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Department of Physics
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Quantum fluctuation relations for generalized Gibbs ensembles

J. Mur-Petit, A. Relaño, R.A. Molina, D. Jaksch

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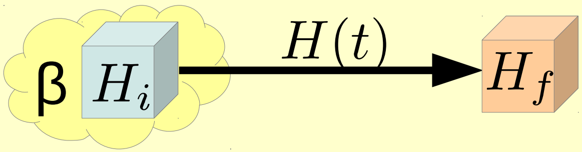
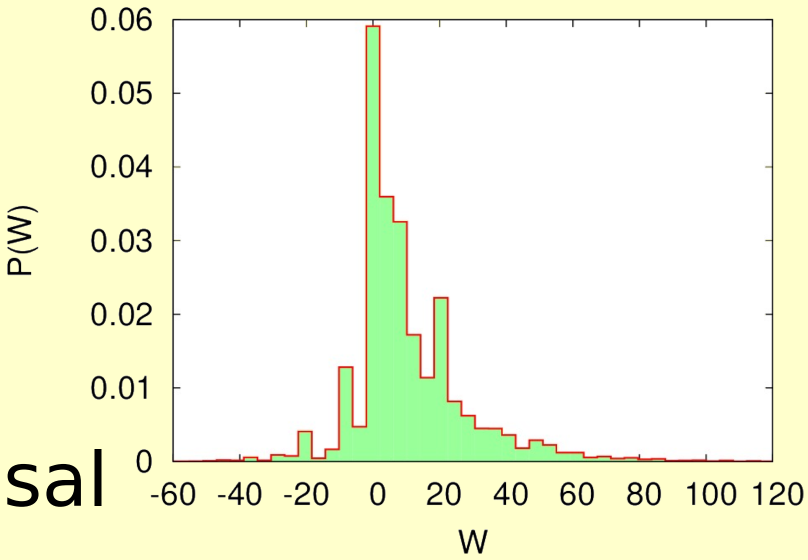
Outline

- From 2^{nd} law to quantum fluctuation relations



- QFRs 4 GGEs
  - Analytical results
  - Numerics & experimental proposal

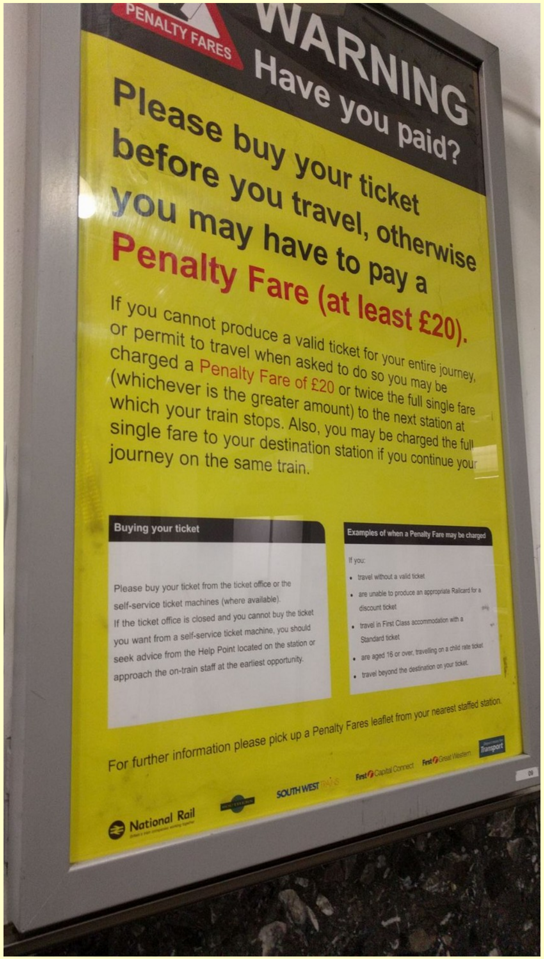


- Summary and Outlook

The character of the Law

Human Law

- Can be violated
- Fines, prisons, judges, police

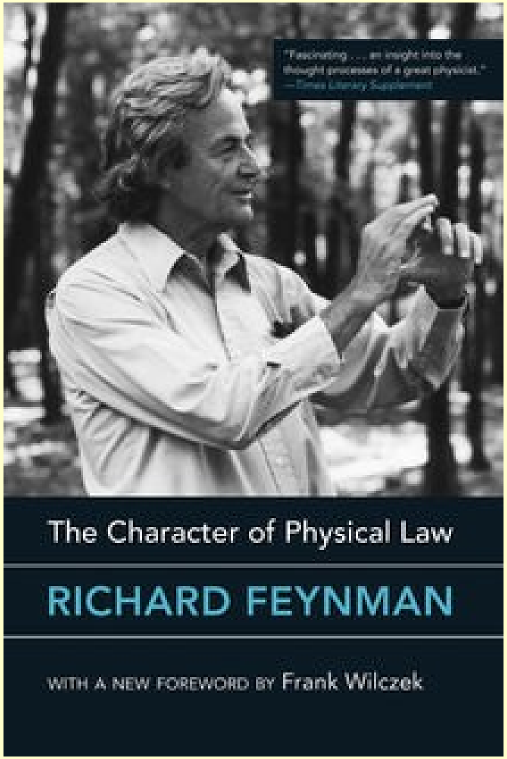


The character of the Law

Human Law

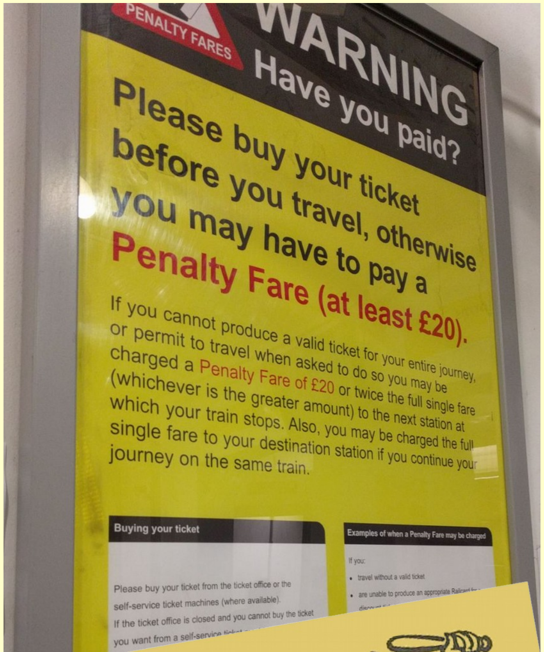
Physical Law

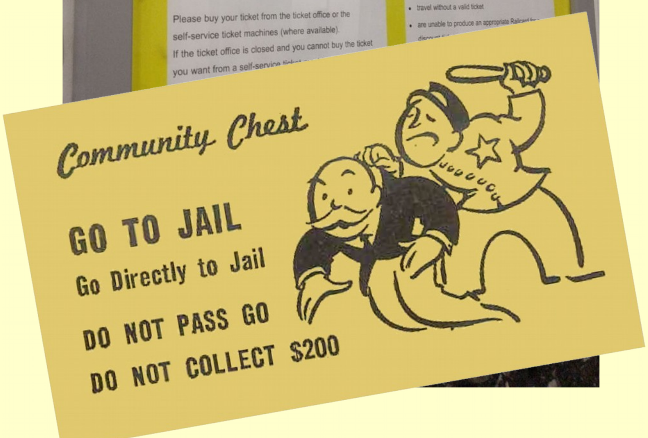
- Can't be violated
- No fines, police…



The character of the Law

Human Law





Physical Law

- Can't be violated
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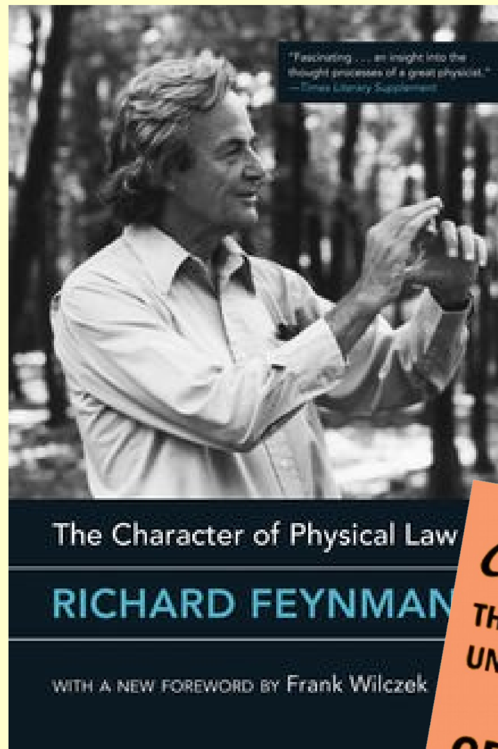
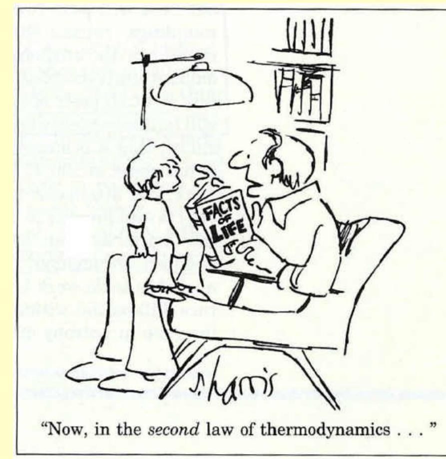


ΔS>=0

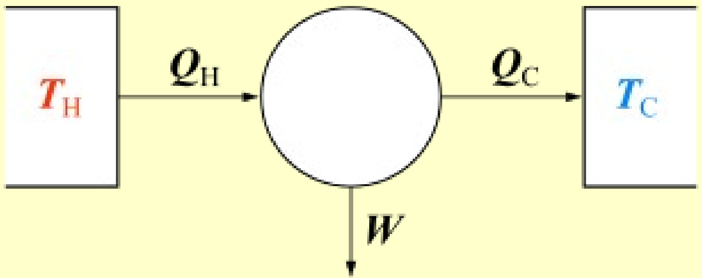
- Or it can...
- For free!



"Now, in the *second* law of thermodynamics . . ."

Better than a bound

2nd law [1824/1851/1854/...]



$$W > 0$$

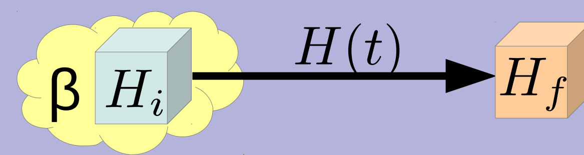
$$\eta = 1 - \frac{T_C}{T_H}$$

$$\Delta S \geq 0$$

$$\oint \frac{\delta Q}{T} \leq 0$$

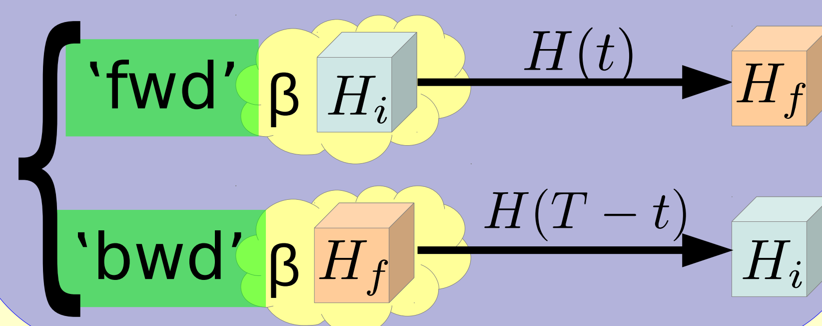
Jarzynski equality [1997]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

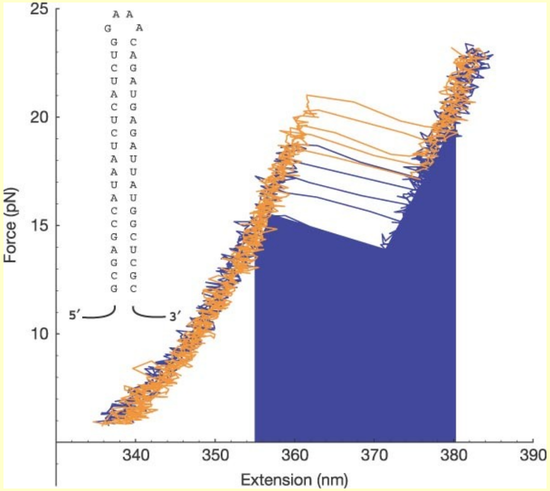


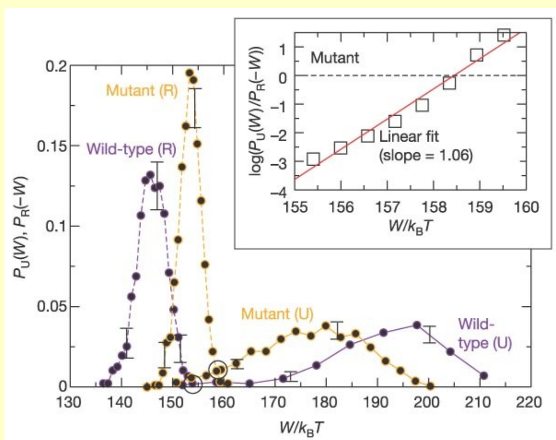
Crooks relation [1999]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$



Better than a bound

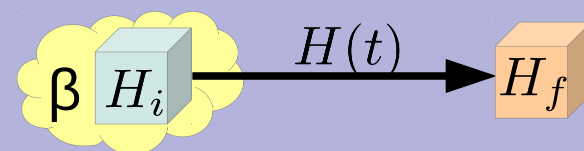




Collin et al., Nature (2005)

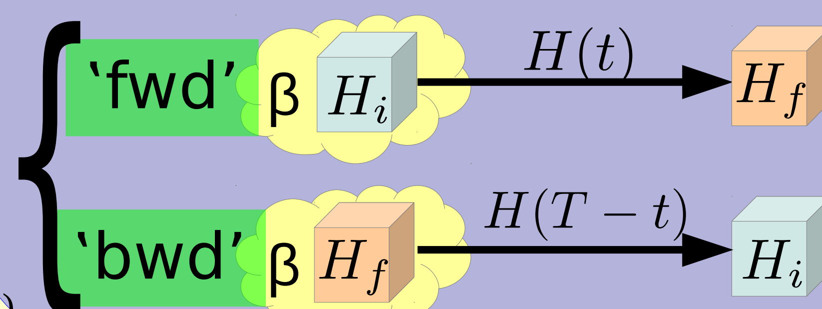
Jarzynski equality [1997]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$



Crooks relation [1999]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$

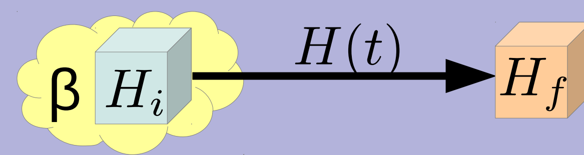


Using Jarzynski for getting ΔF: Liphardt et al., Nature (2001)

Enter the Quantum

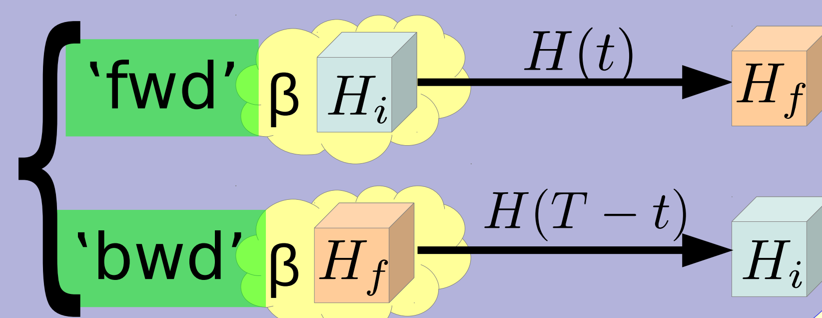
Quantum Jarzynski equality [1999]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$



Tasaki-Crooks relation [2000]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$



Anything under the rug?

Quantum Jarzynski equality [1999]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

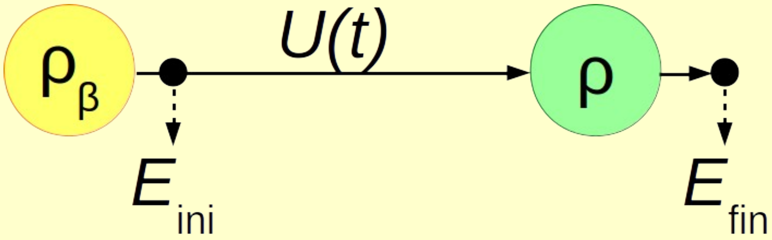
Tasaki-Crooks relation [2000]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$

Underlying assumptions:

i) Work defined via *Two Energy-Measurements protocol*

$$W = E_{\mathrm{fin}} - E_{\mathrm{ini}}$$
$$= \mathrm{Tr}[U \rho_\beta U^{-1} H_{\mathrm{fin}}] - \mathrm{Tr}[\rho_\beta H_{\mathrm{ini}}]$$



Talkner, Hänggi et al., J.Phys.A (2007), PRE (2006), PRE (2016)

Anything under the rug?

Quantum Jarzynski equality [1999]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$

Tasaki-Crooks relation [2000]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w - \Delta F)}$$

Underlying assumptions:

i) Work defined via *Two Energy-Measurements protocol*

ii) Initial state: **canonical (Gibbs) equilibrium state:**

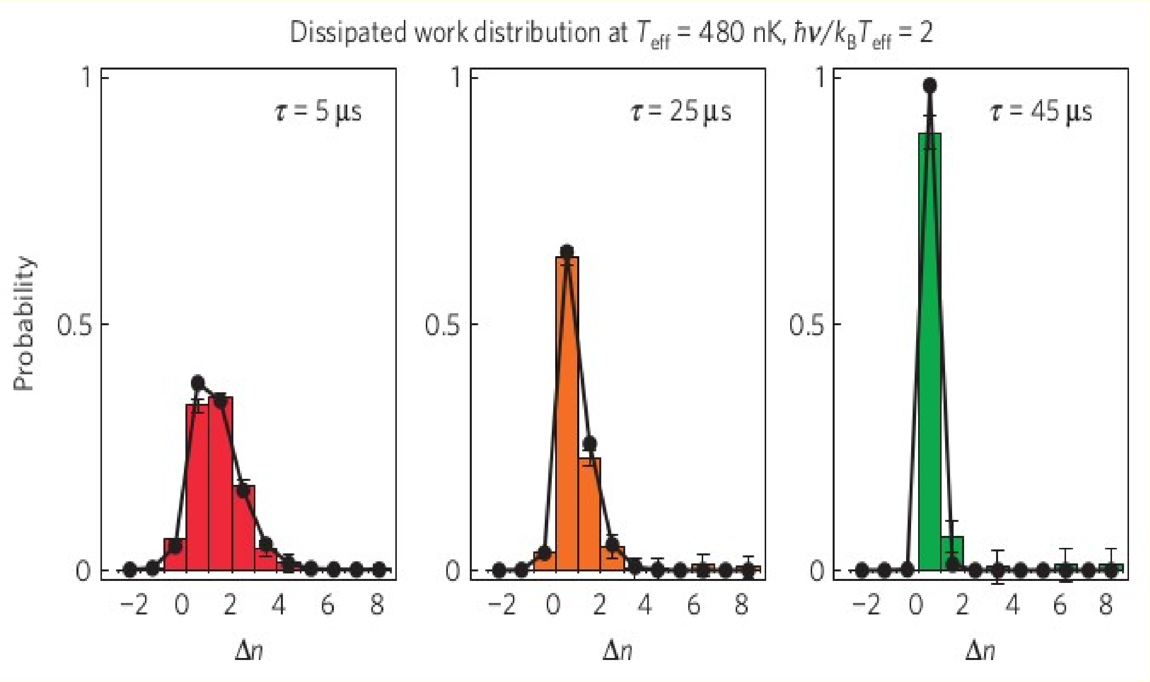
$$\rho(t=0) = \rho_\beta = \frac{1}{Z} e^{-\beta H}$$

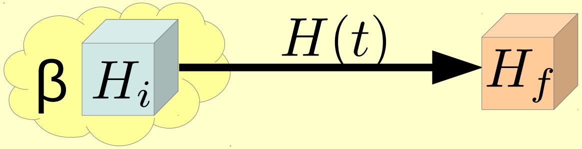
Iii) Principle of microreversibility

Testing the Quantum

Quantum Jarzynski equality [1999]

$$\langle e^{-\beta w} \rangle = e^{-\beta \Delta F}$$





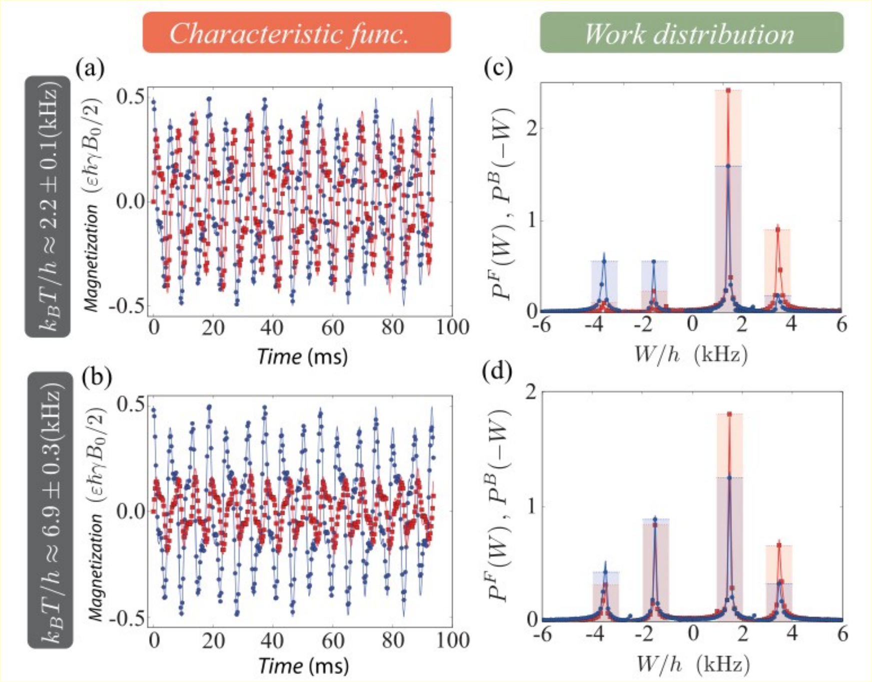
Th: Huber et al., PRL 2008

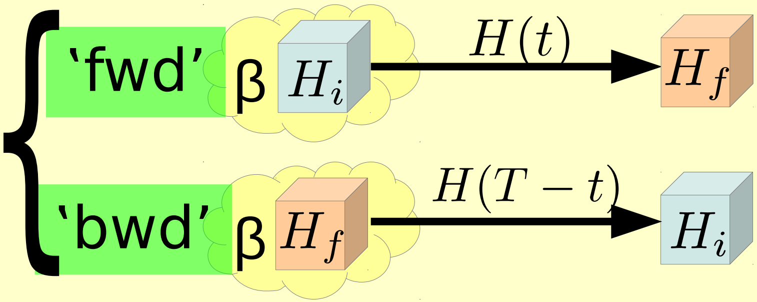
Expt: An et al., Nat. Phys. 2015

Testing the Quantum

Tasaki-Crooks relation [2000]

$$\frac{P_f(w)}{P_b(-w)} = e^{\beta(w-\Delta F)}$$

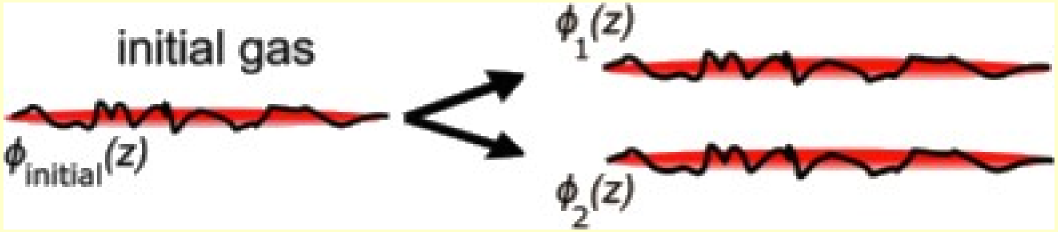




Th: Dorner et al., & Mazzola et al., PRL 2013
Expt: Batalhão et al., PRL 2014

Beyond Gibbs

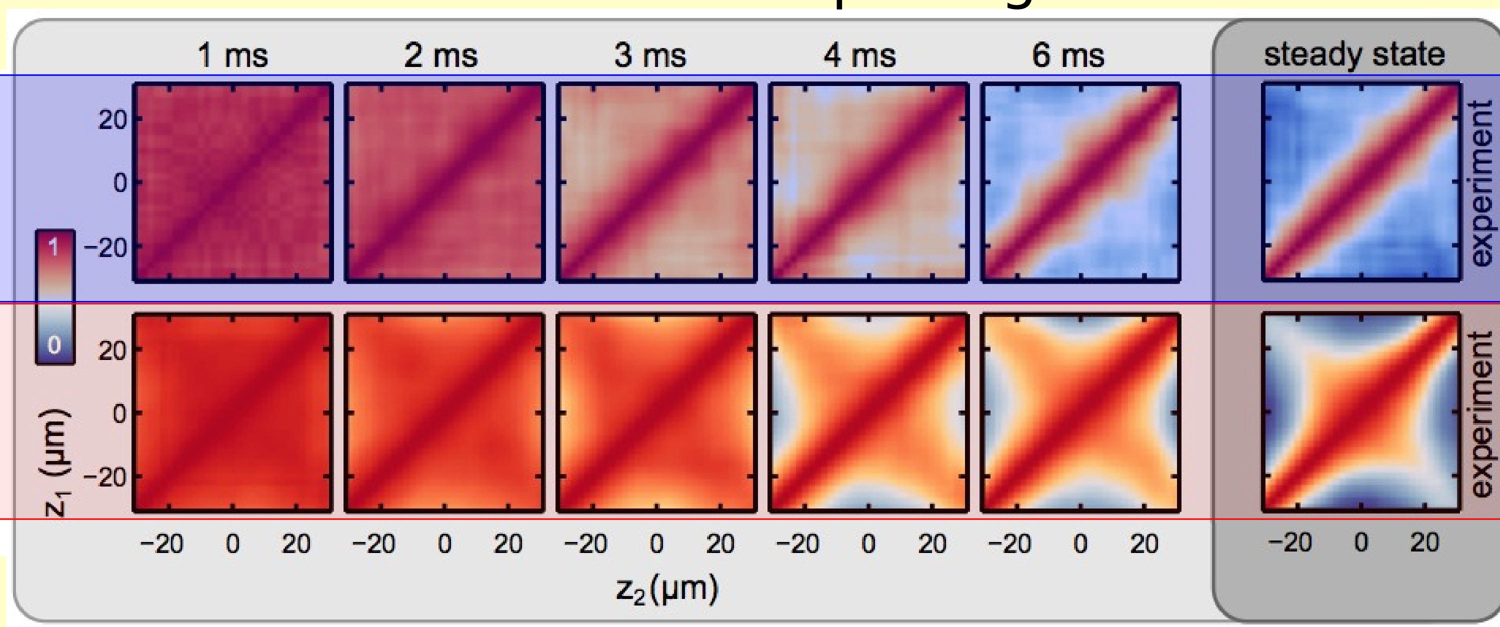
Split a 1D gas non-adiabatically *[Langen et al., Science* ***2015****]*



Two-point correlation function vs. splitting rate:

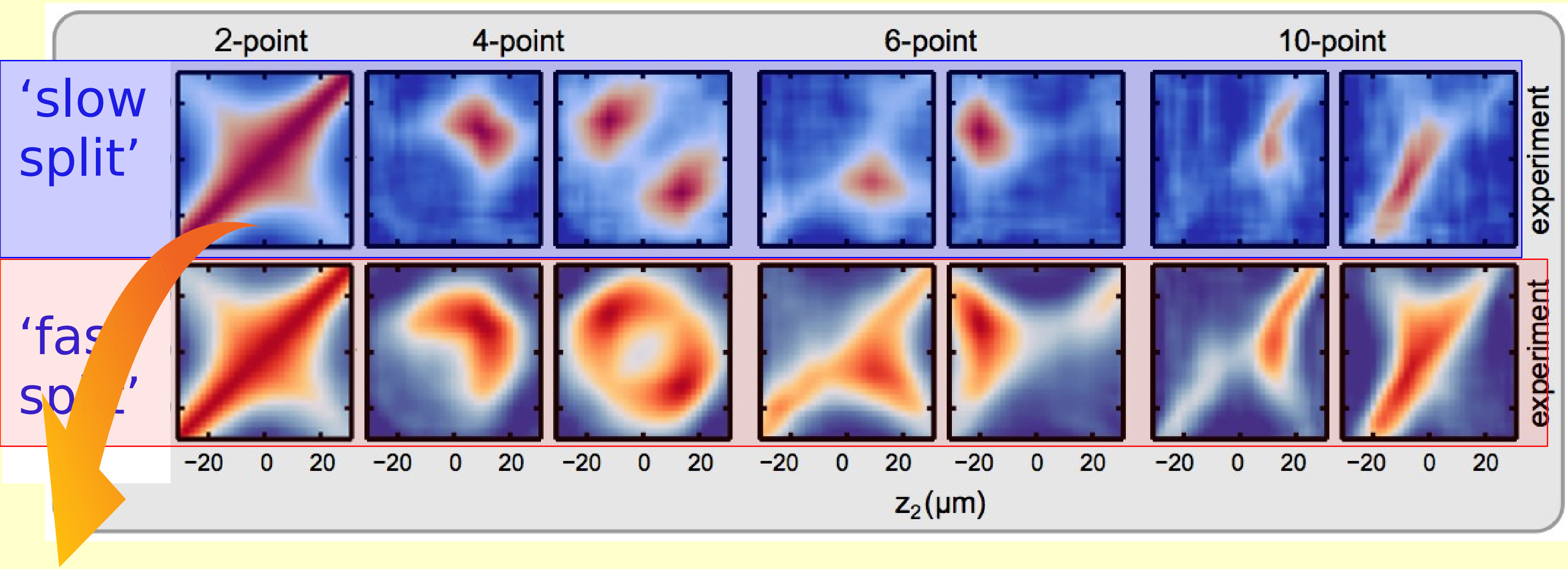
‘slow split’

‘fast split’



Beyond Gibbs

$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



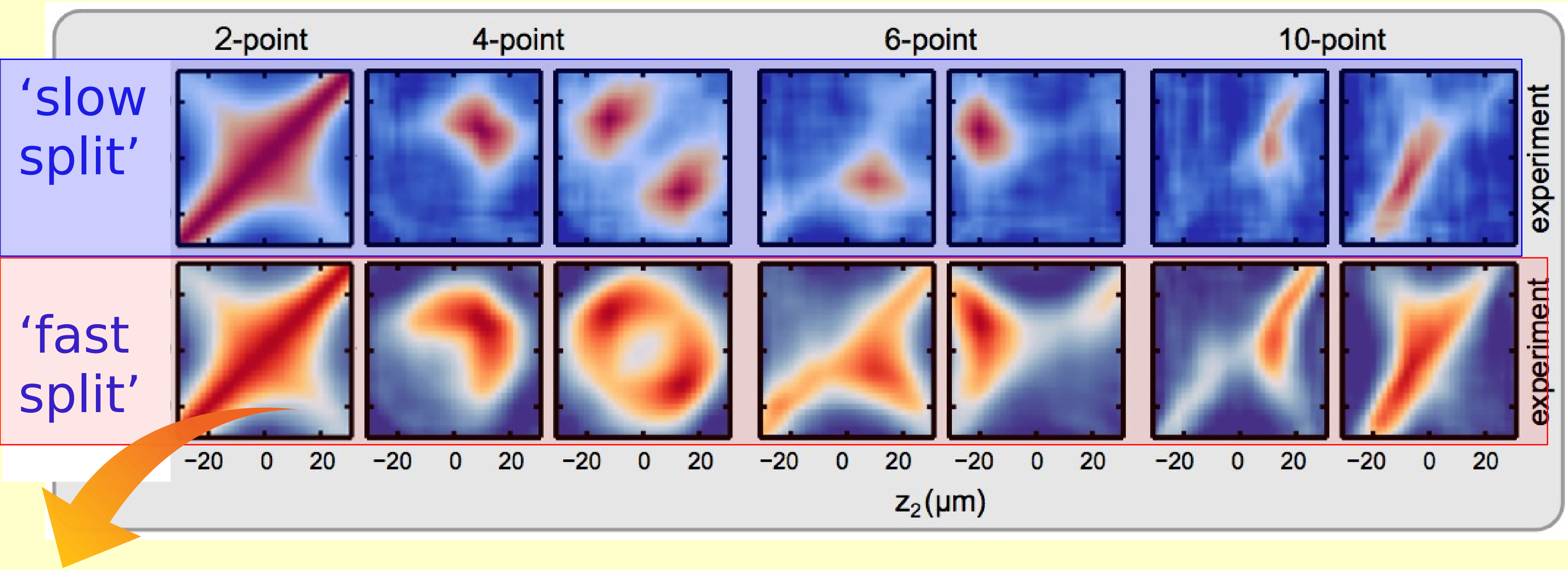
Slow split: Correlations well described with Gibbs distribution with... T_{eff} independent of initial T: Pre-thermalization

$$\rho = \exp\left(-\beta_{\text{eff}} H\right)/Z$$

Gring et al., Science 2012; Langen et al., Science 2015

Beyond Gibbs

$$C(z_1, \ldots, z_N) = \langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_N)\Psi_2(z_N)\rangle$$



Fast split: Need up to 10 different 'temperatures' to fit!
=> 'Memory of conserved quantities': generalized Gibbs ensemble

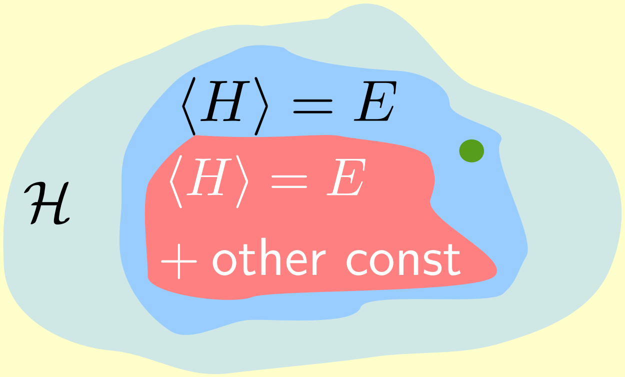
$$\rho_{\mathrm{GGE}} = \exp\left(-\sum \beta_k Q_k\right)/Z$$

Langen et al., Science 2015

QFRs for GGEs

“Can we derive QFRs if initial state = GGE?”

$$\rho_\beta = \frac{e^{-\beta H}}{Z} \to \rho_{\mathsf{GGE}} = \frac{e^{-\beta H - \sum_k \beta_k Q_k}}{Z}\,, \quad [Q_k, H] = 0 \;\forall k$$



E.T Jaynes Phys. Rev. (1957)
M. Rigol et al. PRL (2007)
Guryanova et al.Nat. Comms. (2016)
Halpern et al., Nat. Comms. (2016)

QFRs for GGEs: Analytical

"Can we derive QFRs if initial state = GGE?"

$$\rho_\beta = \frac{e^{-\beta H}}{Z} \to \rho_{\text{GGE}} = \frac{e^{-\beta H - \sum_k \beta_k Q_k}}{Z}, \quad [Q_k, H] = 0 \ \forall k$$

Generalized Q. Jarzynski equality

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

$$\beta W := \beta(\langle H' \rangle_f - \langle H_i \rangle_i) + \sum \beta_k (\langle Q'_k \rangle_f - \langle Q_k \rangle_i)$$

Generalized Tasaki-Crooks relation

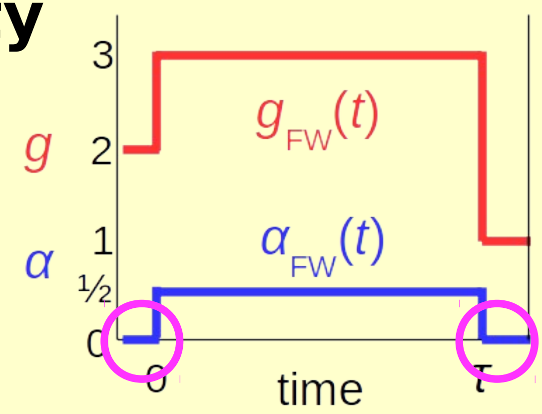
$$e^{-\beta \Delta \mathcal{F}} = e^{-\beta W} \frac{\mathcal{P}_f(W)}{\mathcal{P}_b(-W)} \overset{(*)}{=} e^{-\beta_k w_k} \frac{P_f^{(k)}(w_k)}{P_b^{(k)}(-w_k)} \overset{(**)}{=} e^{-\beta w} \frac{P_f(w)}{P_b(-w)}$$

QFRs for GGEs: Numerics

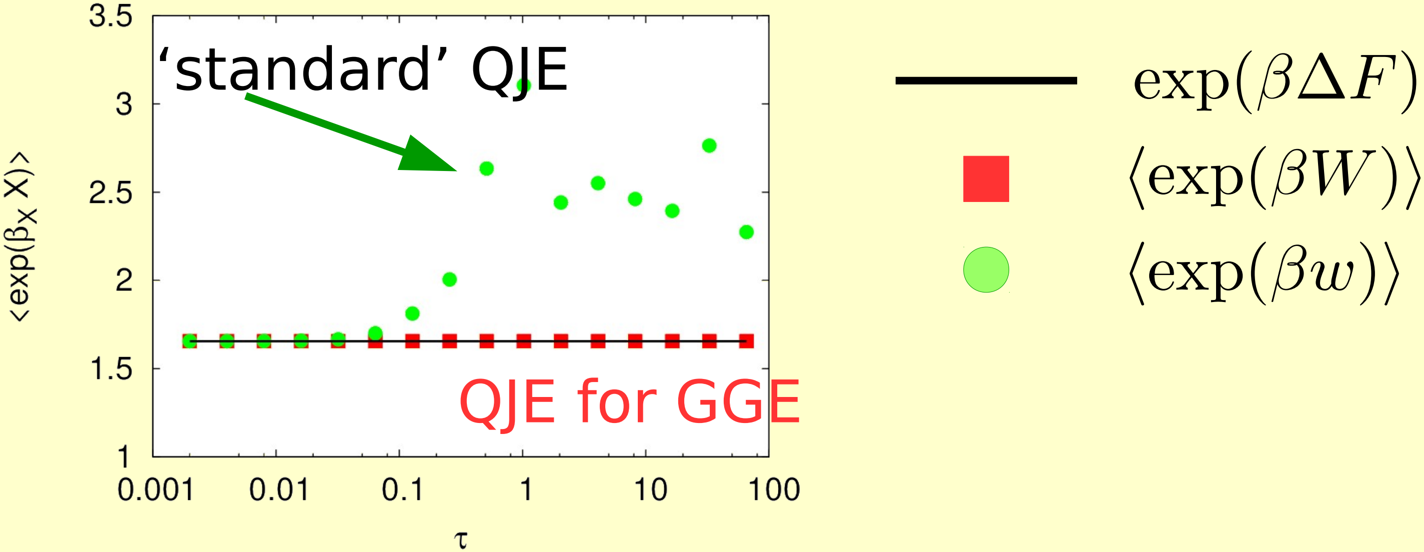
Generalized Q. Jarzynski equality

$$\langle e^{\beta W} \rangle = e^{\beta \Delta F}$$

$$\beta W := \beta w + \beta_Q (\langle Q' \rangle_f - \langle Q \rangle_i)$$



Testing ground: Dicke model



$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_Q = -0.1)$$

QFRs for GGEs: Numerics

Generalized Q. Jarzynski equality

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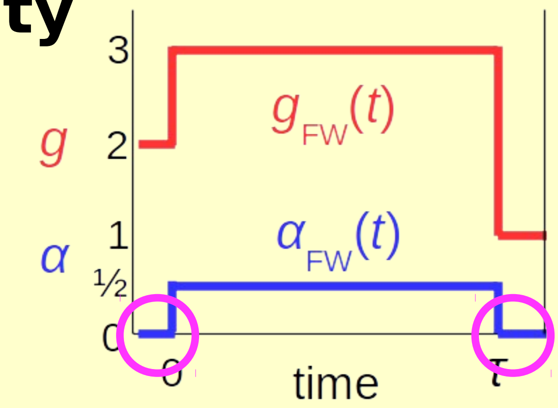
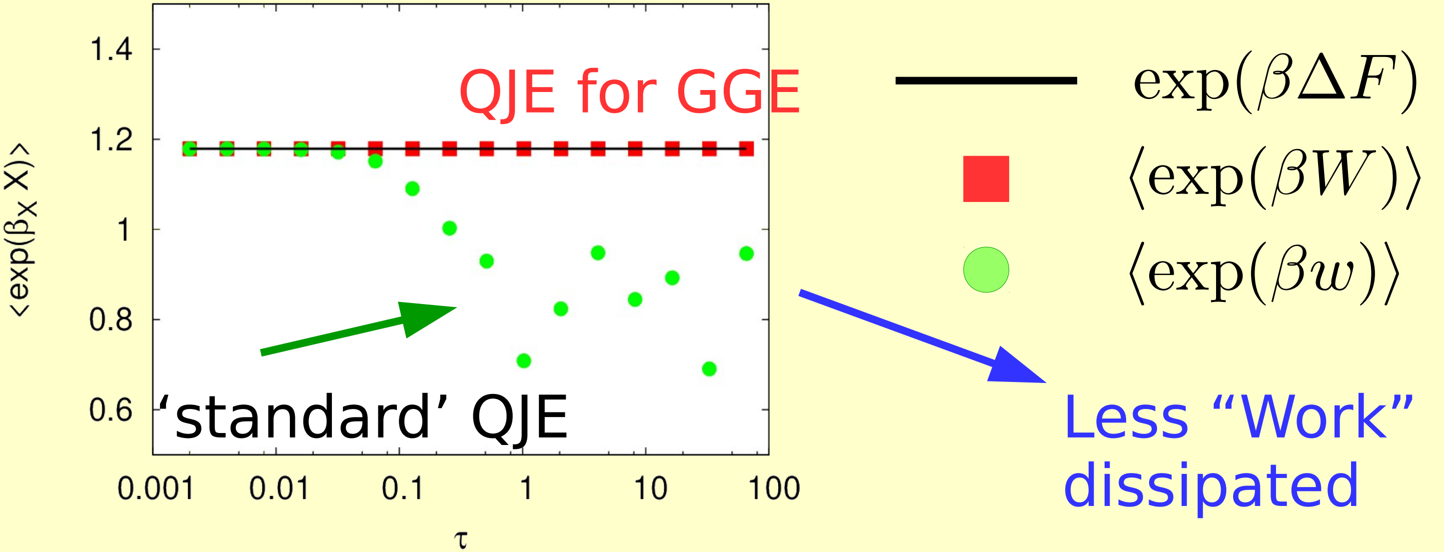


Testing ground: Dicke model



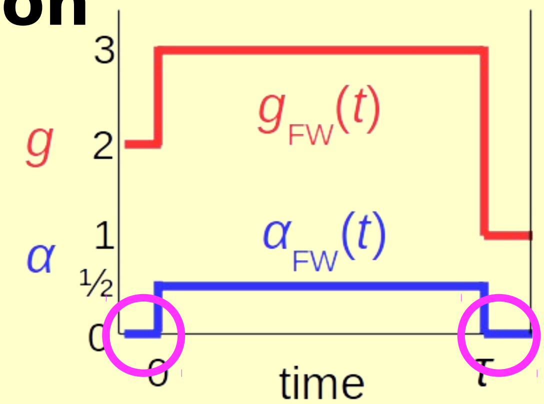
$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_Q = +0.3)$$

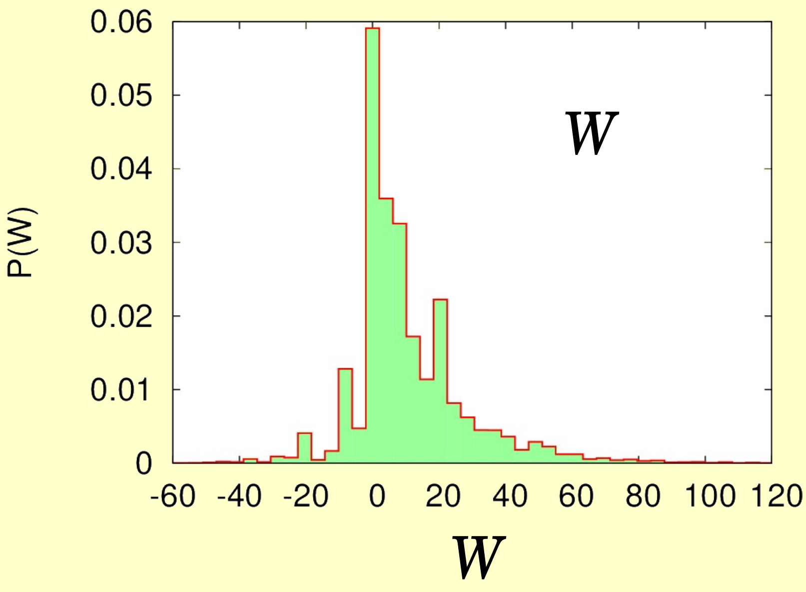
QFRs for GGEs: Numerics

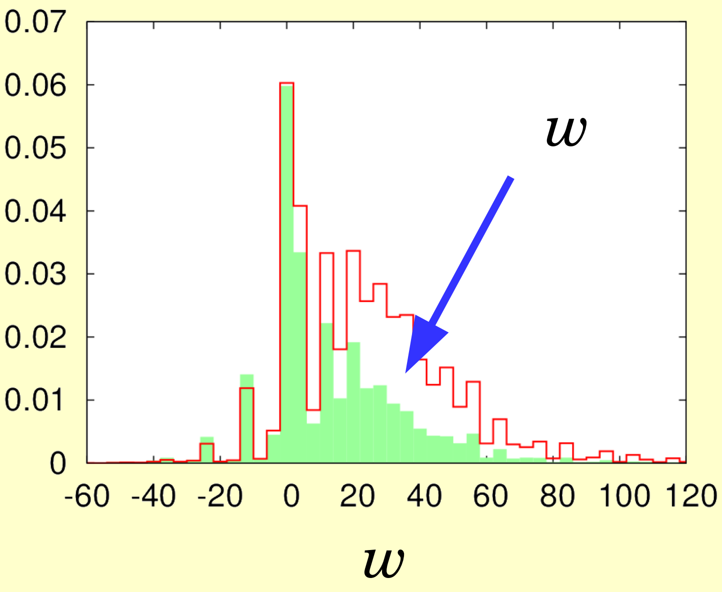
Generalized Tasaki-Crooks relation

$$\mathcal{P}_f(W) = e^{\beta(W-\Delta\mathcal{F})}\mathcal{P}_b(-W)$$

$$(*)\ P_f(w) = e^{\beta(w-\Delta F)}P_b(-w)$$







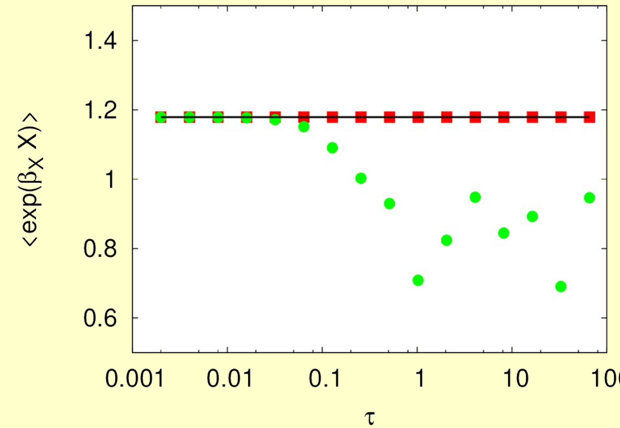
$$\rho_{ini} = \rho_{GGE}(\beta = 0.1, \beta_Q = -0.1)\ , \quad \tau = 1.024$$

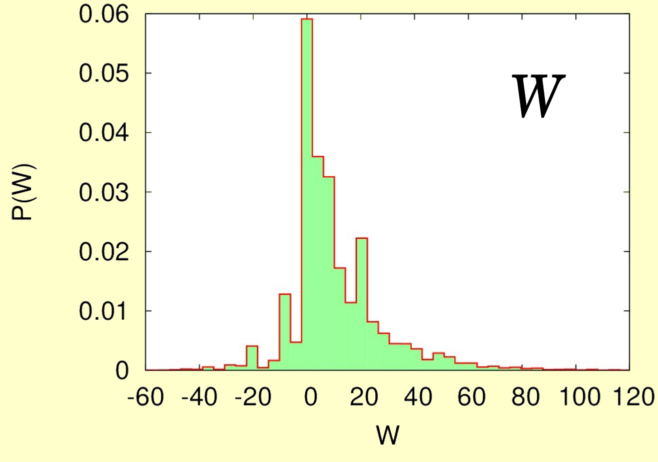
Summary

- Deduced generalized QFRs for processes starting from a GGE
  - Extension of q. thermo. ideas for GGE to non-equilibrium dynamics
- Numerically verified with simple model
  - Possibility of reduced ‘work’ dissipation
- Experimental proposal with trapped ions
- Outlook:
  - How to prepare GGE?
  - Limitations of ETH?

$$\mathcal{P}_f(W) = e^{\beta(W-\Delta\mathcal{F})}\mathcal{P}_b(-W)$$

$$\langle e^{\beta W}\rangle = e^{\beta\Delta F}$$





Vielen Dank!

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