Spin-chain inspired symmetry and many-particle interference

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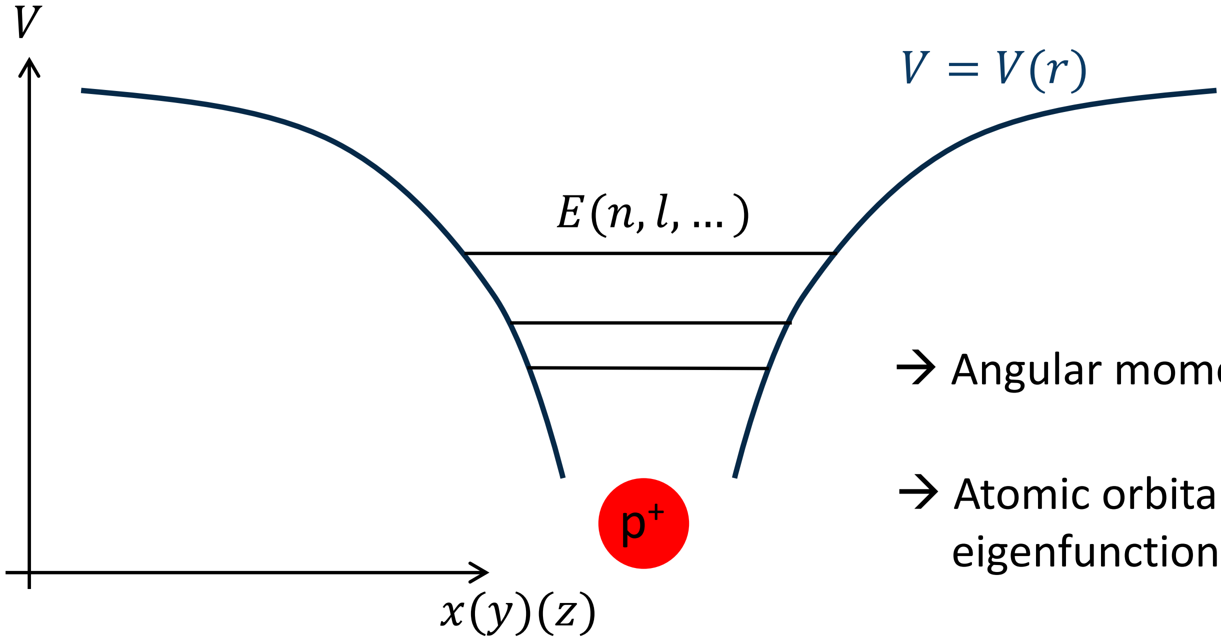
Introduction – Symmetries in Physics

<u>Rotational symmetry:</u>

Potential of atomic nucleus:

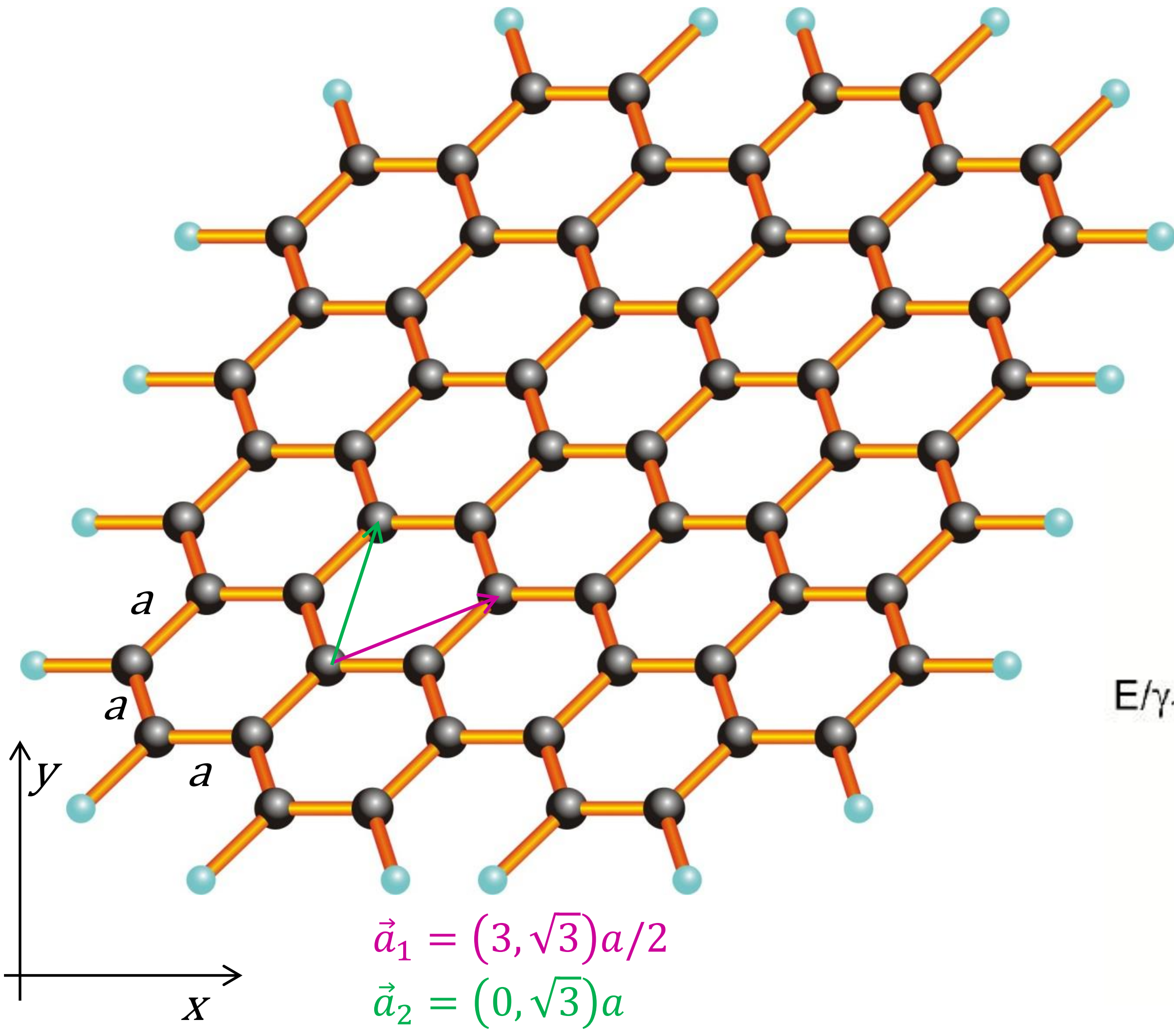


→ Angular momentum conservation

→ Atomic orbitals as common eigenfunctions of H and $\vec{L}$

Introduction – Symmetries in Physics

Translational invariance:

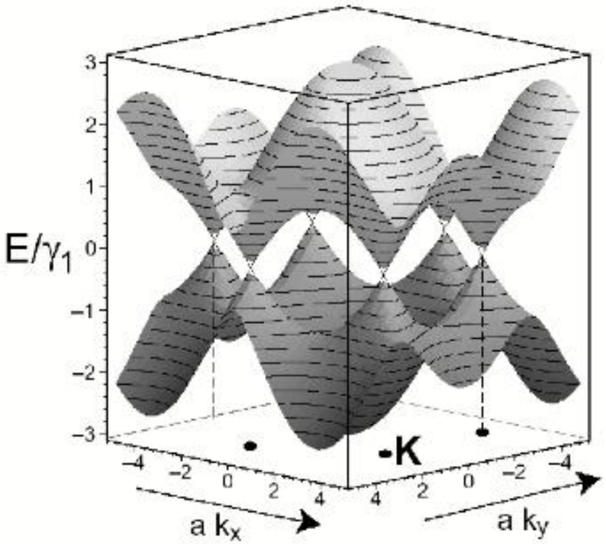


$$\vec{a}_1 = \left(3, \sqrt{3}\right) a/2$$

$$\vec{a}_2 = \left(0, \sqrt{3}\right) a$$

→ Bloch waves

→ Band structure



Novoselov *et al.*, Science **306**, 666 (2004)

Introduction – Symmetries in Biology

Taxonomy of animals

Bilateral Mirror symmetry

(Bilateria)

http://www.starfish.ch/

http://www.weinbergschnecke.info/

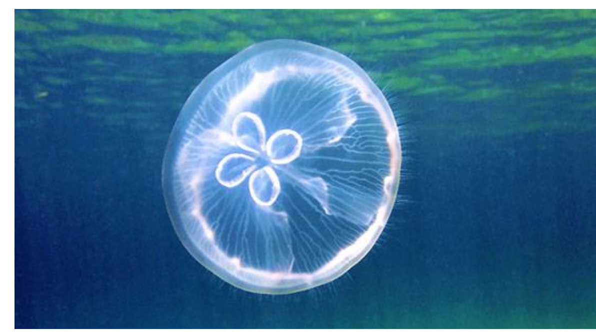
http://www.wirbellosen-aquarium.de/

http://www.india.com/

Radial symmetry

(Cnidaria)

http://www.fotos.sc/



http://www.ostsee-urlaube.de/

→ **Symmetries simplify our description of nature**

Outline

- Introduction – Symmetries
- Symmetries in multi-particle interference
- Spin-chains for perfect state transfer and their optical representation
- Many-photon dynamics in state transfer lattices
  - Suppression law
  - Multi-photon experiments

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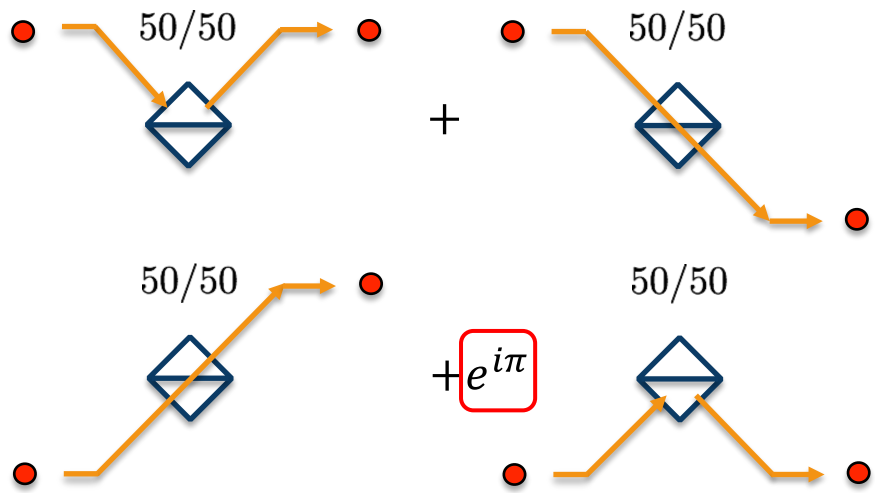
Two-particle interference

Photons on a beam splitter:



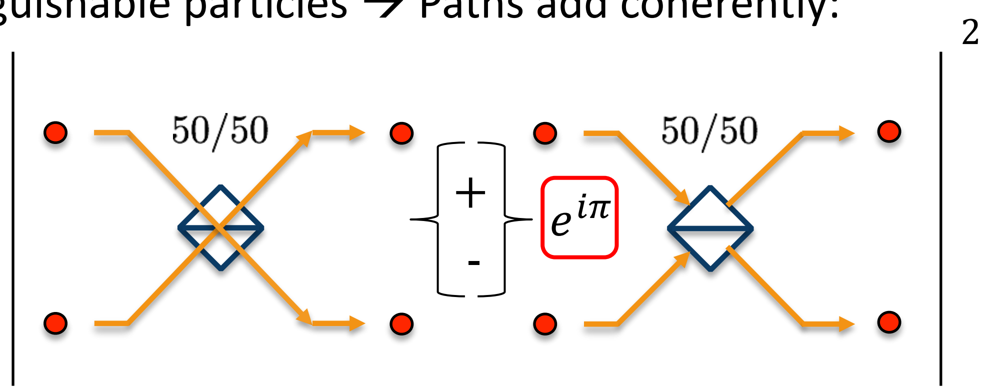
Zeilinger, Am. J. Phys. **49,** 882 (1981)

Campos *et al.* Phys. Rev. A **40,** 1371 (1989)

Two-Particle Interference:

Indistinguishable particles → Paths add coherently:

P_B (•,•)=

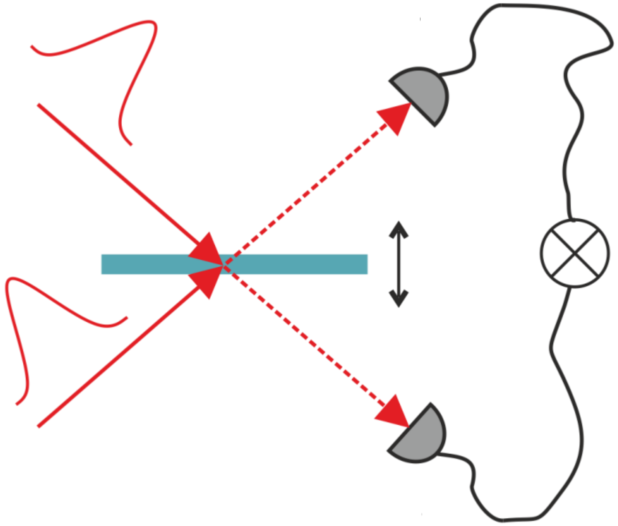


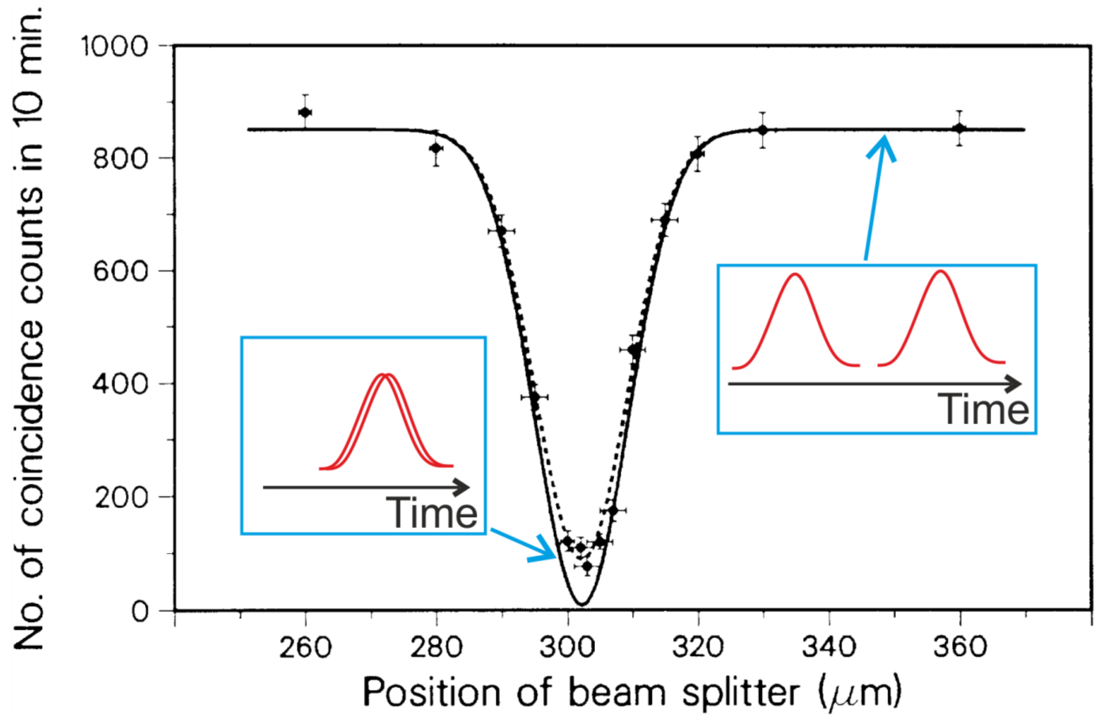
$$= \begin{cases} 0 & \text{bosons} \\ 1 & \text{fermions} \end{cases}$$

Two-particle interference

<u>Hong-Ou-Mandel experiment</u>

- Vary distinguishability of photons (bosons)



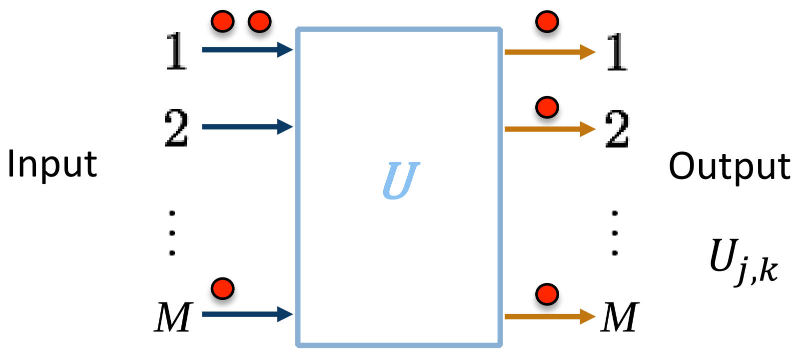


→ Widely used to measure indistinguishability of photons

Hong *et al.*, Phys. Rev. Lett. **59,** 2044 (1987)

Multi-particle interference

N bosons in M-port scattering matrix $\boldsymbol{U}$:



$U_{j,k}$ … amplitude $j \rightarrow k$

- Mode occupation:

  How many particles in each mode?

  $\vec{r} = (2,0,\ldots,1)$ $\vec{s} = (1,1,\ldots,1)$ (length M)

- Mode assignment:

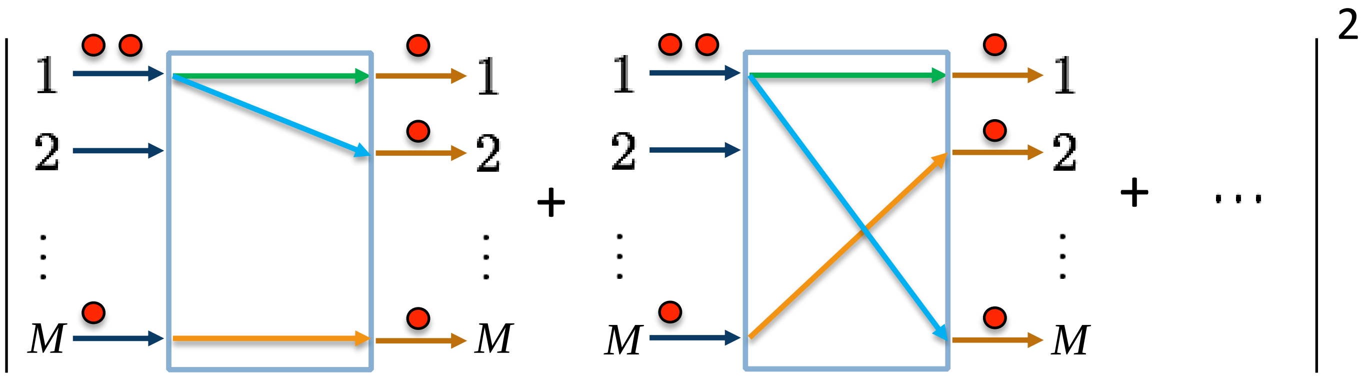
  Which mode occupied by each particle?

  $\vec{d}(\vec{r}) = (1,1,M)$ $\vec{d}(\vec{s}) = (1,2,M)$ (length N)

Tichy *et al.* J. Phys. B: At. Mol. Opt. Phys. **47,** 103001 (2014)

Multi-particle interference

$P_{\mathrm{B}}(\vec{r},\vec{s}) \propto$



$U_{1,1}U_{1,2}\dots U_{M,M}$ (Single-particle amplitudes) $\quad U_{1,1}U_{1,M}\dots U_{M,2}$

→ Sum over all permutations of input-output mode combinations

Bosons:

$$P_{\mathrm{B}}(\vec{r},\vec{s}) \propto \left| \sum_{\vec{\sigma}\in S_{\vec{d}(\vec{s})}} \prod_{j=1}^{N} U_{d_j(\vec{r}),\sigma_j} \right|^2 \propto |\mathrm{perm}(V)|^2$$

Permutations of $\vec{d}(\vec{s})$

Submatrix of occupied input-/output-modes

→ $\mathbf{O}(N!)$ summands

Tichy *et al.* J. Phys. B: At. Mol. Opt. Phys. **47,** 103001 (2014)

Multi-particle interference

Fermions:

$$P_{\mathrm{F}}(\vec{r},\vec{s}) \propto |\det(V)|^2$$

Distinguishable:

$$P_{\mathrm{D}}(\vec{r},\vec{s}) \propto \mathrm{perm}|V|^2$$

Tichy *et al.* J. Phys. B: At. Mol. Opt. Phys. **47,** 103001 (2014)

Bosons in random unitaries:

- High computational complexity (best algorithm $\mathbf{O}(2^N)$/output state)

$\binom{M+N-1}{N}$ output states

→ Boson sampling problem

Aaronson & Arkhipov, Theory Comput. **9,** 143 (2013)

→ Experiments with photons:

Broome *et al.*, Science **339,** 794 (2013)
Spring *et al.*, Science **339,** 798 (2013)
Crespi *et al.*, Nat. Phot. **7**, 545 (2013)
Tillmann *et al.*, Nat. Phot. **7**, 540 (2013)

Certification of indistinguishability:

Carolan *et al.*, Nat. Phot. **8,** 621 (2014)
Spagnolo *et al.*, Nat. Phot. **8,** 615 (2014)
Carolan *et al.*, Science **349,** 711 (2015)

Distinguishability transition:

Tillmann *et al.*, Phys. Rev. X **5**, 041015 (2015)

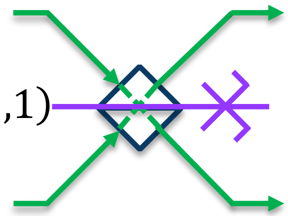
Scalability:

Bentivegna *et al.*, Sci. Adv. **1,** 1400255 (2016)
Loredo *et al.*, arXiv:1603.00054 (2016)
He *et al.*, arXiv:1603.04127 (2016)
Wang *et al.*, arXiv:1612.06956 (2016)

Symmetries in Multi-particle interference

Symmetries in the unitary:

Beam splitter

$\vec{r} = (1,1)$  $\vec{s} = (1,1)$

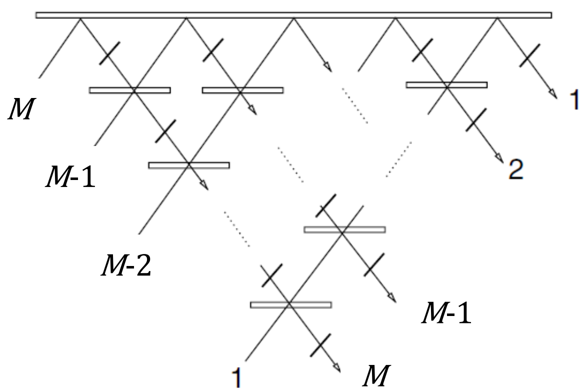
$$\mathbf{U} \propto \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cong \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$P_{\mathrm{B}}(\vec{r}, \vec{s}) = 0$$

→ Destructive interference due to unitary symmetry

→ Generalisation to more complex scenarios?

Multiport beamsplitter:



Lim *et al.*, New J. Phys. **7,** 155 (2005)

Tichy *et al.*, Phys. Rev. Lett. **104,** 220405 (2010)

$$U_{j,k} \propto e^{i\frac{2\pi}{M}jk}$$

+ $\vec{r}$ cyclically symmetric (periodicity)

→ Fourier **suppression law**:

$\sim \dfrac{N-1}{N}$ of output states vanish (know which)

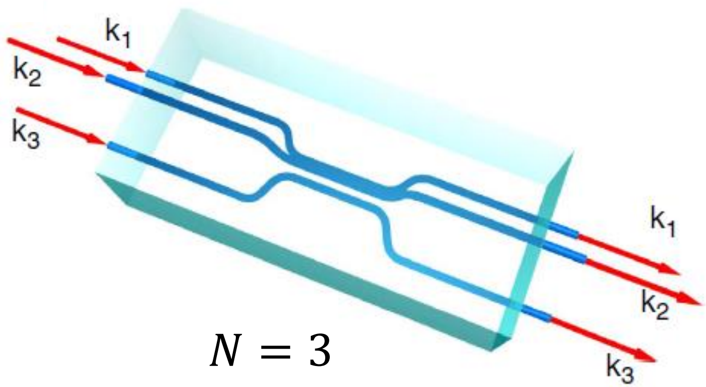
→ Analytic formula for suppressed states

→ Simplifies the general calculation

Tichy *et al.*, New J. Phys. **14,** 093015 (2012)

Symmetries in Multi-particle interference

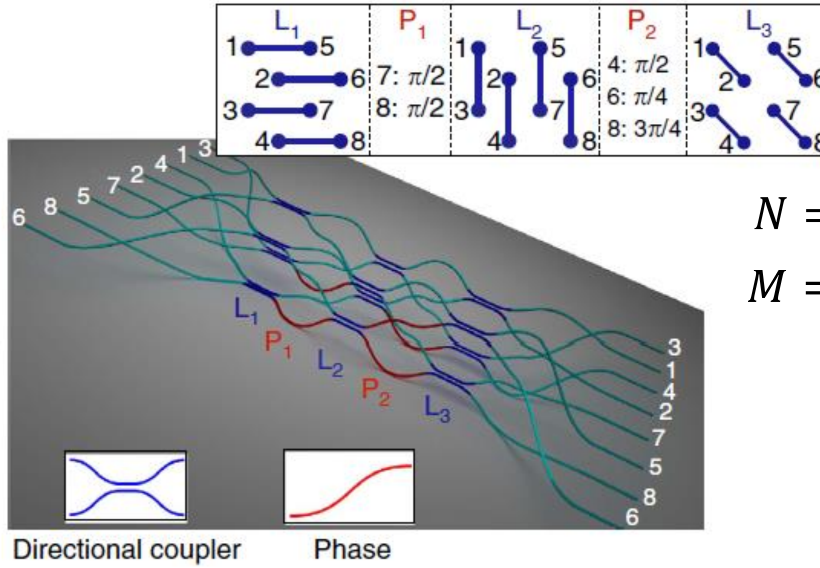
Fourier suppression – Experimental realisation:



$N = 3$

$M = 3$

Spagnolo *et al.*, Sci. Rep. **4,** 1606 (2013)



$N = 2$

$M = 8$

Crespi *et al.*, Nat. Comm. **7,** 10469 (2016)

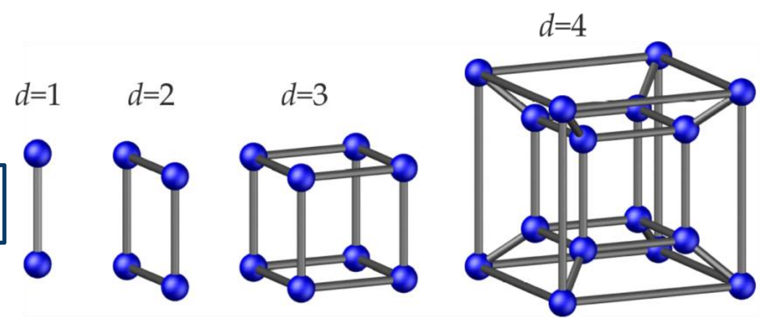
Other known symmetry-induced suppression laws:

→ Sylvester interferometer

Crespi, Phys. Rev. A **91,** 013811 (2015)

→ Hypercube

Dittel *et al.*, Quant. Sci. Technol. **2,** 015003 (2017)



→ see Christoph's talk on Monday

photonik
universität innsbruck

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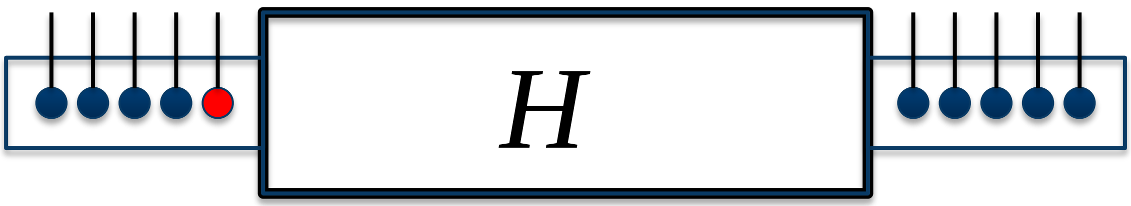
- **Spin-chains for perfect state transfer and their optical representation**
- Many-photon dynamics in state transfer lattices
  - Suppression law
  - Multi-photon experiments

Perfect state transfer

<u>Transport of a quantum state across static system</u>

- Hamiltonian *H* transferring the state by its <u>internal dynamics</u>
- Appropriate choice of *H* → <u>Coherent transport</u>
- <u>No external control</u> in the transfer region → <u>isolation</u> from the environment possible → good <u>coherence</u>



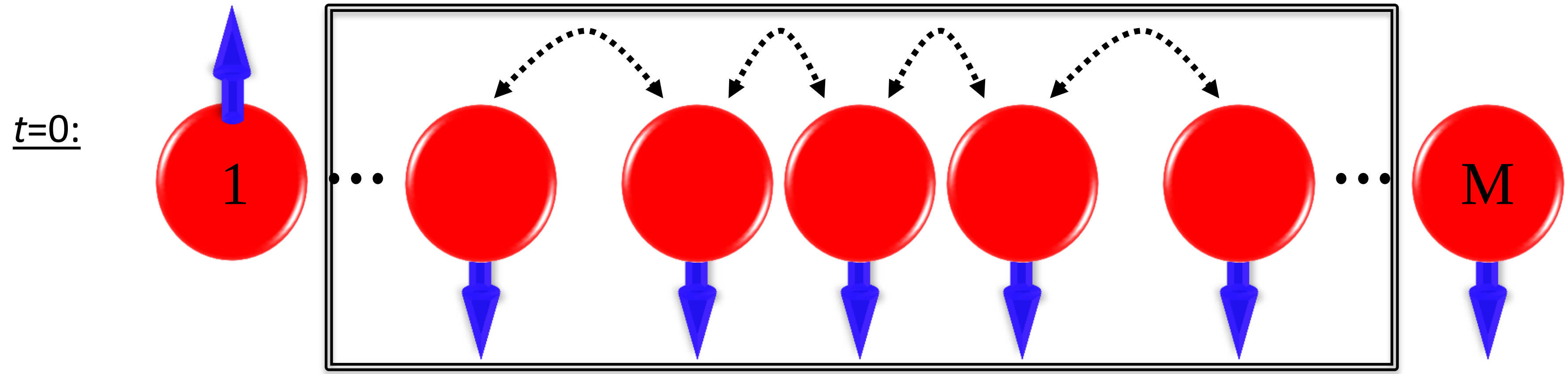
$|\Psi_{\mathrm{in}}\rangle$ $\qquad$ $e^{iHt_f}|\Psi_{\mathrm{in}}\rangle = |\Psi_{\mathrm{out}}\rangle$ $\qquad$ t_f…transfer time

Christandl *et al.*, Phys. Rev. Lett. **92,** 187902 (2004)

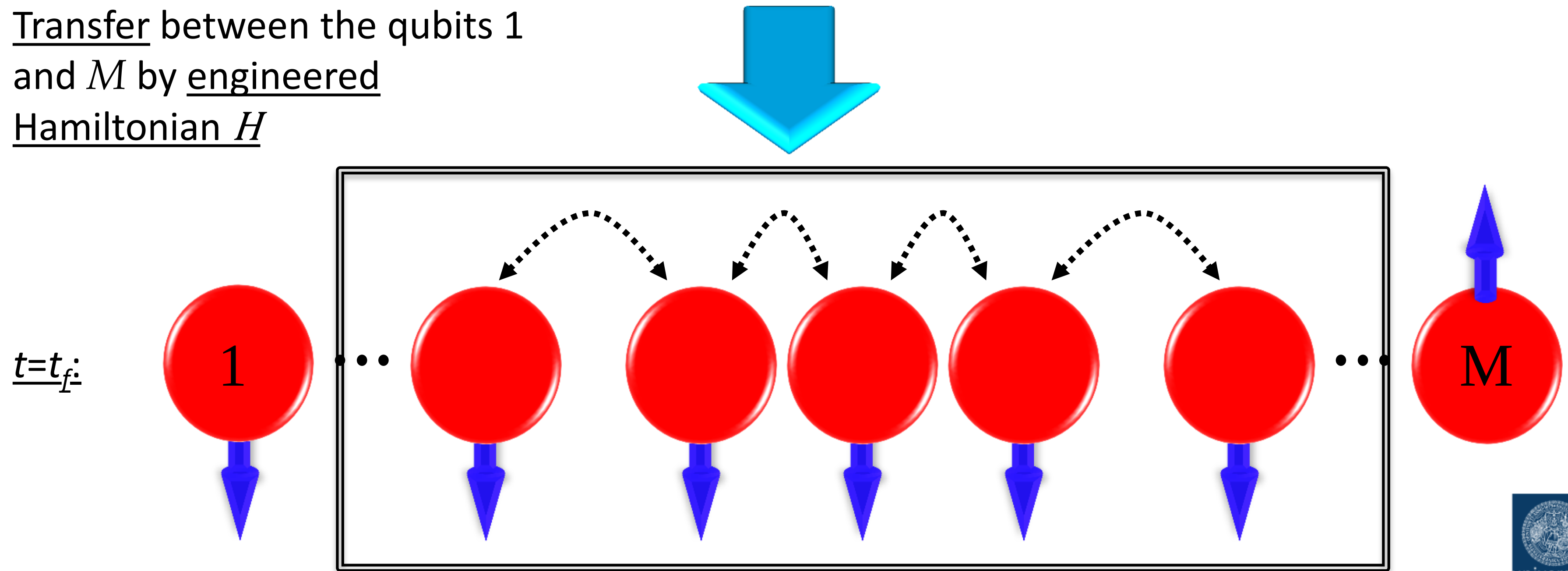
Kay, Int. J. Quant. Inf. **8,** 641 (2010)

Transfer Hamiltonian

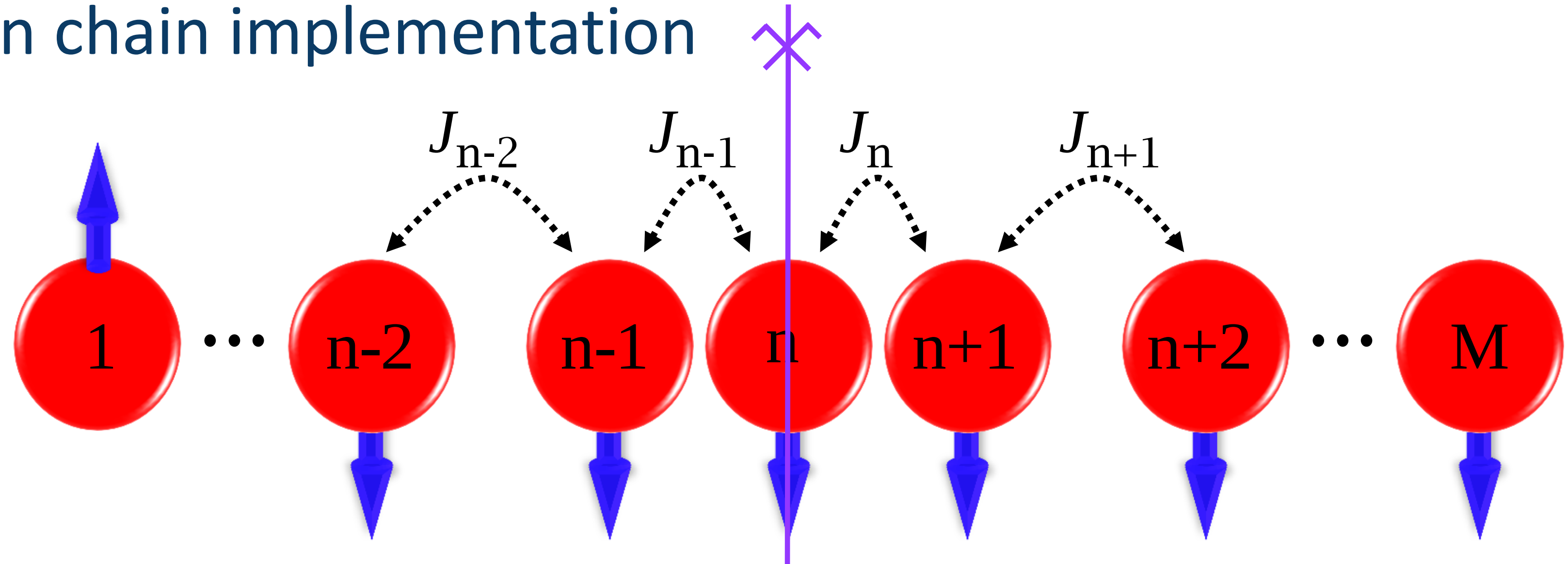
Ferromagnetic coupled spin-1/2-chain:



Transfer between the qubits 1 and M by engineered Hamiltonian H



Spin chain implementation



Single-excitation subspace:

$$i\frac{d\alpha_n}{dt} + \boxed{J_{n-1}\alpha_{n-1} + J_n\alpha_{n+1}} = 0 \qquad \alpha_n \equiv \langle\Psi|n\rangle$$

$|n\rangle$ … excitation of n^{th} spin

Nearest neighbour coupling

Transfer condition:

$$\alpha_n(t=0) = \delta_{n,1} \rightarrow \alpha_n(t=t_\text{f}) = \delta_{n,M}$$

Optimal Hamiltonian provided by coupling distribution:

$$\boxed{J_n = \frac{\pi}{2t_\text{f}}\sqrt{n(M-n)}}$$

Mirror symmetry

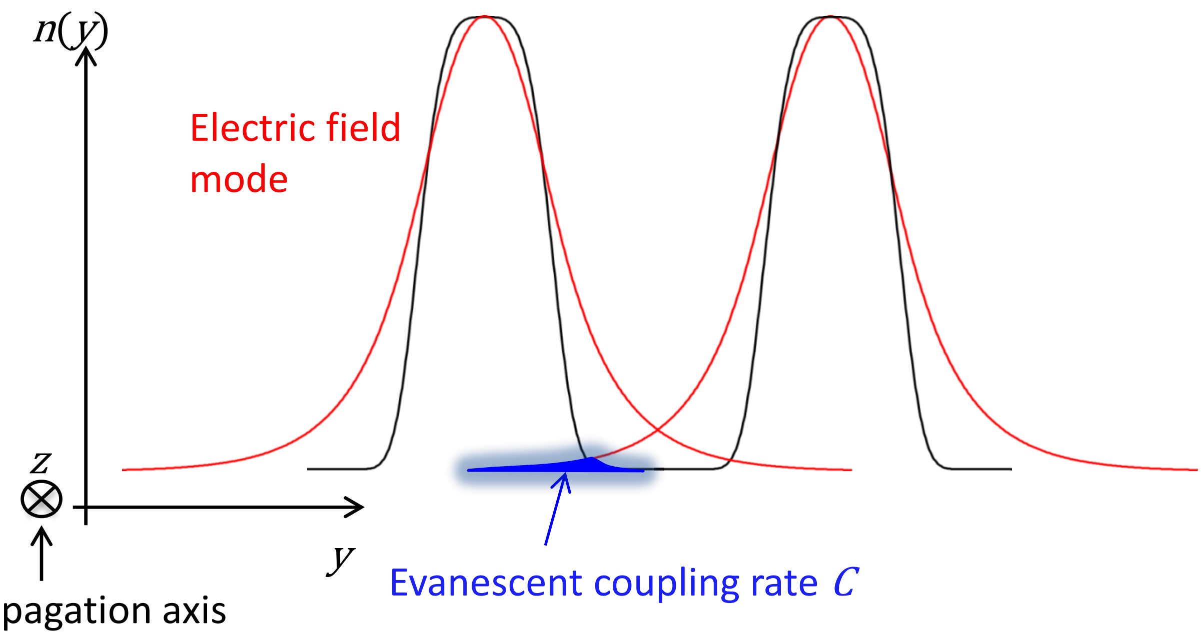
$$n \leftrightarrow M-n$$

Christandl *et al.*, Phys. Rev. Lett. **92,** 187902 (2004)

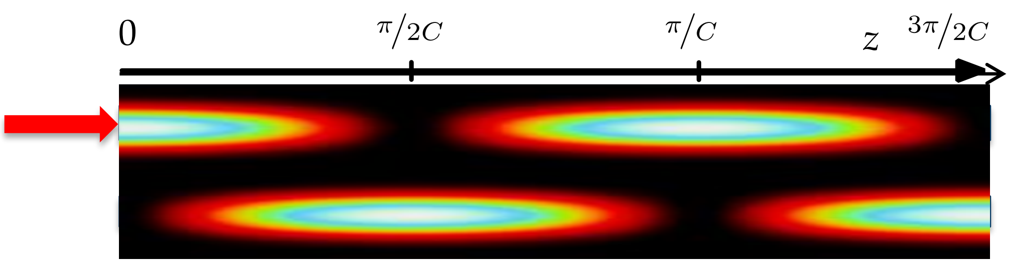
Kay, Int. J. Quant. Inf. **8,** 641 (2010)

Evanescent coupling in optics

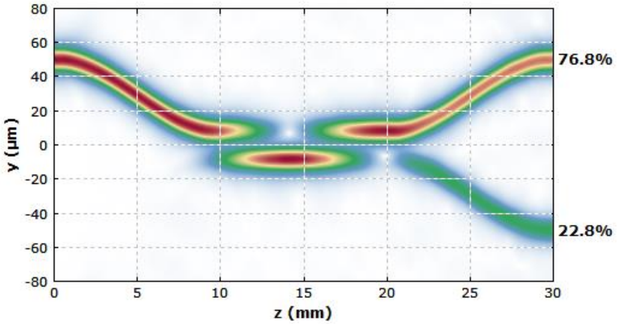
Light guided in optical waveguides with refractive index profile $n(y)$:



→ Directional coupler:



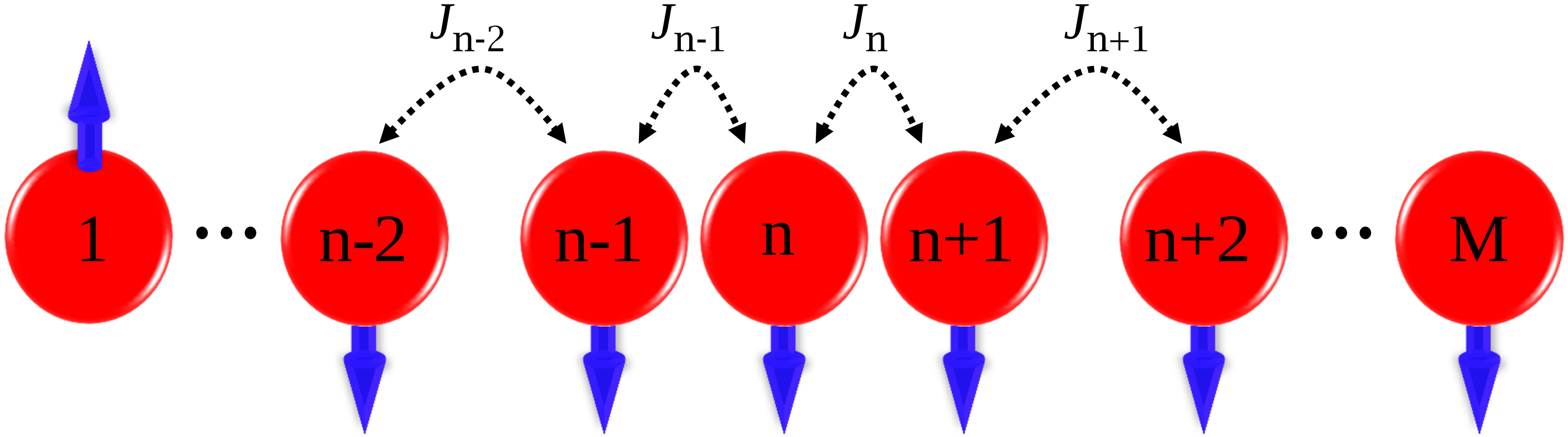
→ Acts as beam splitter:



Jones, J. Opt. Soc. Am. **55,** 261 (1965)

www.rp.photonics.com

Optical implementation

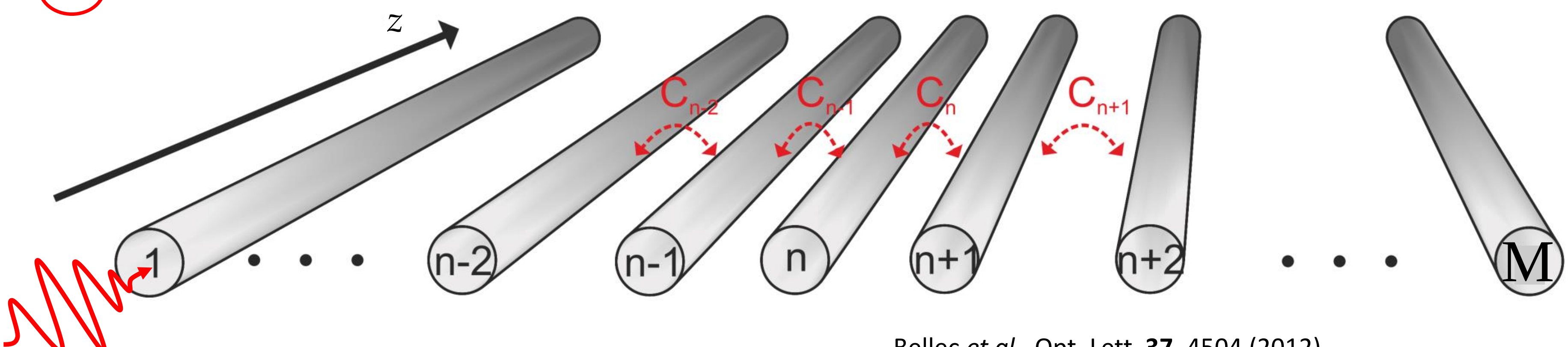


$$i\frac{d\alpha_n}{dt} + J_{n-1}\alpha_{n-1} + J_n\alpha_{n+1} = 0 \qquad J_n \propto \sqrt{n(M-n)}$$

<u>Photons in waveguides:</u>

$$i\frac{da_n}{dz} + C_{n-1}a_{n-1} + C_n a_{n+1} = 0 \qquad C_n \propto \sqrt{n(M-n)}$$

a_n … light amplitude in n^{th} waveguide



Gordon, Opt. Lett. **29,** 2752 (2004)

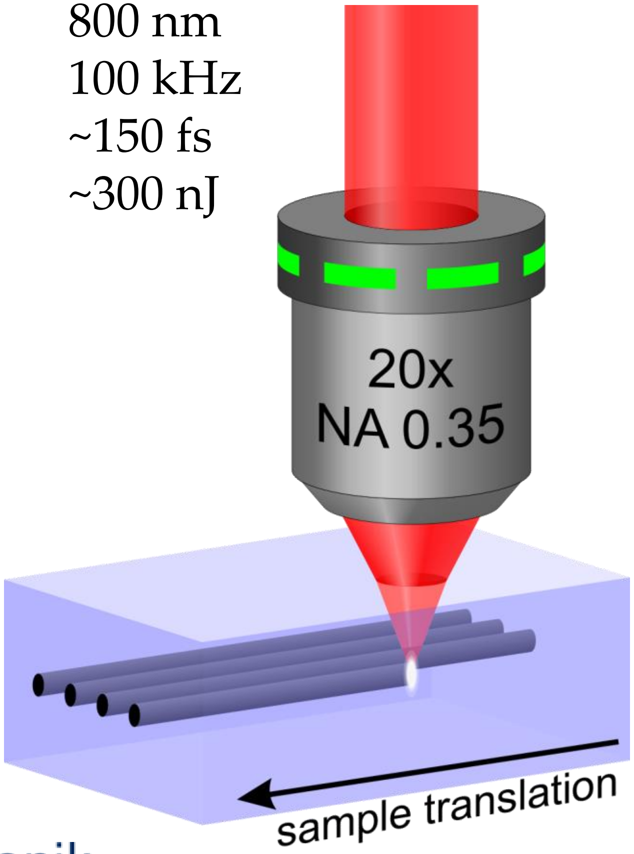
Bellec *et al.*, Opt. Lett. **37,** 4504 (2012)

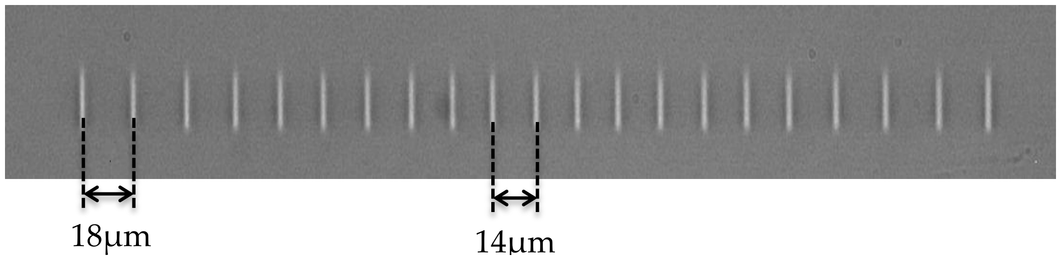
Perez-Leija *et al.*, Phys. Rev. A **87,** 012309 (2013)

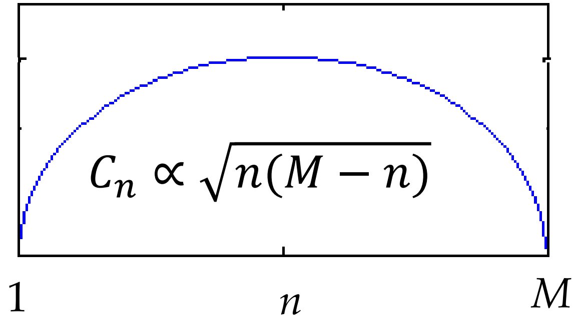
Waveguide fabrication

- **Direct waveguide inscription** by ultrashort laser pulses
- Permanent refractive index increase

800 nm
100 kHz
~150 fs
~300 nJ

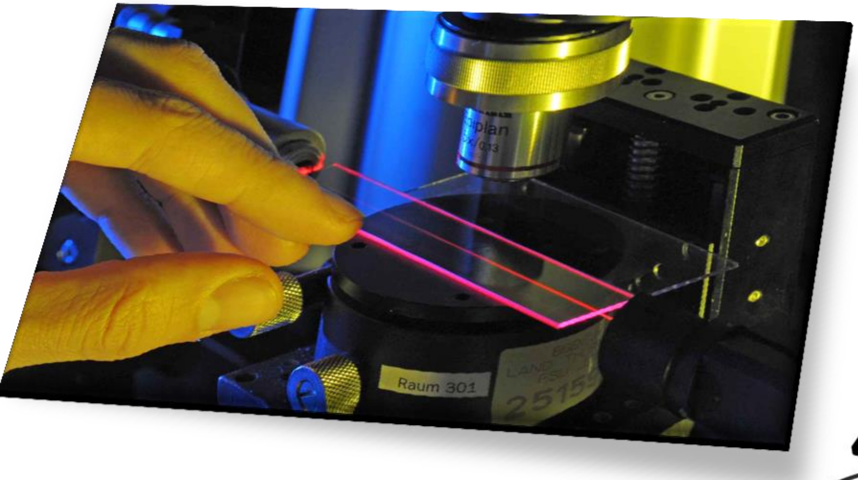






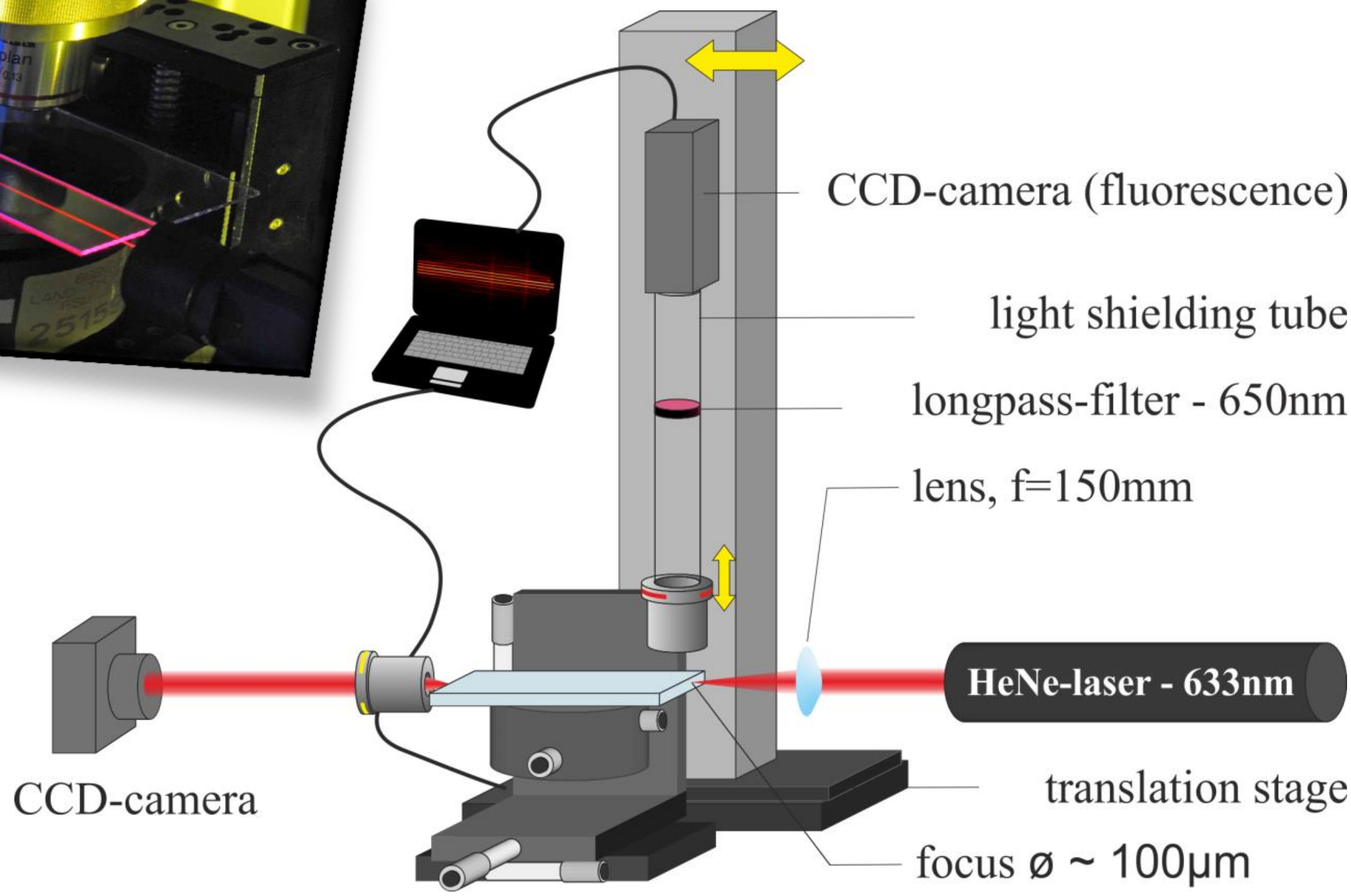
Szameit & Nolte, J. Phys. B: At. Mol. Opt. Phys. **43,** 163001 (2010)

Observation technique for coherent light



Fluorescence images

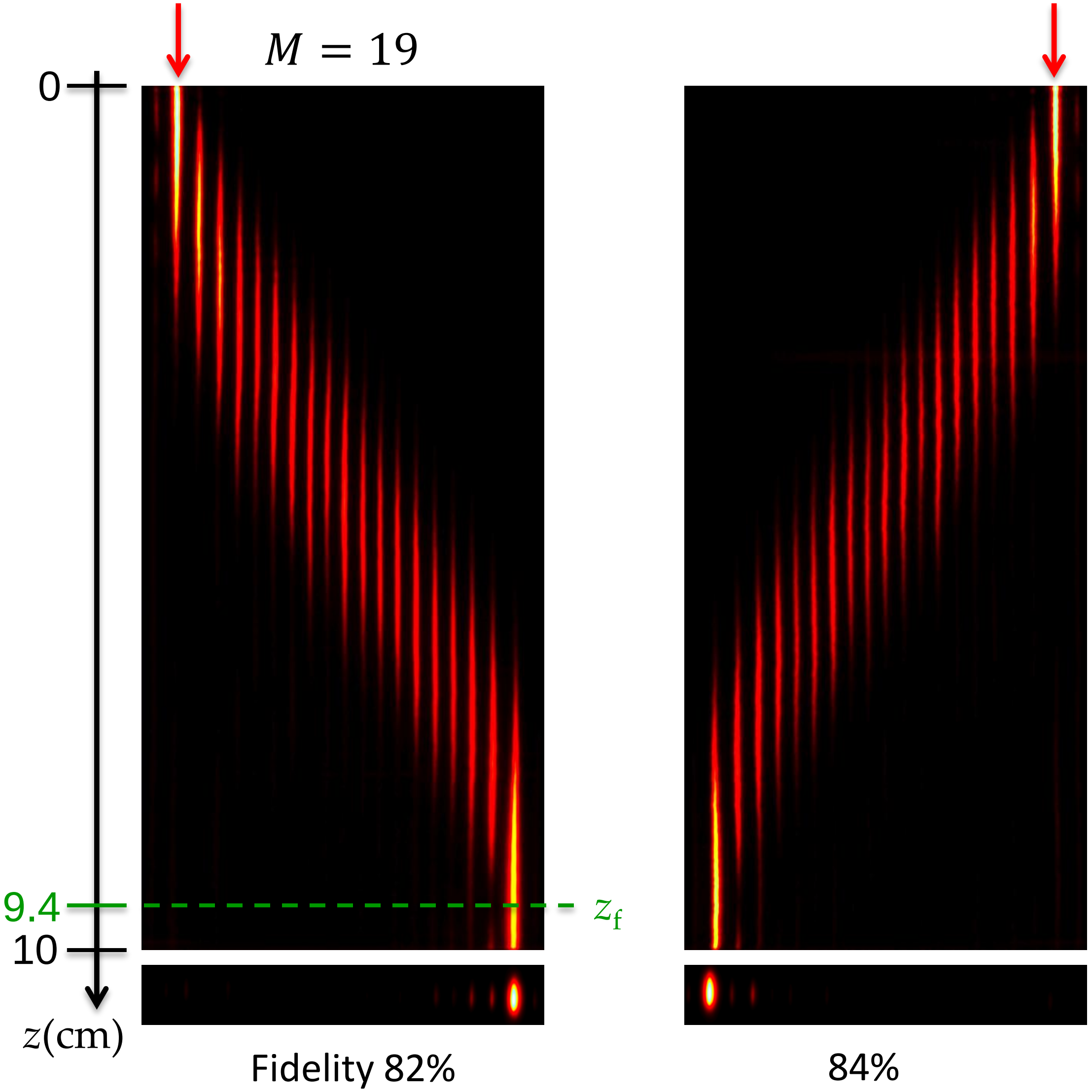
→ Measuring dynamics



→ Measuring **output intensities**

Dreisow *et al.*, Opt. Express **16**, 3474 (2008)

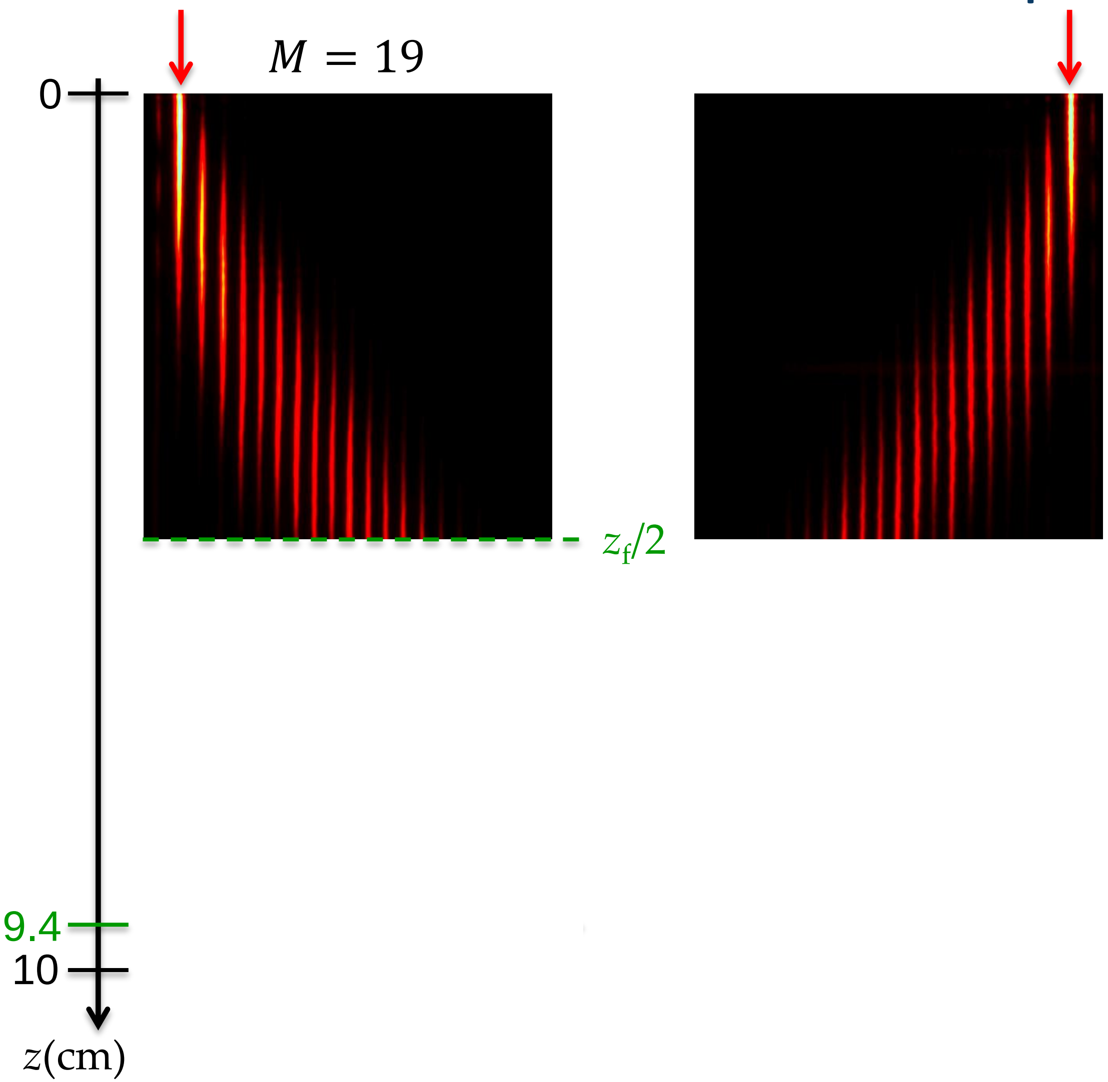
Observation of coherent transport



- Fluorescence signal from coherent light excitation
- Optimal transfer @ $z = z_{\mathrm{f}}$
- Mirror-symmetry

Perez-Leija *et al.*, Phys. Rev. A **87,** 012309 (2013)

Observation of coherent transport

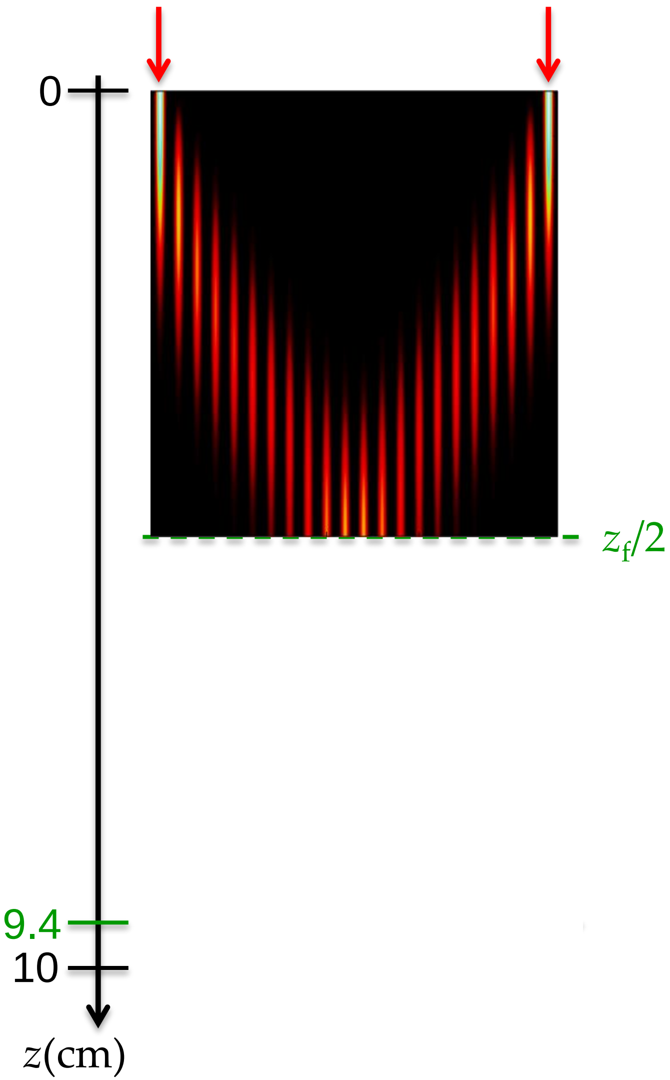


- Fluorescence signal from coherent light excitation
- Optimal transfer @ $z = z_{\mathrm{f}}$
- Mirror-symmetry
- Multi-particle interference @ $z = z_{\mathrm{f}}/2$

  → Which-way interference

Perez-Leija *et al.*, Phys. Rev. A **87,** 012309 (2013)

Observation of coherent transport



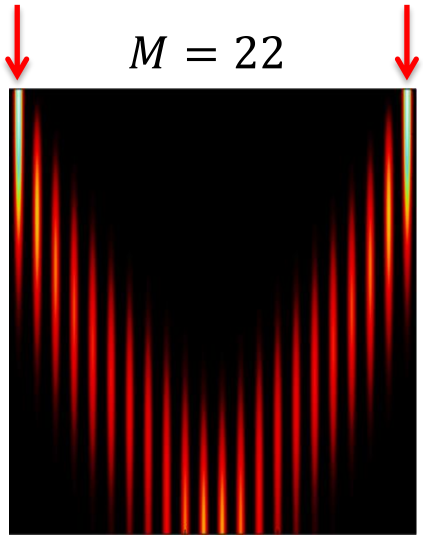
- Fluorescence signal from coherent light excitation
- Optimal transfer @ $z = z_f$
- Mirror-symmetry
- Multi-particle interference @ $z = z_f/2$

  → Which-way interference

Perez-Leija *et al.*, Phys. Rev. A **87,** 012309 (2013)

Two-photon interference

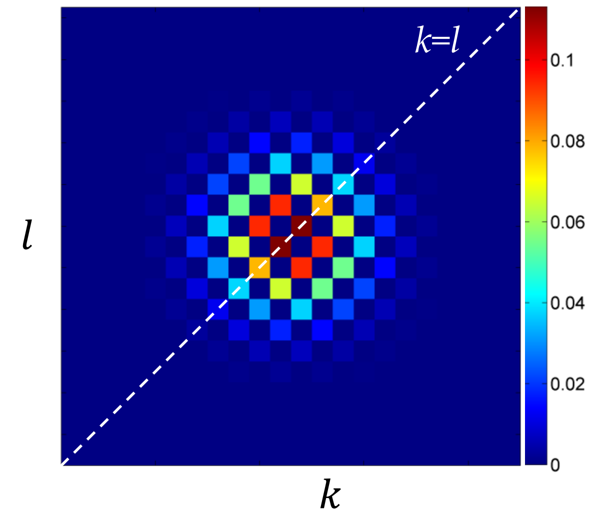
$M = 22$



$\vec{d}(\vec{r}) = (1, M)$

$\vec{d}(\vec{s}) = (k, l)$

Theory:



→ Half of the output states with zero probability

Two-photon correlation function:

$$\Gamma_{k,l} = \langle \hat{a}_k^\dagger \hat{a}_l^\dagger \hat{a}_l \hat{a}_k \rangle = (1 + \delta_{k,l}) P_{\mathrm{B}}(\vec{r}, \vec{s})$$

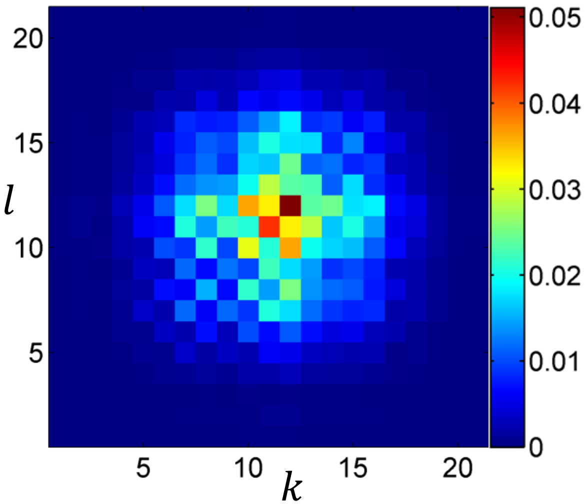
Analytic solution:

$$\Gamma_{k,l} = \begin{cases} 0, k - l \text{ odd} \\ 2^{4-2M} \binom{M-1}{k-1} \binom{M-1}{l-1}, k - l \text{ even} \end{cases}$$

Perez-Leija *et al.*, Phys. Rev. A **87,** 012309 (2013)

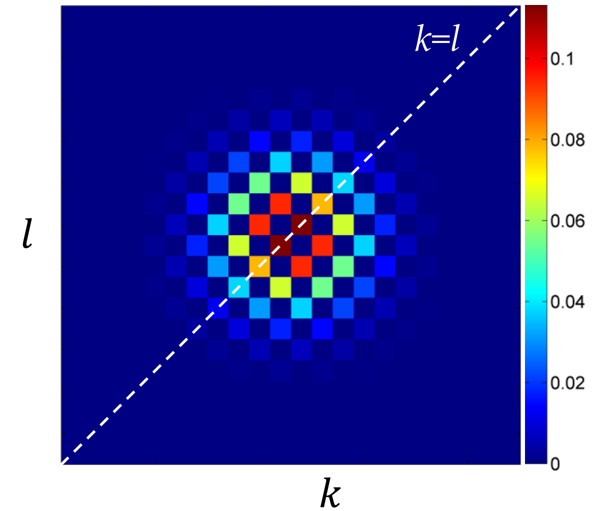
Two-photon correlation

Coherent states, phase randomised:

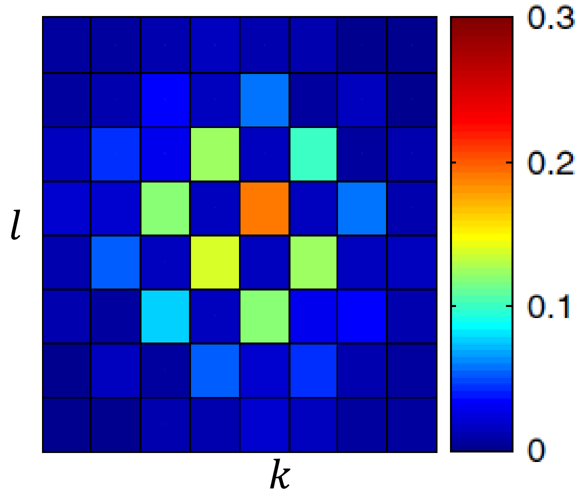


$M = 22$

Theory:



Photon pair (Fock states, N=2):



$M = 8$

→ Half of the output states with zero probability

→ **Suppression law** → How to generalise for N photons and relate to the symmetry?

Perez-Leija *et al.*, Phys. Rev. A **87,** 012309 (2013)

Keil *et al.*, Phys. Rev. A **81,** 023834 (2010)

Weimann *et al.*, Nat. Commun. **7,** 11027 (2016)

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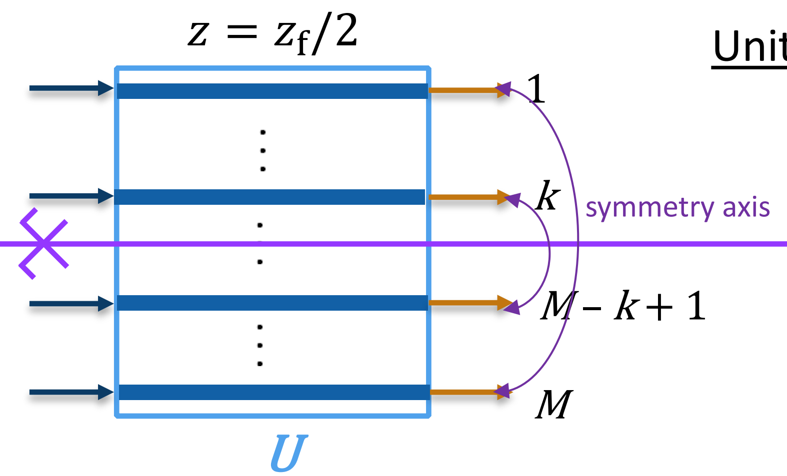
  - **Suppression law**
  - Multi-photon experiments

Symmetry of the state transfer lattice



Unitary of the state transfer lattice:

$$U_{k,m} = e^{i(k-m)\pi/2} u_{k,m}$$

$$u_{k,m} \propto \frac{P_{m-1}^{k-m, M-k+1-m}(0)}{\sqrt{(k-1)!\,(M-k)!}}$$

Jacobi polynomials

Weimann *et al.*, Nat. Commun. **7,** 11027 (2016)

Symmetry relations:

$$k \leftrightarrow M - k + 1$$

$$P_m^{a,b}(0) = (-1)^m P_m^{b,a}(0) \;\Rightarrow u_{M-k+1,m} = (-1)^{m-1} u_{k,m}$$

→ Symmetry of the unitary:

$$\forall k, m:\; U_{M-k+1,m} = e^{i\phi(M)} (-1)^{m-k} U_{k,m}$$

global phase factor

→ Parity dependent mirror (anti-) symmetry

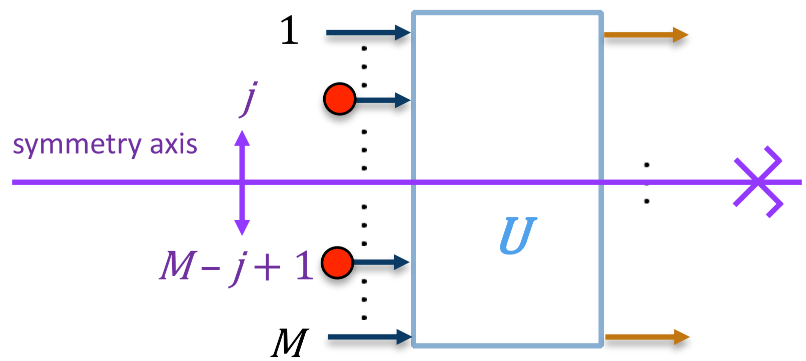
Dittel *et al.*, 31st SFB FoQuS Meeting, 2015

Parity-symmetric arrays

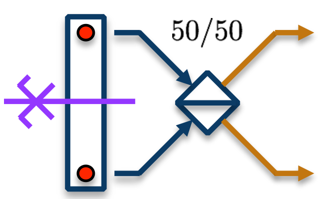
$$U_{M-k+1,m} = e^{i\phi(M)}(-1)^{m-k}U_{k,m}$$

Symmetry of the input state:

$$r_j = r_{M-j+1}$$



Example M=2:



$$\mathbf{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\phi(M) = \pi/2$$

$$\vec{r}^{\mathrm{T}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

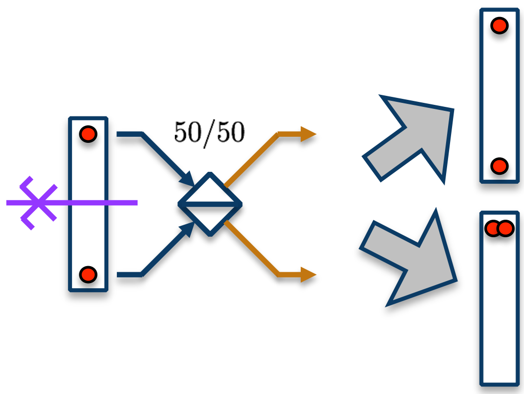
→ Generalisation of the beam splitter symmetry to M modes

Suppression Law

$$\mathrm{mod}\left[\sum_{j=1}^{N} d_j(\vec{s}) + N, 2\right] = 1 \Rightarrow P_{\mathrm{B}}(\vec{r},\vec{s}) = 0$$

Output states with an **odd number of bosons** in **even labelled modes** are strictly suppressed

Example $N = 2, M = 2$:



$\vec{s} = (1,1)$

$\vec{d}(\vec{s}) = (1,2)$

$$\sum_{j=1}^{2} d_j(\vec{s}) + 2 = 5 \Rightarrow P_{\mathrm{B}}(\vec{r},\vec{s}) = 0$$ Suppressed event

$\vec{s} = (2,0)$

$\vec{d}(\vec{s}) = (1,1)$

$$\sum_{j=1}^{2} d_j(\vec{s}) + 2 = 4 \Rightarrow P_{\mathrm{B}}(\vec{r},\vec{s}) = ?$$ Allowed event

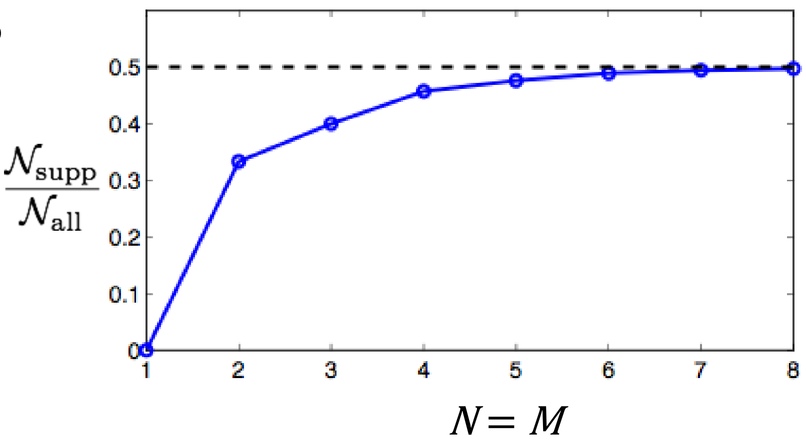
Example $N = 6, M = 4$:

$$\vec{s}_1 = (\overset{1}{0}, \overset{2}{\boxed{5}}, \overset{3}{0}, \overset{4}{\boxed{1}}) \qquad 5 + 1 = 6 \Rightarrow P_{\mathrm{B}}(\vec{r},\vec{s}_1) = ?$$ Allowed event

$$\vec{s}_2 = (1, \boxed{4}, 0, \boxed{1}) \qquad 4 + 1 = 5 \Rightarrow P_{\mathrm{B}}(\vec{r},\vec{s}_2) = 0$$ Suppressed event

Suppression Law - Characteristics

- Fraction of suppressed events: $\frac{N_{\text{supp}}}{N_{\text{all}}} \approx 0.5$



- Suppression relies on N-particle interference

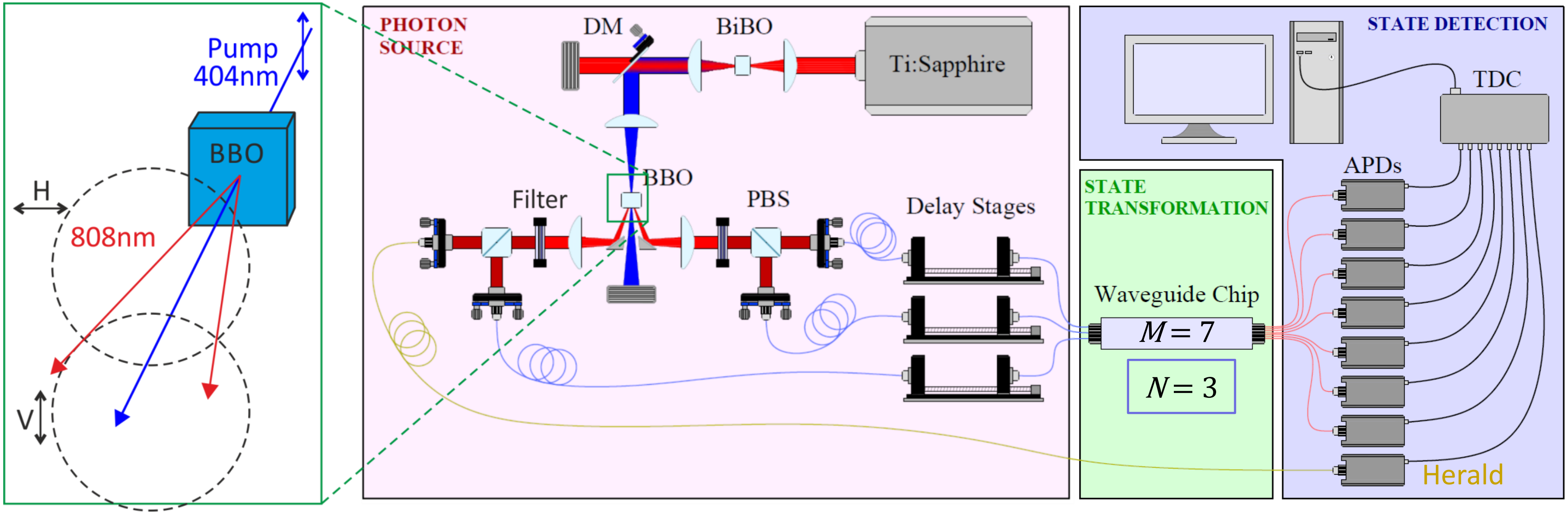
→ **Requires full indistinguishability**

- Analytic result → Computable also for very large systems

Outline

- Introduction – Symmetries
- Symmetries in multi-particle interference
- Spin-chains for perfect state transfer and their optical representation
- **Many-photon dynamics in state transfer lattices**
  - Suppression law
  - **Multi-photon experiments**

Experimental setup

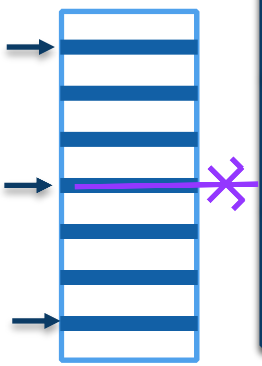


- Pulsed Ti:Sapphire pump laser at 808 nm, 200 fs, 76 MHz
- Frequency doubled , ca. 400 mW @ 404 nm
- BBO crystal for type-II parametric fluorescence (SPDC)
- Distinguishability adjusted by time-delay
- Heralded collection of non-colliding 3-Photon events

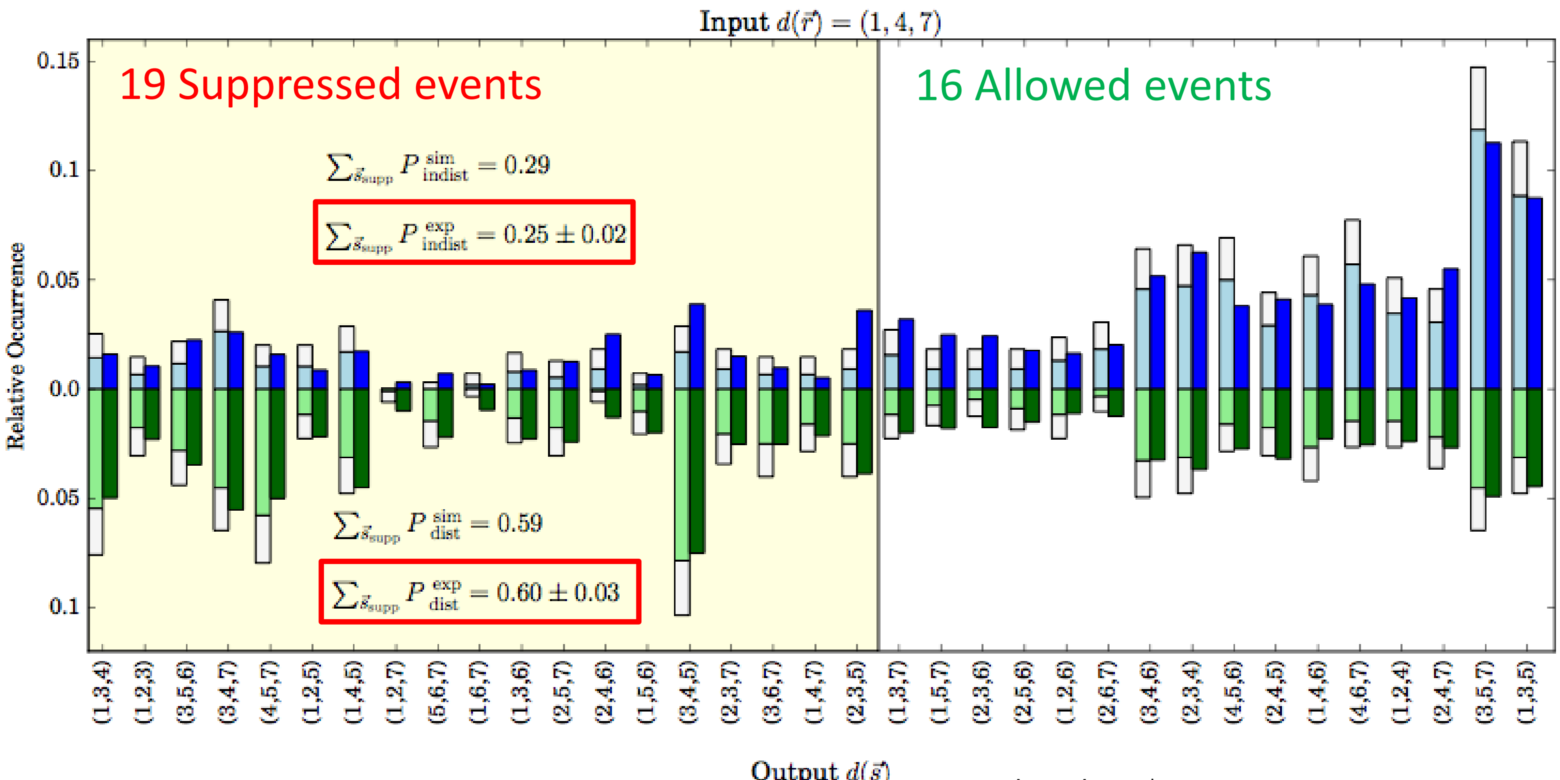
Experimental results

Input:

$$\vec{r}^{\,T} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$



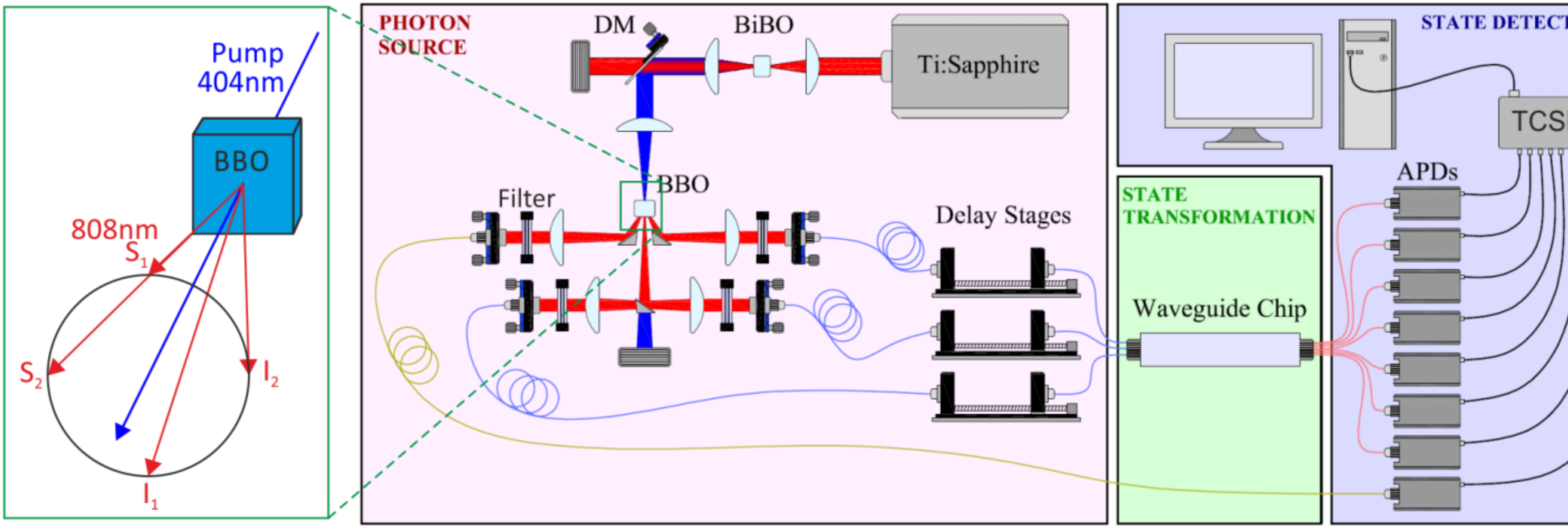
	Simulation (incl. Unitary imperfections)	Experiment
Indistinguishable (80% HOM-visib.)		654 events $\pm\sigma$
Distinguishable		583 events $\pm\sigma$



Dittel *et al.*, 31st SFB FoQuS Meeting, 2015

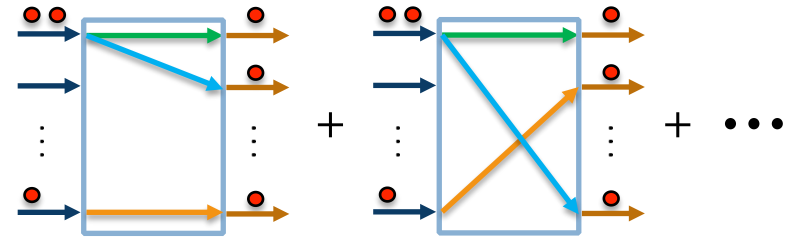
Experimental results

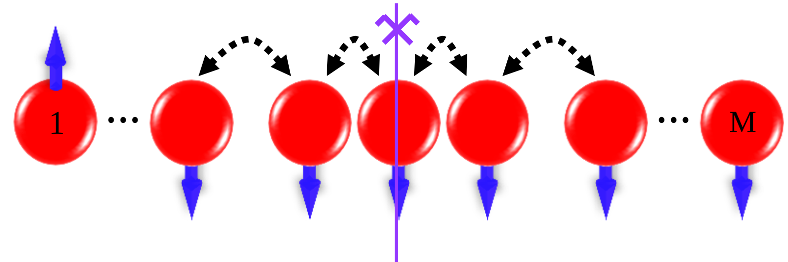
→ <u>Next steps:</u> More precise unitary, $N = 4$ photons from upgraded source (type-I SPDC, brighter, 90% HOM-visibility)

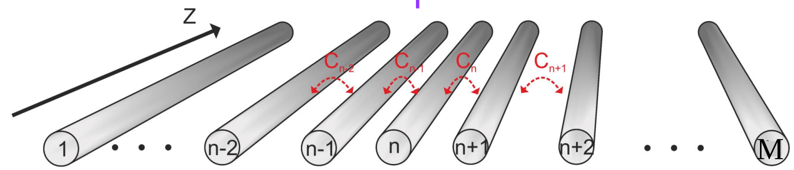


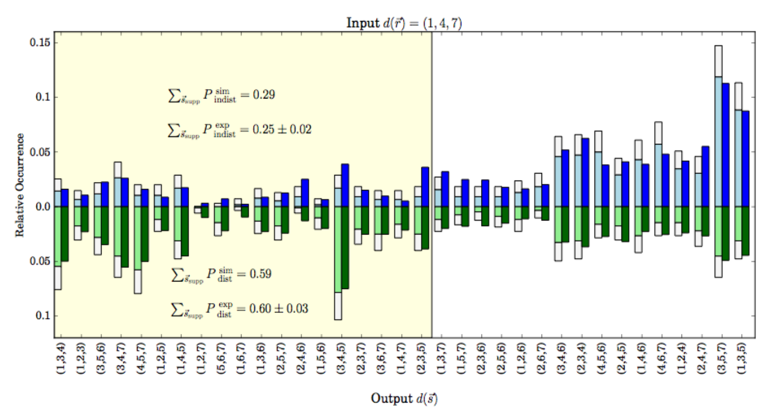
Conclusion

- Multi-particle interference – governed by single-particle dynamics + exchange statistics
- Boson interference hard to calculate classically → Boson sampling
- **Symmetries** can help to **reduce complexity of Boson scattering**
- Spin-chain for perfect state transfer → Mirror symmetry
- Waveguide lattice for multi-photon interference → **Suppression law** for symmetric inputs









Thank you for your attention!