Stability of Alternate Bars and Oblique Dunes

Alternate bars in the Tokachi River, Japan
Image courtesy of V. Langlois
Stability of River Bed Forms
Part III

- Part I – linear stability of roll-waves, dunes & antidunes;
- Part II – linear stability of ripples and dunes;
Stability of River Bed Forms

Part III

- Part I – linear stability of roll-waves, dunes & antidunes;
- Part II – linear stability of ripples and dunes;
- Part III – linear stability of alternate bars & oblique dunes;
Stability of River Bed Forms
Part III

- Part I – linear stability of roll-waves, dunes & antidunes;
- Part II – linear stability of ripples and dunes;
- Part III – linear stability of alternate bars & oblique dunes;
- Part IV – weakly nonlinear analysis
Alternate bars in straight channels

Ingeniería de ríos, Jorge Abad 2011

β = \frac{W_h^*}{D^*}

Lisle et al. (1997)

Naka River, Parker’s e-book

Federici and Colombini (2003)

3-D view

Two-Phase Continuum Models for Geophysical Particle-Fluid Flows
Max Planck Institute for the Physics of Complex Systems
14 March - 15 April 2016
The name game: alternate bars
Alternate bars in a flume.

Image courtesy of S. Ikeda.
The name game: alternate bars

Super fun group game!
"A small dune sings only the few days in which there is no wind and no clouds”

(Andreotti, B. - LiveScience, 2005)
Diagonal bars in a flume.

From Einstein & Shen, JGR 1964
Diagonal bars in a flume.

From Einstein & Shen, JGR 1964

Alternate bars in a flume.

Image courtesy of S. Ikeda.
Diagonal bars in a flume.

From Einstein & Shen, JGR 1964

- “... it is probable that diagonal bars have sometimes been classified as alternate bars by some authors.”
- “... experiments seem to indicate that a grouping of three-dimensional mesoforms [i.e. scaling with flow depth], in which the fronts of the mesoforms were diagonally aligned over the channel width, was responsible for these features.”

From Jaeggi, JHE 1984
JSM: Jaeggi (1984), Sukegawa (1971), Muramoto & Fujita (1978)
GSR: Guy, Simons & Richardson (1966)

Longitudinal scaling: bars & dunes

\[ C = \frac{U^*}{u_f^*} = \sqrt{\frac{8}{f}} = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right) \]  
(Rough regime)
\[ C = \frac{U^*}{u_f^*} = \sqrt{\frac{8}{f}} = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right) \]  

(Rough regime)
Longitudinal wavenumbers for Diagonal Bars are larger than for Alternate Bars

\[ k_x = \frac{2\pi D^*}{L_x} \]
Longitudinal wavenumbers for Diagonal Bars are larger than for Alternate Bars

\[ k_x = \frac{2\pi D^*}{L_x} \]

Transverse wavenumbers for Diagonal Bars are larger than for Alternate Bars

\[ k_y = \frac{2\pi D^*}{L_y} = \frac{2\pi D^*}{4W_h^*} = \frac{\pi}{2\beta} \]
Instability and river channels
R. A. Callander

*Journal of Fluid Mechanics, Volume 36, Issue 03, May 1969, pp 465-480*
doi: 10.1017/S0022112069001765, Published online by Cambridge University Press 29 Mar 2006

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A unified bar–bend theory of river meanders
P. Blondeaux and G. Seminara

*Journal of Fluid Mechanics, Volume 157, August 1985, pp 449-470*
doi: 10.1017/S0022112085002440, Published online by Cambridge University Press 20 Apr 2006

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Finite-amplitude alternate bars
M. Colombini and G. Seminara and M. Tubino

*Journal of Fluid Mechanics, Volume 181, September 1987, pp 213-232*
doi: 10.1017/S0022112087002064, Published online by Cambridge University Press 21 Apr 2006

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Three-dimensional river bed forms
M. Colombini and A. Stocchino

*Journal of Fluid Mechanics, Volume 695, March 2012, pp 63-80*
doi: 10.1017/jfm.2011.556, Published online by Cambridge University Press 07 Feb 2012

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- **Linear**
- **2D Flow model**
- **Free Bars**

- **Linear**
- **2D Flow model**
- **Free and Forced Bars**

- **Weakly Non Linear**
- **2D Flow model**
- **Free Bars**

- **Linear**
- **3D Flow model**
- **Free Bars**
FLOW MODEL

- 3D ROTATIONAL FLOW MODEL (infinitely wide channel)
- BOUSSINESQ’S CLOSURE (algebraic mixing length)
- COORDINATE TRANSFORMATION (rectangular domain)

SEDIMENT TRANSPORT MODEL

- EQUILIBRIUM MODEL (Exner)
- BEDLOAD ONLY (MPM bedload function)
- CORRECTIONS FOR SEDIMENT WEIGHT (x – Fredsøe, y – Engelund)
- CORRECTION FOR BEDLOAD LAYER THICKNESS
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp[i k_x (x - \omega t) + i k_y y + \Omega t] + c.c. \]

- **LINEAR LEVEL**: differential eigenvalue problem
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp[i k_x(x - \omega t) + i k_y y + \Omega t] + c.c. \]

- **LINEAR LEVEL: differential eigenvalue problem**

\[ \Omega = \Omega(k_x, k_y; Fr, C) \]

DUNE FLAVOUR
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z)\exp[ik_x(x - \omega t) + ik_y y + \Omega t] + c.c. \]

- **LINEAR LEVEL**: differential eigenvalue problem

\[ \Omega = \Omega(k_x, k_y; Fr, C) \quad \text{\textit{DUNE FLAVOUR}} \]

\[ \Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \text{\textit{BAR FLAVOUR}} \]
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp[i k_x (x - \omega t) + i k_y y + \Omega t] + c.c. \]

- **LINEAR LEVEL**: differential eigenvalue problem

\[ \Omega = \Omega(k_x, k_y; Fr, C) \quad \Omega = \Omega(\lambda, \beta; \vartheta, d) \]

**DUNE FLAVOUR**

\[ \lambda = k_x \beta \]

**BAR FLAVOUR**

\[ \beta = \frac{\pi}{2k_y} \]

\[ \vartheta \approx 0.14 \frac{Fr^2 e^{kC}}{C^2} \quad C = \frac{1}{k} \ln \left( \frac{11.09}{2.5d} \right) \]
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp\left[ ik_x(x - \omega t) + ik_y y + \Omega t \right] + c.c. \]

- **LINEAR LEVEL**: differential eigenvalue problem

\[
\Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x \beta \quad \beta = \frac{\pi}{2k_y}
\]

\[
\frac{\vartheta}{\vartheta_c} = 3 \quad d = 0.025
\]

\[
Fr = 1 \quad C = 13
\]
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp[ik_x(x - \omega t) + ik_y y + \Omega t] + c.c. \]

- **LINEAR LEVEL:** differential eigenvalue problem

\[ \Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x \beta \quad \beta = \frac{\pi}{2k_y} \]

[Graph showing resonant conditions: \((\lambda_R, \beta_R)\)]
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp[i k_x (x - \omega t) + i k_y y + \Omega t] + c.c. \]

- **LINEAR LEVEL**: differential eigenvalue problem

\[ \Omega = \Omega(\lambda, \beta; \psi, d) \quad \lambda = k_x \beta \quad \beta = \frac{\pi}{2k_y} \]
\[ G(x, y, z, t) = G_0(z) + \varepsilon g_1(z) \exp[i k_x (x - \omega t) + i k_y y + \Omega t] + c.c. \]

**LINEAR LEVEL:** differential eigenvalue problem

\[ \Omega = \Omega(\lambda, \beta; \vartheta, d) \quad \lambda = k_x \beta \quad \beta = \frac{\pi}{2k_y} \]

- **Resonant conditions:** \((\lambda_R, \beta_R)\)
- **Critical conditions:** \((\lambda_C, \beta_C)\)

Alternate Bars do not form in a narrow channel
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad F_r = 1 \quad d = 0.025 \]

Critical conditions - \((\lambda_C, \beta_C)\)

Resonant conditions - \((\lambda_R, \beta_R)\)
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \Leftarrow \quad Fr = 1 \quad d = 0.025 \]
\[
\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \longleftrightarrow \quad Fr = 1 \quad d = 0.025
\]
Alternate Bars: linear stability

\[
\frac{v}{v_C} = 3 \quad C = 13 \quad \longleftrightarrow \quad Fr = 1 \quad d = 0.025
\]

SW MODEL

3D MODEL

Bar instability

Antidune instability
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \longleftrightarrow \quad Fr = 1 \quad d = 0.025 \]

Alternate Bars: linear stability

\[ \omega > 0 \]

**SW MODEL**

\[ \beta = \frac{\lambda}{k_x} \]

**3D MODEL**

\[ \omega > 0 \quad \omega < 0 \]
$$\frac{\vartheta}{\vartheta_c} = 3 \quad C = 13 \quad Fr = 1 \quad d = 0.025$$

$$\beta = \frac{\lambda}{k_x}$$
\[ \frac{\vartheta}{\vartheta_c} = 3 \quad C = 13 \quad \leftrightarrow \quad Fr = 1 \quad d = 0.025 \]
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad Fr = 1 \quad d = 0.025 \]

Alternate Bars: 3D linear stability

\[ k_y = \frac{\pi}{2\beta} \]
\[ \frac{\vartheta}{\vartheta_c} = 3 \quad C = 13 \quad Fr = 1 \quad d = 0.025 \]
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad Fr = 1 \quad d = 0.025 \]

Bar instability

Antidune instability
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \leftrightarrow \quad Fr = 1 \quad d = 0.025 \]
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \longleftrightarrow \quad Fr = 1 \quad d = 0.025 \]
\[ \frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \leftrightarrow \quad Fr = 1 \quad d = 0.025 \]

2D disturbances are stable!
Alternate Bars: 3D linear stability

\[
\frac{\vartheta}{\vartheta_C} = 3 \quad C = 13 \quad \leftrightarrow \quad Fr = 1 \quad d = 0.025
\]

2D disturbances are the most unstable!
Experimental data sets

JSM: Jaeggi (1984), Sukegawa (1971), Muramoto & Fujita (1978)

GSR: Guy, Simons & Richardson (1966)
\[ C = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right) \]

\[ \vartheta \approx 0.14 \frac{Fr^2 e^{\kappa C}}{C^2} \]

Moving up along a vertical line means (same sediment):

- \( C \) constant \( \Rightarrow \) \( d \) constant \( \Rightarrow \) \( D^* \) constant
- \( \vartheta \) increases \( \Rightarrow \) \( Fr \) increases \( \Rightarrow \) \( U^* \) increases
$\phi / \phi_c \approx 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}$

$C = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right)$

Moving up along a vertical line means (same sediment):

1. $C$ constant $\Rightarrow$ $d$ constant $\Rightarrow$ $D^*$ constant
2. $\theta$ increases $\Rightarrow$ $Fr$ increases $\Rightarrow$ $U^*$ increases

$\Rightarrow S \propto Fr^2$ increases
Moving up along a vertical line means (same sediment):

- $C$ constant $\Rightarrow d$ constant $\Rightarrow D^*$ constant
- $\theta$ increases $\Rightarrow Fr$ increases $\Rightarrow U^*$ increases
- Flow rate increases with constant flow depth

\[
C = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right)
\]

\[
\theta \approx 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}
\]
Moving up along a vertical line means (same sediment):

- $C$ constant $\Rightarrow d$ constant $\Rightarrow D^*$ constant
- $\theta$ increases $\Rightarrow Fr$ increases $\Rightarrow U^*$ increases
- Flow rate increases with constant flow depth $\Rightarrow$ Slope increases
- From Alternate Bars to Alternate Bars & Antidunes

\[
C = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right)
\]

\[
\theta \approx 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}
\]
As $\vartheta$ increases:

\[
\frac{\vartheta}{\vartheta_c} = 2
\]
As $\vartheta$ increases:

- A new region of instability appears: 2D antidunes;
- The regions of instability for bars and antidunes are distinct;
- Bars and antidunes linearly coexist: bars are less unstable;
Moving up: $C = 10$

As $\vartheta$ increases:

- A new region of instability appears: 2D antidunes;
- The regions of instability for bars and antidunes are distinct;
- Bars and antidunes linearly coexist: bars are less unstable;
- 3D antidunes become the most unstable;
As $\vartheta$ increases:

- A new region of instability appears: 2D antidunes;
- The regions of instability for bars and antidunes are distinct;
- Bars and antidunes linearly coexist: bars are less unstable;
- 3D antidunes become the most unstable;
- This results in a transition from 2D to 3D antidunes;
Regime diagram: bars & dunes

Moving right along a horizontal line means (same sediment):

- $C$ increases $\Rightarrow$ $d$ decreases $\Rightarrow$ $D^*$ increases
- $\theta$ constant $\Rightarrow$ $U^* \propto C$ increases
- Flow rate increases but $Fr$ decreases
- From Alternate Bars to Diagonal Bars to 2D Dunes

\[
C = \frac{1}{\kappa} \ln \left( \frac{11.09}{2.5d} \right)
\]

\[
\theta \approx 0.14 \frac{Fr^2 e^{\kappa C}}{C^2}
\]
Regime diagram: bars & dunes
Regime diagram: bars & dunes
Regime diagram: bars & dunes
Regime diagram: bars & dunes
Regime diagram: bars & dunes
Regime diagram: bars & dunes
Regime diagram: bars & dunes
As $C$ increases:

- the longitudinal wavenumber of maximum growth rate increases; bars become shorter;
- 2D disturbances become unstable but are less unstable than 3D disturbances;
- 2D disturbances become the most unstable;
- This results in a transition from 3D bars to 2D dunes via diagonal bars (3D dunes).
As $C$ increases:

- the longitudinal wavenumber of maximum growth rate increases; bars become shorter;
- 2D disturbances become unstable but are less unstable than 3D disturbances;
- 2D disturbances become the most unstable;
- This results in a transition from 3D bars to 2D dunes via diagonal bars (3D dunes).