Coherent backscattering in the Fock space of ultracold bosonic atoms

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Outline

- Thermalization and localization in bosonic many-body systems
- Weak localization and coherent backscattering
- Coherent backscattering in Fock space
- Verification with ultracold atoms
- Conclusion
Interacting bosonic many-body systems

We consider a one-dimensional disordered Bose-Hubbard ring:

\[ \hat{H} = \sum_{l=1}^{L} \left[ E_{l} \hat{b}_{l}^{\dagger} \hat{b}_{l} - J \left( \hat{b}_{l}^{\dagger} \hat{b}_{l-1} + \hat{b}_{l-1}^{\dagger} \hat{b}_{l} \right) + \frac{U}{2} \hat{b}_{l}^{\dagger} \hat{b}_{l}^{\dagger} \hat{b}_{l} \hat{b}_{l} \right] \]
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Regime of eigenstate thermalization:
J.M. Deutsch, PRA 43, 2046 (1991); M. Srednicki, PRE 50, 888 (1994);

→ all many-body eigenstates are equidistributed within the classical phase space of the system

→ maximal entanglement entropy for subsystems
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→ Bose-Einstein distribution of the population of individual one-body eigenmodes

PS & D. Shepelyansky, PRE 93, 012126 (2016)
Regime of many-body localization:
I.V. Gornyi, A.D. Mirlin & D.G. Polyakov, PRL 95, 206603 (2005);
V. Oganesyan & D.A. Huse, PRB 75, 155111 (2007); …

→ absence of eigenstate thermalization

→ logarithmic growth of entanglement entropy
  M. Žnidarič, T. Prosen & P. Prelovšek, PRB 77, 064426 (2008);
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→ experimental verification with ultracold bosonic atoms in 2D optical lattices
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→ weak localization in the Fock space of many-body systems?
Coherent backscattering in disordered systems

constructive wave interference between reflected classical paths and their time-reversed counterparts
Coherent backscattering in disordered systems

→ constructive wave interference between reflected classical paths and their time-reversed counterparts

coherent backscattering of laser light in disordered media

M. P. Van Albada and A. Lagendijk, PRL 55, 2692 (1985)
P.-E. Wolf and G. Maret, PRL 55, 2696 (1985)
Coherent backscattering in disordered systems

→ constructive wave interference between reflected classical paths and their time-reversed counterparts

- coherent backscattering of laser light in disordered media
- magnetoresistance within two-dimensional electron gases

A. M. Chang *et al.*, PRL 73, 2111 (1994)
Coherent backscattering in disordered systems

→ constructive wave interference between reflected classical paths and their time-reversed counterparts

- coherent backscattering of laser light in disordered media
- magnetoresistance within two-dimensional electron gases
- coherent backscattering of ultracold atoms in 2D disorder

F. Jendrzejewski et al., PRL 109, 195302 (2012)
(see also N. Cherroret et al., PRA 85, 011604(R) (2012))
Many-body CBS in disordered Bose-Hubbard rings

\[ \hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{b}_l^\dagger \hat{b}_l - J \left( \hat{b}_l^\dagger \hat{b}_{l-1} + \hat{b}_{l-1}^\dagger \hat{b}_l \right) + \frac{U}{2} \hat{b}_l^\dagger \hat{b}_l^\dagger \hat{b}_l \hat{b}_l \right] \]
Many-body CBS in disordered Bose-Hubbard rings

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\hat{H} = \sum_{l=1}^{L} \left[ E_l \hat{b}_l^{\dagger} \hat{b}_l - J \left( \hat{b}_l^{\dagger} \hat{b}_{l-1} + \hat{b}_{l-1}^{\dagger} \hat{b}_l \right) + \frac{U}{2} \hat{b}_l^{\dagger} \hat{b}_l \hat{b}_l^{\dagger} \hat{b}_l \right]
\]

Start with a given initial Fock state, e.g. \( |n^i_i\rangle = |3, 3, 2, 3, 4, 2\rangle \), and determine the probability to reach a given final Fock state \( |n^f_i\rangle = |n^f_1, \ldots, n^f_L\rangle \) at time \( t \) under some disorder average.

Coherent backscattering in Fock space

Semiclassical van Vleck-Gutzwiller approach

Represent the quantum transition amplitude

\[
\langle n^f | \hat{U} | n^i \rangle \equiv \langle n^f | \exp[-\frac{i}{\hbar} t \hat{H}] | n^i \rangle = \sum_{\gamma} A_{\gamma} e^{i R_{\gamma}/\hbar}
\]

in terms of classical (Gross-Pitaevskii) trajectories \( \gamma \) satisfying

\[
i\hbar \frac{\partial}{\partial t} \psi_l(t) = E_l \psi_l(t) - J [\psi_{l+1}(t) + \psi_{l-1}(t)] + U (|\psi_l(t)|^2 - 1) \psi_l(t)
\]

with \( \psi_l(0) = \sqrt{n_l^i + 0.5} e^{i \theta_l^i} \) and \( \psi_l(t) = \sqrt{n_l^f + 0.5} e^{i \theta_l^f} \)

for all \( l = 1, \ldots, L \) with some arbitrary phases \( 0 \leq \theta_l^i/\theta_l^f < 2\pi \)

\( R_{\gamma} = \) classical action of the trajectory \( \gamma \)

\( A_{\gamma} = \) stability amplitude (related to Lyapunov exponent) of \( \gamma \)
Semiclassical van Vleck-Gutzwiller approach

Represent the quantum transition amplitude
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for all \( l = 1, \ldots, L \) with some arbitrary phases \( 0 \leq \theta^i_l / \theta^f_l < 2\pi \)

Average detection probability of the Fock state \( |n^f \rangle \):
\[
|\langle n^f | \hat{U} | n^i \rangle|^2 = \sum_{\gamma, \gamma'} A_\gamma A_{\gamma'} e^{i(R_\gamma - R_{\gamma'}) / \hbar}
\]
\[
= 0 \quad \text{if} \quad R_\gamma \neq R_{\gamma'}
\]
Semiclassical van Vleck-Gutzwiller approach

Represent the quantum transition amplitude

\[
\langle n^f | \hat{U} | n^i \rangle \equiv \langle n^f | \exp[-i\frac{\hbar}{\tau} \hat{H}] | n^i \rangle = \sum_{\gamma} A_{\gamma} e^{iR_{\gamma}/\hbar}
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in terms of classical (Gross-Pitaevskii) trajectories \( \gamma \) satisfying

\[
i\hbar \frac{\partial}{\partial \tau} \psi_l(t) = E_l \psi_l(t) - J [\psi_{l+1}(t) + \psi_{l-1}(t)] + U (|\psi_l(t)|^2 - 1) \psi_l(t)
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Average detection probability of the Fock state \( |n^f\rangle \):

\[
|\langle n^f | \hat{U} | n^i \rangle|^2 = \sum_{\gamma} |A_{\gamma}|^2 \text{ if } n^f \neq n^i
\]

\[
|\langle n^i | \hat{U} | n^i \rangle|^2 = 2 \sum_{\gamma} |A_{\gamma}|^2 \text{ due to CBS}
\]
Comparison with numerical data

Initial state:

Detection probability

\[
\langle n_f | \hat{U} | n_i \rangle \big|_{\text{classical}}^2 = \sum_{\gamma} |A_\gamma|^2
\]

\[
= \int_0^{2\pi} \frac{d\theta_2}{2\pi} \cdots \int_0^{2\pi} \frac{d\theta_L}{2\pi} \prod_{l=2}^{L} \delta \left( n_f^l + 0.5 - |\psi_l(t; n_1^i, 0, n_2^i, \theta_2^i \ldots n_L^i, \theta_L^i)|^2 \right)
\]

in the presence of classical chaos and ergodicity


Coherent backscattering in Fock space

Comparison with numerical data

Initial state:

Detection probability

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\langle n^f | \hat{U} | n^i \rangle \rvert^2_{\text{classical}} = \sum_\gamma |A_\gamma|^2 \\
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\]

\[\rightarrow \text{ significant deviation from quantum ergodicity}\]

\[\rightarrow \text{ observable in a cold-atom experiment?}\]
Proposal for a few-body CBS experiment

Consider an isolated 2D sheet of a 3D optical lattice within which you isolate a single plaquette (e.g. by means of a focused red-detuned laser beam)
Proposal for a few-body CBS experiment

Experimental procedure:

1. Load the plaquette with a well-defined number of bosonic atoms in the deep Mott-insulator regime

2. Add some disorder (by means of an optical speckle field) and/or randomly displace the focus of the red-detuned laser beam
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3. Switch on the inter-site hopping and let the atoms move . . .
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4. Quench back to the Mott regime after a given evolution time and detect the atomic population on each site

W. Bakr et al., Nature 462, 74 (2009)
(→ beyond binary populations)
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(→ beyond binary populations)
5. Repeat the experiment with the same initial state but for a different disorder configuration
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Proposal for a few-body CBS experiment

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Coherent backscattering in Fock space

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$.
Proposal for a few-body CBS experiment

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200 measurements

Coherent backscattering in Fock space

Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$

![Graph showing final state probabilities and a diagram of a Fock space]
Proposal for a few-body CBS experiment

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Coherent backscattering in Fock space

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some 1000 repetitions of the experiments are required in order to see the signature of coherent backscattering

Coherent backscattering in Fock space

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Coherent backscattering in Fock space

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Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar / J$

$$J e^{i\varphi} \rightarrow \text{induce a synthetic gauge field in order to break time-reversal invariance}$$

N. Goldman et al., PRL 105, 255302 (2010)
Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$

$\psi = 0.01\pi$

→ induce a synthetic gauge field in order to break time-reversal invariance

N. Goldman et al., PRL 105, 255302 (2010)
Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$

![Graph showing probabilities and final states](graph.png)

$\rightarrow$ induce a synthetic gauge field in order to break time-reversal invariance

N. Goldman et al., PRL 105, 255302 (2010)
Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10\hbar/J$

\[ J e^{i\varphi} \quad J e^{-i\varphi} \]

induce a synthetic gauge field in order to break time-reversal invariance

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\[ \text{final state} \]

\[ \text{probability} \]

\[ \text{single-site population} \]

\[ \varphi = 0.1\pi \]

\[ \text{slightly inhomogeneous single-site populations} \]

Coherent backscattering in Fock space

Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 10J/\hbar$

$\rightarrow$ slightly inhomogeneous single-site populations due to limited classical ergodicity at $t = 10J/\hbar$

Coherent backscattering in Fock space
Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = J$, $0 < E_l < 4J$ and the evolution time $t = 100\hbar/J$

$$
\begin{array}{c}
\text{final state} \\
0122 \\
0212 \\
0221 \\
1022 \\
1202 \\
1220 \\
12220 \\
2012 \\
2021 \\
2102 \\
2120 \\
2201 \\
2210 \\
\end{array}
$$

- $\times$ exact probabilities
- $\diamond$ classical prediction
- $\varphi = 0.1\pi$

→ slightly inhomogeneous single-site populations
due to limited classical ergodicity at $t = 10J/\hbar$ and
due to quantum (few-body) localization at $t = 100\hbar/J > \tau_H$

Coherent backscattering in Fock space

Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = 4J$, $0 < E_l < 10J$ and the evolution time $t = 10\hbar/J$

![Graph showing single-site population as a function of site index.

Coherent backscattering in Fock space

Proposal for a few-body CBS experiment

Numerical simulation of the experiment for the parameters $U = 4J$, $0 < E_l < 10J$ and the evolution time $t = 100\hbar/J$

Coherent backscattering in Fock space

Coherent backscattering in Fock space

- privileges, on average, the initial Fock state as compared to other states with comparable energy;
- significantly affects quantum ergodicity in finite systems, in a regime where the classical dynamics is fully ergodic;
- can be experimentally detected using ultracold atoms in optical lattices;
- relies on time-reversal invariance and can therefore be switched off with a synthetic gauge field;
- can be generalized to fermions T. Engl et al., arXiv:1409.5684;
- may act as a precursor to “strong” many-body localization? D. Vollhardt & P. Wölfle, PRL 48, 699 (1982)