FACULTY OF PHYSICS

UNIVERSITÄT DUISBURG ESSEN

*Open-Minded*

# Periodic Orbits in Quantum Many-Body Systems


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with Maram Akila, Boris Gutkin, Petr Braun, Thomas Guhr


Quantum-Classical Transition in Many-Body Systems: Indistinguishability, Interference and Interactions,

MPI Dresden, 16. February 2017

# Outline

- Semiclassical connection for the short-time behaviour of a quantum many-body system
- Connection established for experimentally and theoretically topical system of a spin chain
- Establish a quantum evolution of reduced dimension
- Impact of collective dynamics on the quantum spectrum

# Motivation

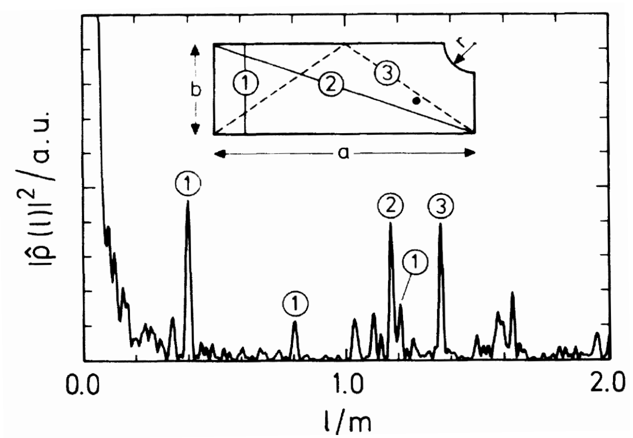
Semiclassical connection for a single particle:

Gutzwiller trace formula:

$$\rho(E) = \underbrace{\sum_n \delta(E - E_n)}_{\text{quantum level density}} \sim \overline{\rho}(E) + \underbrace{\sum_\gamma A_\gamma e^{iS_\gamma/\hbar}}_{\text{sum over classical orbits with action } S_\gamma \text{ and stability coefficient } A_\gamma}$$

Single-particle systems:

- Billiards: $S_\gamma = \hbar k l_\gamma$: Fourier-transform with respect to $k$:
  Spectrum of the classical orbits $\delta(l - l_\gamma)$
  Stöckmann, Stein (1990)




- Kicked top: Fourier-transform with respect to spin quantum number $s$ Kuś, Haake, Delande (1993)

# Kicked Top

Hamiltonian:

$$\hat{H}(t) = \frac{4J\left(\hat{s}_z\right)^2}{(s+1/2)^2} + \frac{2\mathbf{b}\cdot\hat{\mathbf{s}}}{(s+1/2)} \sum_{n=-\infty}^{\infty} \delta(t-n)$$

Kick part of kicked top:

**Quantum**

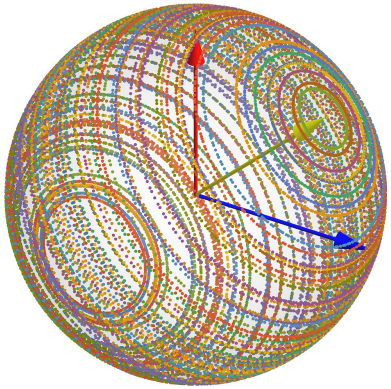
$$\hat{H}_K = \frac{2\mathbf{b}\cdot\hat{\mathbf{s}}}{s+1/2}$$

$$\hat{U}_K = \exp\left(-i(s+1/2)\hat{H}_K\right)$$

with

- magnetic field $\mathbf{b} = (b^x, 0, b^z)$
- spin vector $\hat{\mathbf{s}} = (\hat{s}_x, \hat{s}_y, \hat{s}_z)$
- spin quantum number $s$

**Classical**



$$\mathbf{n}(t+1) = R_{\mathbf{b}}(2|\mathbf{b}|)\mathbf{n}(t)$$

- unit vector $\mathbf{n}(t)$
- rotation around $\mathbf{b}$ with angle $2|\mathbf{b}|$: $R_{\mathbf{b}}(2|\mathbf{b}|)$

# Kicked Top

“Ising” part of kicked top:

**Quantum**

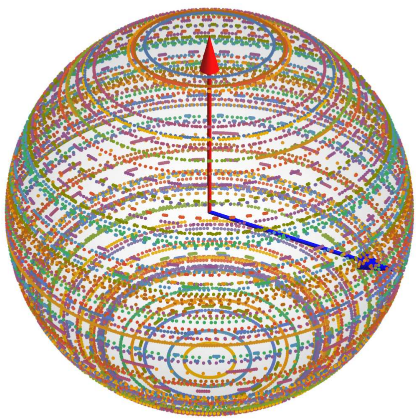
$$\hat{H}_I = \frac{4J(\hat{s}_z)^2}{(s+1/2)^2}$$

$$\hat{U}_I = \exp\left(-i(s+1/2)\hat{H}_I\right)$$

with

- “Ising” coupling $J$
- spin vector $\hat{\mathrm{s}} = (\hat{s}_x, \hat{s}_y, \hat{s}_z)$
- spin quantum number $s$

**Classical**



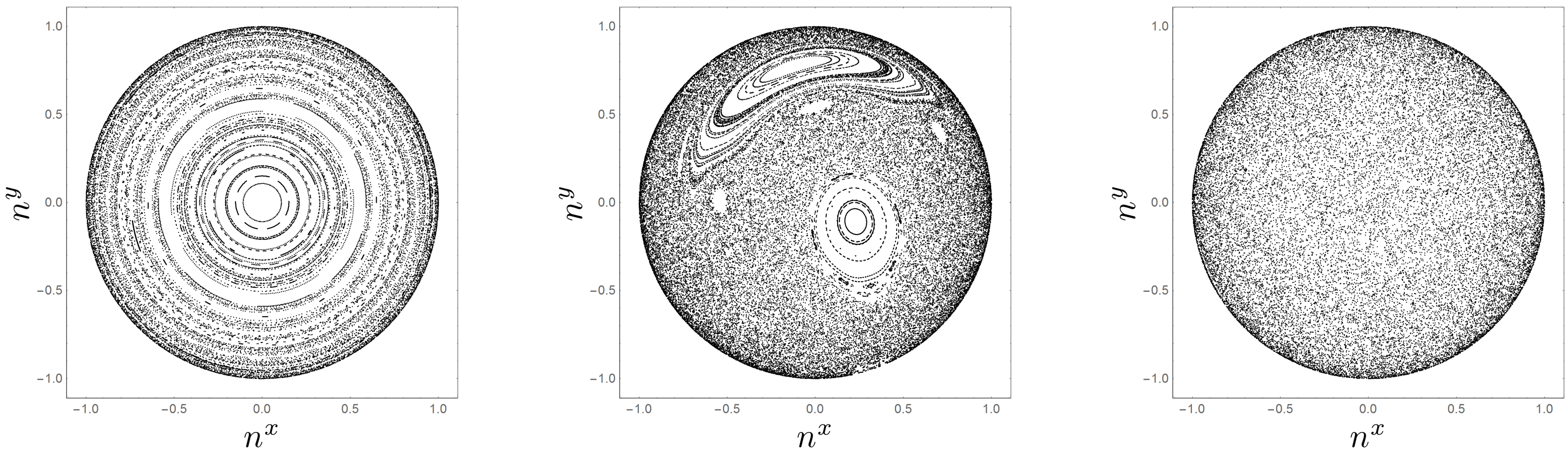
$$\mathbf{n}(t+1) = R_{\mathbf{z}}(8Jn^z)\mathbf{n}(t)$$

- unit vector $\mathbf{n}(t)$
- rotation around $\mathbf{z}$ with angle $8Jn^z$: $R_{\mathbf{z}}(8Jn^z)$

# Kicked Top - Classical Dynamics

Combination of kick and Ising part: $\hat{U} = \hat{U}_I \hat{U}_K$

Parameters: $\tan\beta = b^x/b^z$, $|\mathbf{b}| = 1.27$, $J = 0.7$




$\beta = 0$ $\beta = 0.2$ $\beta = \pi/4$

**regular** **mixed** **chaotic**

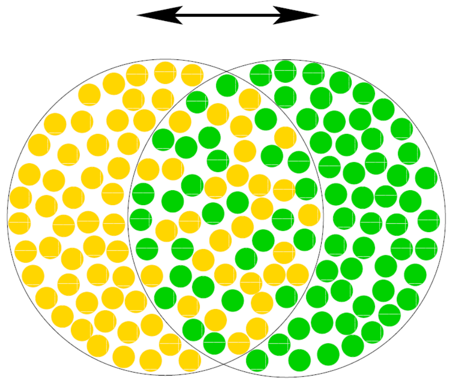
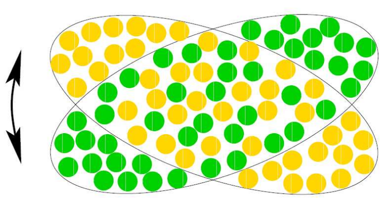
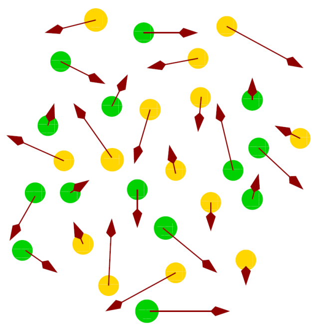
# Motivation

Many-particle system: Two limit parameters particle number $N$ and spin quantum number $s$

# Classical Motion

Many-particle systems: relative motion of particles provides additional degree of freedom

Nuclear physics:

| Coherent (collective) motion | Incoherent single particle motion |
|---|---|
| Giant-Dipole Resonance: Baldwin, Klaiber (1947)  Scissor Mode: Bohle, et al. (1984)  |  |

⇒ Description by effective models

# Previous Studies and Aims

Semiclassics for many-particle systems:

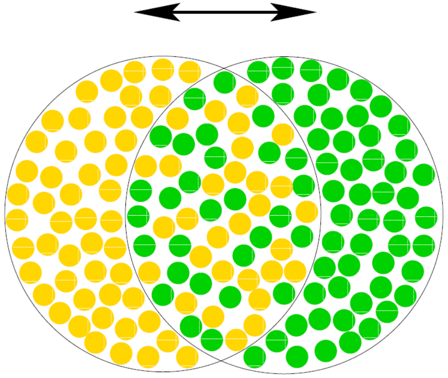
- Propagator, trace formula for bosonic many-particle systems

  Engl et al. (2014); Engl, Urbina, Richter (2015); Dubertrand, Müller (2016)

- Classical dynamics in spin chains for $s \gg 1$ with variable interaction range Gessner, Bastidas, Brandes, Buchleitner (2016)

Aims:

- Quantum many-body systems: identify classical periodic orbits and their impact on the quantum spectrum
- Consider no effective degrees of freedom: coordinates in a real physical system (kicked spin-chain)
- Identify impact of high energy and short time collective motion



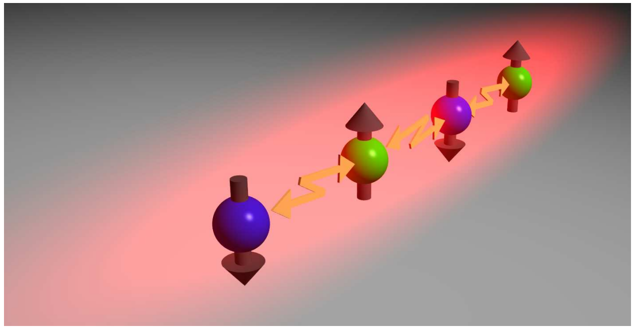
# Kicked Spin Chain

Quantum System: kicked spin chain consisting of $N$ coupled spin-$s$-particles:

$$\hat{H}(t) = \underbrace{\sum_{n=1}^{N} \frac{4J\hat{s}^z_{n+1}\hat{s}^z_n}{(s+1/2)^2}}_{} + \underbrace{\frac{2}{s+1/2}\sum_{n=1}^{N} \mathbf{b}\cdot\hat{\mathbf{s}}_n}_{} \sum_{\tau=-\infty}^{\infty} \delta(t-\tau)$$

next nearest neighbor Ising interaction $\hat{H}_I$

local kick part $\hat{H}_K$



Periodic boundary conditions: $\hat{\mathbf{s}}_{N+1} = \hat{\mathbf{s}}_1$

# Kicked Spin Chain

Motivation:

- Many-particle generalization of the kicked top
- Such systems are in the center of experimental studies:
  - Ytterbium experiments with $s = 5/2$: Immanuel Bloch group (Munich)
  - ion traps with $\approx 10$ spins: Christopher Monroe group (Maryland)
  - ultracold fermionic atoms in optical traps: Selim Jochim group (Heidelberg)
  - Bose-Einstein condensate formed by two-level systems: Markus Oberthaler's talk

# Semiclassics

Identification of many-body periodic orbits:

Trace formula ($s \gg 1$):

$$\mathrm{Tr} U^T = \int da \, \langle a \left| U^T \right| a \rangle \sim \sum_{\gamma(T)} A_\gamma \mathrm{e}^{isS_\gamma}$$

for isolated periodic orbits $\gamma(T)$, stability prefactor $A_\gamma$, classical action $S_\gamma$

for non isolated orbit $A_\gamma$ diverges

Fourier-transform yields action spectrum:

$$\rho(S) \propto \sum_{s=1}^{s_{\mathrm{cut}}} \mathrm{e}^{-isS} \mathrm{Tr} U^T \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$$

Waltner, Braun, Akila, Guhr (2017)

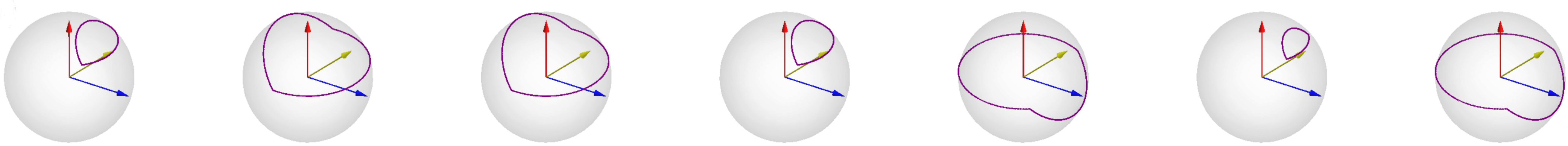
# Classical Dynamics

Classical state represented as unit vector $\mathbf{n}_m(t)$ on the Bloch sphere for spin $m$

Dynamics:

$$\mathbf{n}_m(t+1) = R_{\mathbf{z}}(4J\chi_m)R_{\mathbf{b}}(2|\mathbf{b}|)\mathbf{n}_m(t)$$
$$\chi_m = n^z_{m-1} + n^z_{m+1}$$

Periodic orbits for $T = 1$:



# Classical Dynamics

Problems specific for many-body system:

$$\rho(S) \propto \sum_{s=1}^{s_{\text{cut}}} \mathrm{e}^{-isS} \underbrace{\mathrm{Tr} U^T}_{(2s+1)^N} \sim \underbrace{\sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)}$$

$\times (2s+1)^N-$ dimensional

number of orbits grows grows exponentially with $N$

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$s = 100 \quad N = 1 \quad \rightarrow \quad \text{dim } U^T = 201$

$s = 100 \quad N = 2 \quad \rightarrow \quad \text{dim } U^T = 40401$

$s = 1/2 \quad N = 14 \quad \rightarrow \quad \text{dim } U^T = 16384$

$s = 100 \quad N = 14 \quad \rightarrow \quad \text{dim } U^T = 1.76 \cdot 10^{32}$

# Duality Relation

Aim: Reduce dimension of $U^T$

Time propagation:

$$|\psi(t+1)\rangle = U\,|\psi(t)\rangle$$

Particle propagation:

$$\left|\widetilde{\psi}(n+1)\right\rangle = \widetilde{U}\left|\widetilde{\psi}(n)\right\rangle$$

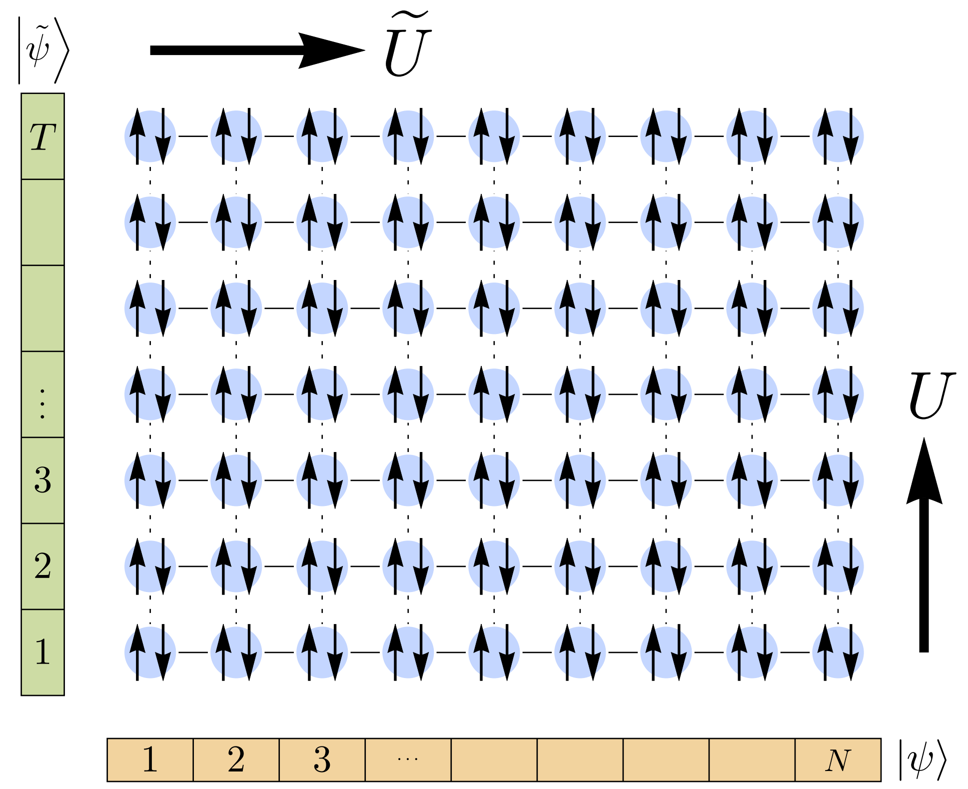
Duality:

$$\mathrm{Tr}U^T = \mathrm{Tr}\widetilde{U}^N$$

Dimensions:

$$\dim U = (2s+1)^N \times (2s+1)^N,$$

$$\dim \widetilde{U} = (2s+1)^T \times (2s+1)^T$$



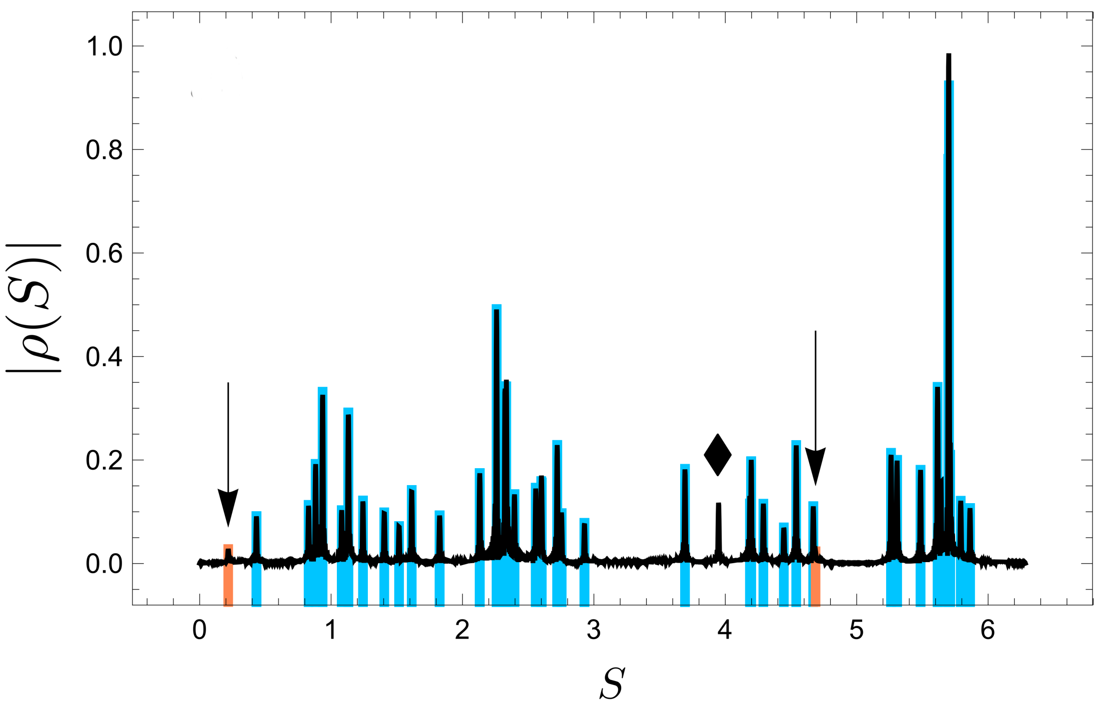

$\Rightarrow$ Analogy between time- and particle-propagation

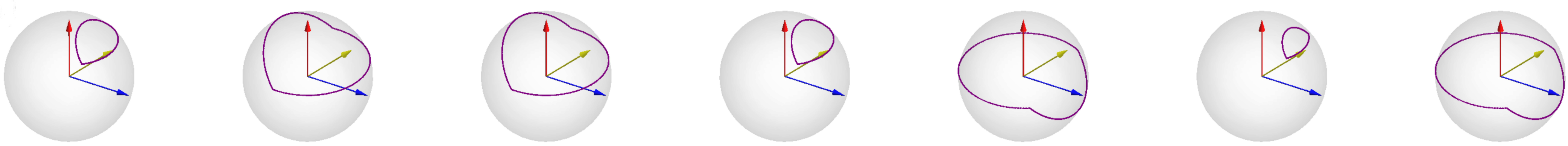
Gutkin, Osipov (2016); Akila, Waltner, Gutkin, Guhr (2016)

# Action Spectrum $T = 1$

Identification of individual peaks in $\rho(S) \sim \sum_{\gamma(T)} A_\gamma \delta(S - S_\gamma)$

Parameters: $N = 7$, $J = 0.75$, $b^x = b^z = 0.9$, $s_{\mathrm{cut}} = 800$



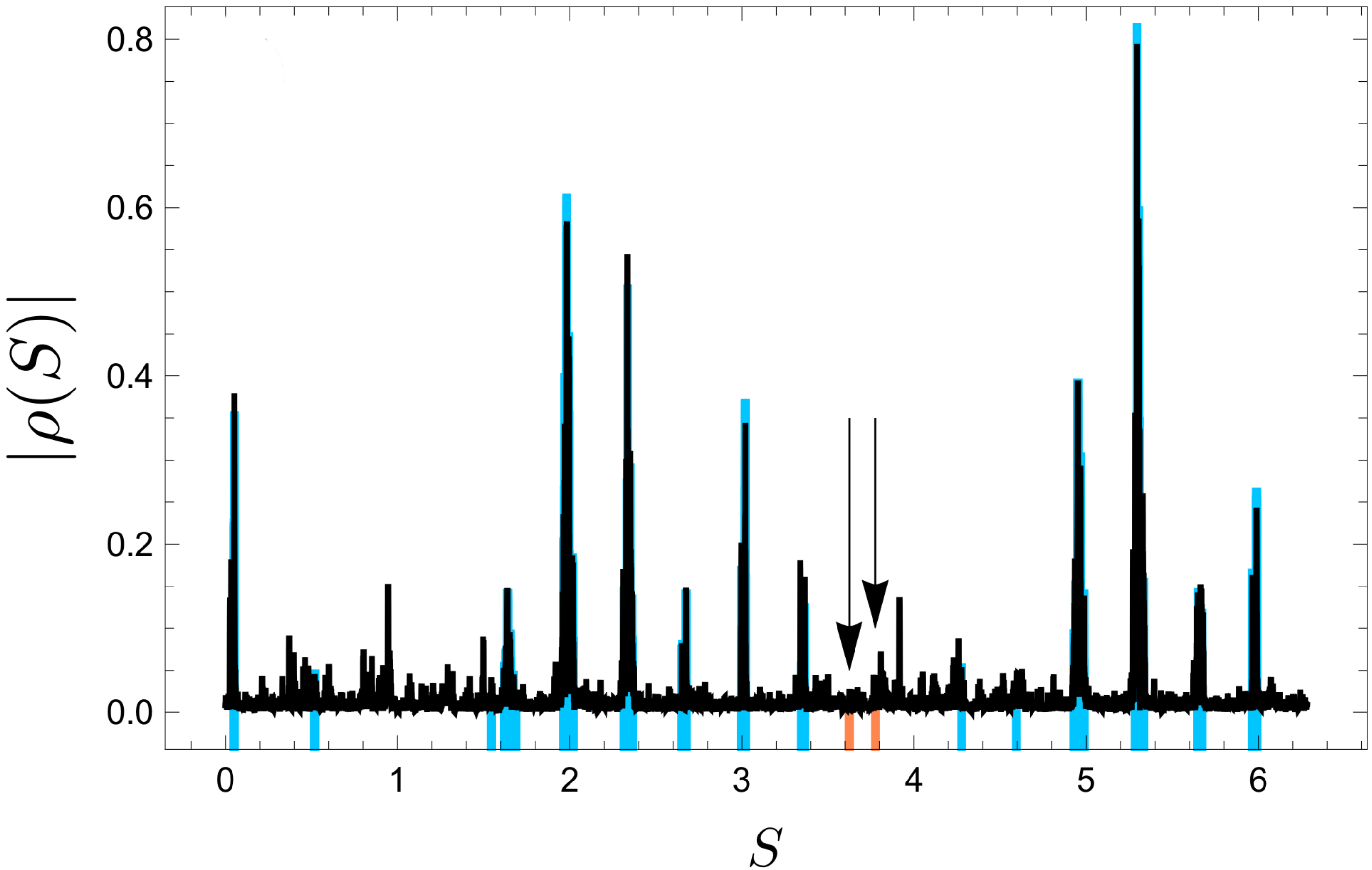



Akila, Waltner, Gutkin, Braun, Guhr (2016)

# Action Spectrum $T = 1$

Larger $N$: number of orbits competes with resolution

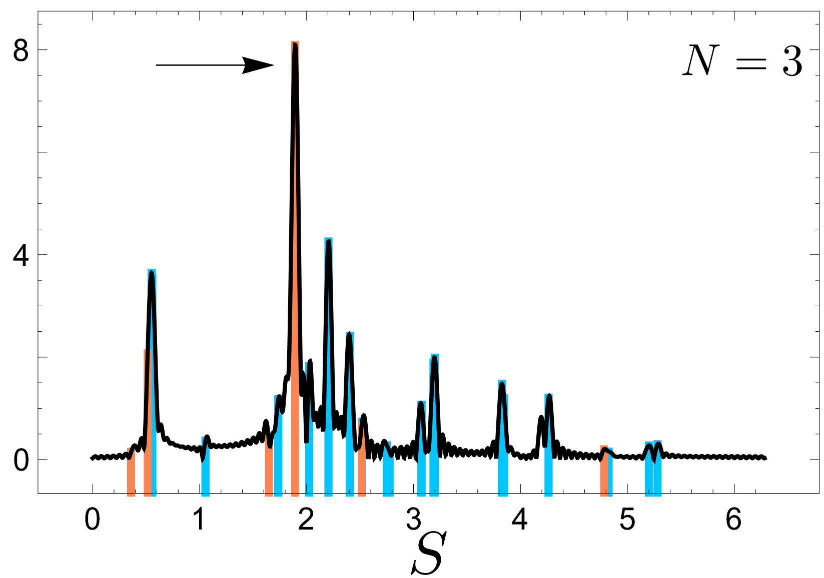
Parameters: $N = 19$, $J = 0.7$, $b^x = b^z = 0.9$, $s_{\text{cut}} = 4650$



# Dominance of Collectivity

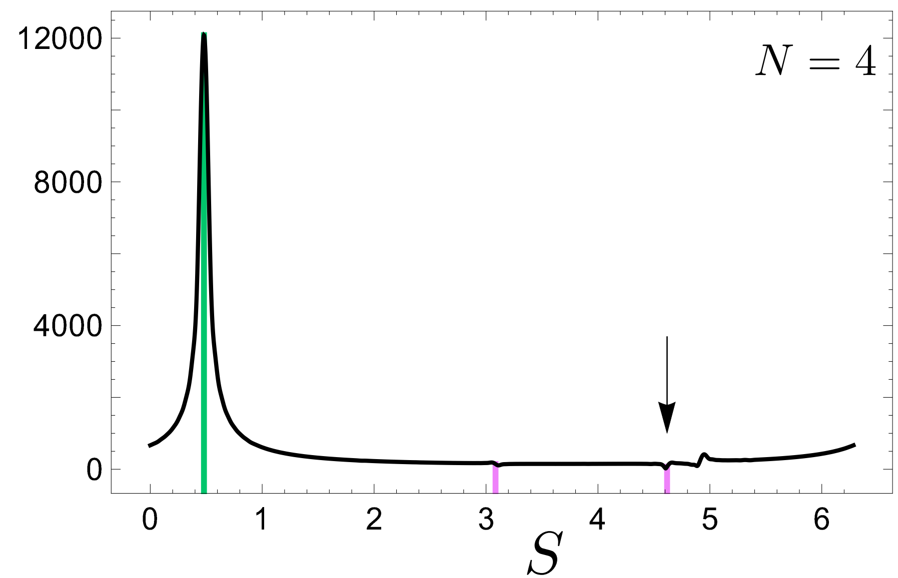
Parameters: $T = 2$, $J = 0.7$, $b^x = b^z = 0.9$, $s_{\mathrm{cut}} = 114$

$N = 3$




Peaks resulting from repetitions
not negligible

$N = 4$

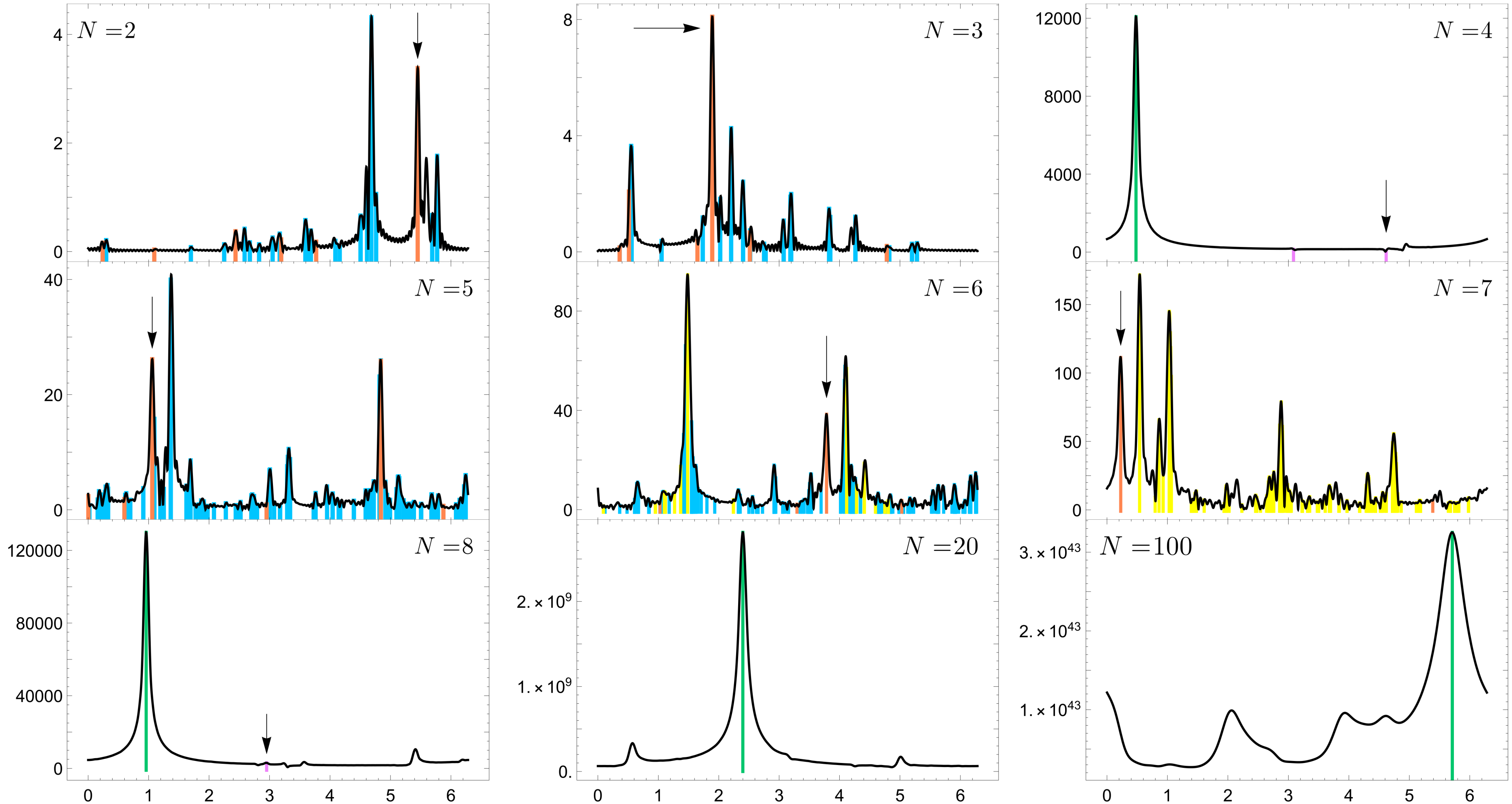



Large peaks dominate
the spectrum

Akila, Waltner, Gutkin, Braun, Guhr (2016)

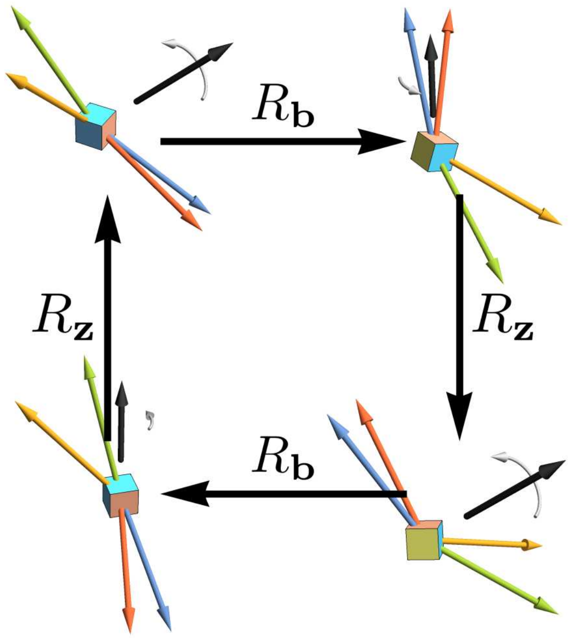
# Dominance of Collectivity

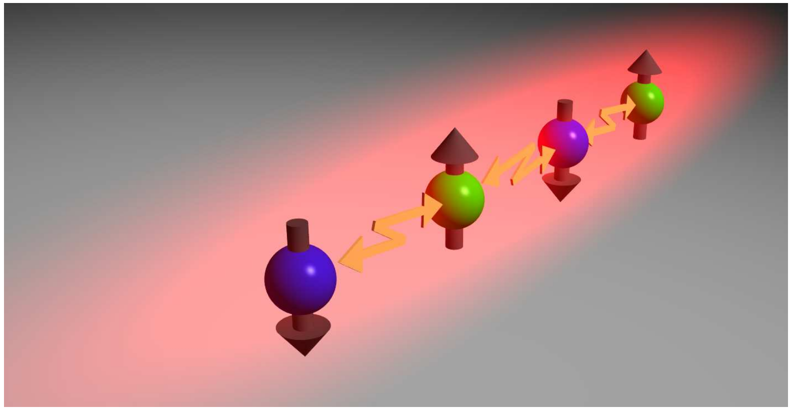
Observation generalizes to $N = 4k$ ($k \in \mathbb{N}$):



# Classical Collective Dynamics

4-dimensional manifold of periodic orbits for $N = 4$:
identified by $\chi_m = \chi$ such that $(R_{\mathbf{z}}(4J\chi)R_{\mathbf{b}}(2|\mathbf{b}|))^2 = \mathbb{1}$



blue spins influenced by
the green and vice versa

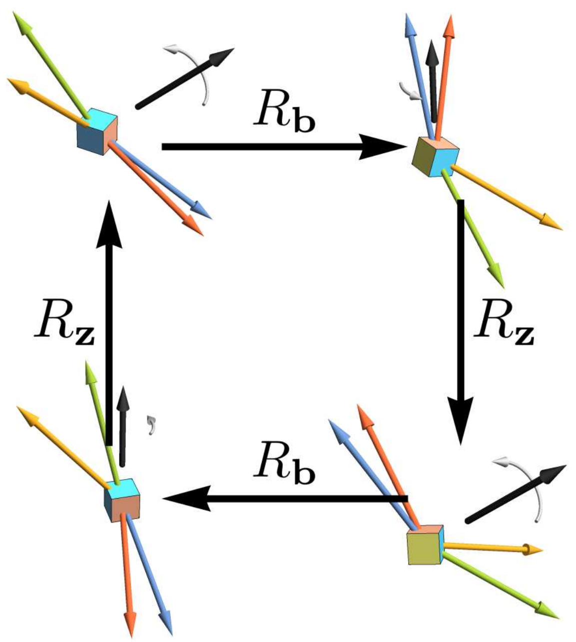
# Scaling

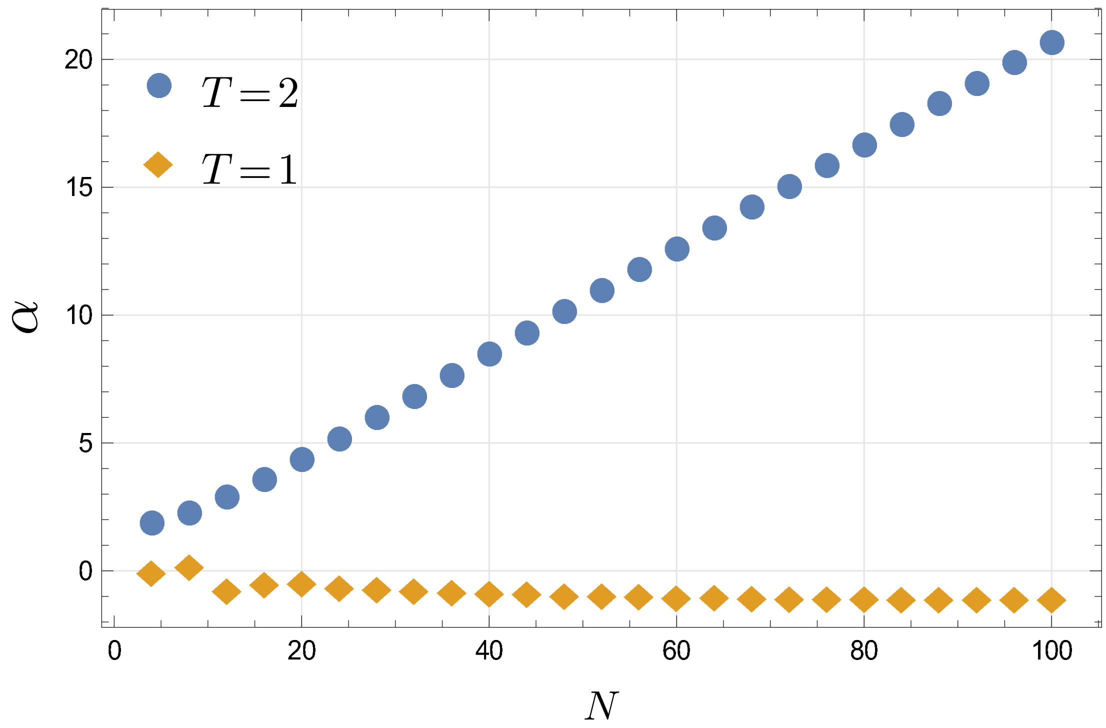
Dominant for large $N$:

$A_\gamma$ diverges for orbits forming manifold

$\Rightarrow$ Study scaling of the largest peak of $\rho(S)$ with $s_{\text{cut}}$:

$$|\rho(S_\gamma)| \propto (s_{\text{cut}})^\alpha, \qquad \alpha \sim N/5$$



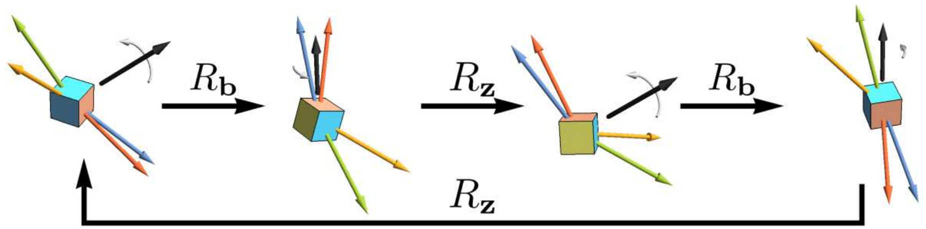

$\rightarrow$ Poster by M. Akila: “Semi-classics in many-body spin chains”

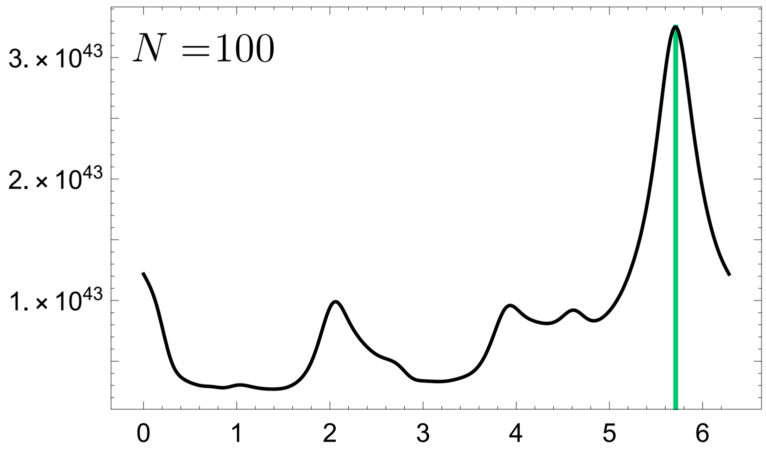
# Conclusions

- Established method to compute classical orbits in quantum many-particle system and identified impact on quantum spectrum for a spin chain

- Duality reduces dimension of $U^T$ by an exchange of $N$ and $T$

- Collective dynamics dominates the quantum spectrum



# Conclusions

- Established method to compute classical orbits in quantum many-particle system and identify impact on quantum spectrum for a spin chain
- Duality reduces dimension of $U^T$ by an exchange of $N$ and $T$
- Collective dynamics dominates the quantum spectrum

Thank you for your attention!

Akila, Waltner, Gutkin, Braun, Guhr, `arXiv 1611.05749`

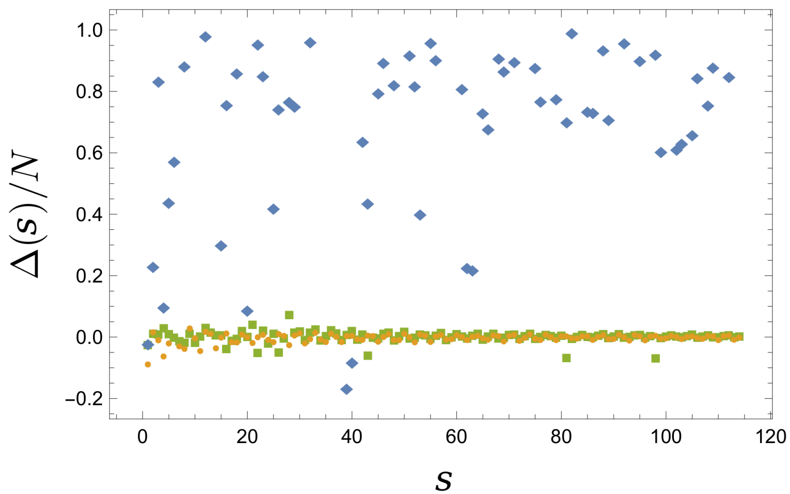
# Dominance of Collectivity

Orbits on manifold dominate spectrum for specific $s$

$$\mathrm{Tr}U^T \sim \sum_{\gamma(T)} A_\gamma \mathrm{e}^{isS_\gamma} \approx A_{\mathrm{man}} \mathrm{e}^{isS_{\mathrm{man}}}$$

Difference of the phase:

$$\Delta(s) = \mathrm{Im}\mathrm{Log}\mathrm{Tr}U^T - sS_{\mathrm{man}}$$




$\Rightarrow$ $\mathrm{Tr}U^T$ dominated by a type of collective motion

Akila, Waltner, Gutkin, Braun, Guhr (2016)