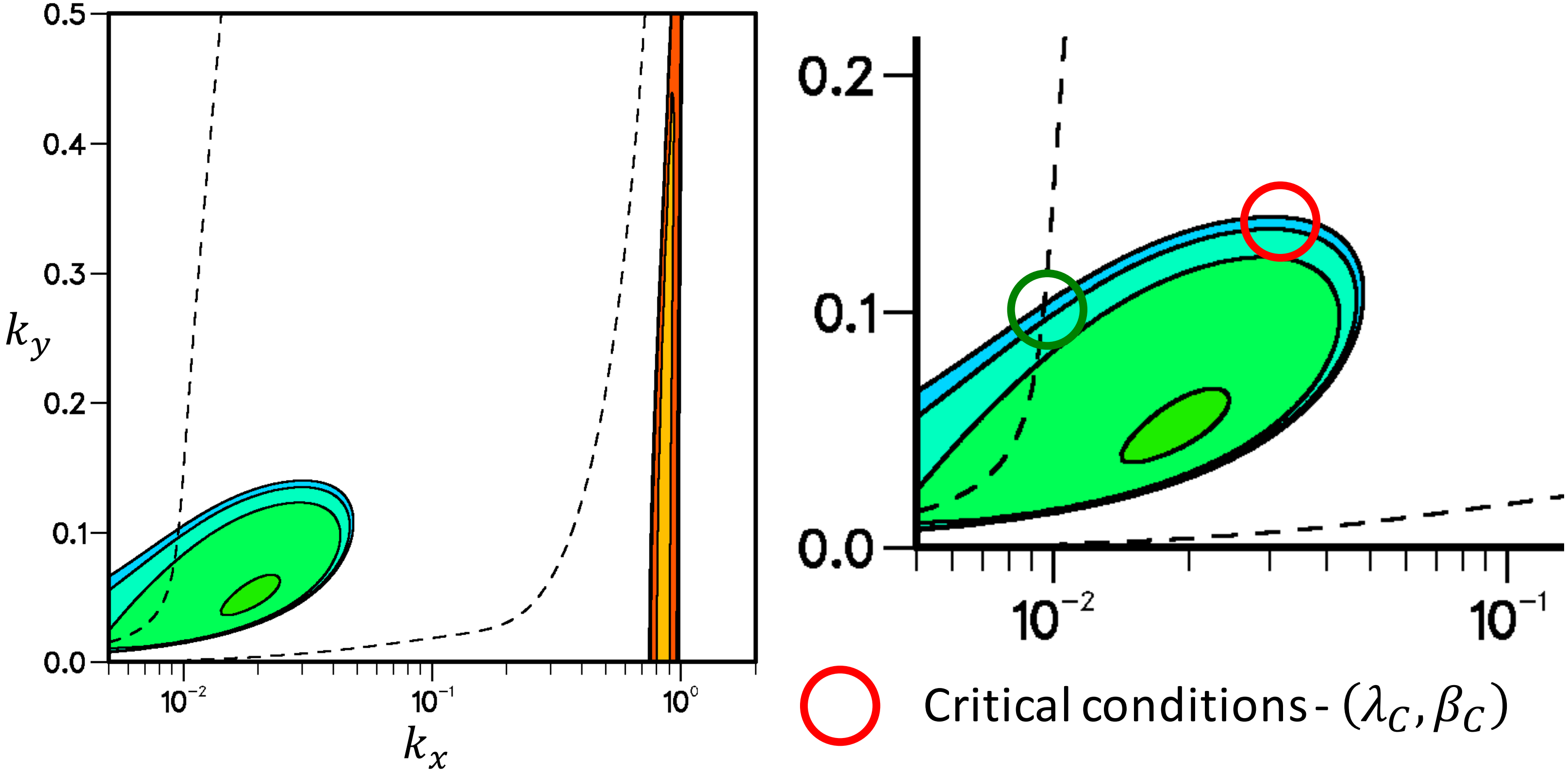
Alternate Bars: 3D linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$



Alternate Bars: 3D linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

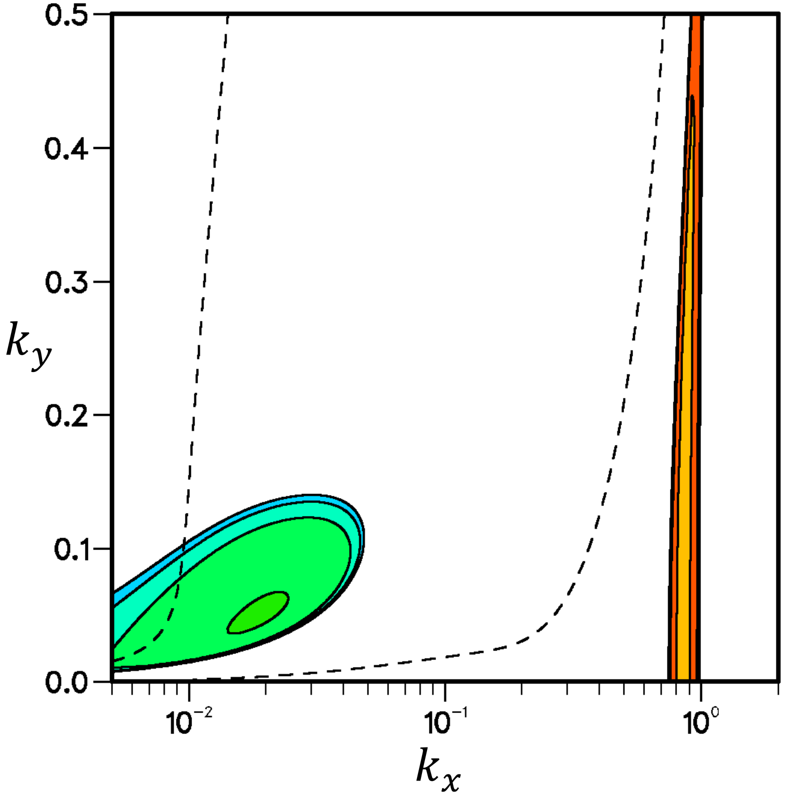


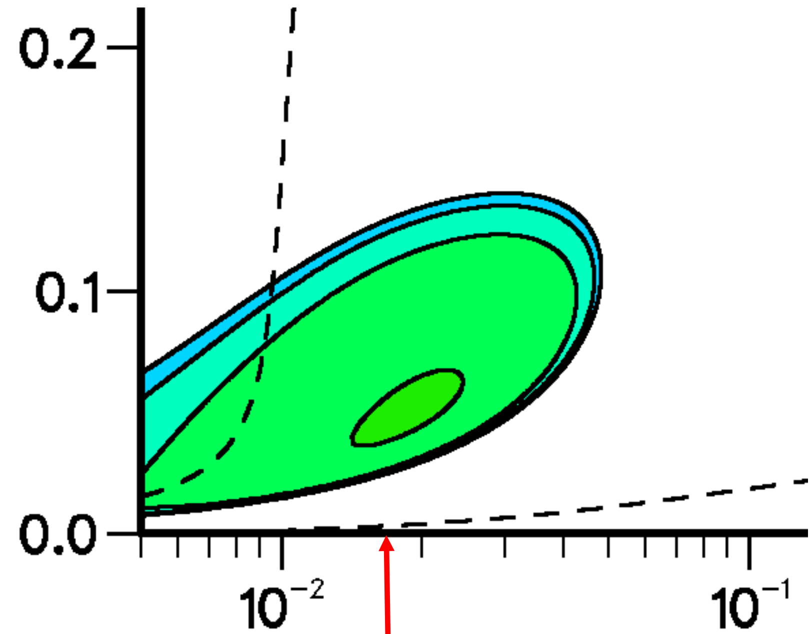
Critical conditions - (λ_C, β_C)

Resonant conditions - (λ_R, β_R)

Alternate Bars: 3D linear stability

$$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Longleftrightarrow \quad Fr = 1 \quad d = 0.025$$

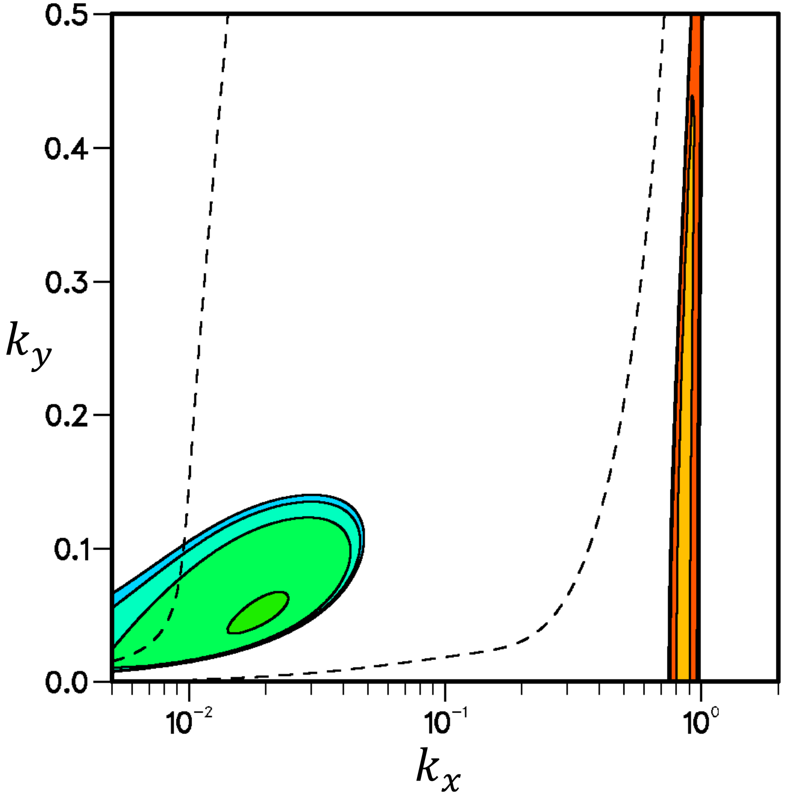


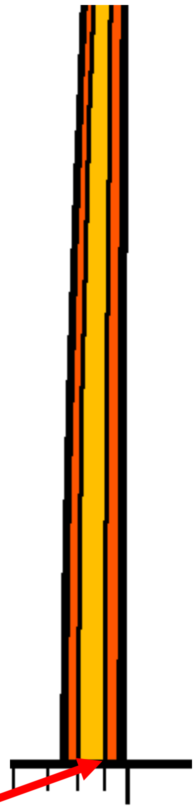


2D disturbances are stable!

Alternate Bars: 3D linear stability

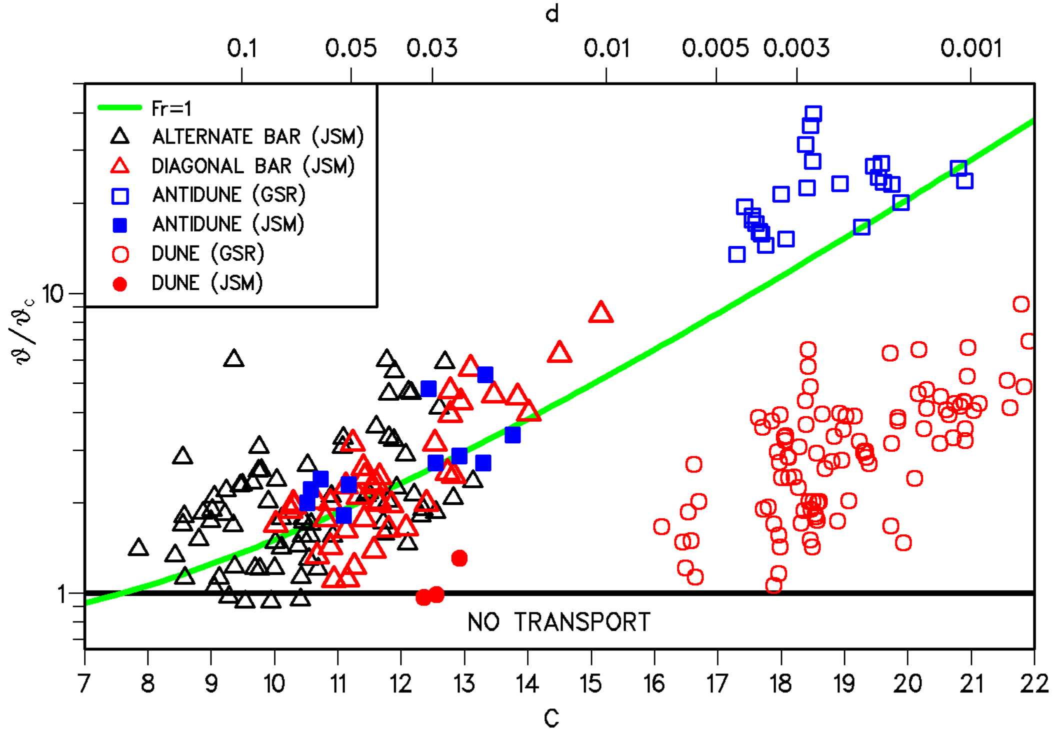
$\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13$ ⟷ $Fr = 1 \quad d = 0.025$





2D disturbances are the most unstable!

Regime diagram: bars & dunes



$$C = \frac{1}{\kappa} \ln\left(\frac{11.09}{2.5d}\right)$$

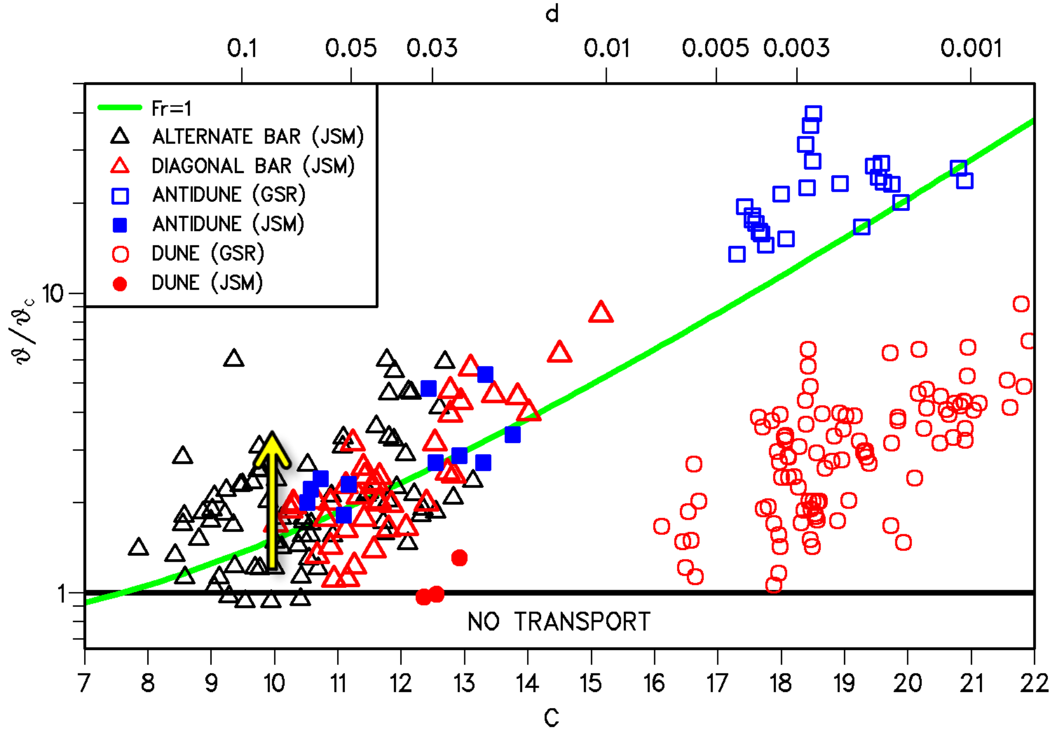
$$\vartheta \cong 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}$$

Experimental data sets

JSM: Jaeggi (1984), Sukegawa (1971), Muramoto & Fujita (1978)

GSR: Guy, Simons & Richardson (1966)

Regime diagram: bars & dunes



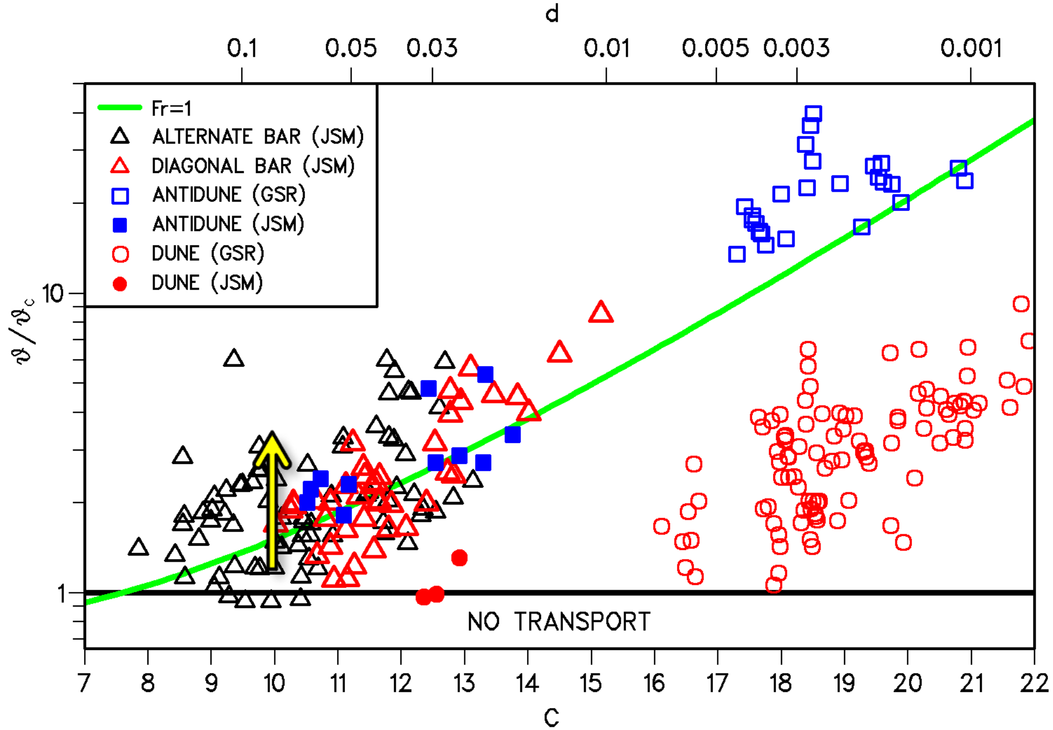
$$C = \frac{1}{\kappa} \ln\left(\frac{11.09}{2.5d}\right)$$

$$\vartheta \cong 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}$$

Moving up along a vertical line means (same sediment):

- C constant $\Rightarrow d$ constant $\Rightarrow D^*$ constant
- θ increases $\Rightarrow Fr$ increases $\Rightarrow U^*$ increases

Regime diagram: bars & dunes



$$C = \frac{1}{\kappa}\ln\left(\frac{11.09}{2.5d}\right)$$

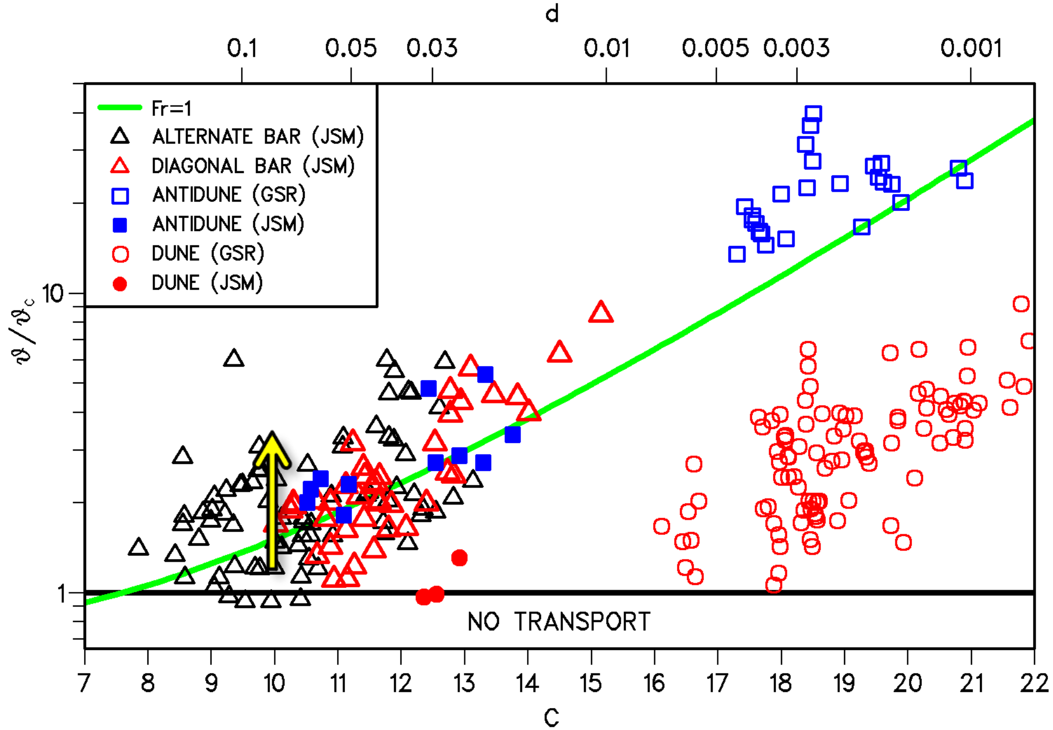
$$\vartheta \cong 0.14\,\frac{Fr^2 e^{\kappa C}}{C^2}$$

Moving up along a vertical line means (same sediment):

- C constant $\Rightarrow d$ constant $\Rightarrow D^*$ constant
- θ increases $\Rightarrow Fr$ increases $\Rightarrow U^*$ increases

$\Big\} \Rightarrow S \propto Fr^2$ increases

Regime diagram: bars & dunes



$$C = \frac{1}{\kappa} \ln\left(\frac{11.09}{2.5d}\right)$$

$$\vartheta \cong 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}$$

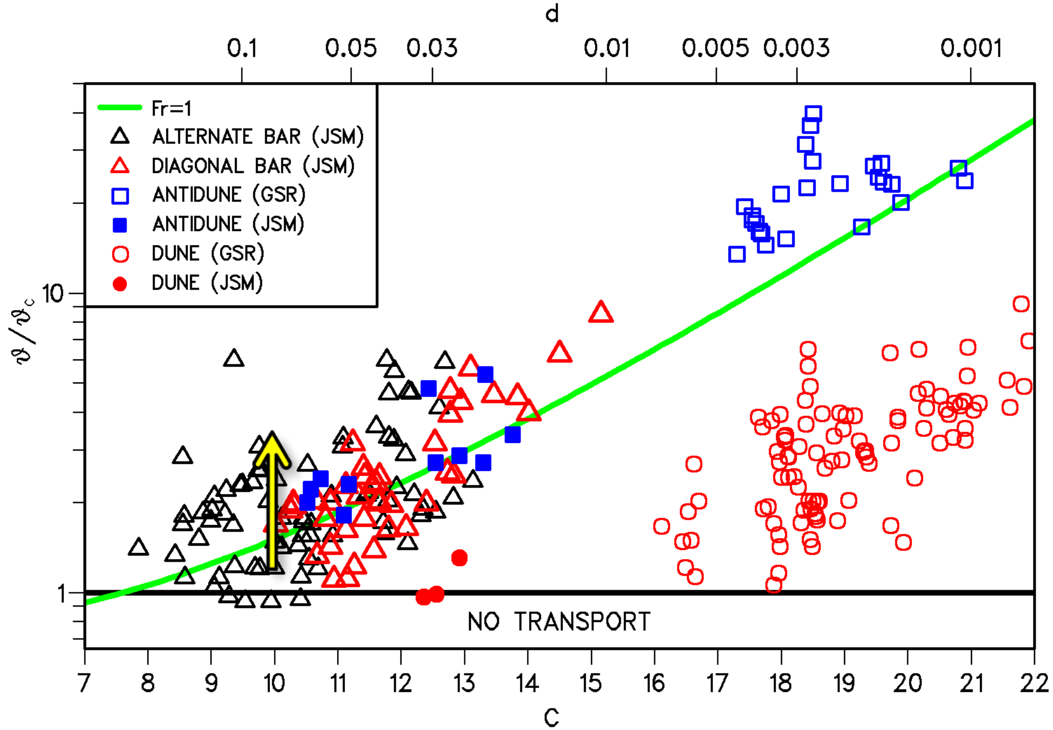
Moving up along a vertical line means (same sediment):

- C constant $\Rightarrow d$ constant $\Rightarrow D^*$ constant
- θ increases $\Rightarrow Fr$ increases $\Rightarrow U^*$ increases

$\Rightarrow S \propto Fr^2$ increases

- Flow rate increases with constant flow depth

Regime diagram: bars & dunes



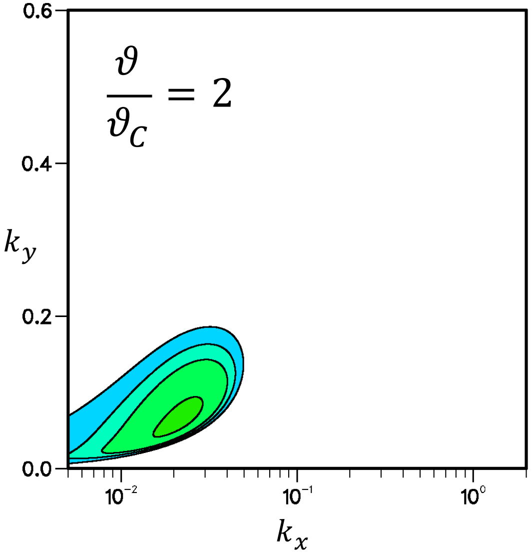
$$C = \frac{1}{\kappa} \ln\left(\frac{11.09}{2.5d}\right)$$

$$\vartheta \cong 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}$$

Moving up along a vertical line means (same sediment):

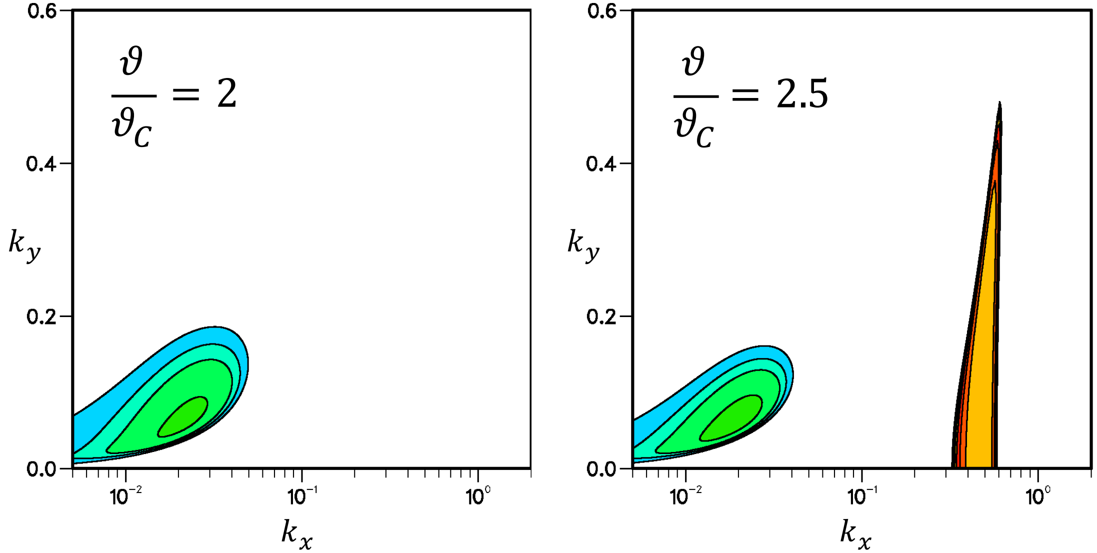
- C constant $\Rightarrow$ d constant $\Rightarrow$ D^* constant
- θ increases $\Rightarrow$ Fr increases $\Rightarrow$ U^* increases
- Flow rate increases with constant flow depth $\Rightarrow$ Slope increases
- From Alternate Bars to Alternate Bars & Antidunes

Moving up: $C = 10$



As ϑ increases:

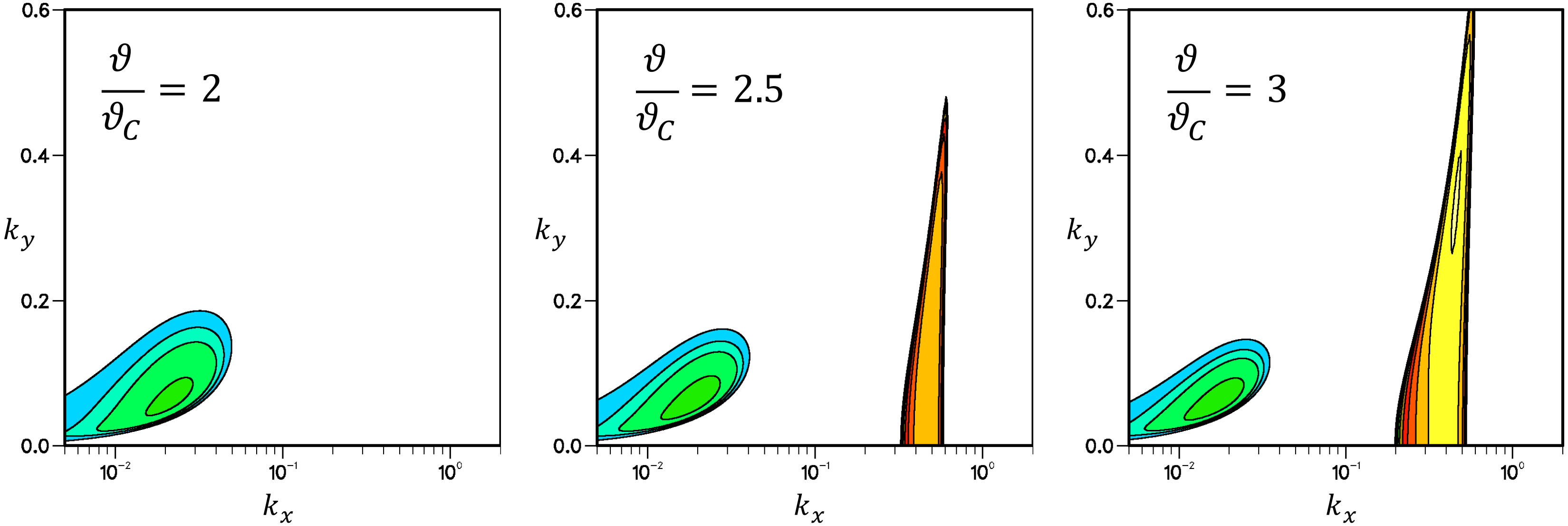
Moving up: $C = 10$



As ϑ increases:

- A new region of instability appears: 2D antidunes;
- The regions of instability for bars and antidunes are distinct;
- Bars and antidunes linearly coexist: bars are less unstable;

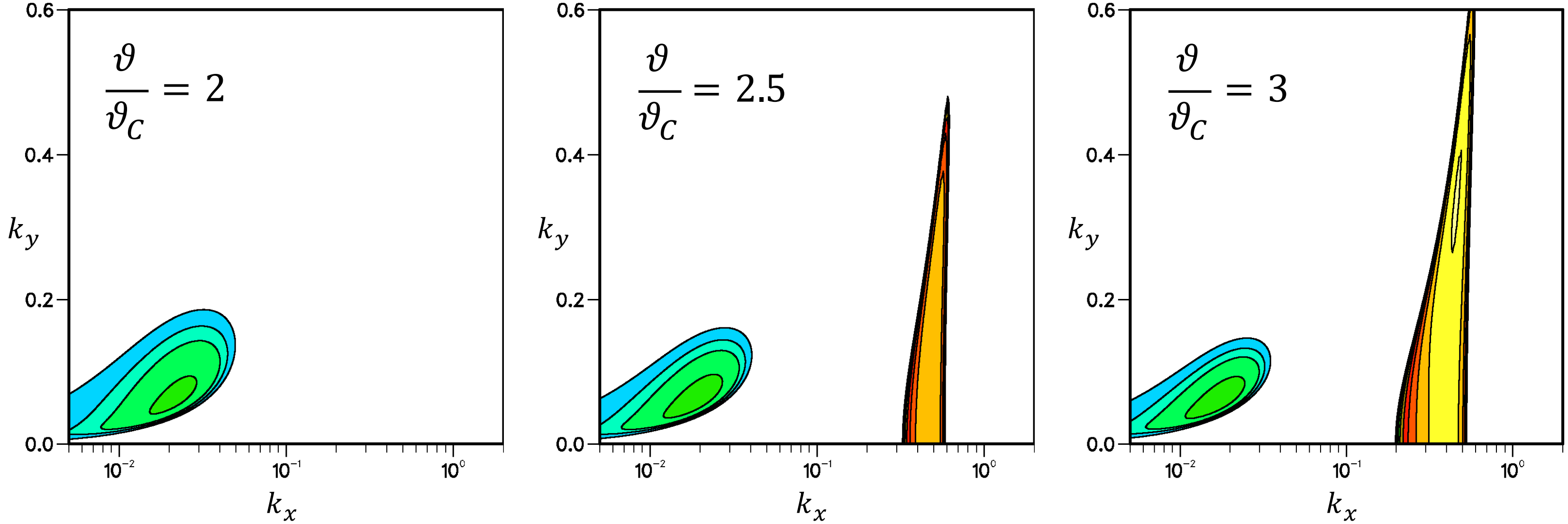
Moving up: $C = 10$



As ϑ increases:

- A new region of instability appears: 2D antidunes;
- The regions of instability for bars and antidunes are distinct;
- Bars and antidunes linearly coexist: bars are less unstable;
- 3D antidunes become the most unstable;

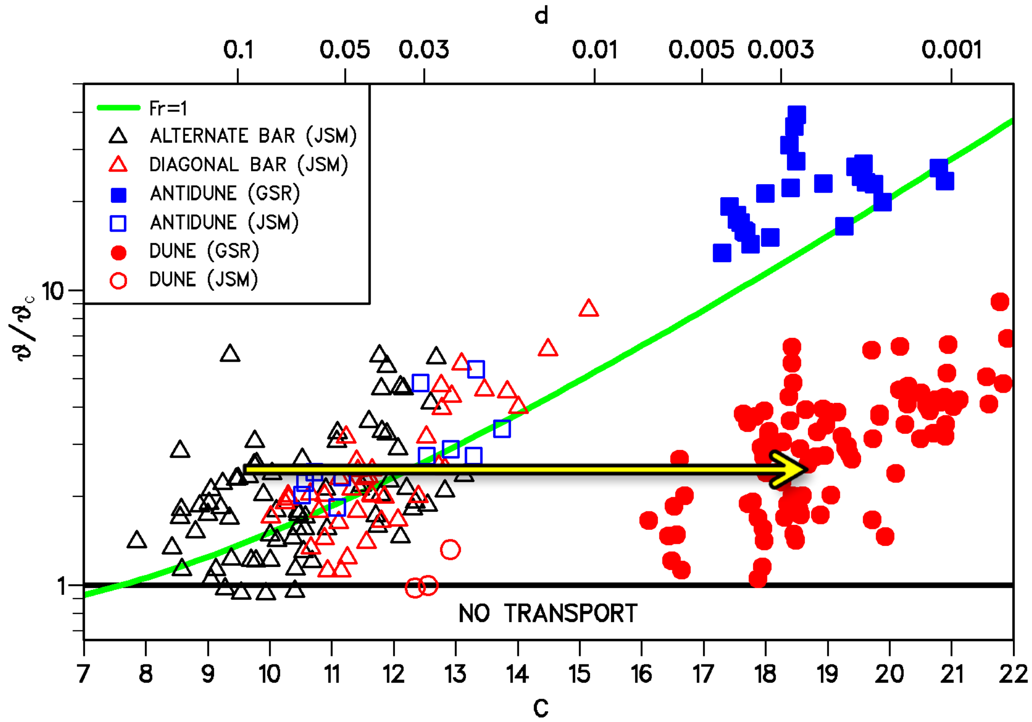
Moving up: C = 10



As ϑ increases:

- A new region of instability appears: 2D antidunes;
- The regions of instability for bars and antidunes are distinct;
- Bars and antidunes linearly coexist: bars are less unstable;
- 3D antidunes become the most unstable;
- This results in a transition from 2D to 3D antidunes;

Regime diagram: bars & dunes



$$C = \frac{1}{\kappa} \ln\left(\frac{11.09}{2.5d}\right)$$

$$\vartheta \cong 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}$$

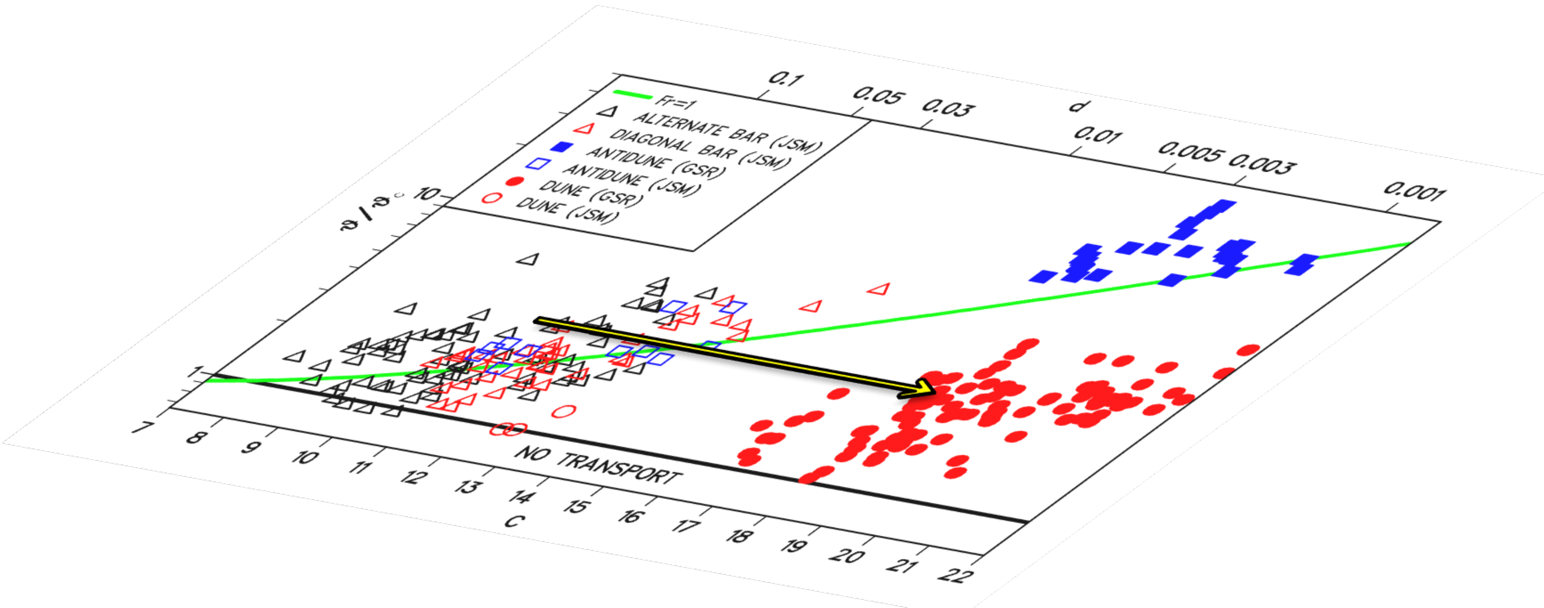
Moving right along a horizontal line means (same sediment):

- C increases $\Rightarrow d$ decreases $\Rightarrow D^*$ increases
- θ constant $\Rightarrow U^* \propto C$ increases

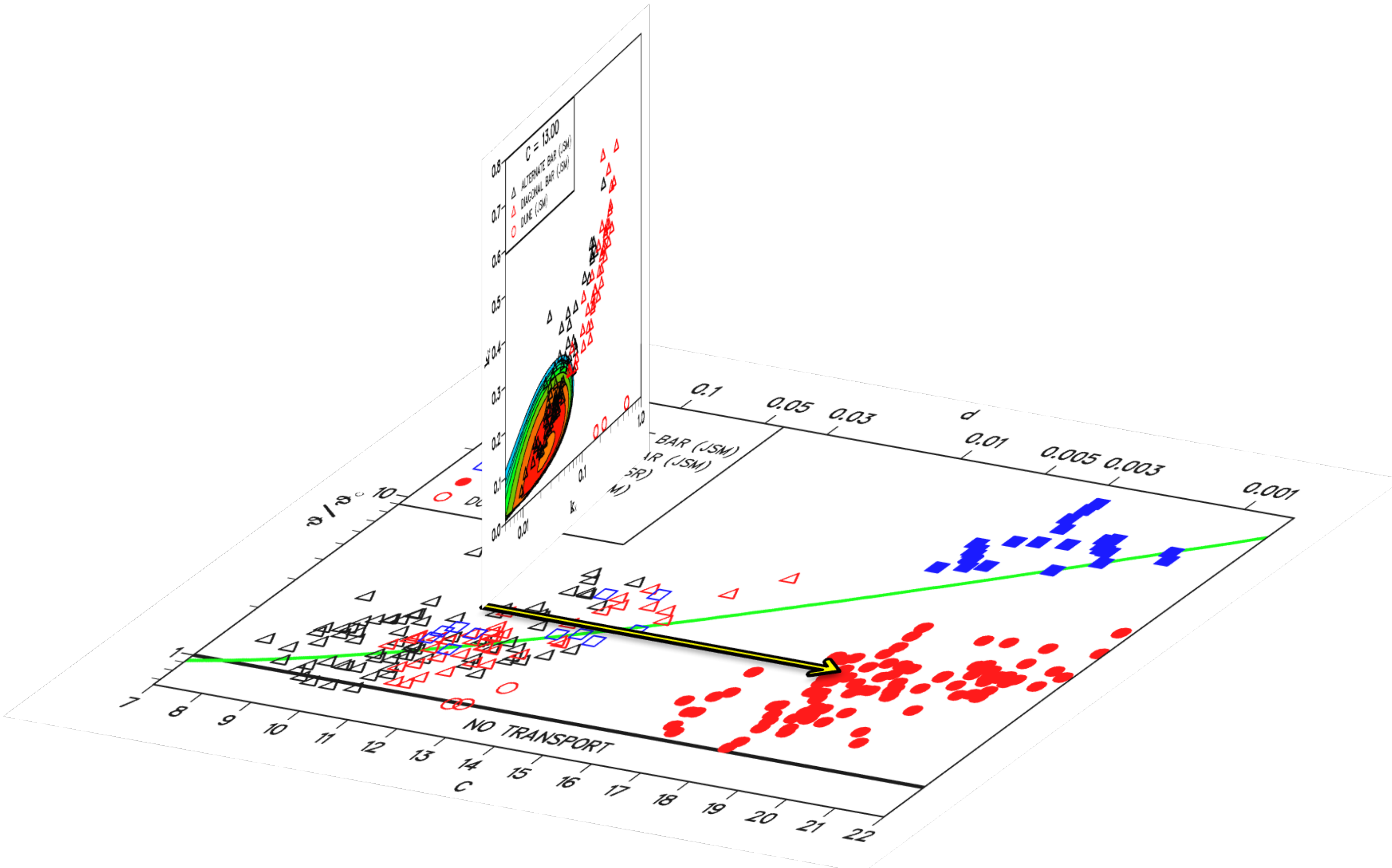
$\} \Rightarrow S \propto 1/D^*$ decreases

- Flow rate increases but Fr decreases
- From Alternate Bars to Diagonal Bars to 2D Dunes

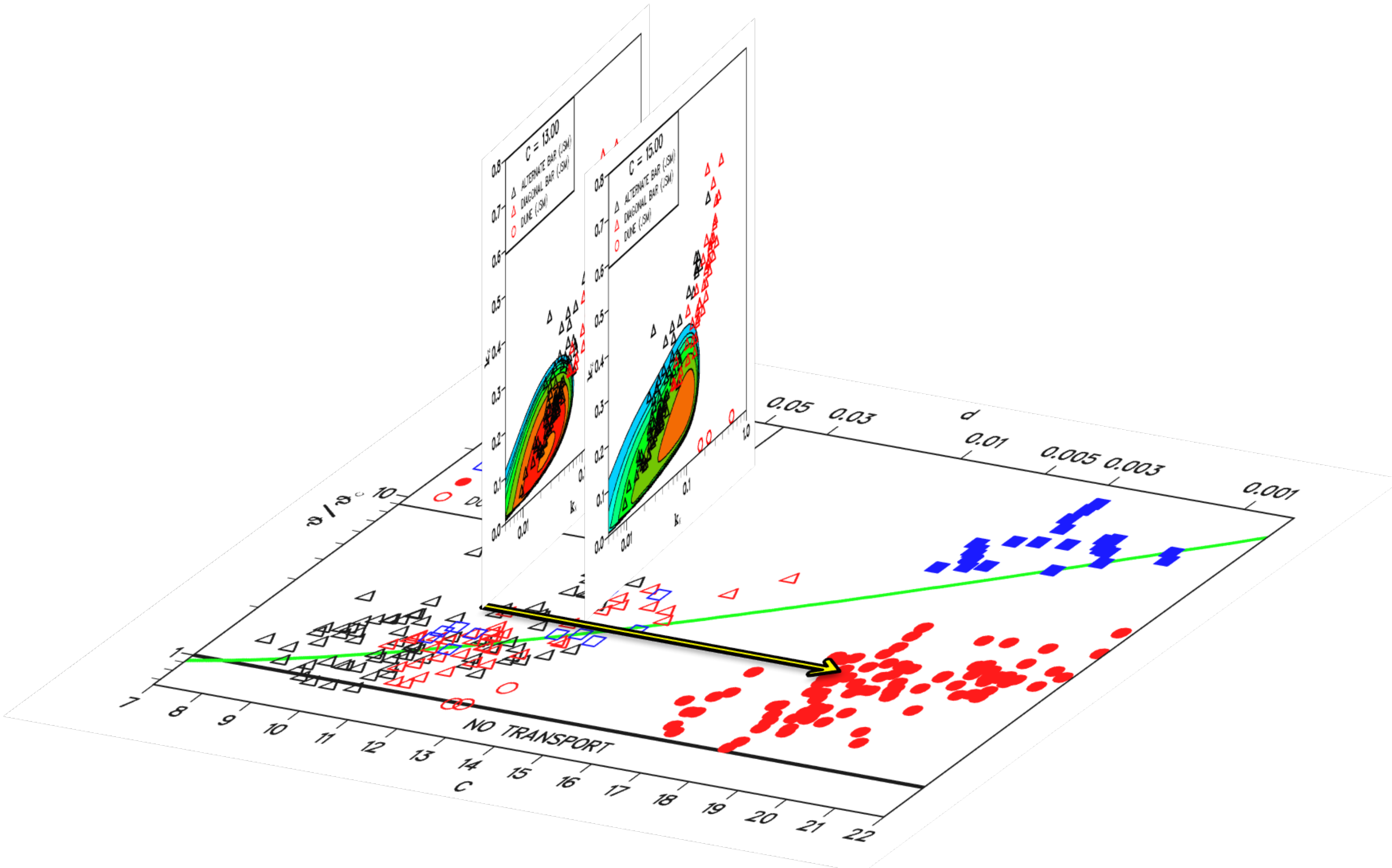
Regime diagram: bars & dunes



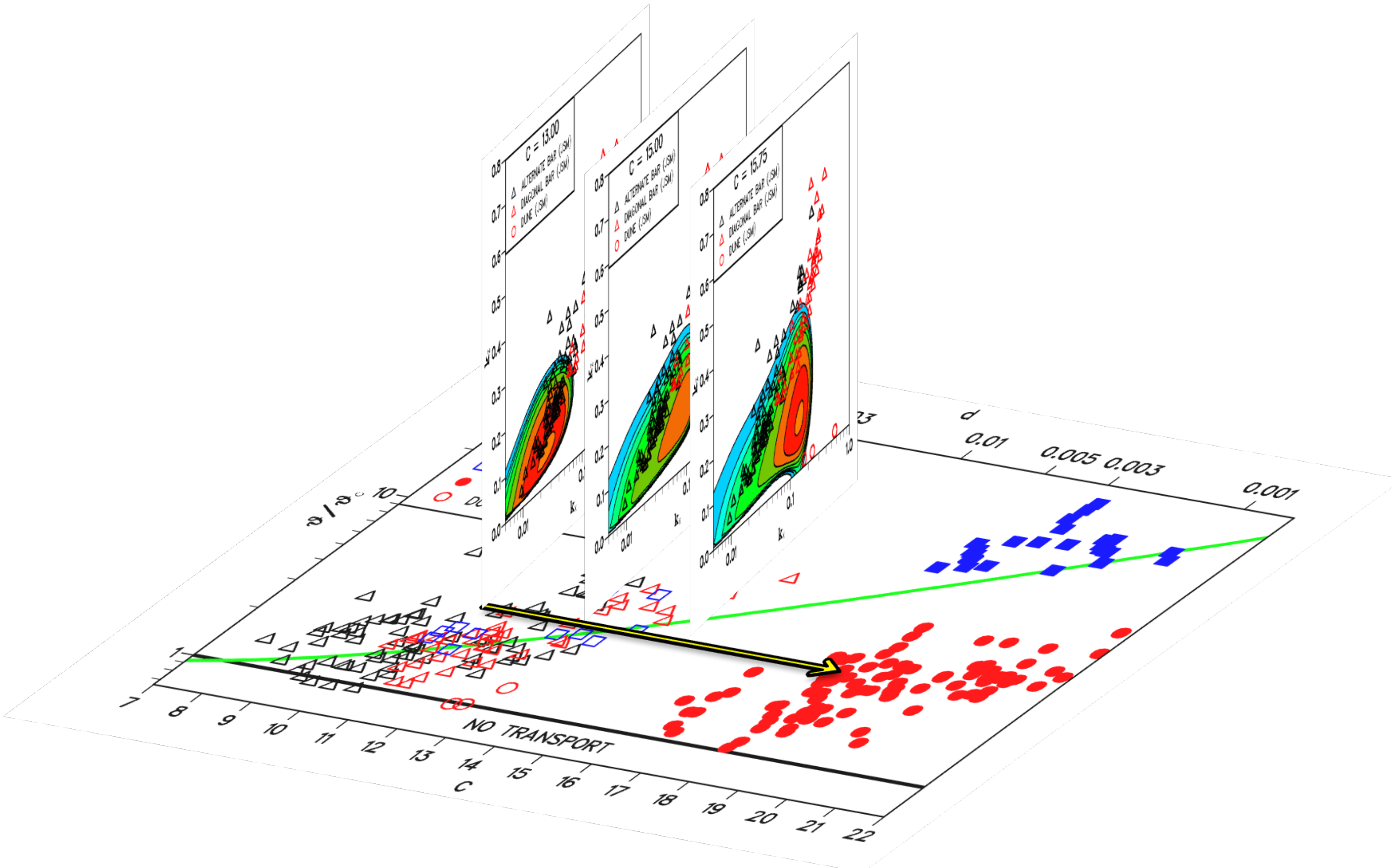
Regime diagram: bars & dunes



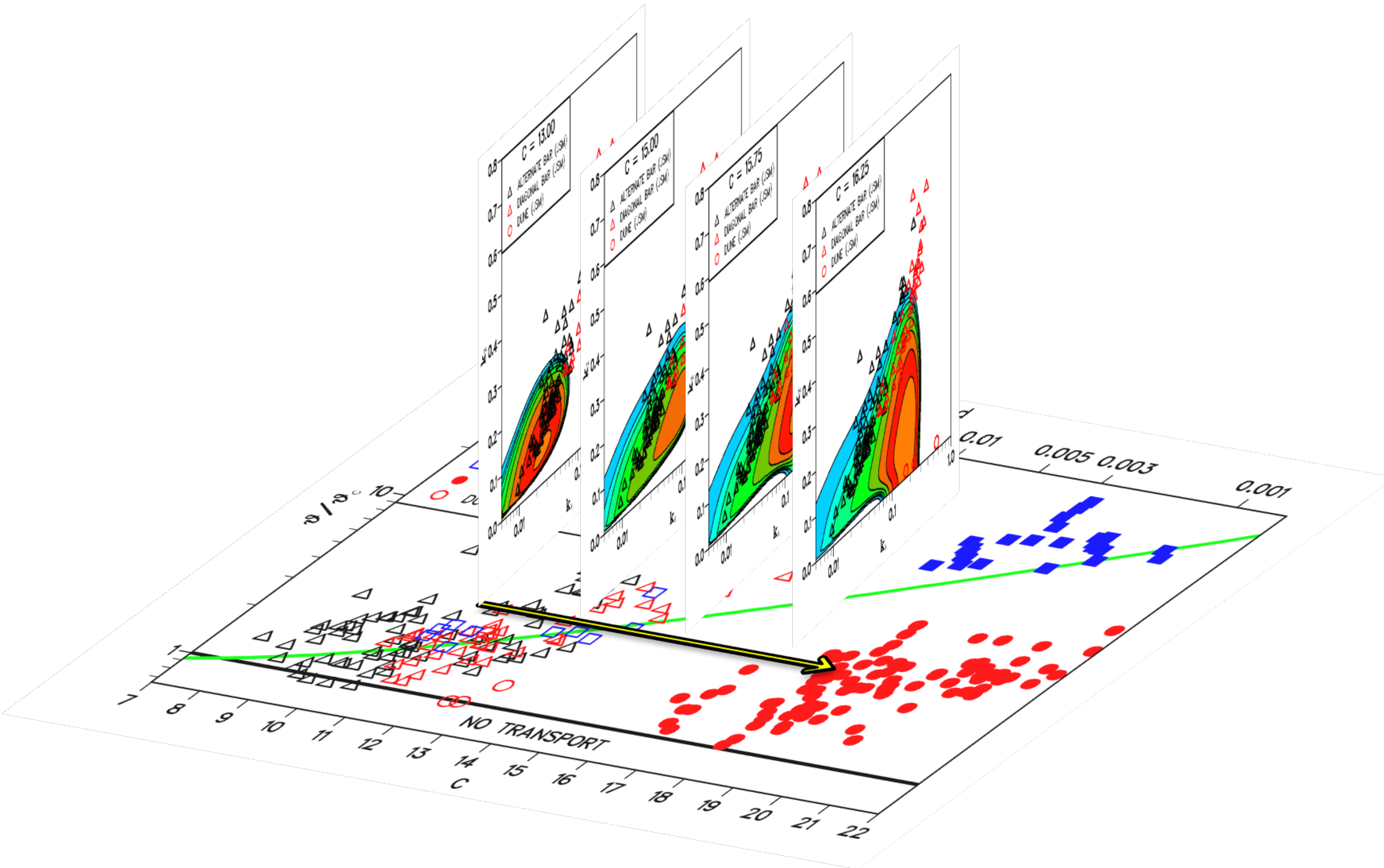
Regime diagram: bars & dunes



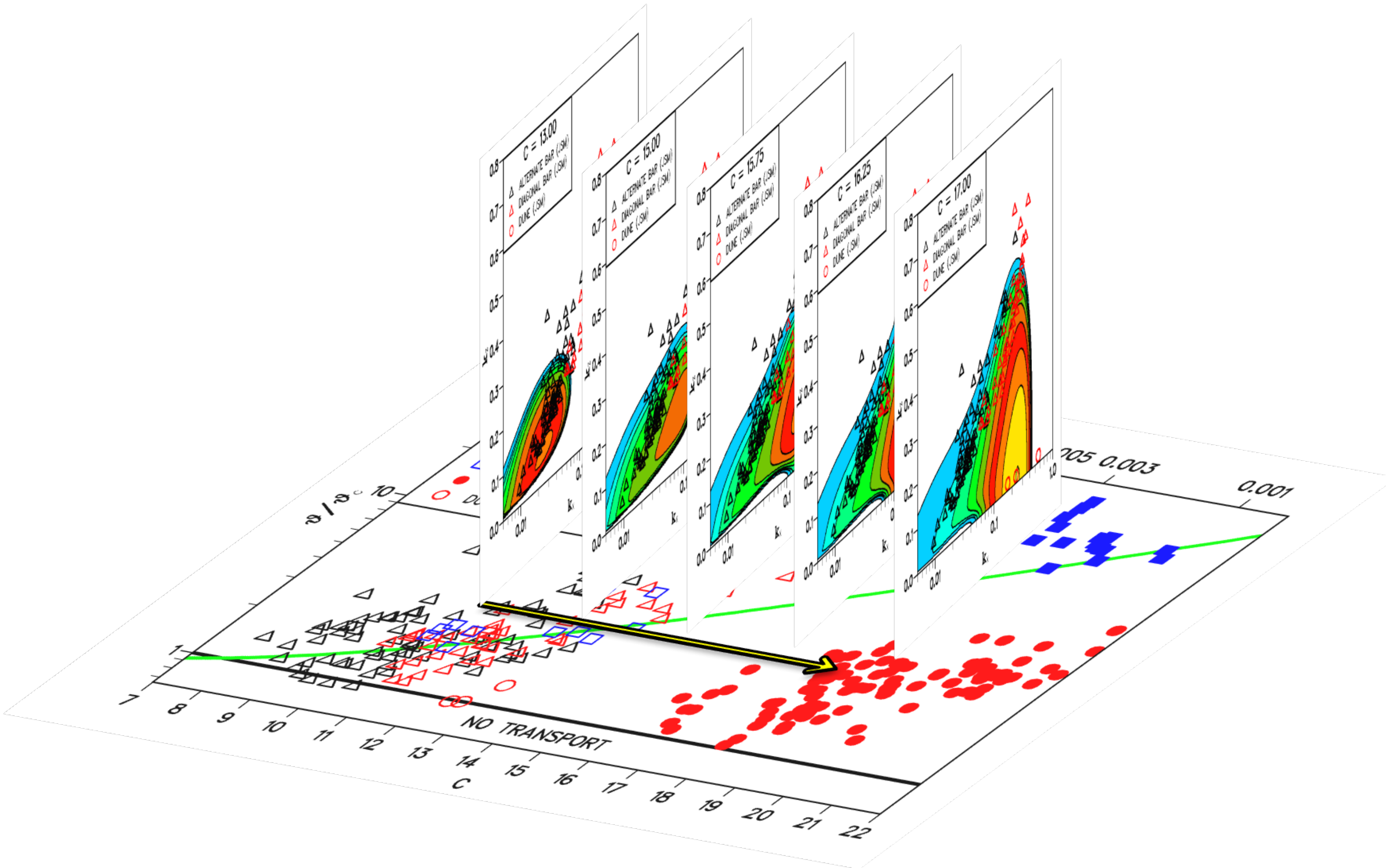
Regime diagram: bars & dunes



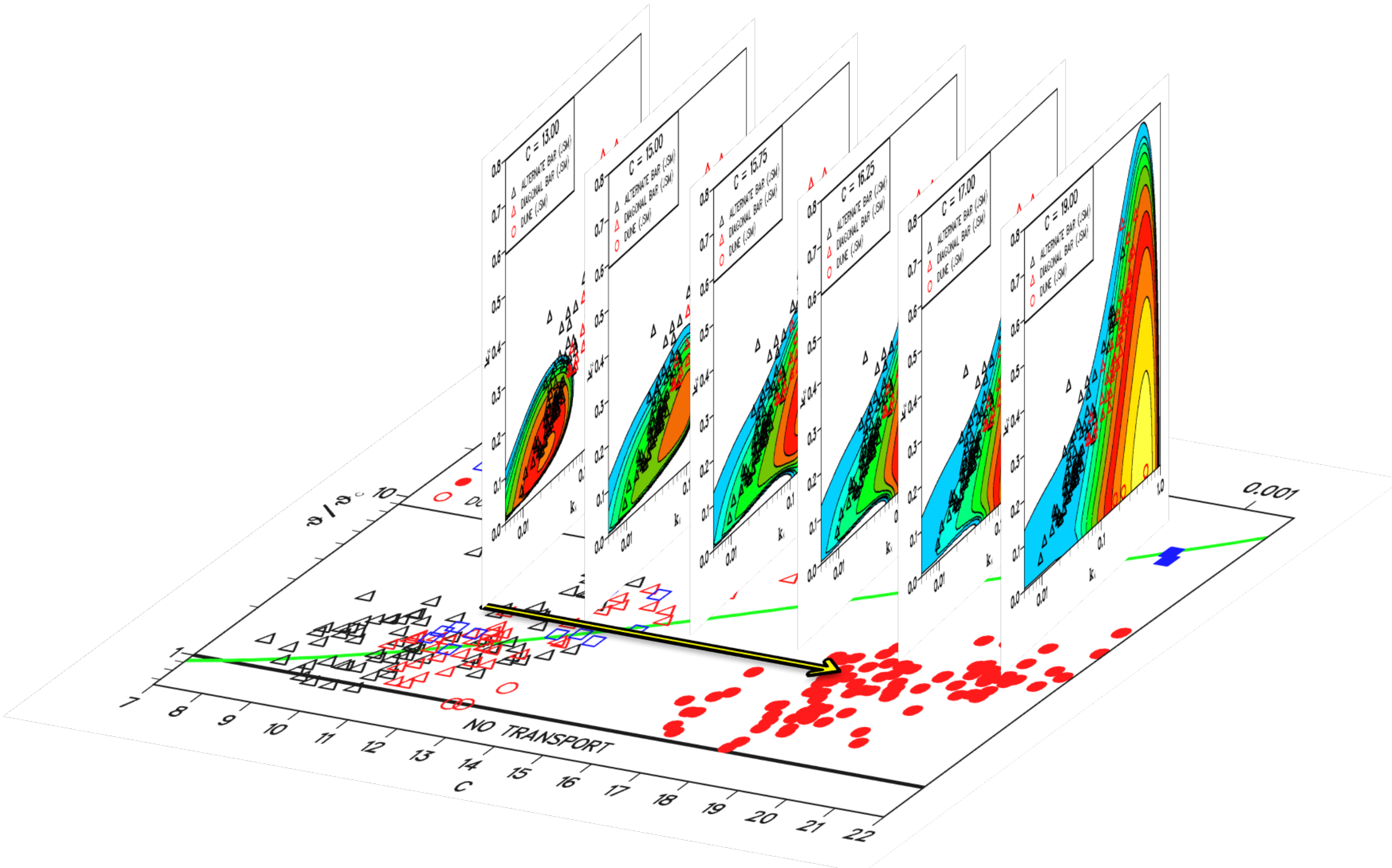
Regime diagram: bars & dunes



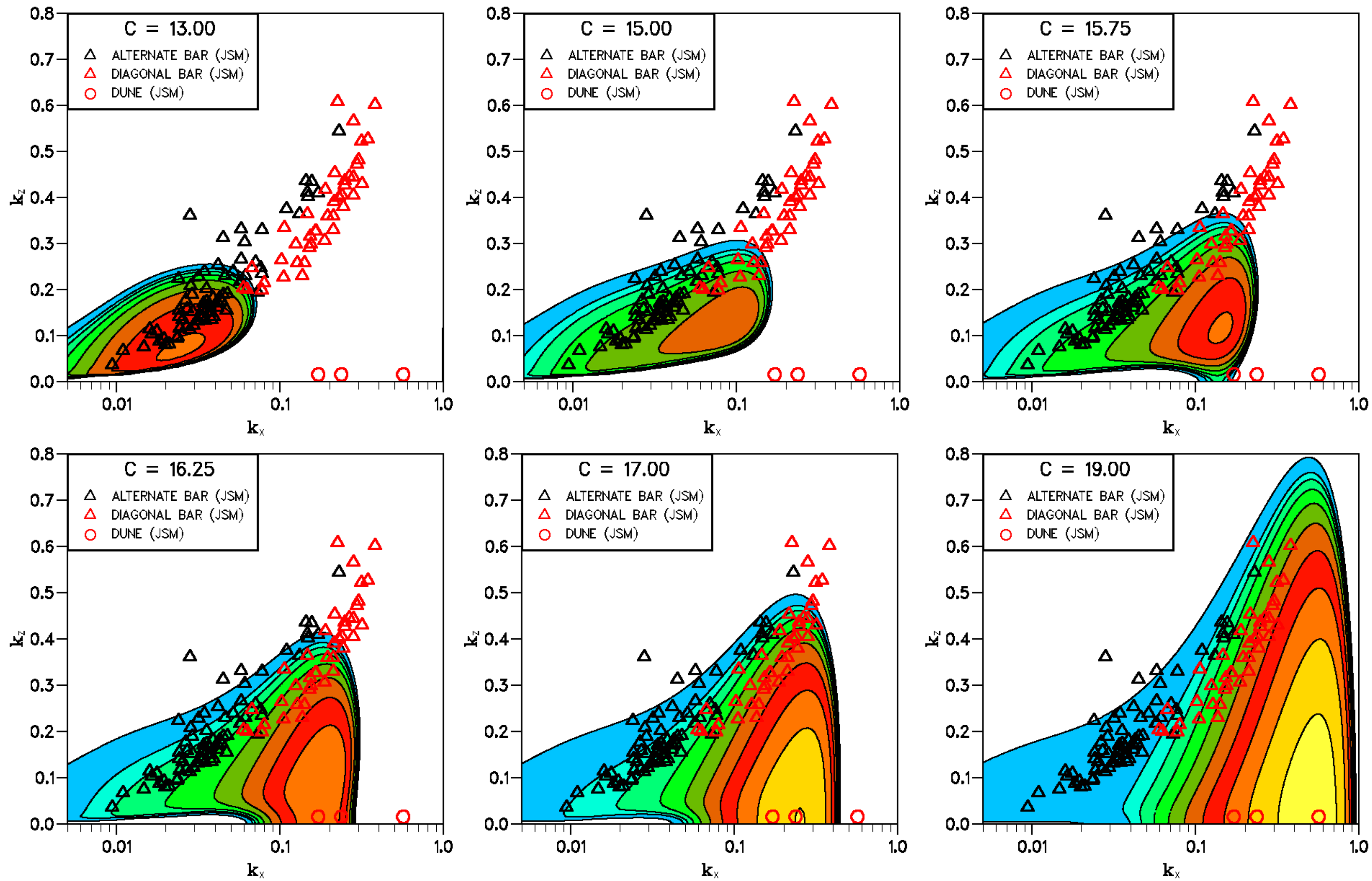
Regime diagram: bars & dunes



Regime diagram: bars & dunes



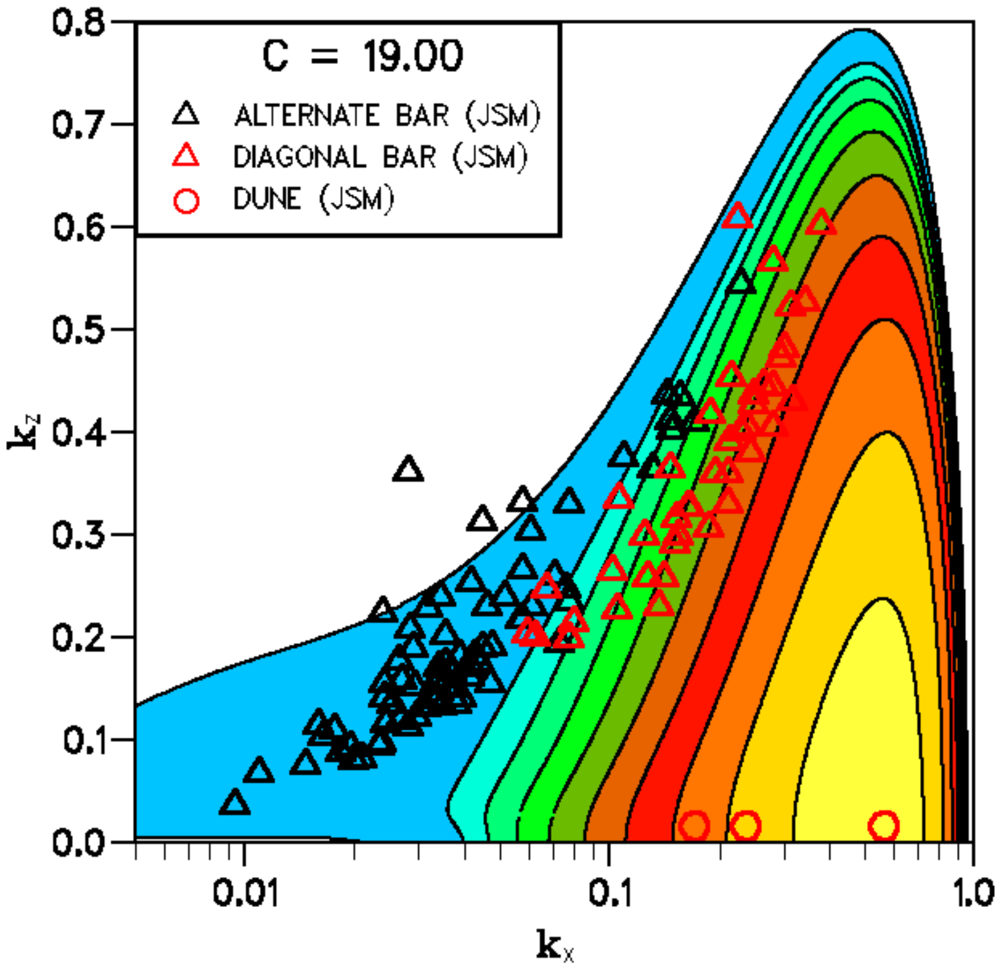
Dune-ripple transition



As C increases:

- the longitudinal wavenumber of maximum growth rate increases: bars become shorter;
- 2D disturbances become unstable but are less unstable than 3D disturbances;
- 2D disturbances become the most unstable;
- This results in a transition from 3D bars to 2D dunes via diagonal bars (3D dunes);

Dune-ripple transition



As C increases:

- the longitudinal wavenumber of maximum growth rate increases: bars become shorter;
- 2D disturbances become unstable but are less unstable than 3D disturbances;
- 2D disturbances become the most unstable;
- This results in a transition from 3D bars to 2D dunes via diagonal bars (3D dunes);