Anticipating critical transitions of chaotic attractors through boundary crises

Adamson, Matthew
(Universität Osnabrück, Institut für Umweltsystemforschung, Osnabrück, Germany)

A critical transition in a system, such as an ecological community or ecosystem, is a sudden shift from a favourable system state to an undesirable one, or vice-versa. For example, an animal population which is apparently healthy may suddenly collapse without warning. Such critical transitions are often irreversible, so that their consequences are permanent, and their sudden nature makes them extremely difficult to predict. For this reason, much research in recent years has been devoted towards the construction of generic methods for detecting critical transitions before they happen so that steps can be taken to prevent them. The aim of such methods is to predict a critical transition based purely on time series data, by analysing how properties such as autocorrelation and variance change in time. So far, such research into early warning signals has almost exclusively focused on critical transitions of stable equilibria, such as a saddle-node bifurcation, in which a favourable stable equilibrium collides with a saddle point, causing both to disappear and resulting in a plateau-shaped collapse as the slowly changing stable ecosystem suddenly transitions to a completely different state.

Despite this near-exclusive focus on smooth, plateau-like collapses, however, many critical transitions in ecology are of an erratic nature, in which populations undergo large fluctuations before collapsing. The most likely explanation for such critical transitions is a boundary crisis in which a chaotic attractor collides with the basin of attraction of a less favourable attractor. There has been considerably little research carried out towards trying to detect early warning signals for such critical transitions, largely because the global bifurcations involved are much harder to analyse than local bifurcations—especially if we want to consider properties which are generic and not system-specific. In this talk, we'll consider a generic method for anticipating these critical transitions through boundary crises. As well as signalling when a critical transitions is likely, the new method has the advantage that it can also predict when the collapse will take place, by predicting the probability of the population persisting at any given time.

Dependence of extreme events on location

Amritkar, Ravindra E.
(Institute of Infrastructure, Technology, Research and Management, Ahmedabad, India)

Extreme events are very common in nature and many times they are associated with some calamities such as typhoons, earthquakes, floods etc. It is well known that the frequency of extreme events depends on the location, e.g. some regions are prone to frequent earthquakes.

We use the model of a brownian particle in a potential and show that the probability of extreme events depends on the local potential. We study the dependence of the probability of extreme events on the size of the local region and the potential in the region. We find that except for very small and very large sizes of the local region, the probability of extreme events near a maximum of potential is larger than that near a minimum.

Qualitative stability of nonlinear networked systems

Angulo, Marco Tulio
(Universidad Nacional Autónoma de México (UNAM), Institute of Mathematics, Queretaro, Mexico)

In many large systems, such as those encountered in biology or economics, the dynamics are nonlinear and are only known very coarsely. It is often the case, however, that the signs (excitation or inhibition) of individual interactions are known. In this talk, I will discuss the extension to nonlinear systems of the classical criteria of linear sign stability introduced in the 70's. The method yields simple sufficient conditions to determine stability using only the sign patterns of the interactions.
Joint work with Jean-Jacques Slotine (MIT)

Pattern switching and multistability on complex networks of excitable units

Ansmann, Gerrit
(Rheinische Friedrich-Wilhelms-Universität Bonn (University of Bonn), Klinik und Poliklinik für Epileptologie, Universitätsklinikum, Bonn, Germany)

We study deterministic, multistable systems of FitzHugh--Nagumo units coupled on small-world networks. These systems exhibit very long transients during which the dynamics switches between three different collective dynamical patterns. The multiple attractors of the system can be grouped into different types, each of which corresponds to one of the patterns. We discuss how the different attractors gain and lose stability upon parameter changes and how their coexistence relates to the switching of patterns.

Understanding tipping behaviour for parameter shifts

Ashwin, Peter
(University of Exeter, Centre for Systems, Dynamics and Control, Mathematics, Exeter, United Kingdom)

I will outline some recent work on sing pullback attractors to classify and predict the presence of critical transitions/tipping points of different types, in particular to understand the appearance of rate-induced tipping for systems that are asymptotically autonomous both for forwards and backwards time. In the case we can associate a pullback attractor with each attractor for the past-time limit system and understand rate-induced tipping in terms of the forward limit of the pullback attractor. (Work with Sebastian Wieczorek and Clare Perryman)

Inverse stochastic resonance in neuronal systems

Barreto, Ernest
(George Mason University, Krasnow Institute for Advanced Study, School of Physics, Astronomy, and Computational Sciences, Fairfax, VA 22030, USA)

Big data approaches to abrupt climate change

Bathiany, Sebastian
(Wageningen University, Environmental Sciences, Netherlands)

It has been widely discussed whether future climate change will be a linear response to anthropogenic forcing, or whether it will be punctuated by abrupt shifts, often called tipping points. As it would be difficult for societies and ecosystems to cope with such events, it is important to aim for an assessment of the associated uncertainty. However, the list of potential climate tipping points often put forward in scientific studies mainly results from idealised models, qualitative arguments and inspection by eye. We propose to turn this strategy around and use the exploding amount of climate model output to get a more systematic picture of where and why abrupt change may occur in the future. To this end, we apply tailored automatic algorithms to detect abrupt changes, classify them and test hypotheses regarding their causes.

Using such Big Data approaches would also allow to quantify the uncertainty of abrupt change in intermediate complexity models by performing a perturbed-parameter sensitivity analysis. We will present this work in progress on the workshop to discuss relevant links to other scientific disciplines and learn from each other.

Extreme multistability in coupled systems

Dana, Syamal Kumar
(CSIR-Indian Institute of Chemical BIology, Kolkata, India, CSIR-Indian Institute of Chemical Biology, Central Instrumentation, Kolkata, India)

Multistability is a known feature of many dynamical systems where many attractors coexist for a set of system parameters. On the other hand, we define extreme multistability as coexistence of infinitely many attractors for a given set of parameters. We obtained extreme multistability in two dynamical systems when a type of partial synchronization was established by a design of mutual coupling based on Lyapunov function stability and it is characterized by the emergence of a conserved quantity. The method is very general, in the sense, that the choice of coupling is flexible and applicable in different models, the paradigmatic Rössler system and a chemical oscillator. We tested the robustness of extreme multistability with respect to parameter mismatch and evidence extreme multistability in electronic experiments. We have generalized the coupling to obtain spontaneous emergence of extreme events.

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Basin entropy: a new tool to analyze unpredictability

Daza, Álvar
(Universidad Rey Juan Carlos, Departamento de Biología y Geología, Física y Química Inorgánica, Madrid, Spain)

The idea of basins of attraction has its origin in nonlinear dynamics, but it has been proved prolific in a broad range of scientific disciplines. Named after the fluvial analogy, basins refer to the set of initial conditions leading to a particular destination. Intuitively, the more intricate the basins, the more unpredictable the evolution of the system. Other tools like the basin stability and the uncertainty exponent often fail to quantify this unpredictability properly, resulting in vague affirmations in many research works.

We introduce the notion of basin entropy, which is a natural measure of this unpredictability. Using the basin entropy framework we are able to identify the different factors that hinder the prediction of the final state of the system. Numerical simulations of paradigmatic dynamical systems are provided to illustrate and to test the theoretical analysis.

Additionally, the basin entropy provides a sufficient condition to test the fractality at a given resolution: if the boundary basin entropy is above log2 then the boundaries are fractal. This criterion brings new possibilities to detect fractal structures in experimental settings where resolution cannot be arbitrarily changed.

Is it potentially possible to predict a tipping point in the climate?

Ditlevsen, Peter
(University of Copenhagen, Niels Bohr Institute, Copenhagen, Denmark)

We know from the paleoclimate records that the climatic response to perturbations and internal fluctuations can be very abrupt and non-linear.
If such abrupt changes, or tipping points are to be foreseen in the future we need to know if there are early warning signals in the variations prior to a tipping point. This will depend on the dynamical nature of the tipping:

In the most simple scenarios we can distinguish between b-tipping, n-tipping and r-tipping, referring to bifurcation induced -, noise induced - and rate induced tipping respectively. These have distinctly different early warning signals. This difference can be used to identify the type of tipping in the Dansgaard-Oeschger events observed in the paleoclimatic record. These are n-tipping events, which could hint to the reason why they are difficult to simulate in climate models.

Multistability of convection driven dynamos in rotating spherical shells

Feudel, Fred
(Universität Potsdam, Institut für Physik, Theoretical Physics, Potsdam, Germany)

We study bifurcations of buoyancy-driven dynamos in a rotating spherical shell which represents an idealized model for the generation of magnetic fields in a number of astrophysical applications. An electrically conductive fluid in a spherical shell heated from the inner sphere by imposing a constant temperature difference between its boundaries and driven by the influence of the gravity force can enable the enhancement and maintenance of a magnetic field via electromagnetic induction. The interaction of the convective flow and the generated magnetic field is modeled by the nonlinear system of magnetohydrodynamic (MHD) equations and is known as dynamo effect. Hopf bifurcations break the SO(2) symmetry of the conductive state in a primary bifurcation and generate convective rotating waves (RWs) with a cyclic symmetry. The magnetic field branches bifurcate subcritically in seconday bifurcations and are stabilized subsequently via saddle-node bifurcations. Using a path-following technique both stable and unstable branches can be traced and a complex network of simultaneously existing solution branches with stable portions have been computed.

Compressive sensing based prediction of complex dynamics and complex networksS

Grebogi, Celso
(University of Aberdeen, Institute for Complex Systems and Mathematical Biology (ICSMB), King's College, Aberdeen, United Kingdom)

In the fields of complex dynamics and complex networks, the reverse engineering, systems identification, or inverse problem is generally regarded as hard and extremely challenging mathematically as complex dynamical systems and networks consists of a large number of interacting units. However, our ideas based on compressive sensing, in combination with innovative approaches, generates a new paradigm that offers the possibility to address the fundamental inverse problem in complex dynamics and networks. In particular, in this talk, I will argue that evolutionary games model a common type of interactions in a variety of complex, networked, natural systems and social systems. Given such a system, uncovering the interacting structure of the underlying network is key to understanding its collective dynamics. Based on compressive sensing, we develop an efficient approach to reconstructing complex networks under game-based interactions from small amounts of data. The method is validated by using a variety of model networks and by conducting an actual experiment to reconstruct a social network. While most existing methods in this area assume oscillator networks that generate continuous-time data, our work successfully demonstrates that the extremely challenging problem of reverse engineering of complex networks can also be addressed even when the underlying dynamical processes are governed by realistic, evolutionary-game type of interactions in discrete time. I will also touch on the issue of detecting hidden nodes, on how to ascertain its existence and its location in the network, this being highly relevant to metabolic networks.

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Optimizing controllability of complex networks by minimum structural perturbations, W.-X. Wang, X. Ni, Y.-C. Lai, and C. Grebogi, Phys. Rev. E 85, 026115 (2012)
Uncovering hidden nodes in complex networks in the presence of noise, R.-Q. Su, Y.-C. Lai, X. Wang, and Y. Do, Nature Sci. Rep. 4, 3944 (2014)

Experimental study of bistable images perception and stochastic-based decision-making model

Grubov, Vadim
(Saratov State Technical University, Scientific educational center , Saratov, Russian Federation)

Visual perception is often studied through perceptual alternations while observing ambiguous (multistable) images. This phenomenon is also related to the problem of the categorical perception. Though the underlying mechanisms of image recognition are not yet well understood, the metastable visual perception is known to engage a distributed network of occipital, parietal and frontal cortex. The generally accepted concept of this phenomenon is based on inherent neuron activity noise originated from random neuron spikes. Internal noise seems to play a crucial role in brain dynamics related not only to the perception activity but also to other brain functions. Different manifestations of stochastic processes in brain were extensively studied, including the perception of ambiguous images, in terms of simple stochastic processes like the Wiener process from the point of statistical properties. At present moment, one of the most important problems is to develop methods for quantitative measuring of noise characteristics that can provide new opportunities both in studying of the brain functionality and diagnosis of its pathologies.

In the present work, we develop the quantitative theory and propose the experimental technique for measuring noise intensity related to the perception of ambiguous images. Both our theoretical findings and the proposed experimental approach are proved by psychological experiments.

The experimental studies were performed in accordance to all ethical standards. Participants were healthy males and females in the age from 20 to 45 with normal or corrected-to-normal visual acuity; all participants were unpaid volunteers.

As a bistable image, we used the Necker cube illusion which is represented with flat image of cube's skeleton with ribs and without faces. The contrast of the three middle lines centered in the left middle corner, I \in [0; 1], was considered as a control parameter. During the experiment Necker cube images with different line contrasts, i.e. with the different values of the control parameter I, were repeatedly showed to a person in a random sequence, with each cube being placed in the middle of a computer screen as black lines on a white background. All participants were instructed about two possible orientations of the Necker cube, and both were really seen by all of them. All participants were instructed to press either the left or the right key on the control panel according to their first visual impression (left-oriented cube or right-oriented cube, respectively). Both the image presentation and the recording of personal responses as EEG signals were accomplished with Electroencephalograph-recorder complex "Encephalan-EEGR-19/26" (Medicom MTD).

Contrary to the traditional approach in our study we mainly focused on the theoretical and quantitative description as well as on the experimental measurement of the particular relevant factor of the brain activity, namely, on the noise intensity characterizing the stochastic processes in the brain. Based on the methods of statistical physics, we develop a theory which helped us to derive the analytical (not empirical) expression for the experimental data and measure the noise intensity characterizing the stochastic processes in the brain.

Also EEG recordings were studied with continuous wavelet transform. Analysis of time-frequency structure of EEG signals revealed characteristic patterns for the perception of the Necker cube with different parameter I. The developed theory provides the solid experimentally approved basis for further understanding of brain functionality. We expect that our work will be interesting and useful for scientists carrying out interdisciplinary research at the cutting edge of physics, neurophysiology and medicine.

This work has been supported by the Russian Science Foundation (grant 16-12-10100).

Spatial synchrony and critical points in ecological systems

Hastings, Alan
(University of California - Davis, Environmental Science and Policy, Davis, USA)

I will present results on spatial synchrony and phase transitions in ecological models that are shown to be equivalent to the Ising model. I will emphasize models for masting of plants (synchronous fruit production in different years) and present both general results and results that match models to data from yields of Pistachio trees. The basic result is that our stochastic model of this system near the point of the first period doubling belongs to the Ising universality class. I will discuss how these results provide insight into explanations for masting in plants.

Emergence of extreme multistability in linearly coupled chaotic oscillators

Hens, Chittaranjan
(Bar-Ilan University, Mathematics Department, Ramat Gan, Israel)

We report emergence of an extreme kind of multistability in two coupled chaotic oscillators. The coupling function is purely linear. We analytically derive how such multistability appear in the coupled system under certain coupling strength when few components of the oscillators are completely synchronized to each other. For demonstration we use chaotic Rössler systems.

Control of fluctuating populations: leading to or coming from alternative attractors

Hilker, Frank
(Osnabrück University, Institute of Environmental Systems Research, Osnabrück, Germany)

Ecosystems are affected by humans in many ways, one of which is by exploitation (harvesting, fishing) or by the control of pests and protected species. Such management interventions can have a two-way interaction with the ecological system. On the one hand, they can profoundly change their dynamic complexity, e.g., leading to (de-)stabilization or inducing multistability - which may be favourable or undesirable from the management point of view. On the other hand, ecosystems by themselves can exhibit complex dynamics - so management interventions should account for and react to possible regime shifts in the ecoligcal system.

In this presentation, I will consider both aspects of the two-way interaction between ecology and management interventions. This will be based on simple population models that show complex dynamics, and I will focus first on restocking schemes and second on pest control of forest insects. The management interventions share analogies to chaos control methods from physics, but they are applied in an ecological context characterized by high uncertainty, variability and limited opportunity for intervention.

Taming high-dimensional dynamical systems

Kaneko, Kunihiko
(University of Tokyo, Center for Complex Systems Biology, Tokyo, Japan)

Biological systems, sat cells, consist of a huge degrees of freedom, and thus the dynamics are high-dimensional. In spite of the difficulty in controlling such high-dimensional dynamical systems, they somehow find a solution through evolution. Here I discuss a general consequence of such high-dimensional evolving system, If time is allowed I will also discuss some recent findings in high-dimensional dynamical systems, e.g., chaotic itinerancy in slow-fast systems, chaotic Griffiths phase in network of chaotic elements, and so forth。

Perpetual points and hidden attractors

Kapitaniak, Tomasz
(Technical University of Lodz, Division of Dynamics, 90-924 Lodz, Poland)

We discuss the use of perpetual points for tracing the hidden and the rare attractors of dynamical systems. The analysis of perpetual points and their co-existence due to the parameters values is presented and the impact of these points on the behavior of the systems is shown. The results are obtained for continuous (single as well as coupled externally excited van der Pol–Duffing oscillators) and discrete (maps) systems.

Order parameters for chimera states with arbitrary coupling strength and topology

Kemeth, Felix
(Technische Universität München, Physics Department, Germany)

Chimera states, the counterintuitive coexistence of synchronized and desynchronized regions in an otherwise isotropic system, have received considerable interest during the last decade. While early studies concentrated on chimera states in ensembles of phase oscillators, meanwhile a variety of chimeras in different settings, including Stuart-Landau oscillators with strong nonlocal or global coupling, and oscillatory networks with different coupling topologies have been reported.
In the talk, I will present a universal characterization scheme for chimera states applicable to any numerical or experimental data. The scheme is based on two order parameters that enable a meaningful definition of chimera states as well as their classification into different categories. Furthermore, I will demonstrate that extracting probability density functions in terms of suitably chosen coordinates from given data sets allows for an adequate representation of the data in a reduced state and yields valuable insight into the underlying dynamics.

A Survey of the Mathematical Theory of Early-Warning Signs

Kuehn, Christian
(Vienna University of Technology, Institute for Analysis and Scientific Computing, Vienna, Austria)

In this talk, I am going to illustrate current techniques based upon multiple time scale dynamical systems and stochastic analysis that can be used to detect early-warning signs for drastic transitions (or tipping points). In particular, the role of scaling laws and a quantitative view of bifurcation theory will be emphasized. Several examples will be given including theoretical modelling components as well as data analysis. Furthermore, I shall also highlight several open problems, which arose during the last few years, and which provide challenges for a broad range of scientific backgrounds.

Basin stability in multistable systems and its application

Kurths, Jürgen
(Humboldt Universität Berlin, Potsdam-Institut für Klimaforschung e.V., Transdisziplinäre Konzepte und Methoden, Potsdam, Germany)

The human brain, power grids, arrays of coupled lasers and the Amazon rainforest are all characterized by multistability. The likelihood that these systems will remain in the most desirable of their many stable states depends on their stability against significant perturbations, particularly in a state space populated by undesirable states. Here we claim that the traditional linearization-based approach to stability is in several cases too local to adequately assess how stable a state is. Instead, we quantify it in terms of basin stability, a new measure related to the volume of the basin of attraction. Basin stability is non-local, nonlinear and easily applicable, even to high-dimensional systems. It provides a long-sought-after explanation for the surprisingly regular topologies of neural networks and power grids, which have eluded theoretical description based solely on linear stability.
Specifically, we employ a component-wise version of basin stability, a nonlinear inspection scheme, to investigate how a grid's degree of stability is influenced by certain patterns in the wiring topology. Various statistics from our ensemble simulations all support one main finding: The widespread and cheapest of all connection schemes, namely dead ends and dead trees, strongly diminish stability. For the Northern European power system we demonstrate that the inverse is also true: `Healing' dead ends by addition of transmission lines substantially enhances stability. This indicates a crucial smart-design principle for tomorrow's sustainable power grids: add just a few more lines to avoid dead ends. Further, we analyse the particular function of certain network motifs to promote the stability of the system. Here we uncover the impact of so-called detour motifs on the appearance of nodes with a poor stability score and discuss the implications for power grid design.
Moreover, it will be shown that basin stability enables uncovering the mechanism for explosive synchronization and understanding of evolving networks.

Quantifying the likelihood of meridional overturning circulation collapse using non-stationary data-driven modelling

Kwasniok, Frank
(University of Exeter, College of Engineering, Mathematics and Physical Sciences, Mathematics Research Institute, Exeter, United Kingdom)

The problem of quantifying the likelihood of meridional overturning circulation collapse from observed time series data is discussed. A one-dimensional non-stationary potential model is estimated from data and its probability density propagated beyond the learning data window in order to make probabilistic predictions of future critical transitions. The method is exemplified on a scalar time series from the Stommel box model and a spatio-temporal data set from the fully coupled climate model FAMOUS.

Multistability in nano systems

Lai, Ying-Cheng
(Arizona State University, Arizona State University, School of Electrical, Computer and Energy Engineering, Tempe, USA)

This talk will address the emergence and dynamics of multistability in three nanoscale systems: a nanowire, a coupled system of ferromagnet and topological insulator, and a semiconductor superlattice.

Multiple stable states of tree cover in a global land surface model due to fire - vegetation feedback

Lasslop, Gitta
(MPI for Meteorology, Land,)

The presence of multiple stable states has far reaching consequences for a system's susceptibility to disturbances, including the possibility of abrupt transitions between stable states. The occurrence of multiple stable states of vegetation is supported by ecological theory, models and observations.
Here we describe the occurrence of multiple stable states of tree cover in a a global dynamic vegetation model and provide the first global picture on multiple stable states of tree cover due to a fire-vegetation feedback.
The multiple stable states occur in the transition zones between grasslands and forests, mainly in Africa and Asia. By sensitivity simulations and simplifying the relevant model equations we show that the occurrence of multiple states is caused by the sensitivity of the fire disturbance rate to woody species abundance.

Epileptic seizures and multistability in the human brain

Lehnertz, Klaus
(University of Bonn, Dept of Epileptology, Bonn, Germany)

Epilepsy is one of the most common serious neurological disorders, affecting approximately 65 million people worldwide.
Epileptic seizures are the cardinal symptom of this multi-facetted, dynamic disease and are usually characterized by an overly synchronized firing of neurons. Seizures cannot be controlled by any available therapy in about 25% of individuals.
Knowledge about mechanisms underlying generation, spread, and termination of the extreme event seizure in humans is still fragmentary. Alterations within and between different types of healthy physiologic and pathophysiologic activities as well as the co-existence of such activities within the same brain network are very suggestive of an underlying multistable dynamical system. In this talk, I will present and discuss recent findings from data analyses and modeling approaches that can help to answer the pressing questions as to why, when, and where seizures start and terminate as well as to further improve prediction and prevention techniques.

Multistability of the climate system and melancholia states

Lucarini, Valerio
(University of Hamburg, Institute of Meteorology, Hamburg, Germany)

The Earth supports - for a considerable range of values of the solar intensity - two stable states, a warm climate and a snowball climate, where the bistability results from the ice-albedo feedback. Characterizing the unstable solution lying in-between the two climates is key for having a global view on the stability properties of the system and for understanding the processes leading to critical transitions, triggered, e.g. by noise of modulation of the system's parameters. We find the unstable "Melancholia" states by applying for the first time in a geophysical context the so-called edge tracking method, which has been used for studying multiple coexisting states in shear flows. The unstable states are relative attractors living on the separatrix between the basin of attraction of the two climates. We examine robustness, efficiency, and accuracy properties of the edge tracking algorithm. We perform our analysis using a simplified yet Earth-like climate model based on the primitive equations on the sphere for the atmosphere and a diffusive ocean. The model features sensitive dependence on initial conditions and variability on all time scales in the climate states. We discover for the first time an unstable state characterised by complex dynamics and living on a strange geometrical set.

Inferring regional communities and time-scales of interactions in climate networks

Masoller, Cristina
(Universitat Politecnica de Catalunya, Departament de Fisica, Terrassa, Spain)

Climate networks defined over a regular grid of geographical locations (nodes) covering the Earth surface, have proven to be a valuable analysis tool for advancing our understanding of climate dynamics. In this talk I will discuss climate networks constructed by using different methods for assessing statistical similarities and interdependencies: ordinal symbolic analysis, Hilbert transform, mutual information, conditional mutual information and Granger causality [1-3]. I will also present two techniques for unveiling the underlying community structure of climate networks [4]:

the first one is based on identifying mutual lags among time-series of climatic variables recorded at the nodes, while the second one is based on performing a statistical similarity analysis of the symbolic dynamics at the nodes. These methods extract meaningful regional communities, which uncover different aspects of large-scale climate phenomena.

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Transitory trapping phenomenon attenuates the consequences of tipping points for limit cycles

Medeiros, Everton
(Universidade de São Paulo (University of São Paulo), Institute of Physics, Appled Physics, Brazil)

Nonlinear dynamical systems may be exposed to tipping points, critical thresholds at which small parameter changes abruptly shift the system to a contrasting dynamical regime. Although diverse mechanisms have been proposed to foresee such tipping events, not much is known about the dynamics succeeding tipping points. Here, we investigate the consequences of tipping points delimiting parameter regions of coexistence of two stable limit cycles. Specifically, for the parameter region after the tipping point which destroys one stable limit cycle in a fold bifurcation, we analyze transient properties of trajectories crossing a state space channel towards the remaining limit cycle. For a noise-free ecological model, we find that the intrinsic rotations of transient trajectories crossing the state space channel are similar to those around the limit cycle extinct in the tipping point. Thus, the ecological model in a noisy environment has tipped trajectories crossing the channel with analogous dynamical features of trajectories occurring before the tipping point. As a consequence, we report that the abrupt shift characteristic of tipping points may not occur in the case of limit cycles, i.e., the dynamical behavior corresponding to the limit cycle disappear smoothly in the state space as the tipping point is achieved and crossed. Furthermore, we observe differences in the distribution of time intervals spent by trajectories around the limit cycle and the channel that reveals the smooth disappearance of the dynamical behavior linked to the extinct limit cycle.

From neurons to networks: bifurcations, phase transitions and critical slowing down in neural systems

Meisel, Christian
(National Institute of Mental Health, Section on Critical Brain Dynamics, Bethesda, USA)

The general idea that computational capabilities are maximized at or nearby critical states related to phase

transitions or bifurcations led to the hypothesis that neural systems in the brain operate at or close to a critical state.
Near phase transitions, a system is expected to recover more slowly from small perturbations, a phenomenon called critical slowing, and observables typically exhibit power-law scaling relationships. In particular the dynamical systems' approach has provided a firm theoretical background on scaling laws of critical slowing down as a function of distance to a bifurcation and, consequently, how to determine whether a system is poised in the vicinity of a phase transition or not.

In this talk I will discuss recent experimental work in neural systems, from individual neurons to networks of neurons, investigating these signatures of criticality. On the level of individual neurons two elementary modes of functioning can be observed: quiescence and spiking. Using the whole-cell patch clamp technique we observed the subthreshold membrane potential in neurons to exhibit critical slowing down near the onset of spiking. The associated scaling laws suggest a saddle-node bifurcation governing this transition in pyramidal neurons and fast-spiking interneurons.
When neurons connect to form networks, their individual bistability extends to the emergent network behavior. Here, the ability of individual neurons to homeostatically control their firing rates provides the mechanism for local self- organisation towards a critical network state. At the level of large neuronal ensembles we analyze patient data obtained from invasive EEG of epilepsy patients. Near the seizure onset we find oscillatory behavior and scaling laws of critical slowing down which substantiate the conjecture that a Hopf bifurcation could be involved governing the transition to seizure dynamics. These findings indicate critical transitions to be relevant on different spatial and temporal scales and highlight their importance for the understanding of individual or collective neural dynamics.

Spatial dynamics of regime shifts in dryland ecosystems

Meron, Ehud
(Ben-Gurion University of the Negev, Blaustein Institute for Desert Research, Environmental Physics, Midreshet Ben Gurion, Israel)

Ecosystem regime shifts are commonly regarded as abrupt global transitions from one stable state to another, induced by slow environmental changes or by global disturbances. Spatially extended ecosystems, however, are often subjected to strong local disturbances that result in confined domains of the alternative state. The behaviour of a locally disturbed ecosystem depends on the dynamics of the transition zones or fronts that separate adjacent alternative-state domains. According to pattern formation theory, when one of the alternative states is spatially patterned, these fronts can be pinned in place, forming a multitude of additional hybrid states, i.e. states consisting of fixed domains of the patterned state in an otherwise uniform state and vice versa. A possible realization of this case is the Namibian Fairy-Circle ecosystem, which consists of a uniform grass punctured by circular gaps of sandy bare soil - the fairy circles. We investigated the dynamics of this ecosystem using satellite images, rainfall data and model studies. We first found that the rainfall time series and the fairy-circle dynamics are correlated, suggesting a causal relation between the two. We further identified hybrid states in satellite images as nearly periodic gap patterns with missing gaps (fairy circles). Finally, we compared satellite images that show birth and death of fairy circles in response to droughts and spates, with model simulations, and associated these processes with dynamic transitions between different hybrid states. According to these results, a series of droughts in a disturbed uniform grass can induce a cascade of hybrid-state transitions that results in a partial or complete regime shift to the periodic gap pattern. That regime shift, however, is gradual rather than abrupt and cannot be detected by the early warning signals that have been developed for abrupt shifts.
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The impact of multistability in oscillatory and excitable systems

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We study multistability in oscillatory and excitable systems, in which the multistability results from frustrated bonds. For Kuramoto oscillators and classical rotators, the multistability can be extremely rich even for small lattice sizes, particularly in combination with lattice symmetries.

Under the action of noise the oscillator phases migrate through a rich attractor landscape, leading to signatures of physical aging in autocorrelation functions. We shall give an outlook to other systems, for which we expect physical aging, based on different realizations of the same mechanism that is known from spin glasses. For coupled genetic circuits we observe multistable regimes already when just two circuits are coupled to a toggle switch. The collective behavior simplifies in large sets of coupled circuits, in which we can control the duration of collective oscillations by tuning a single bifurcation parameter.

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Characteristics of noised-induced intermittency: theory and its verification

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Intermittency is an ubiquitous phenomenon in nonlinear science [P. Berge, Y. Pomeau, and C. Vidal, Order within Chaos, New York, 1984]. It is observed in different systems including the physical, physiological and biological ones. It manifests itself as alternation of the episodes of periodic and chaotic regimes [P. Manneville and Y. Pomeau, Phys. Lett. A 75, 1 (1979)] or different forms of the chaotic motion [C. Grebogi, E. Ott, F. J. Romeiras, and J. A. Yorke, Phys. Rev. A 36, 5365 (1987).]. It can also be observed near the boundaries of the different synchronous regimes demonstrating the interchange of the phases of synchronous and asynchronous behavior [S. Boccaletti, E. Allaria, R. Meucci, and F. T. Arecchi, Phys. Rev. Lett. 89, 194101 (2002); A. S. Pikovsky, G. V. Osipov, M. G. Rosenblum, M. Zaks, and J. Kurths, Phys. Rev. Lett. 79, 47 (1997); A. E. Hramov, A. A. Koronovskii, M. K. Kurovskaya, and S. Boccaletti, Phys. Rev. Lett. 97, 114101 (2006)].
Several types of the intermittent behavior are traditionally distinguished, among which there are type I-III, on-off, eyelet and ring intermittencies or their common coexistence [A. E. Hramov, A. A. Koronovskii, O. I. Moskalenko, M. O. Zhuravlev, V. I. Ponomarenko, and M. D. Prokhorov, CHAOS 23, 033129 (2013); A. A. Koronovskii, A. E. Hramov, V. V. Grubov, O. I. Moskalenko, E. Sitnikova, and A. N. Pavlov, Phys. Rev. E 93, 032220 (2016)]. Each of types of intermittency mentioned above is characterized by its own specific mechanism as well as its own statistical characteristics. One can say that such characteristics allows to define unambiguously the type of intermittency realized in the system. Recently the concept of intermittency has been extended to multistable systems. In such case the alternation between coexisting periodic or chaotic regimes can also be observed in the system[A.N. Pisarchik, R. Jaimes-Reategui, R. Sevilla-Escoboza, and G. Huerta-Cuellar, Physical Review E 86, 056219 (2012); R. Sevilla-Escoboza, J. M. Buldu, A. N. Pisarchik, S. Boccaletti, and R. Gutierrez, Phys. Rev. E 91, 032902 (2015)]. At that, the switches between coexisting regimes can be induced by noise. Therefore, the system under study demonstrates the so-called noise-induced intermittency or noise-

induced attractor hopping [S. Kraut and U. Feudel, Phys. Rev. E 66, 015207 (2002); A. N. Pisarchik, R. Jaimes-Reategui, R. Sevilla-Escoboza, G. Huerta-Cuellar, and M. Taki, Physical Review Letters 107, 274101 (2011)].
Despite of a great interest to the problem of noise-induced intermittency there is a number of questions demanding consideration and discussion. One of such problems consists in the fact that there is no appropriate theory allowing to obtain the characteristics of noise-induced intermittency even in the case of two different regime coexistence. In the present report the theory of noise-induced intermittency in bistable dynamical systems would be proposed. We would show that the residence time distributions for every coexisting regime should obey the exponential law. The main results would be illustrated using the examples of Duffing oscillator, coupled Lorenz systems and erbium-doped fiber laser.

Suppression of noise during high-speed switching of multistable systems

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The aim of the present talk is to give a short review of recently observed effect of suppression of noise in nonlinear systems subjected to above-threshold periodic or pulse driving [1]. The effect shows up as a minimum of the standard deviation of the switching time (jitter) versus a pump signal frequency of a pulse width, either through existence of a signal-to-noise ratio maximum as a function of a driving frequency. The applications to the important problem of high-speed switching of electronic devices are considered. In particular, minimization of noise-induced errors during high-speed switching of ac SQUIDs [2], Josephson junctions [3,4], magnetic dipoles [5,6], and quantum bits [7,8] are analyzed.
This work is supported by Russian Science Foundation grant 16-19-10478.

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Noise-modified spiking in neural networks with various types of cell-to-cell communication

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We study the role of noise in formation of repeating spike sequences in multielement neuron-like systems. We demonstrate that for the structural elements with bistable type of behavior, the change of noise intensity leads to non-monotonic resonance-like behavior with maximum of noise-to-signal ratio (the phenomenon of noise-delayed spiking). For various coupling schemes: diffusive (electrical) coupling and two types of chemical communication (purely inhibitory coupling and hybrid coupling with both inhibitory and excitatory synapses), we study the properties of the system's oscillatory regimes.

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Discontinuous transitions to large contagion in networks

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The spread of social phenomena such as behaviours, ideas or products is an ubiquitous but remarkably complex phenomenon. A successful avenue to study the spread of social phenomena relies on epidemic models by establishing analogies between the transmission of social phenomena and infectious diseases. Such models typically assume simple social interactions restricted to pairs of individuals;

effects of the context are often neglected. Such models systematically predict that the number of individuals affected by the spreading phenomenon increases smoothly with increase of the transmission rate between pairs of individuals. This corresponds to a second-order phase transition from non-invasive to invasive regimes.

Our models show that the scenario can be very different when local synergistic effects associated with acquaintances of pairs of individuals are taken into account. In particular, we find that the number of individuals accepting the social spreading phenomenon can become a multi-stable function of the transmission rate. As a consequence, the typically smooth transitions towards large social contagion can become explosive (discontinuous). We find that explosive contagion should be expected when the acquaintances of ignorant receiver individuals are reluctant to accept new social phenomena. This seemingly paradoxical result is especially relevant to social contexts in which individuals hesitate joining a collective movement, e.g. a strike, fearing the risk of becoming part of a minority that can eventually be punished.

Synchronization and multistability in the ring of modified Rössler oscillators

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We discuss the occurrence of various synchronous states in a ring of unidirectionally coupled modified Rössler oscillators. When systems are uncoupled we observe, single node has an infinite number of different states. When the coupling strength increases the infinitely many synchronous states appear. We show that all synchronous solutions are different and change with varying initial conditions. The analysis is performed for three and four coupled oscillators. At the end of the paper we discuss possible synchronization scenarios for larger networks with ring topology.

Multiplicity of micro and macrostates in populations of coupled oscillators

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I discuss partially synchronized states in populations of phase oscillators coupled in a way more complex than in the standard Kuramoto model. Such systems may possess a huge diversity of microscopic and macroscopic states. I describe both a situation where these states can be characterized in the thermodynamic limit, and a case where synchrony is due to finite-size effects and disappears if the number of units is large.

Parameter plane analysis and arithmetic progressions of spiking and bursting in neuronal map-based models

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We study a two-dimensional map-based neuronal model that describes the behavior of a neuron. Keeping constant one of the three parameters, we obtain parameter planes showing well-defined periodicities. We show the importance of the parameter μ identifying the periodicities and the number of spikes per burst, quantities that are related between them by a linear relationship. On the other hand, We report the discovery of nested arithmetic progressions among pulsing and bursting phases of the above mentioned model. The intricate nestings of progressions found are robust and can be observed abundantly in several control parameter planes that we describe in detail. The detailed determination and description of the periodicity zones could be the basis for a further study on synchronization of nonidentical neurons in regions in which the domain of the parameter values ensure the fact to work with the same periodicity.

Chimera patterns in networks with fractal connectivities

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Chimera states consist of coexisting domains of spatially coherent and incoherent dynamics in systems of nonlocally coupled oscillators, and they have recently been discussed in diverse classical and quantum systems (SCH16).
They are usually spatio-temporal patterns of high multistability, coexisting with the completely synchronized state.

Here we focus on the patterns which arise if one goes beyond the Kuramoto phase oscillator model with simple ring topologies and considers coupled amplitude and phase dynamics, and complex fractal (hierarchical) network connectivities
(OME15,ULO16). This is of particular relevance in neuroscience since there exists evidence that the connectivity in the brain has a fractal structure.

We investigate the stepwise transition from a nonlocal to a hierarchical coupling topology, and propose the network clustering coefficient as a measure to establish a link between the existence of chimera states and the compactness of the initial base pattern of a hierarchical topology; we show that a large clustering coefficient promotes the occurrence of chimeras (ULO16).

Depending on the level of hierarchy and base pattern, we obtain chimera states with different numbers of incoherent domains. We investigate the chimera regimes as a function of coupling strength and nonlinearity parameter of the individual Van der Pol oscillators. The analysis of a network with larger base pattern resulting in larger clustering coefficient reveals two different types of chimera states and highlights the increasing role of amplitude dynamics.

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Bistability, spatial interaction and the distribution of tropical forests and savannas

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Recent work has indicated that tropical forest and savanna can be alternative stable states under a range of climatic conditions. However, based on dynamical systems theory it may be expected that in case of strong spatial interactions between patches of alternative stable states, their coexistence becomes unstable. Boundaries between forest and savanna would then only be stable at conditions where the two states have equal potential, called the 'Maxwell point'.

Under different conditions, the state with the lowest potential would not be resilient against invasion of the state with highest potential. We used frequency distributions of MODIS tree-cover data at 250 m resolution to estimate Maxwell points with respect to the amount and seasonality of rainfall in both South America and Africa. We tested on a 0.5° scale whether there is a larger probability of local coexistence of forests and savannas near the estimated Maxwell points. Maxwell points for South America and Africa were estimated at 1760 mm and 1580 mm mean annual precipitation and at Markham's Seasonality Index values of 50% and 24%. Although the probability of local coexistence was indeed highest around these Maxwell points, local coexistence was not limited to the Maxwell points. We conclude that critical transitions between forest and savanna may occur when climatic changes exceed a critical value. However, we also conclude that spatial interactions between patches of forest and savanna may reduce the hysteresis that can be observed in isolated patches, causing more predictable forest-savanna boundaries than continental-scale analyses of tree cover indicate. This effect could be less pronounced in Africa than in South America, where the forest-savanna boundary is substantially affected by rainfall seasonality.

Death and revival of chaos

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Resilience indicators for complex systems

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We define resilience as the ability of a system to persist despite perturbations. Although resilience has been a highly influential concept, its interpretation has often remained rather qualitative. In this presentation I will give an introduction to methods for quantifying resilience on the basis of observations. Most methods are based on the phenomenon of critical slowing down, which implies that recovery of perturbations becomes slower if a system approaches a bifurcation point. This slowing down leads to measurable changes in time series, such as increased autocorrelation and variance. A second group of methods uses massive data for instance of setellite images, to estimate the resilience in terms of the potential and the probability of shifting.

Multistability generated by quorum sensing in population of synthetic genetic oscillators

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Synthetic genetic oscillators [1] have become popular since 2000 when a simple ring circuit with unidirectional repression, called the Repressilator, was engineered [2] and realized inside bacterial cells. The oscillators are considered to be an effective test bed for assessing ideas about gene expression regulation, as well as possible instruments for gene therapy. However, the effectiveness of oscillators depends on how they can function collectively, thereby requiring some coupling method. An almost obvious suggestion was to use natural bacterial quorum sensing (QS) as the method for synchronization [3, 4].

The core of QS is the production of small molecules (autoinducer) which can, first, easily diffuse across the cell membrane and, second, work to activate target gene transcription. Manipulating the gene positions and QS-sensitive promoters in the genetic circuits, one can obtain different coupling types and, as a result, different sets of collective modes in bacterial populations. The effective synchronization of relaxation QS-dependent oscillators was recently demonstrated in an impressive experiment [5].
We use the reduced Repressilator model with the QS mechanism added in a way that autoinducer diffusion provides for repulsive coupling [6]. This type of coupling of two identical Repressilators leads to the domination of the antiphase limit cycle, the formation of stable inhomogeneous stationary states and inhomogeneous limit cycles, as well as to the emergence of chaos in a wide range of parameters. The phase diagram structure and the degree of stable attractor overlapping in the parameter space depends strongly on the degree of cooperativity (steepness of nonlinear terms in the ODE system) in the regulation of gene expression: the higher the cooperativity the larger the parameter area occupied by the chaotic regime. Apart from usual chaos emerging via torus bifurcation, we found the asymmetric chaotic regime in which the amplitudes of oscillations differ significantly. This chaos is located far from torus bifurcation and includes the set of "periodic windows" with asymmetric regular oscillations. The main part of the described dynamics has been reproduced in measurements using the electronic version of the coupled Repressilators [7] (except the most delicate regimes inside chaos (work in progress)).

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Dynamics on basin boundaries

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The behaviors of multistable dynamical systems depends strongly on the dynamics on the boundaries between basins. As parameters are varied, tipping points occur when attractors come close to their boundaries. The existence of chaotic repellers within a basin can drastically change the tipping dynamics.

Analysis of multistate intermittency in erbium-doped fiber laser

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Multistability phenomenon is observed in almost all areas of science and technology, including electronics [Maurer J., Libchaber A., J. Phys. Lett. (Paris) 41 (1980) L515–L518], optics [Brun E., Derighetti B., Meier D., Holzner R., Ravani M., J. Opt. Soc. Amer. B 2 (1985) 156–167], mechanics [Thompson J.M.T., Stewart H.B., Nonlinear Dynamics and Chaos, Wiley, Chichester, 1986.] and biology [Foss J., Longtin A., Mensour B., Milton J., Phys. Rev. Lett. 76 (1996) 708–711.].

However, despite of the large number of works in this area, a series of questions still remain unsolved. One from them is the study of the behavior of erbium-doped fiber laser with modulated parameter which is known to be able to demonstrate the coexistence of several oscillating modes with different amplitude and frequency of the generated radiation [A. N. Pisarchik et. al. // Phys. Rev. E. 86 (2012) 056219]. The influence of fluctuations can lead to the fact that initially multistable system becomes the metastable one, and the dynamics of the system would demonstrate the intermittent behavior, at which the noise-induced switches between different attractors would be observed. This results in the change of the frequency and power of the laser radiation [A. N. Pisarchik et. al. // Phys. Rev. E. 86 (2012) 056219; A. N. Pisarchik et. al. // Phys. Rev.Lett. 107 (2011) 274101]. At that, since the presence of several coexisting regimes in the system under study, the switches between attractors lead to the intermittency, the type of which can not be classified at present, since the characteristics of intermittent behavior do not match with any of the currently known types of intermittency.
Usually, in order to determine what type of intermittent behavior is realized in the system, it is necessary to define its statistical characteristics. Thus, the important problem in the study of intermittency is the problem of detection of time intervals corresponding to the different types of the system dynamics (in fact, the problem of the detection of the laminar and turbulent phase lengths). This time, there are a lot of methods allowing to define the phases of the laminar behavior in time series, however, due to the specificity of the system under study none of the existing methods does not allow solving the problem associated with the laminar phase detection in the erbium-doped optical fiber laser with modulated parameter.
In this report we propose the method for detection and localization of different types of coexisting oscillatory regimes which alternate with each other leading to multistate intermittency. Our approach is based on consideration of wavelet spectrum energies [B. Torresani, Continuous Wavelet Transform, Savoire, Paris, 1995]. The new technique is tested in an erbium-doped fiber laser with four coexisting periodic orbits, where external noise induces intermittent switches between coexisting states. Statistical characteristics of multistate intermittency, such as the mean duration of the phases for every oscillation type, are examined with the help of the developed method. We demonstrate strong advantages of the proposed technique over previously used amplitude methods.