Multiple scattering of interacting bosons in random potentials

Thomas Wellens

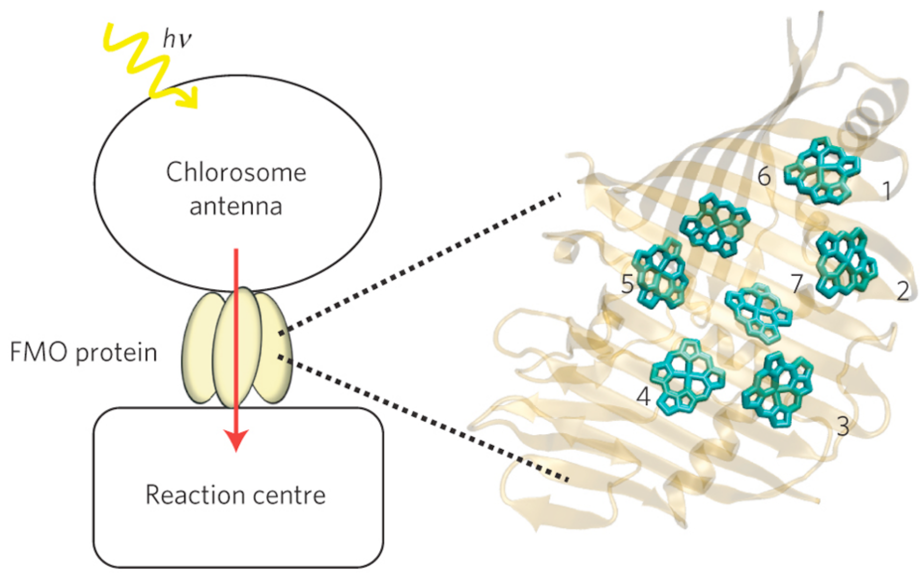
Albert-Ludwigs-Universität Freiburg

Dresden, QCTMBS, February 17, 2017

Motivation

Quantum transport in complex/disordered environments

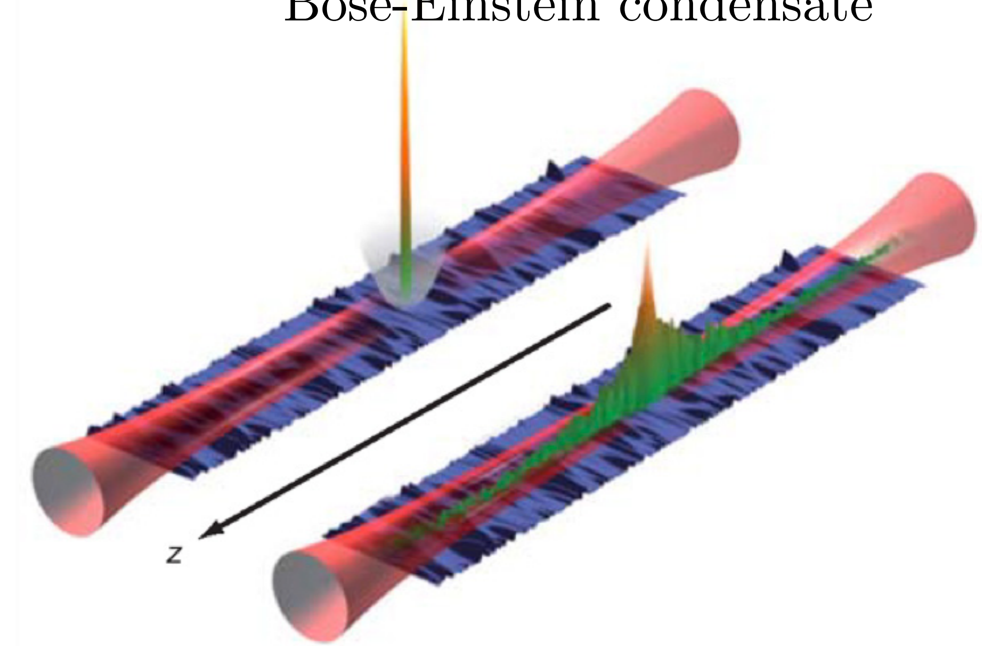
Photosynthesis



M. Sarovar et al., Nature Physics ***6****, 462 (2010)*

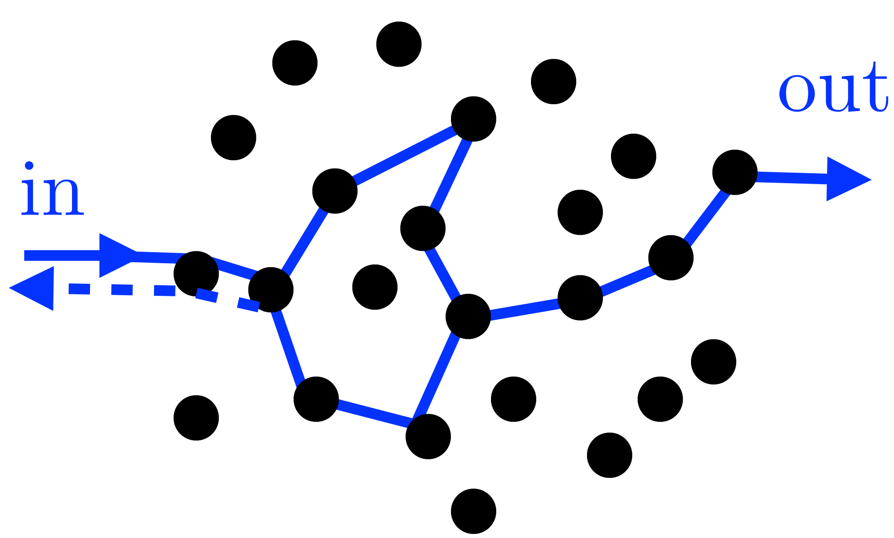
Solar cell

Bose-Einstein condensate



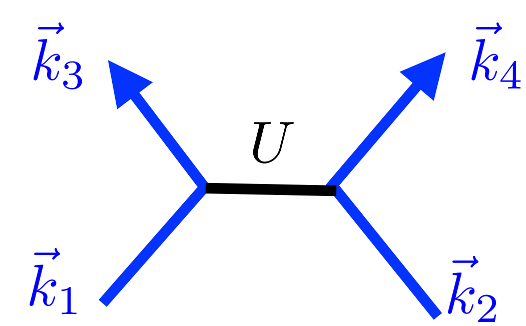
J. Billy et al., Nature ***453****, 891 (2008)*

Interference



$\rightsquigarrow$ constructive or destructive?

Interactions



$\rightsquigarrow$ decoherence, thermalization, many-body localization,...?

F. Jörder et al., PRL ***113****, 063004 (2014)*

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Outline

I) Introduction - single particle & weak disorder

II) Multiple scattering theory for interacting bosons

III) Excitation transport in ultracold Rydberg gases

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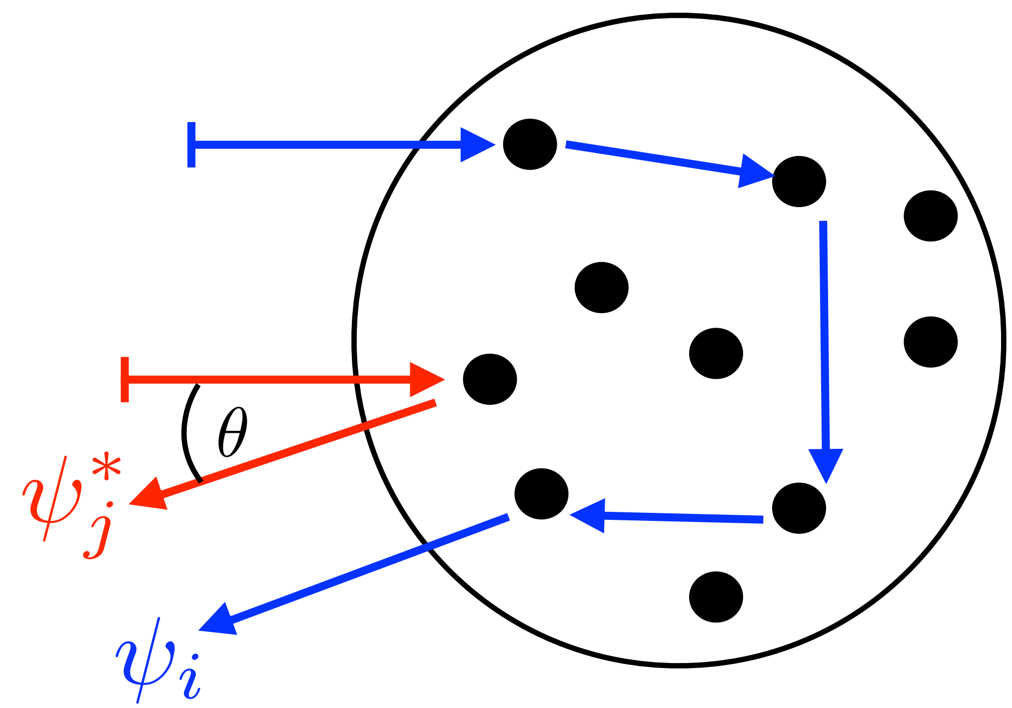
Multiple scattering of a single particle: Interference

$$\psi = \sum_{\text{paths } i} \psi_i$$

Born series

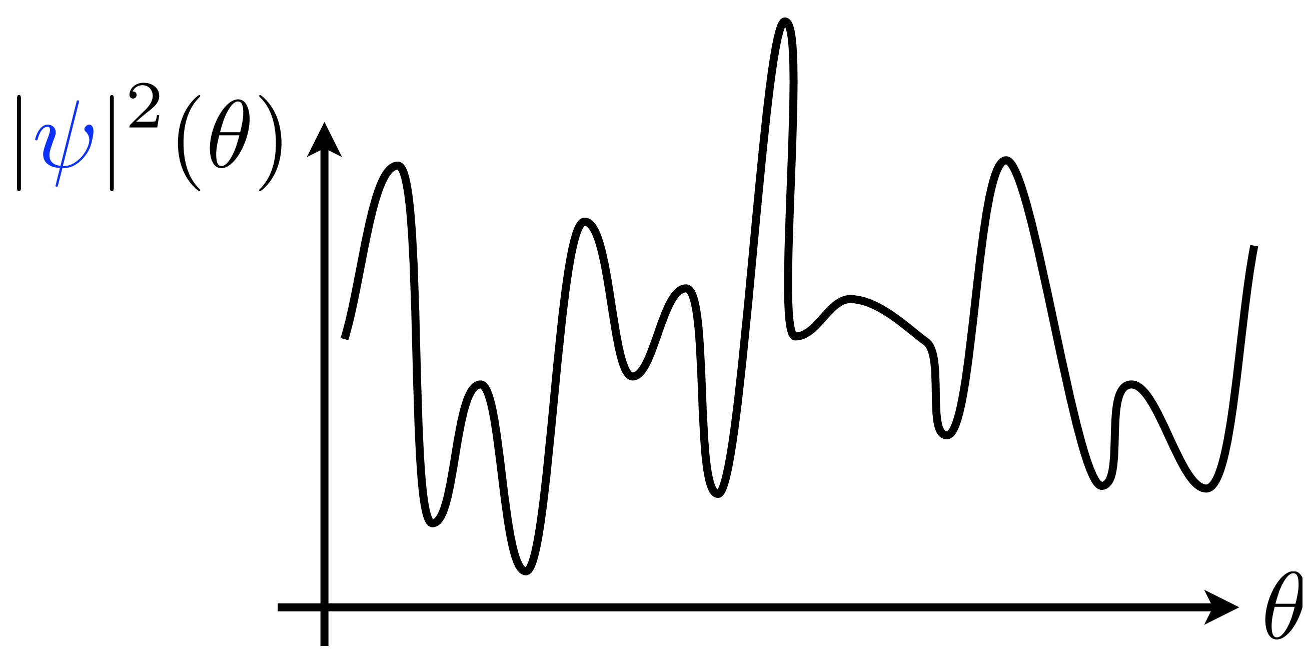


$$|\psi|^2 = \sum_{i,j} \psi_i \psi_j^*$$

$$= \sum_i |\psi_i|^2 + \sum_{i \neq j} \psi_i \psi_j^*$$

Interferences:

$\rightsquigarrow$ speckle



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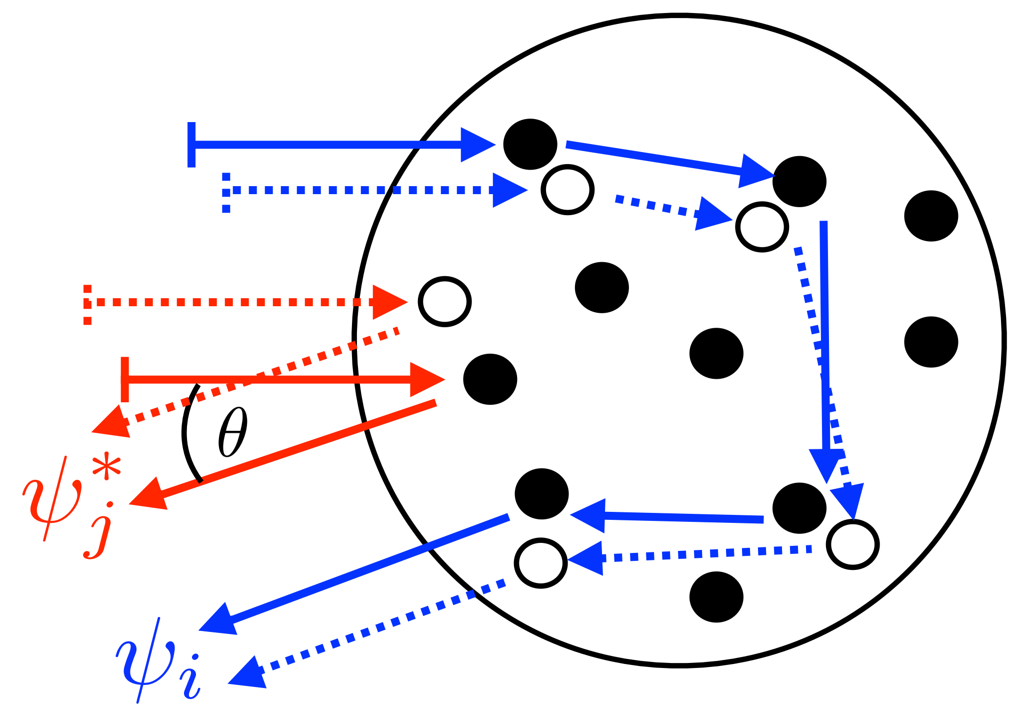

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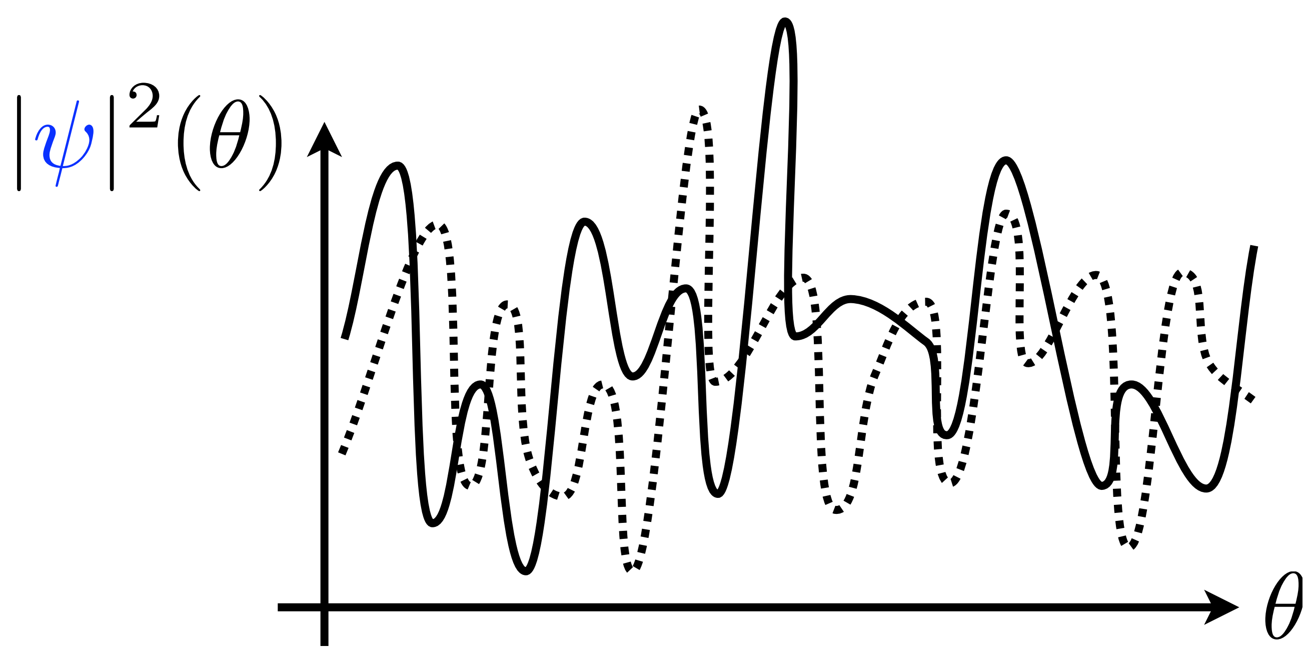

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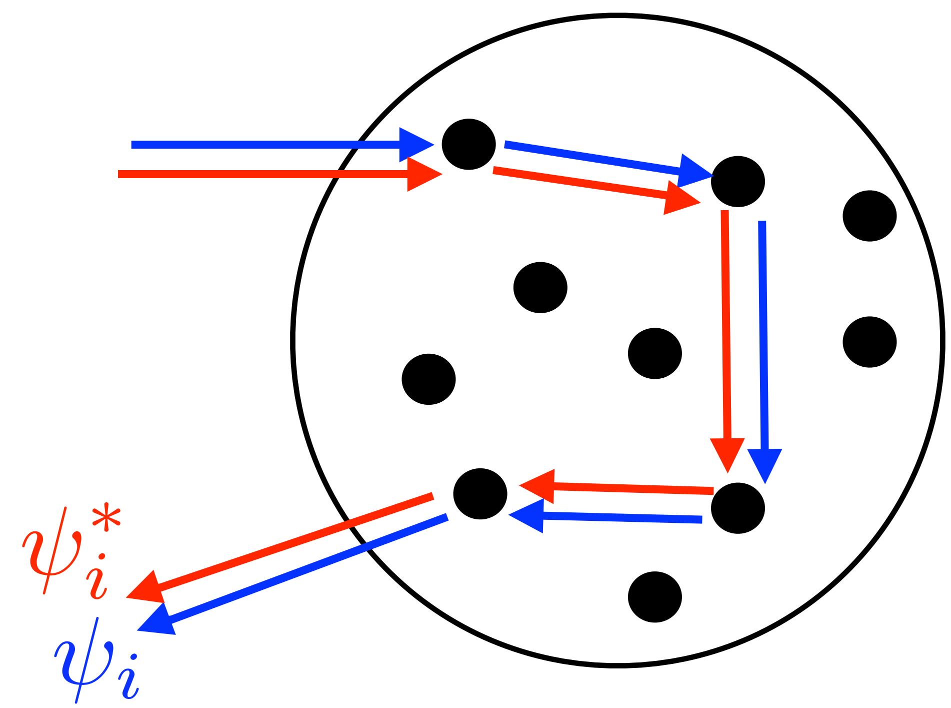
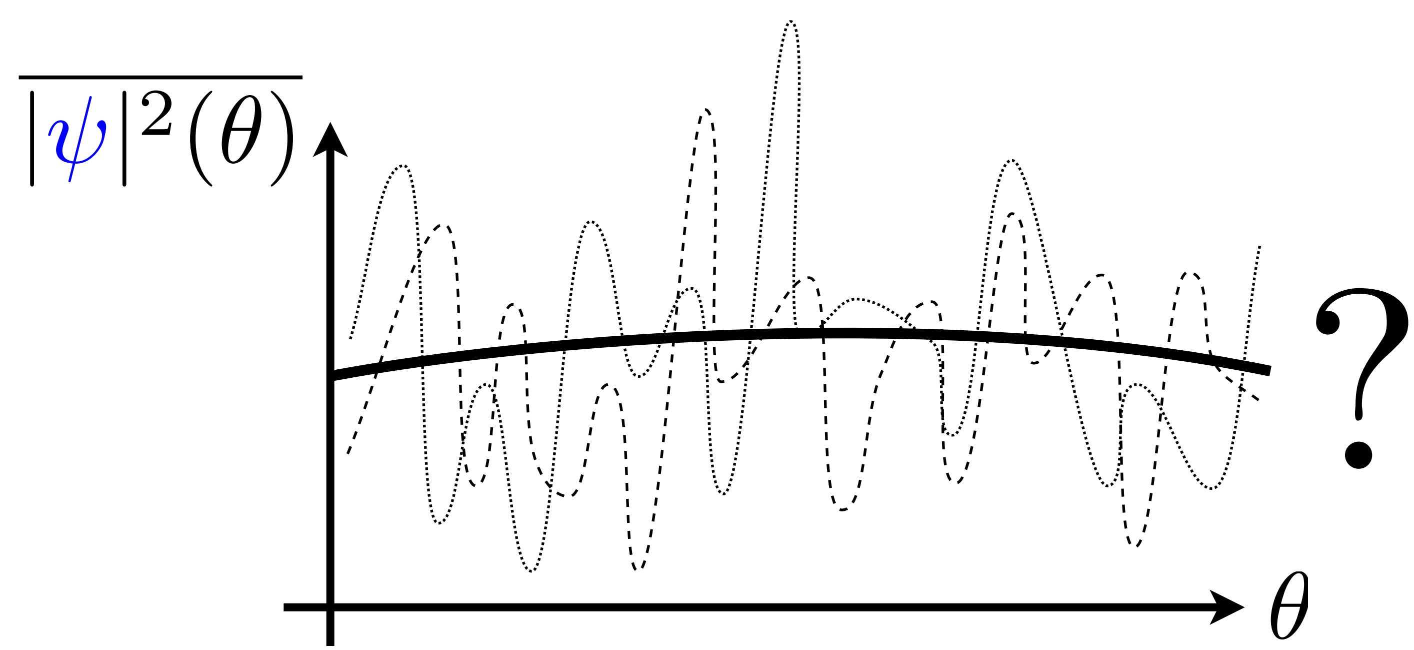


$$\overline{|\psi|^2} = \sum_{i,j} \overline{\psi_i \psi_j^*}$$

$$= \sum_i \overline{|\psi_i|^2} + \sum_{i \neq j} \overline{\psi_i \psi_j^*}$$

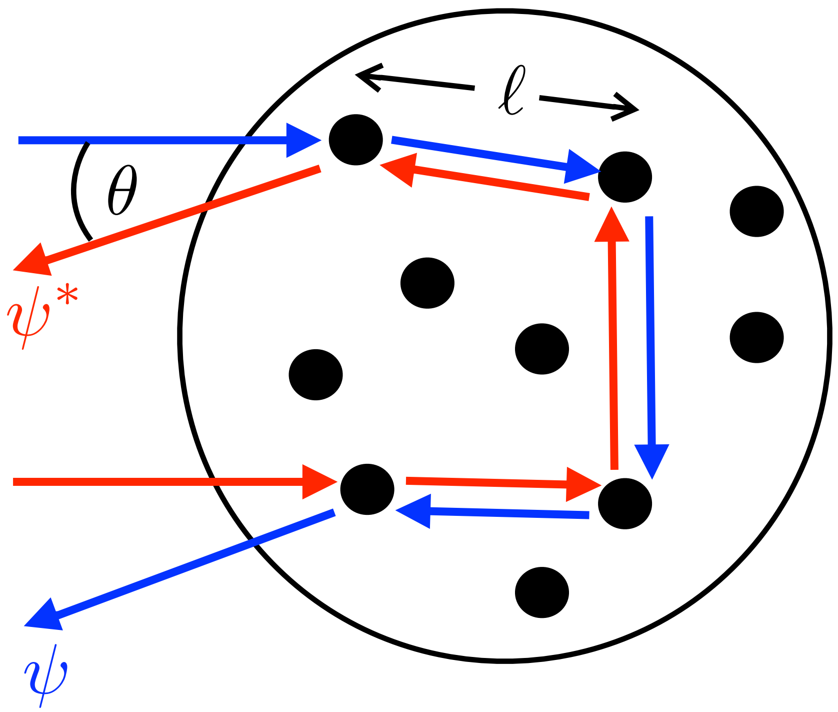


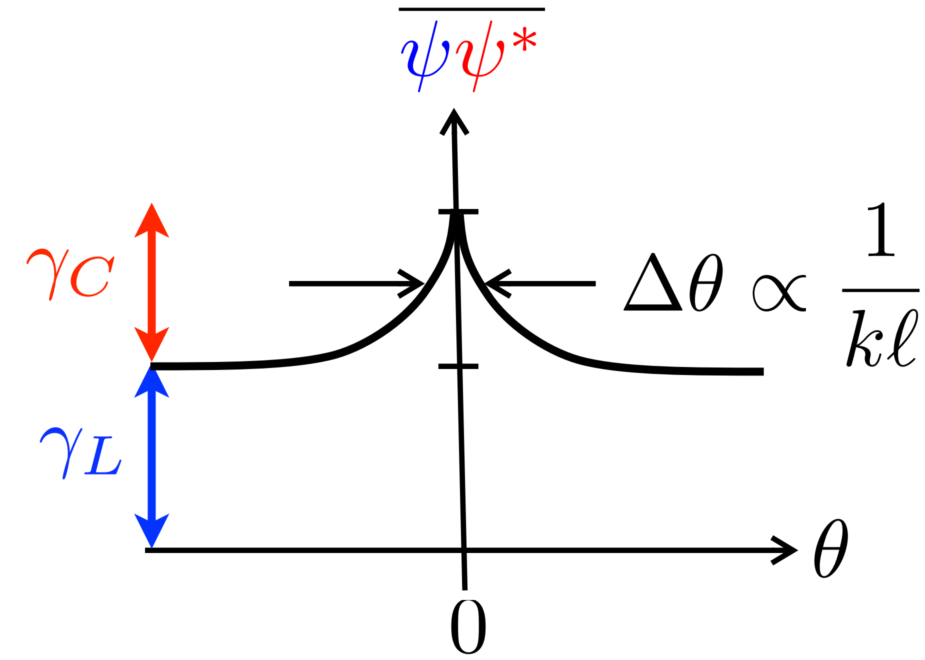
Ensemble average:

Interferences vanish?

Coherent backscattering (CBS)

Interference between reversed paths
survives disorder average!





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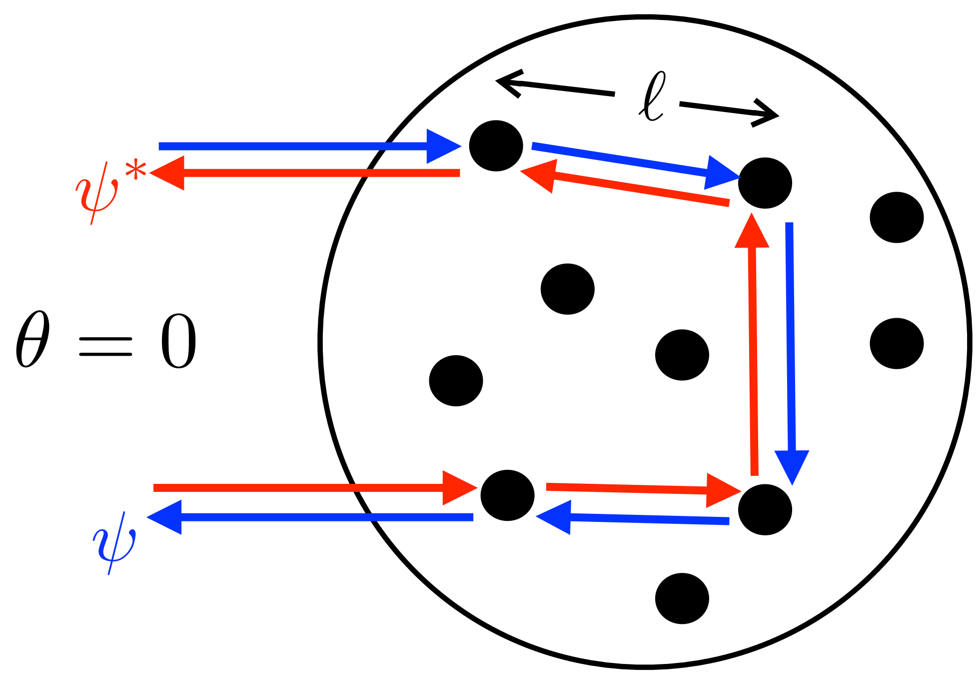

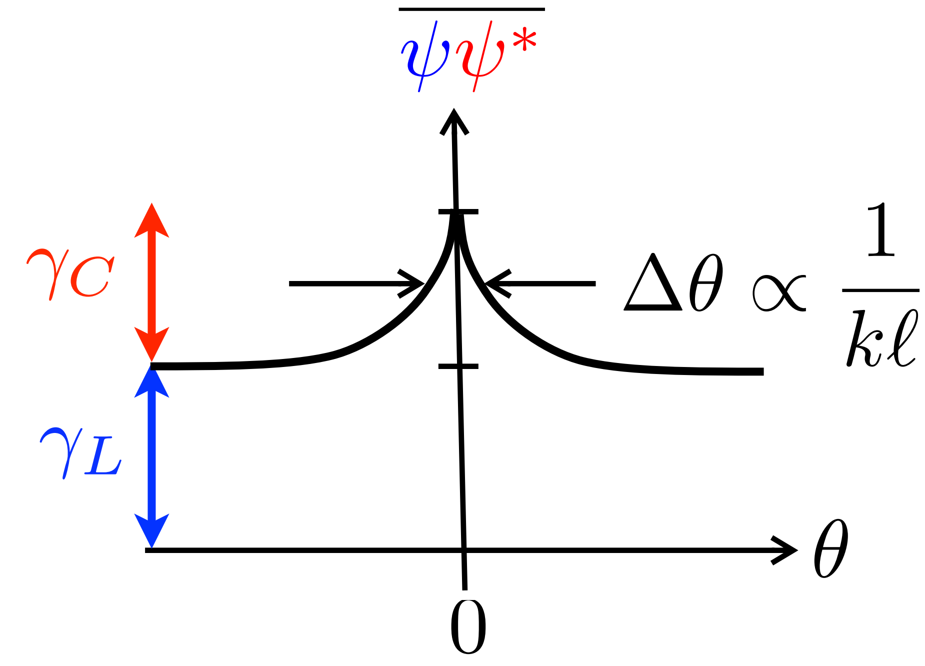

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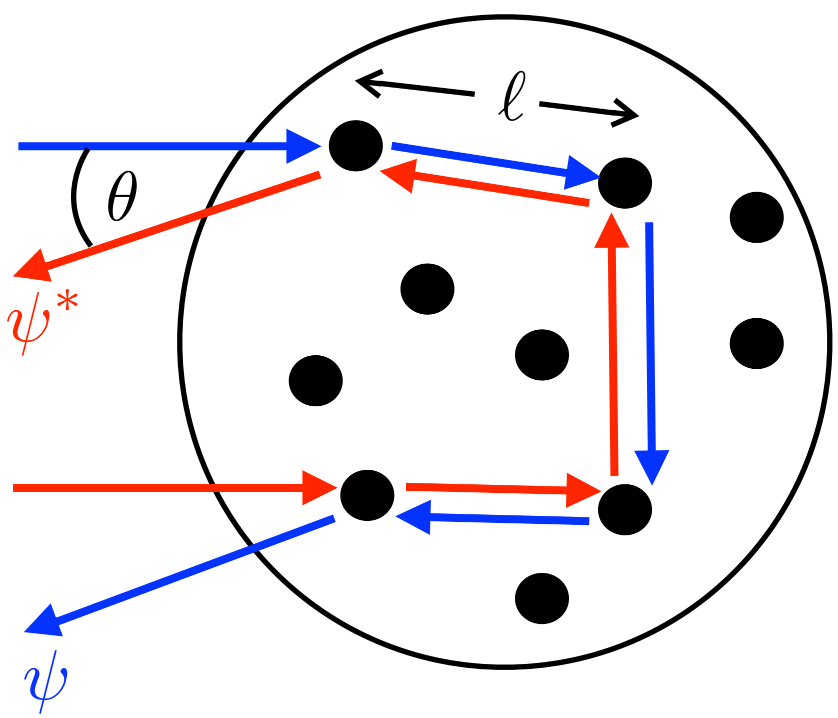
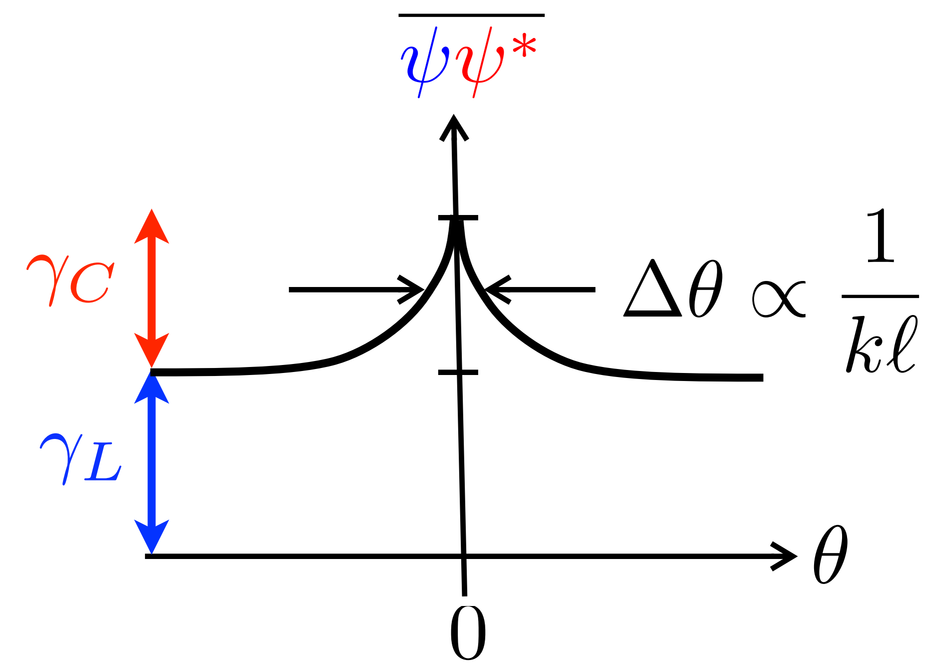
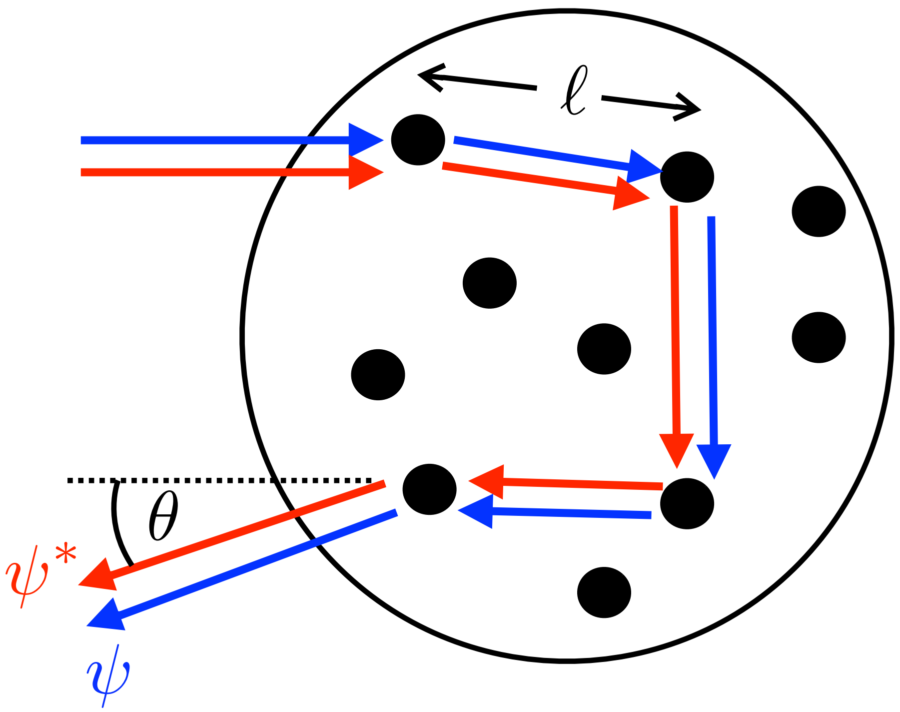
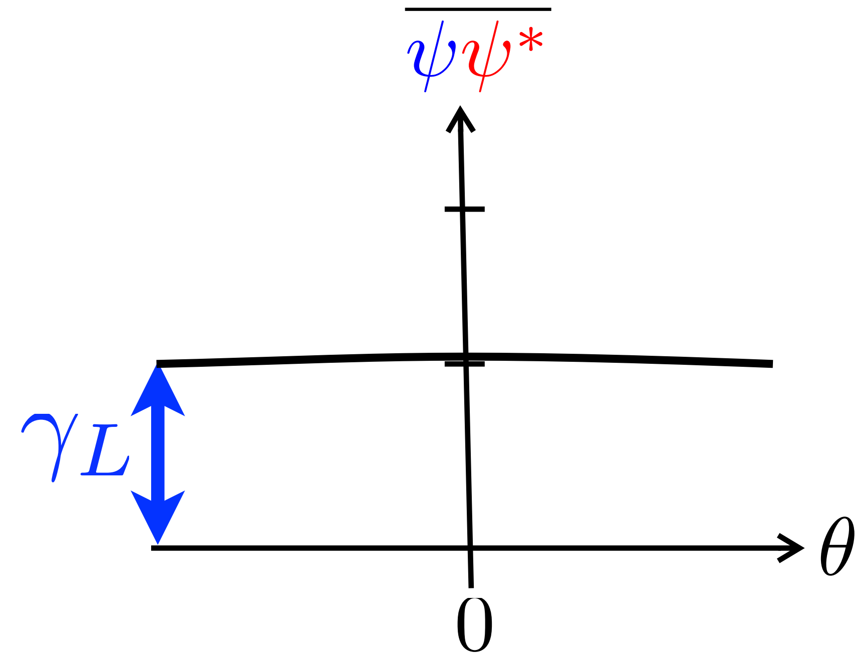




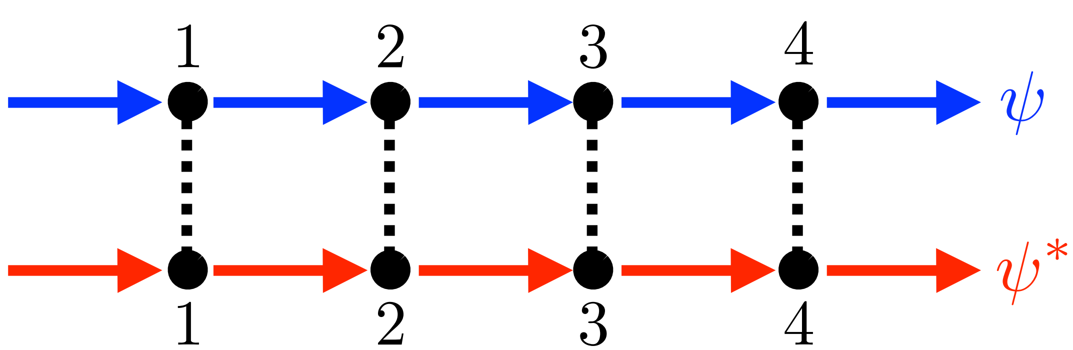
Coherent backscattering (CBS)





Weak disorder: $\frac{1}{k\ell} \ll 1$

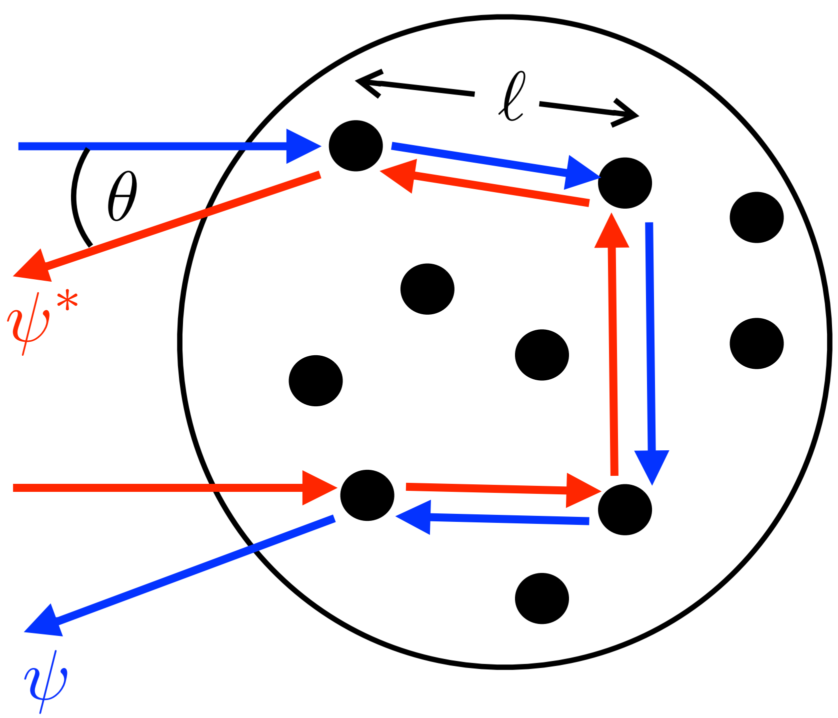
Ladder diagrams:

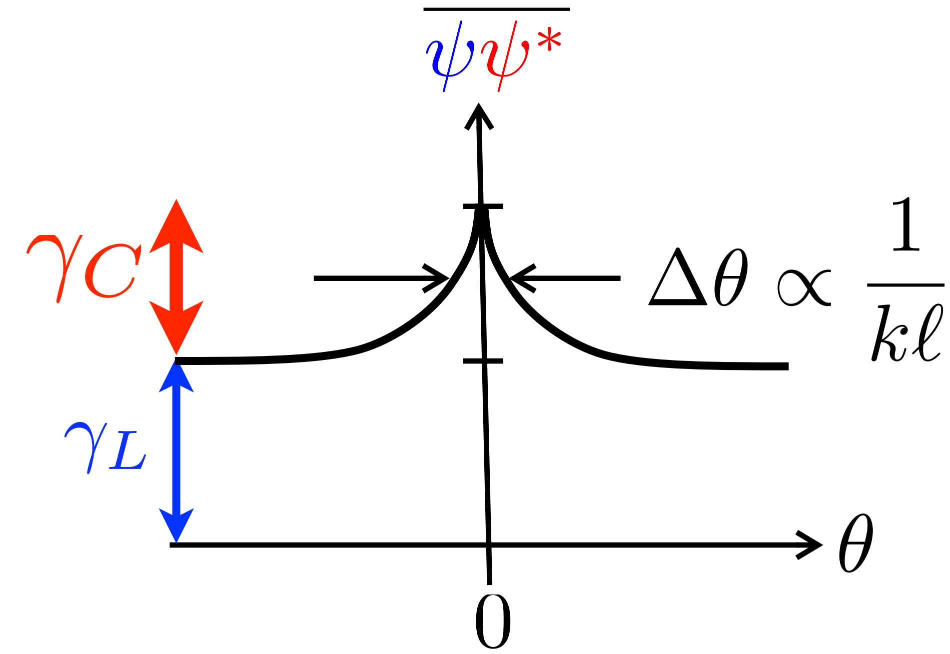


Diffusion γ_L

Coherent backscattering (CBS)

Interference between reversed paths
survives disorder average!





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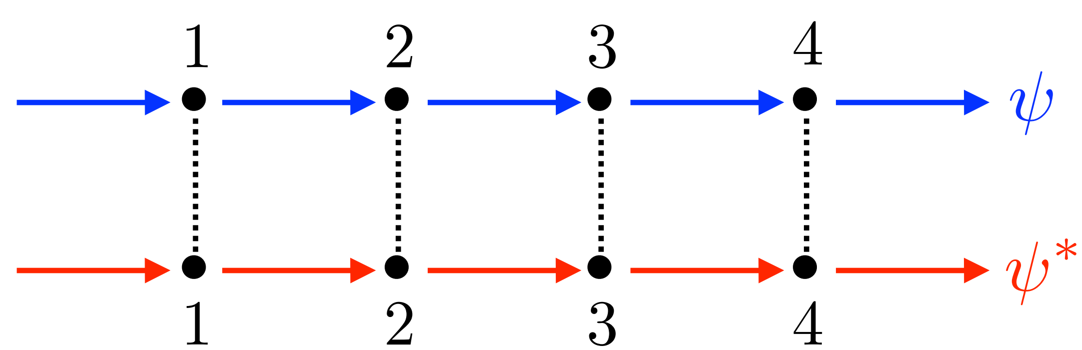
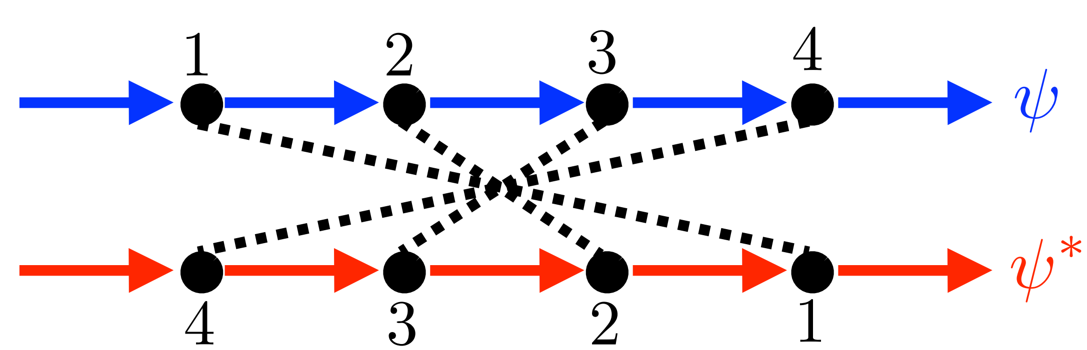


Diffusion γ_L

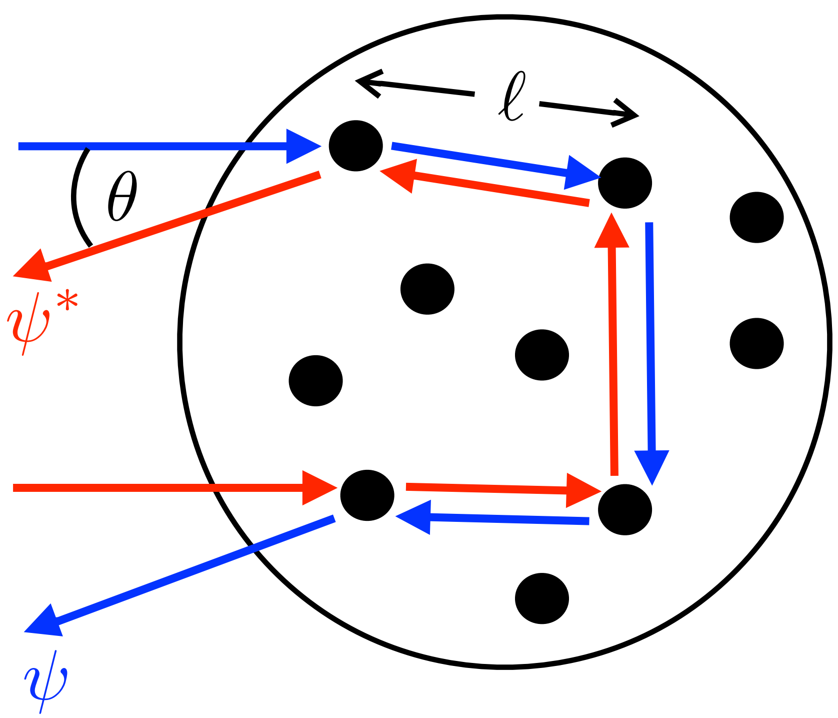
Crossed diagrams:

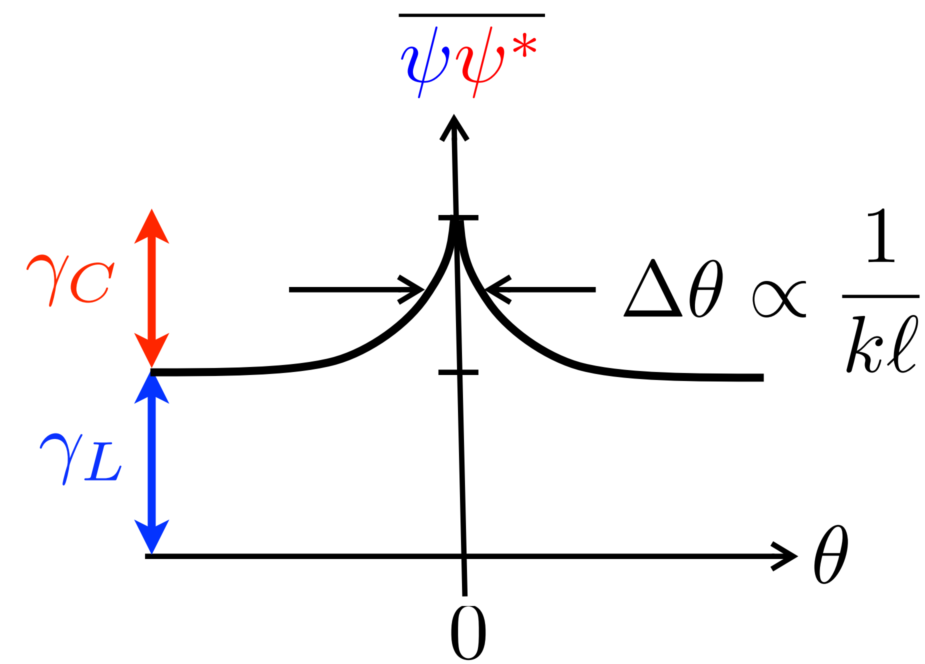


CBS γ_C

Coherent backscattering (CBS)

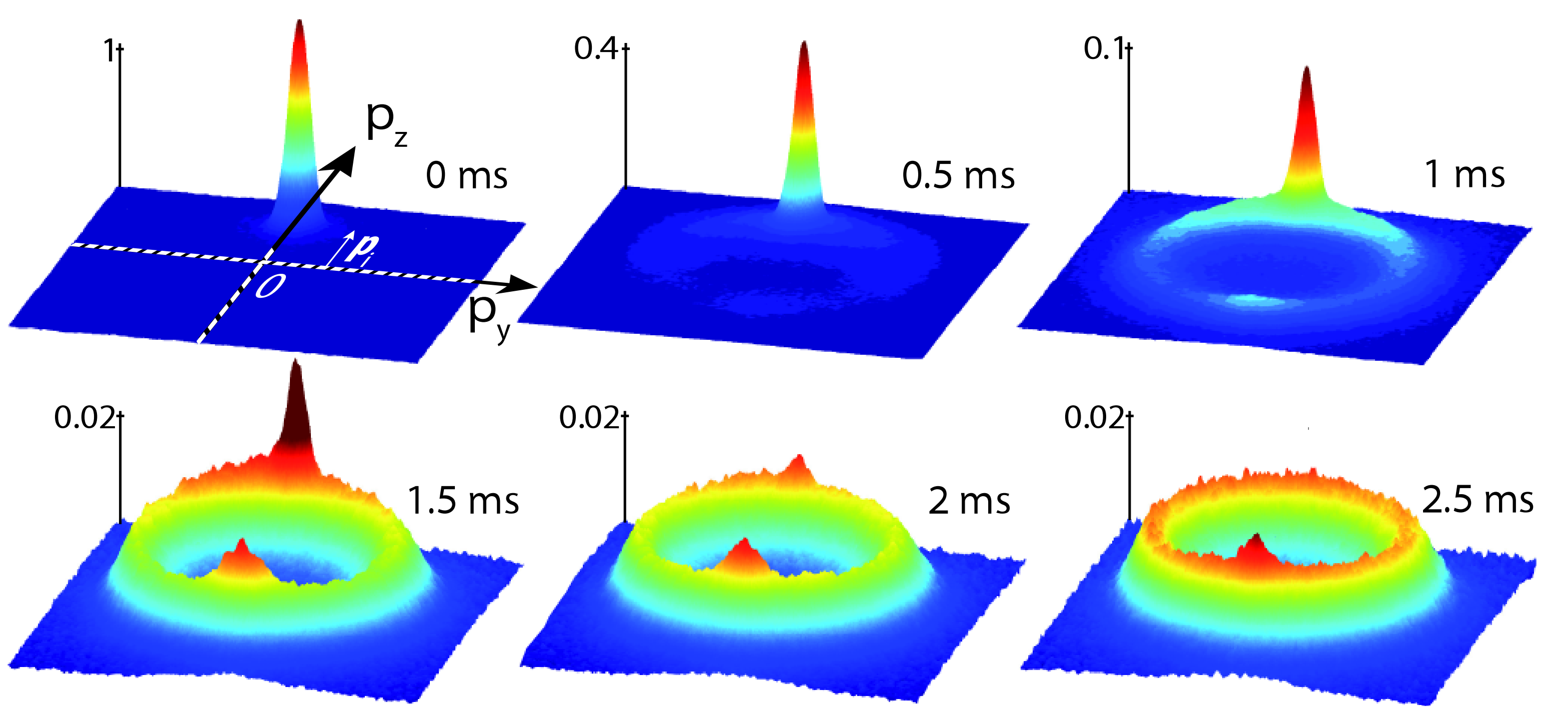
Interference between reversed paths survives disorder average!





Weak disorder: $\frac{1}{k\ell} \ll 1$

Experiments on CBS with Bose-Einstein condensates



*F. Jendrzejewski et al., PRL **109**, 195302 (2012)*

*G. Labeyrie et. al., EPL **100**, 66001 (2012)*

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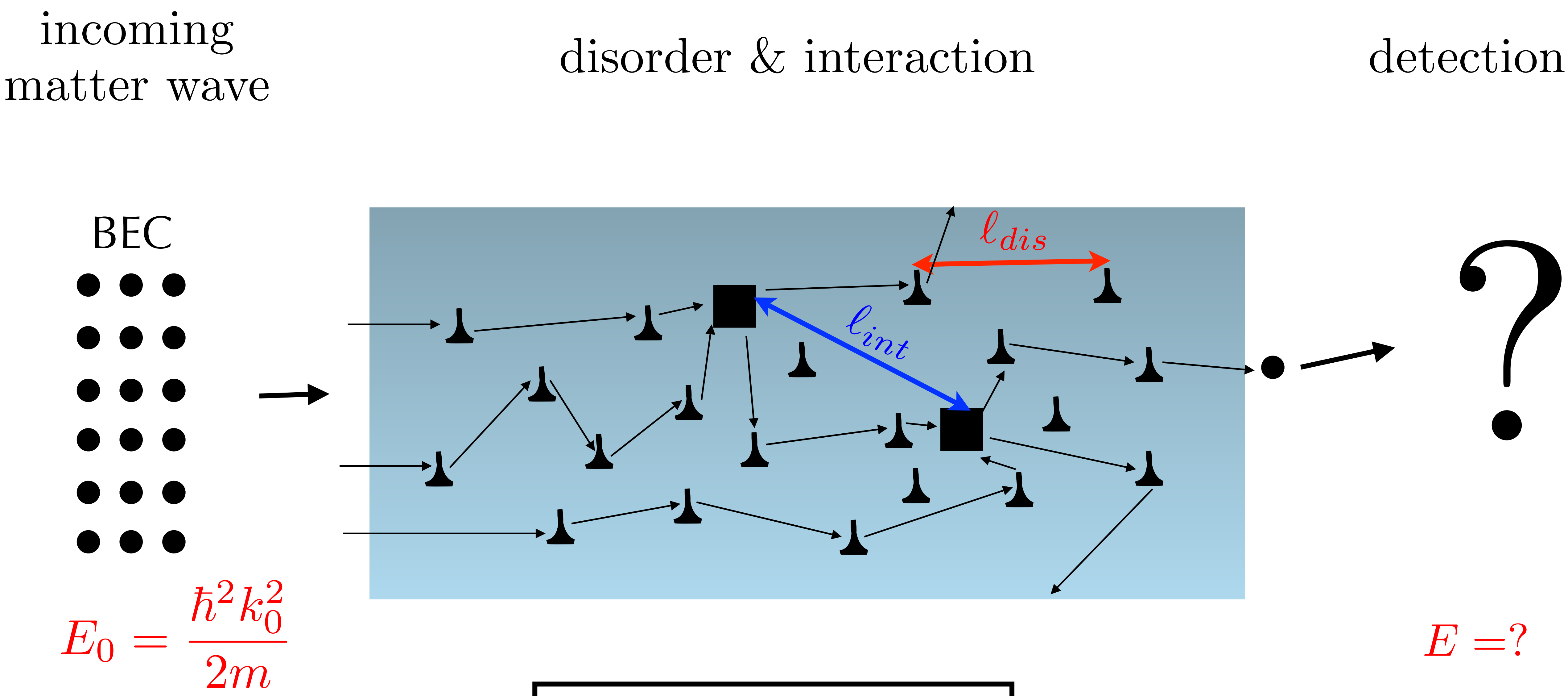
N-particle scattering scenario



$$\frac{1}{k_0} \ll \ell_{\text{dis}} \ll \ell_{\text{int}}$$

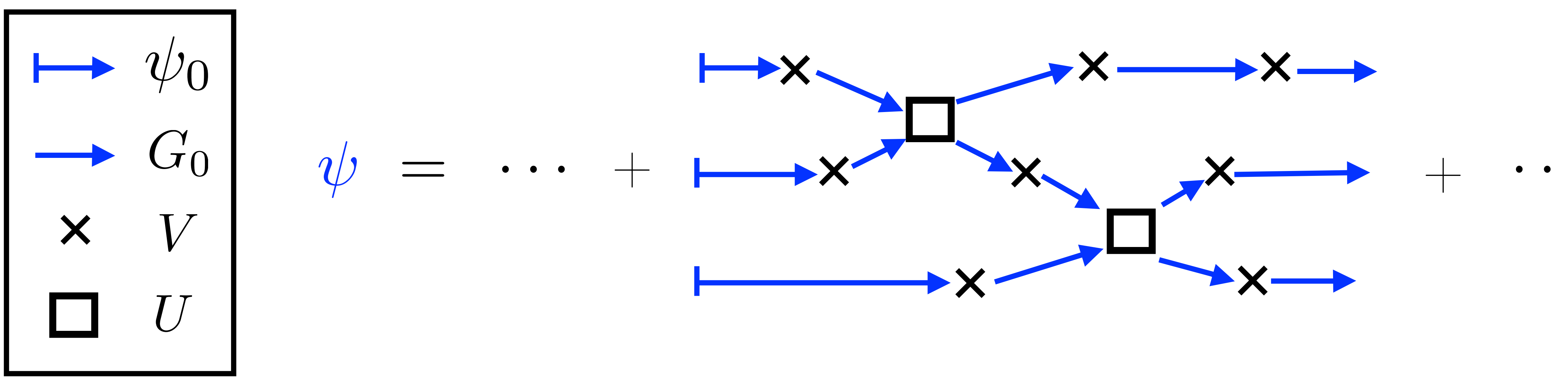
N-particle scattering: Theoretical background

Hamiltonian: $H = H_0 + V + U$ $\qquad$ $G_0(E) = \dfrac{1}{E - H_0 + i\epsilon}$

Initial state: $|\psi_0\rangle = |N\vec{k}_0\rangle$ $\qquad$ $E = NE_0 \qquad E_0 = \dfrac{\hbar^2 k_0^2}{2m}$

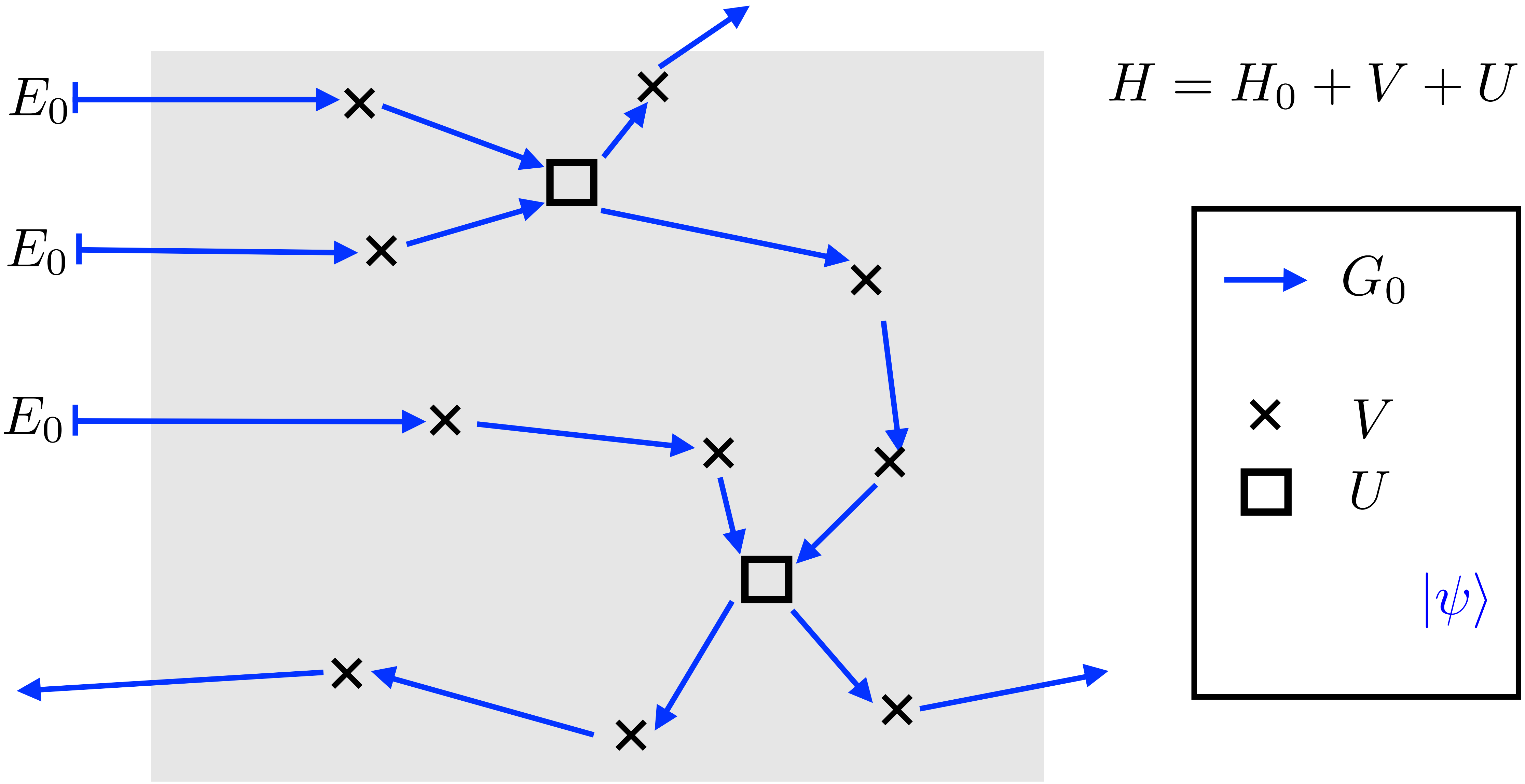
Stationary scattering state: $$|\psi\rangle = |\psi_0\rangle + G_0(E)\big(V + U\big)|\psi\rangle$$

Iteration:
$$|\psi\rangle = |\psi_0\rangle + G_0(E)V|\psi_0\rangle + G_0(E)U|\psi_0\rangle + \\ + G_0(E)VG_0(E)V|\psi_0\rangle + G_0(E)UG_0(E)V|\psi_0\rangle + \ldots$$



T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. **15**, 115015 (2013)

N-particle scattering: diagrammatic approach



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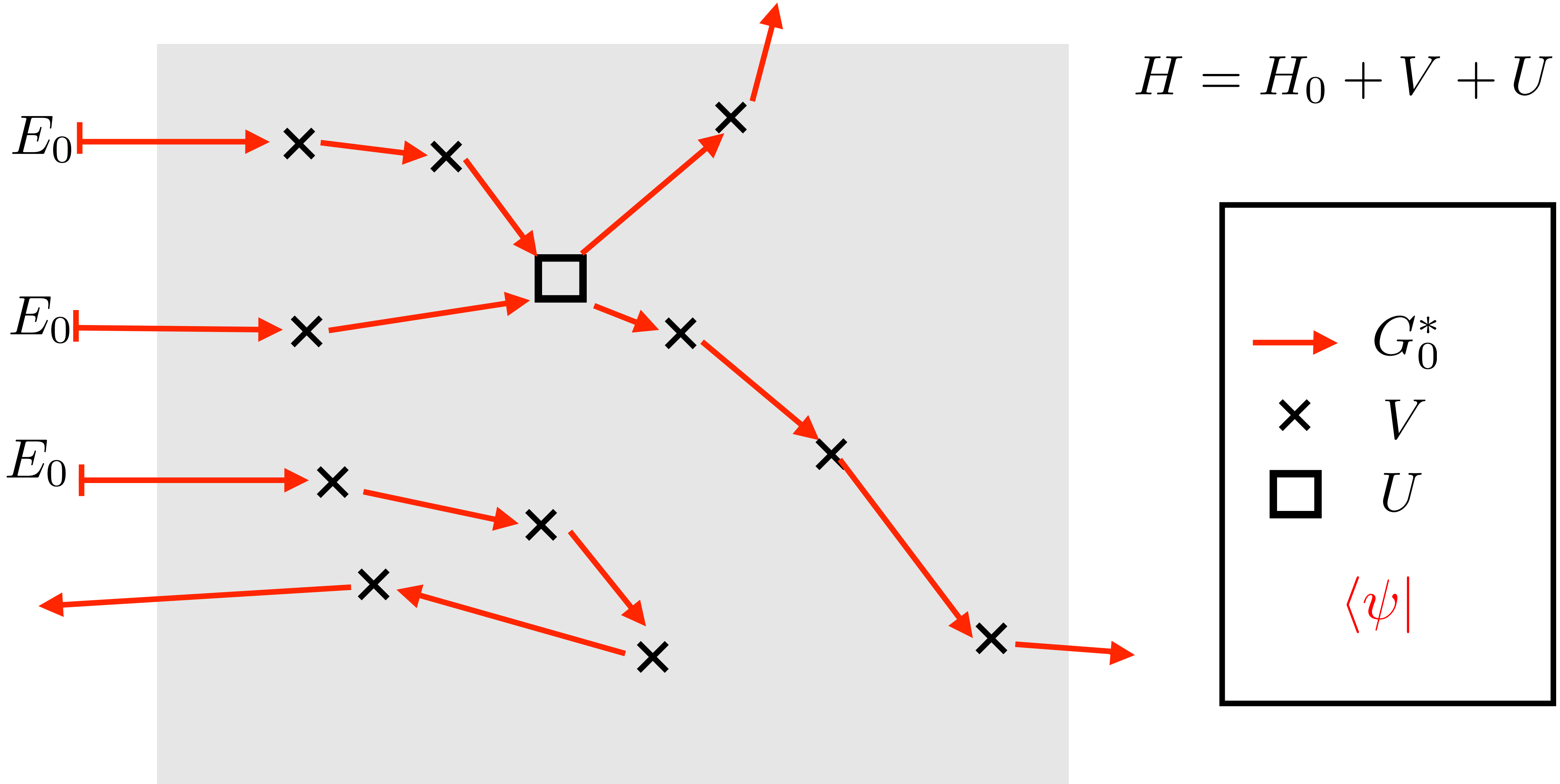

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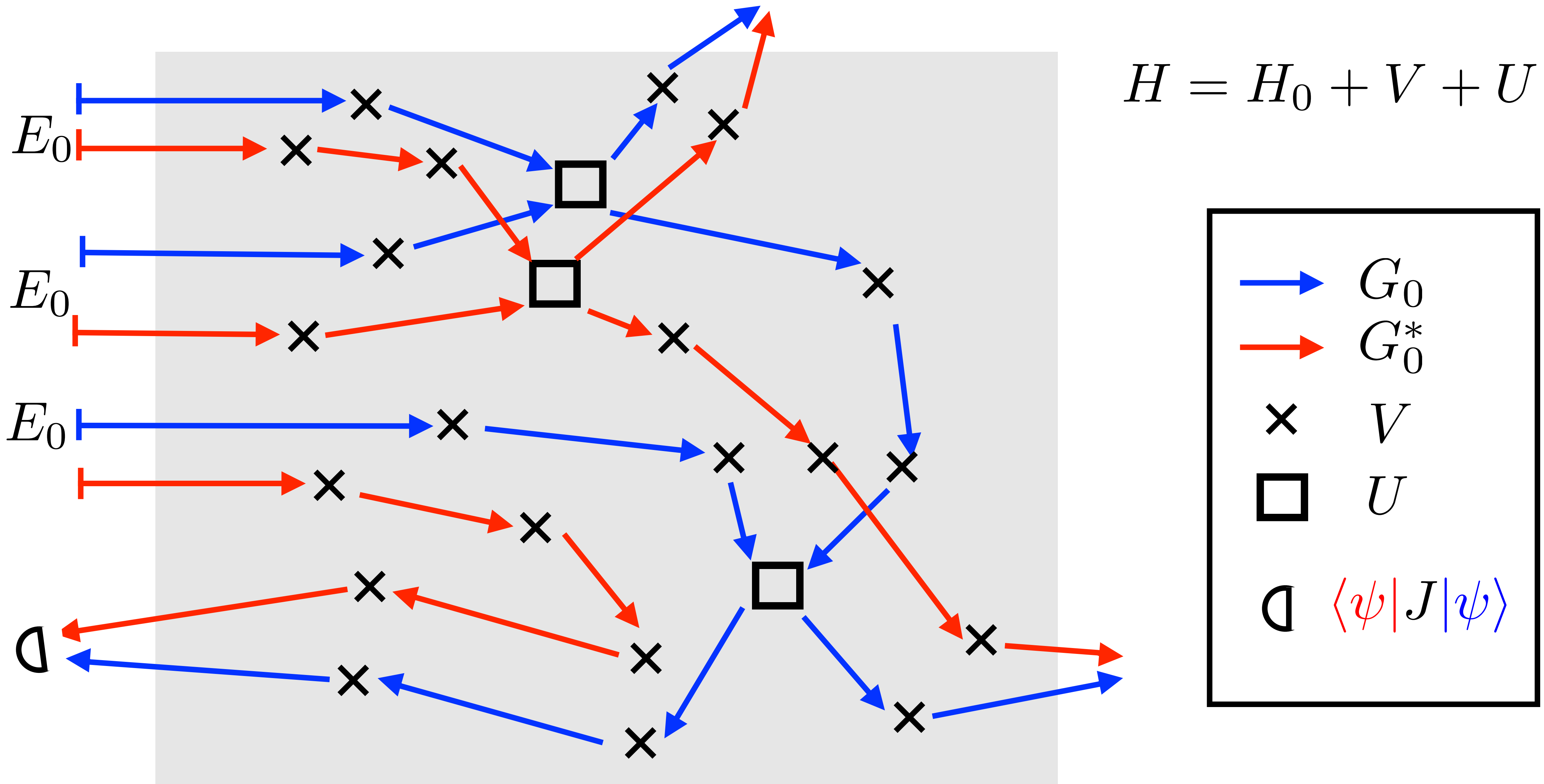
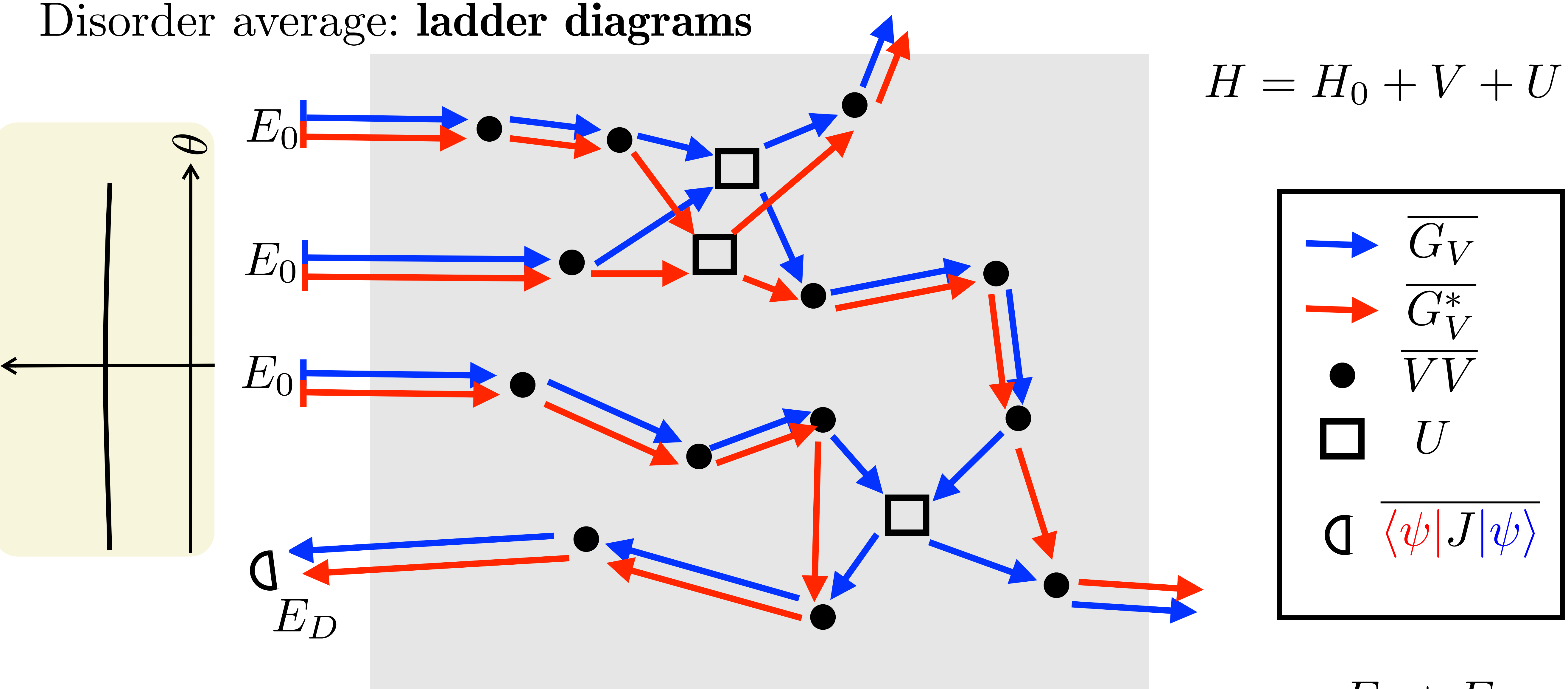


T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. **15**, 115015 (2013)

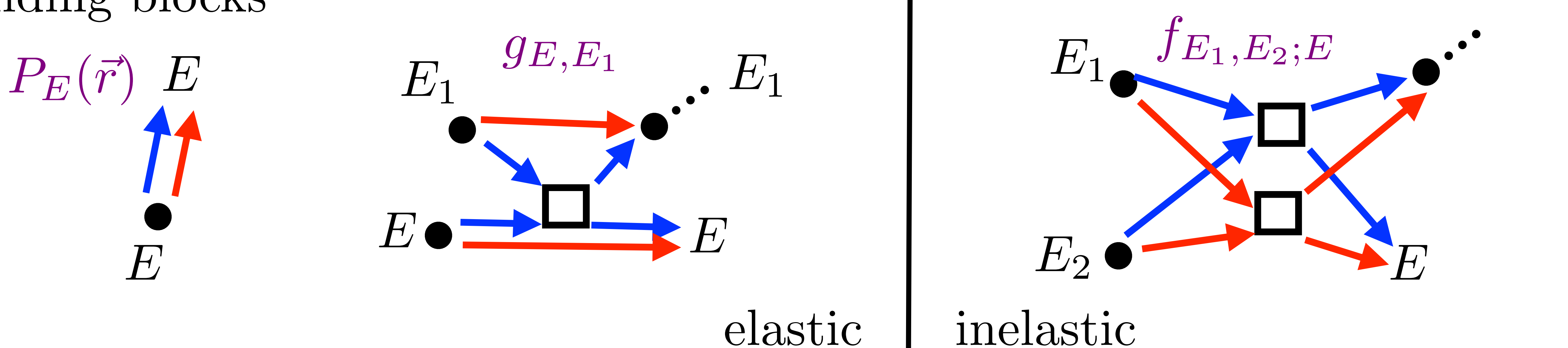
N-particle scattering: diagrammatic approach

Disorder average: **ladder diagrams**

$$H = H_0 + V + U$$



Building blocks



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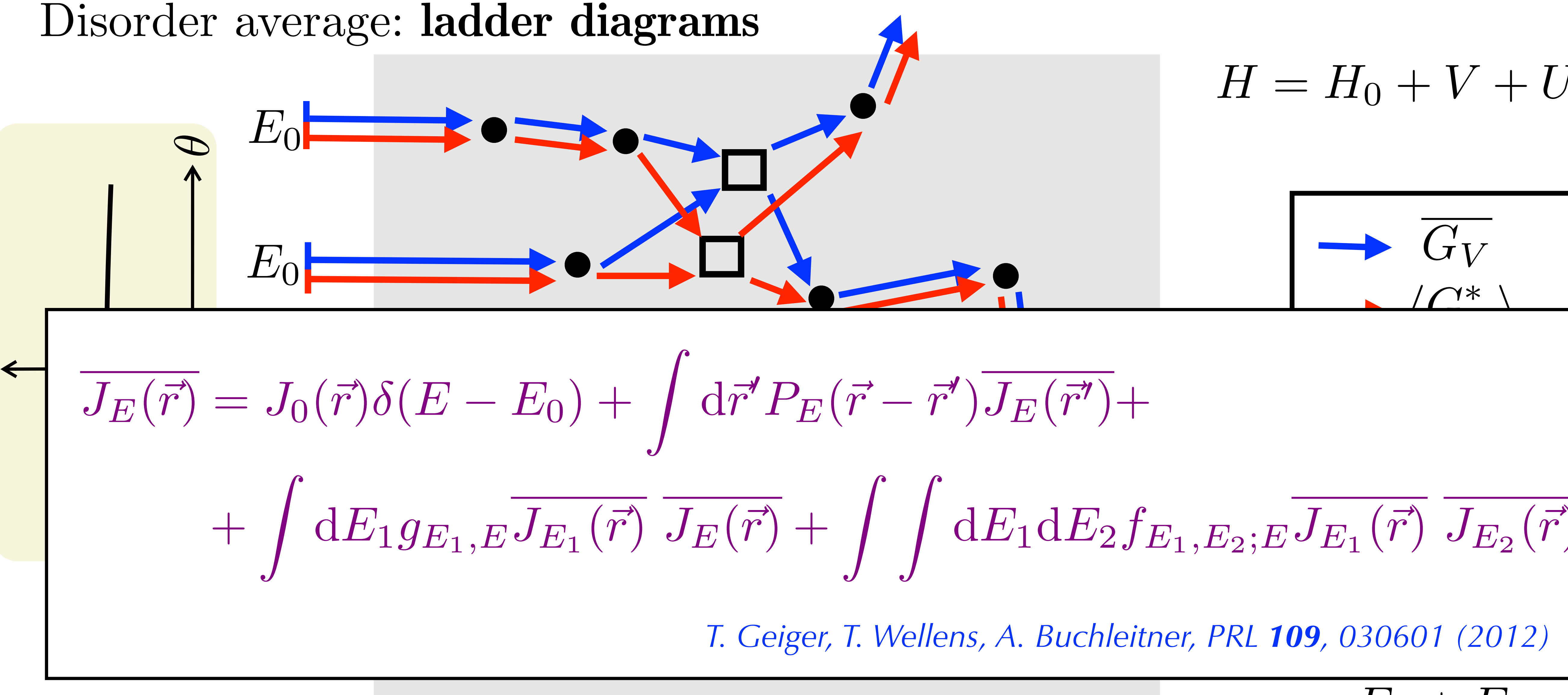
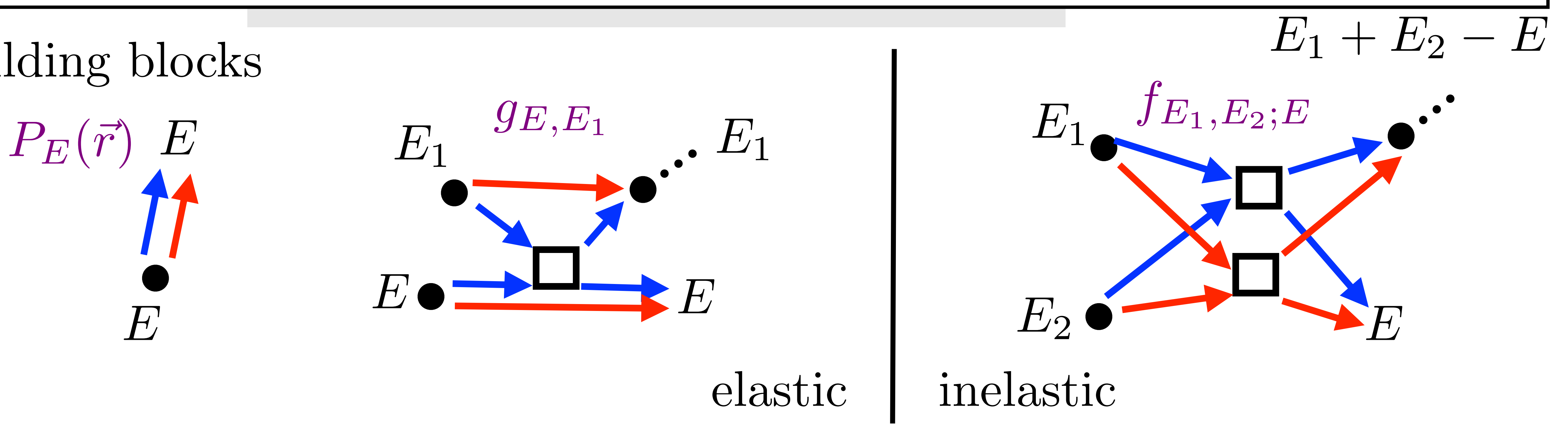


$$\overline{J_E(\vec{r})} = J_0(\vec{r})\delta(E-E_0) + \int \mathrm{d}\vec{r}'\, P_E(\vec{r}-\vec{r}')\overline{J_E(\vec{r}')} +$$

$$+ \int \mathrm{d}E_1 g_{E_1,E}\overline{J_{E_1}(\vec{r})}\;\overline{J_E(\vec{r})} + \int\int \mathrm{d}E_1 \mathrm{d}E_2 f_{E_1,E_2;E}\overline{J_{E_1}(\vec{r})}\;\overline{J_{E_2}(\vec{r})}$$

T. Geiger, T. Wellens, A. Buchleitner, PRL ***109****, 030601 (2012)*

Building blocks



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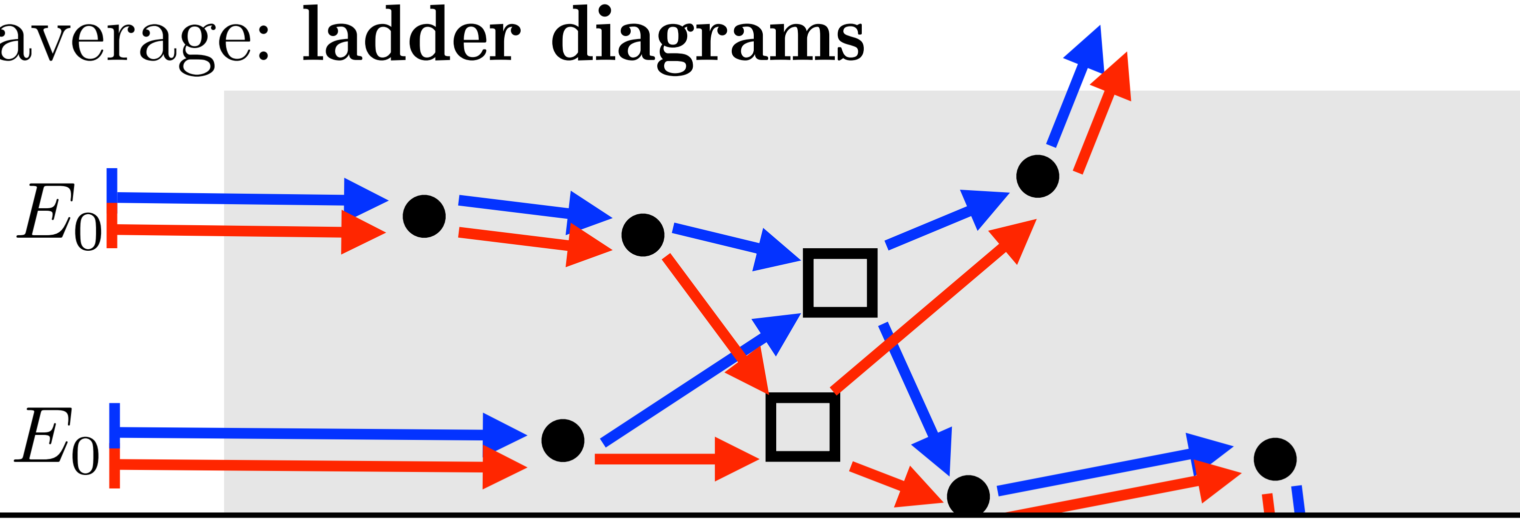
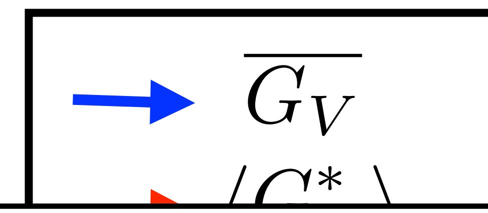
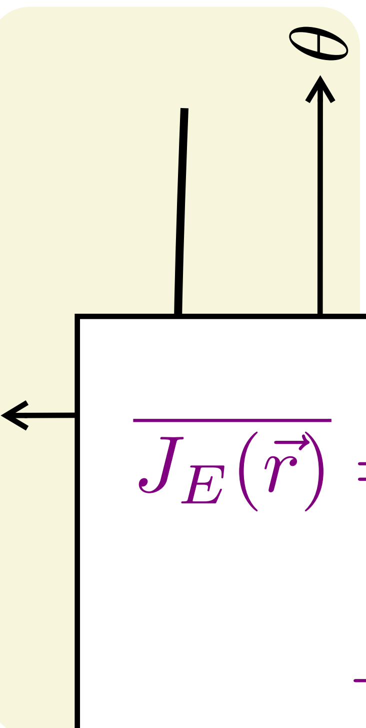


$$H = H_0 + V + U$$





$$\overline{J_E(\vec{r})} = J_0(\vec{r})\delta(E - E_0) + \int \mathrm{d}\vec{r}' P_E(\vec{r} - \vec{r}')\overline{J_E(\vec{r}')} +$$
$$+ \int \mathrm{d}E_1 g_{E_1,E} \overline{J_{E_1}(\vec{r})}\ \overline{J_E(\vec{r})} + \int\int \mathrm{d}E_1 \mathrm{d}E_2 f_{E_1,E_2;E} \overline{J_{E_1}(\vec{r})}\ \overline{J_{E_2}(\vec{r})}$$

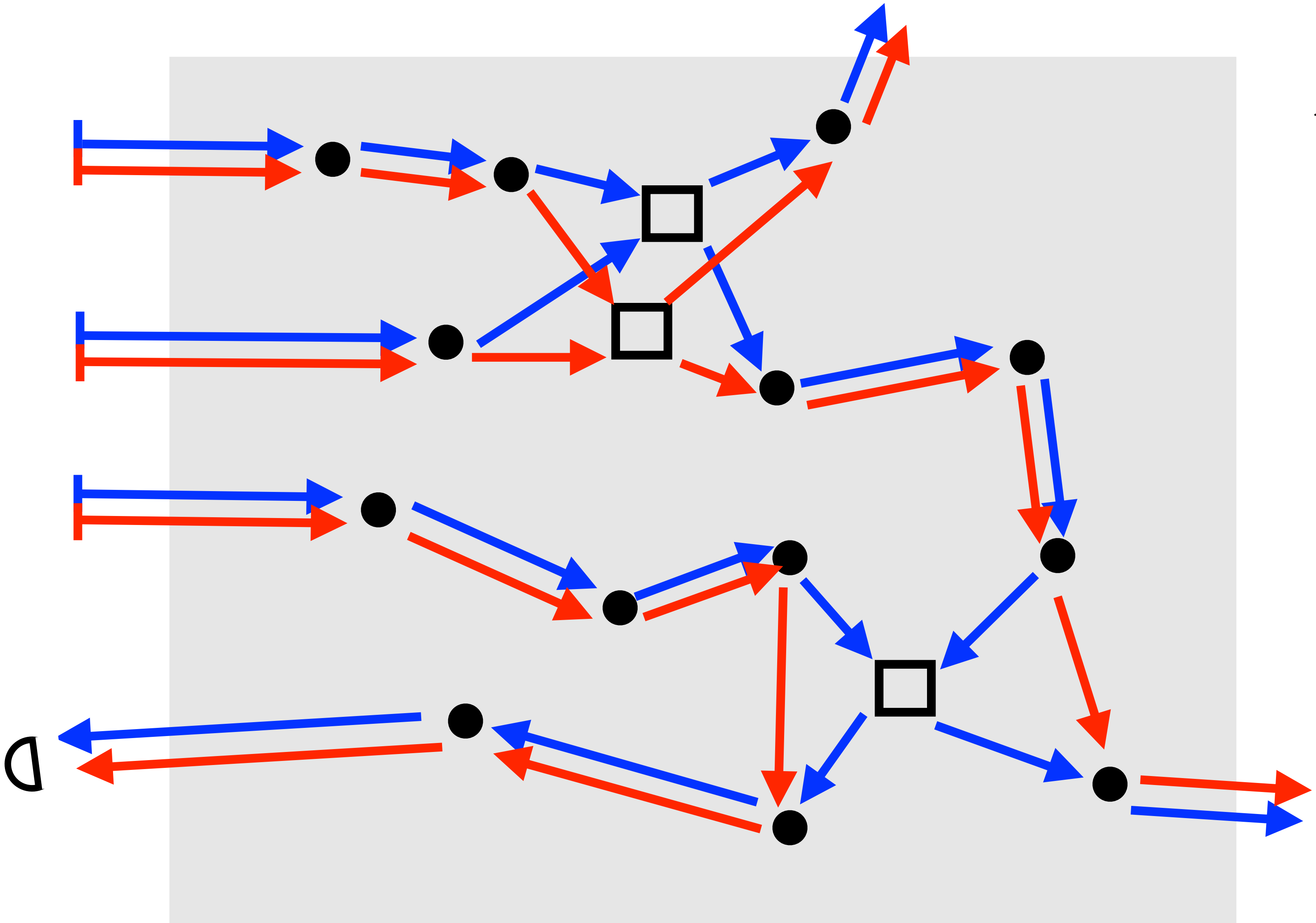
$$\vec{r} \to \infty : \quad \overline{J_E} \to E \exp(-2E/E_0)$$ Maxwell-Boltzmann

$\rightsquigarrow$ Thermalization!

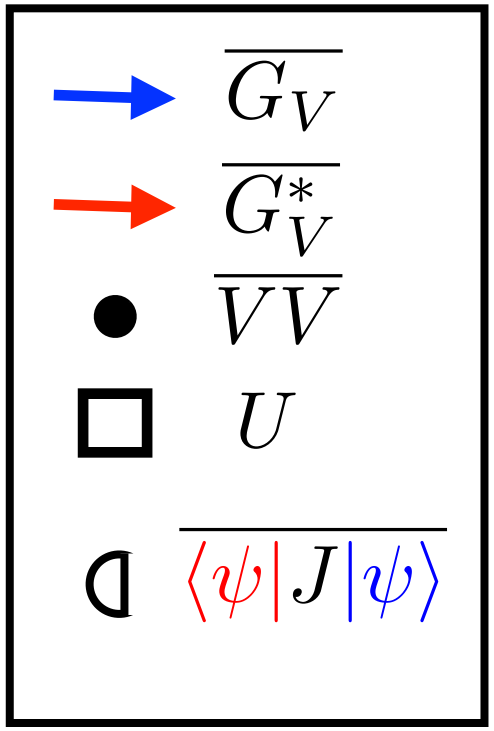
T. Geiger, T. Wellens, A. Buchleitner, PRL ***109****, 030601 (2012)*

elastic | inelastic

$\boldsymbol{N}$-particle scattering: diagrammatic approach

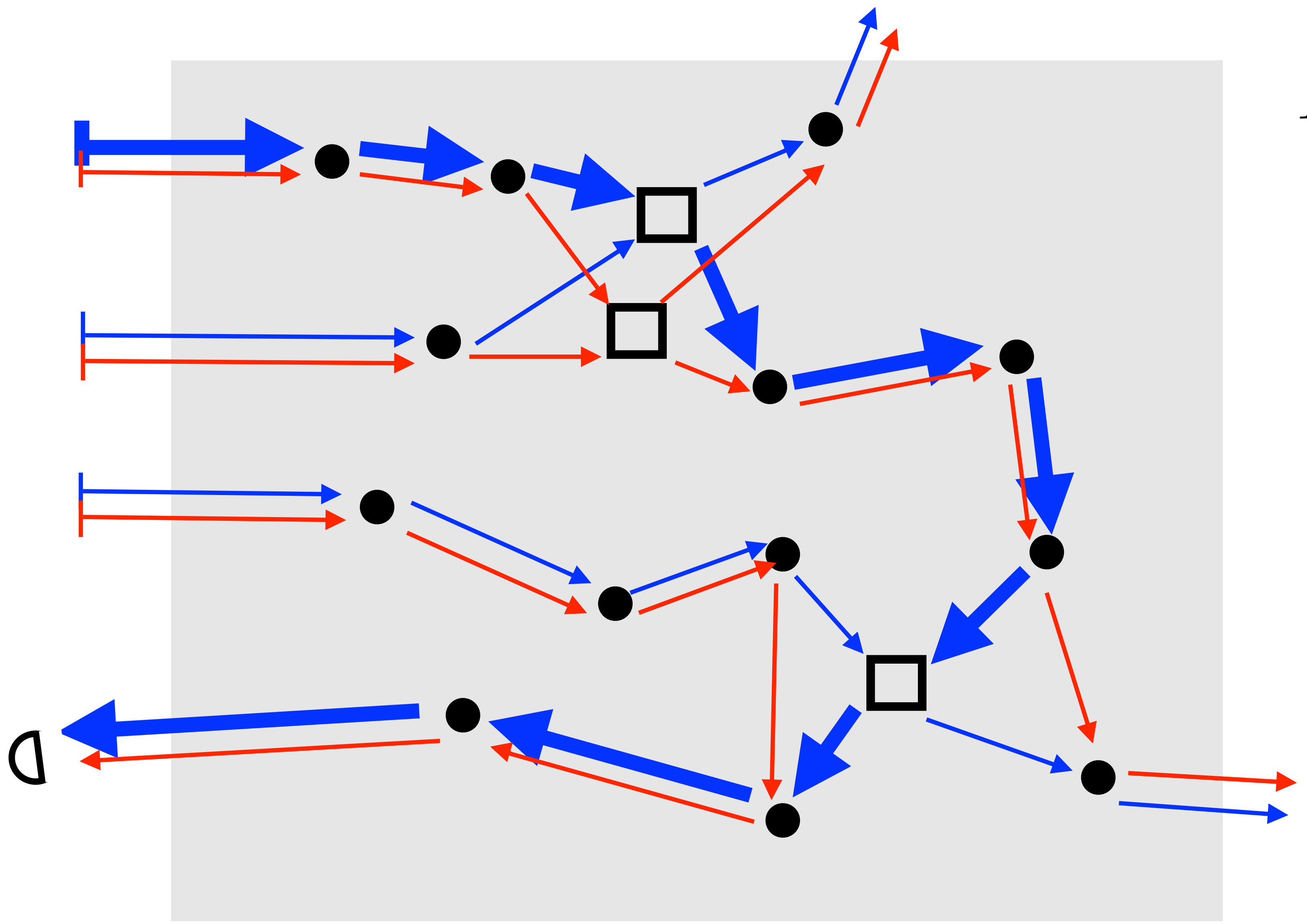


$$H = H_0 + V + U$$



T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. ***15****, 115015 (2013)*

N-particle scattering: diagrammatic approach



$$H = H_0 + V + U$$

→	$\overline{G_V}$
→	$\overline{G_V^*}$
●	$\overline{VV}$
□	U
◖	$\overline{\langle\psi\|J\|\psi\rangle}$

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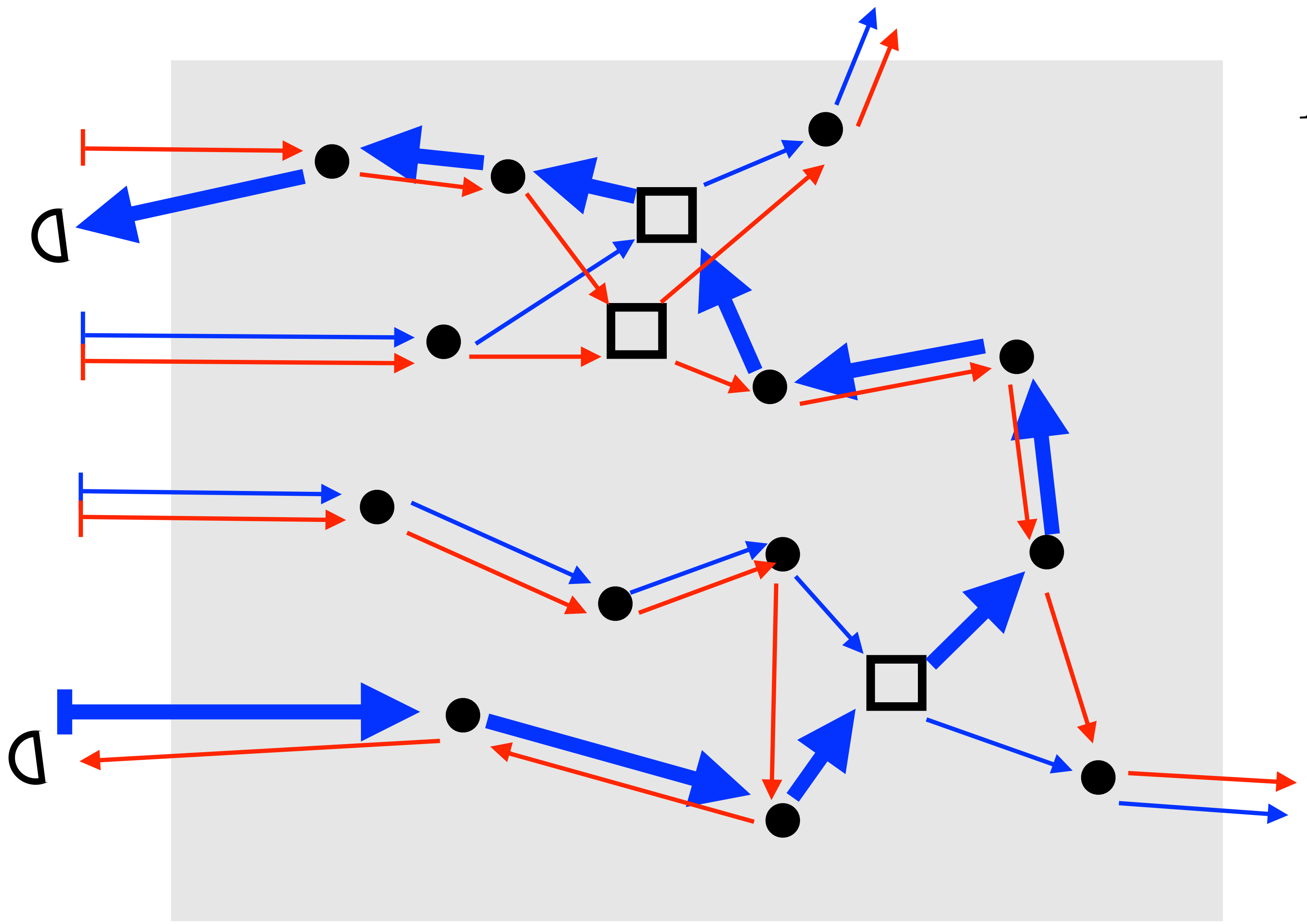

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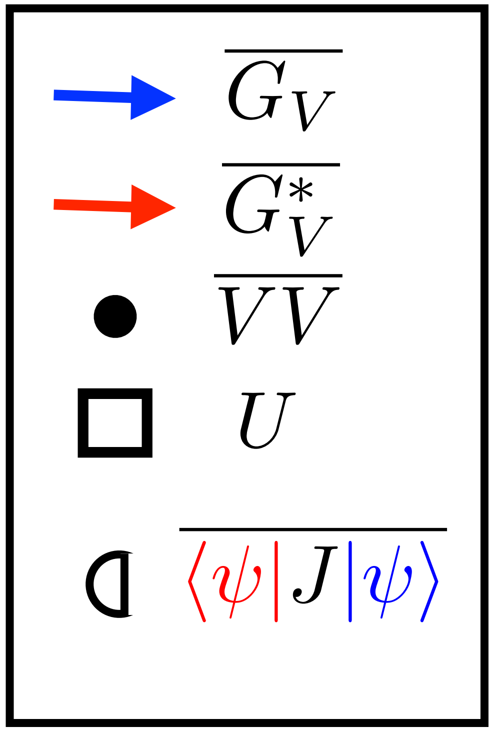

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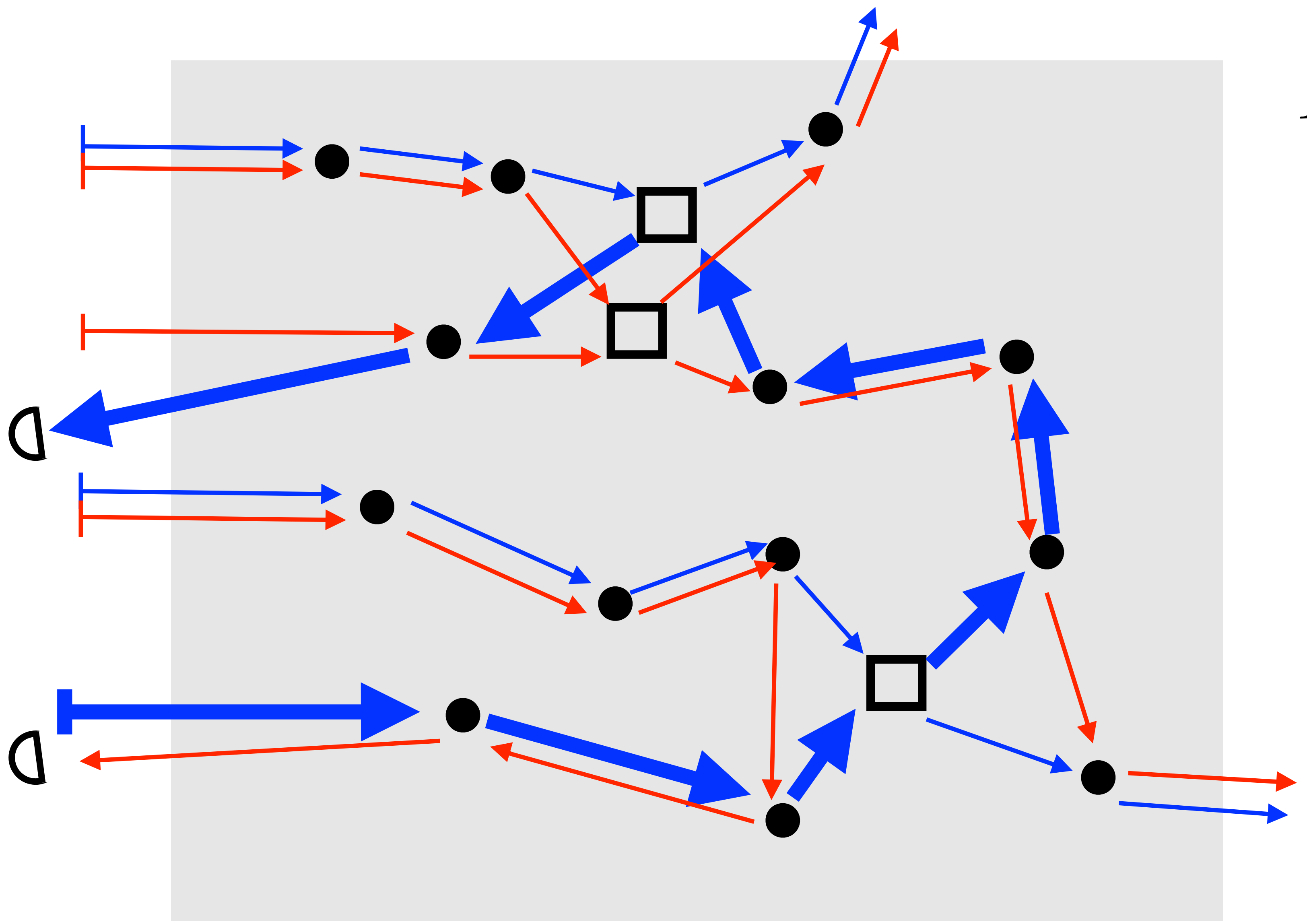

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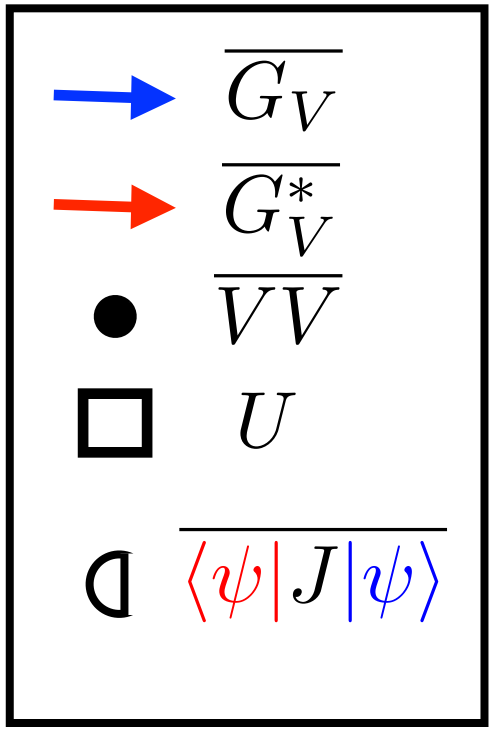

T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. ***15****, 115015 (2013)*

N-particle scattering: diagrammatic approach

Crossed diagrams

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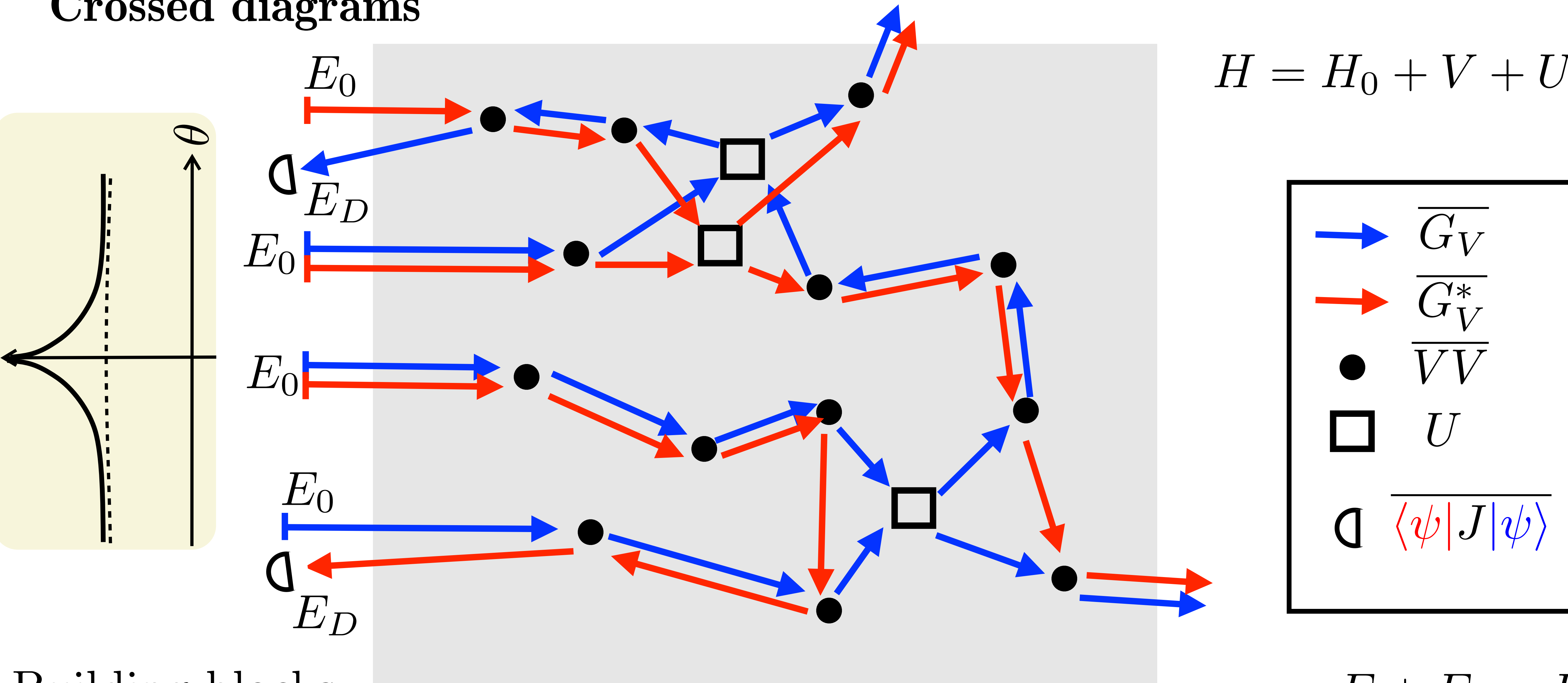
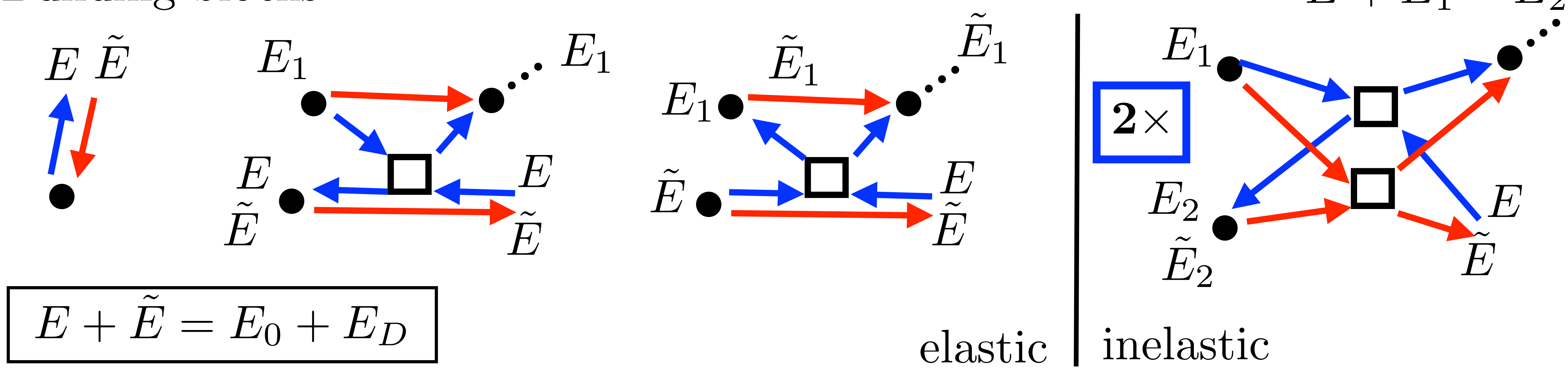
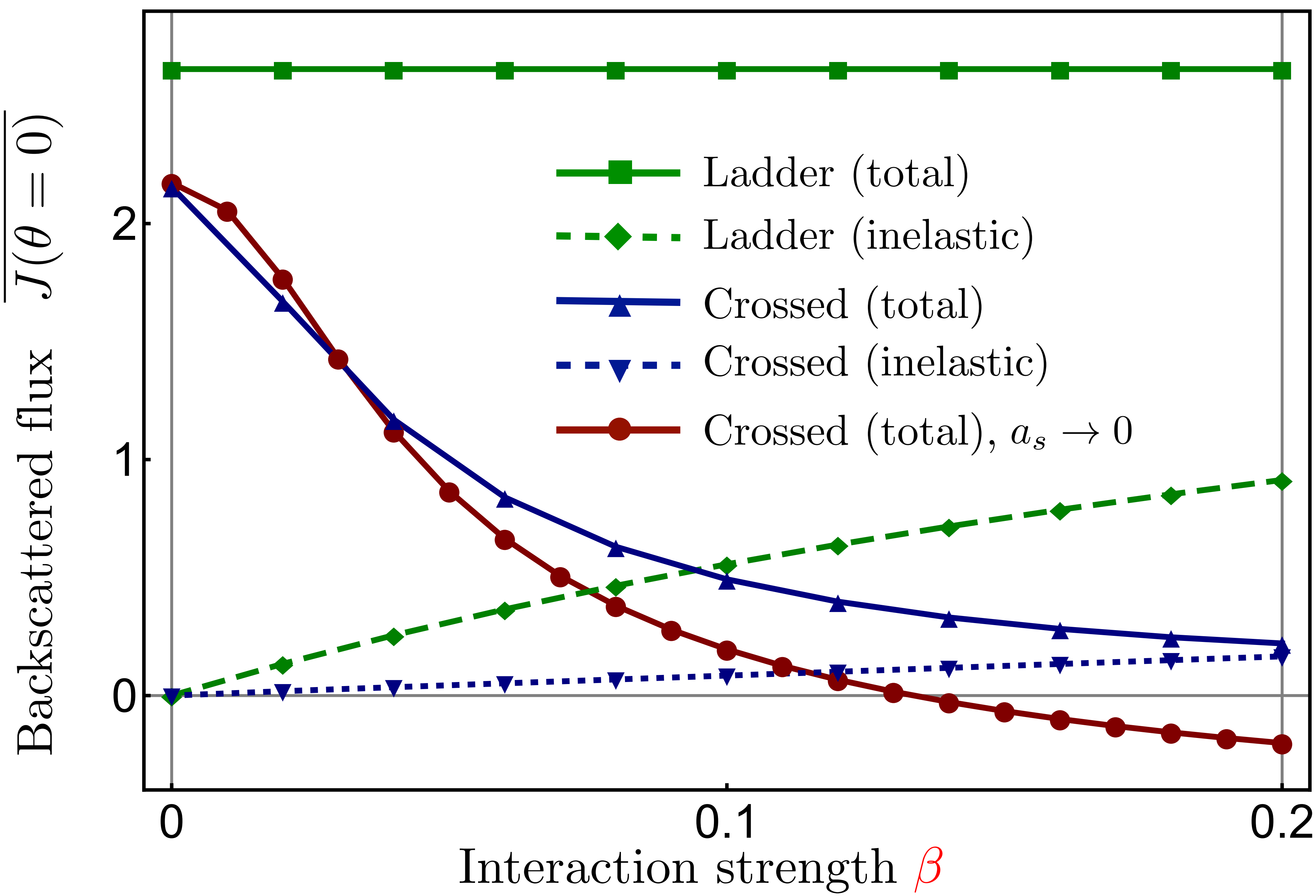


Building blocks



$$E + \tilde{E} = E_0 + E_D$$

N-particle coherent backscattering



$$\beta = \frac{8\pi a_s \ell \rho_0}{k_0}$$

$$k_0 a_s = 1/10$$

$$k_0 \ell = 10$$

$$L = 5\ell$$

- Weak anti-localization for $\beta > 0.13$ in mean field limit ($a_s \to 0$)

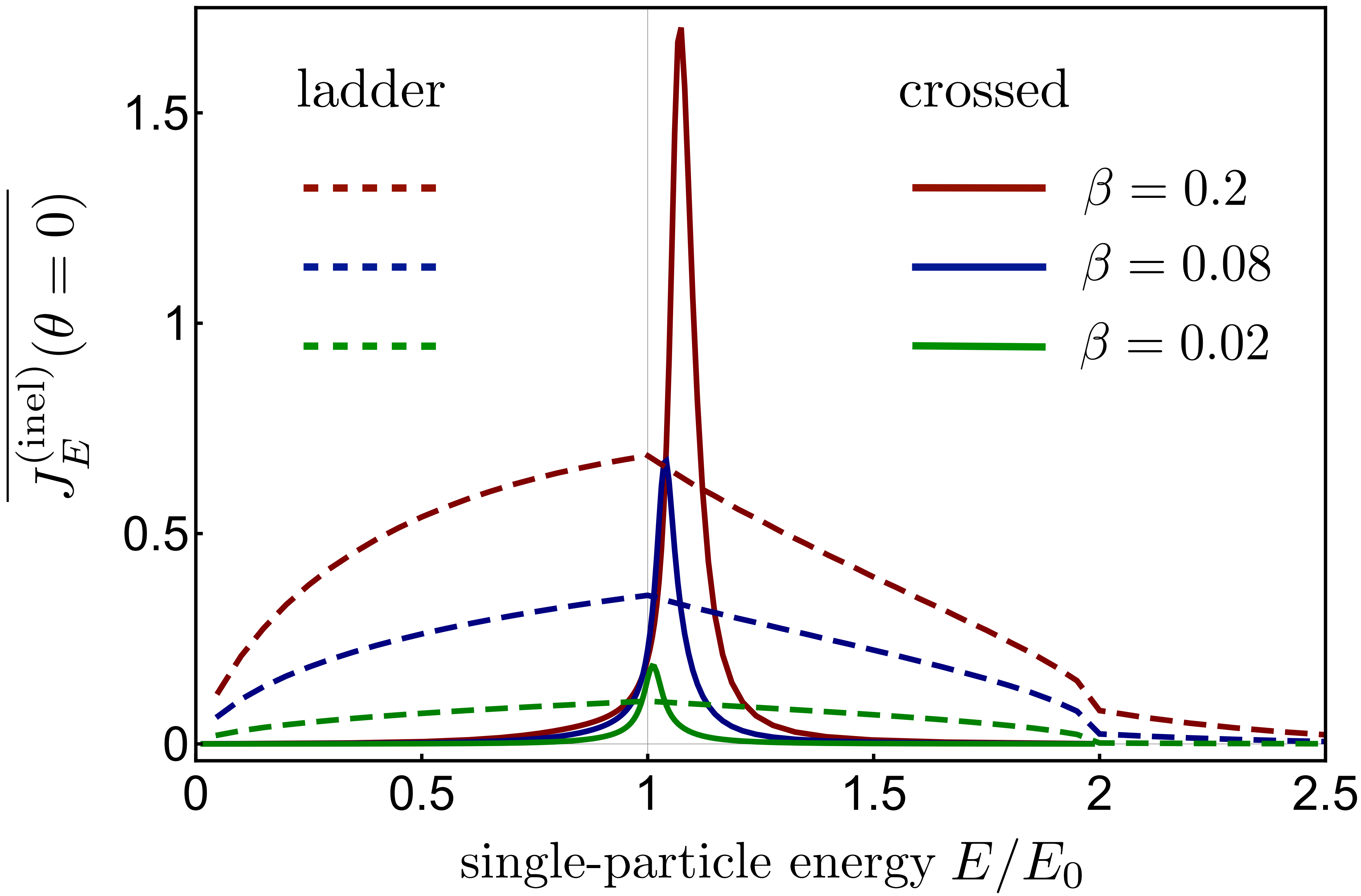
T. Wellens, Appl. Phys. B ***95****, 189 (2009); T. Hartmann et. al., Ann. Phys.* ***327****, 1998 (2012)*

- Decrease of CBS for larger β slowed down by inelastic collisions

T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. ***15****, 115015 (2013)*

N-particle coherent backscattering

Inelastic spectral crossed and ladder flux density at $\theta = 0$



$$\beta = \frac{8\pi a_s \ell \rho_0}{k_0}$$

$$k_0 a_s = 1/10$$

$$k_0 \ell = 10$$

$$L = 5\ell$$

CBS enhancement factor >2 in certain spectral windows

T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. **15**, 115015 (2013)

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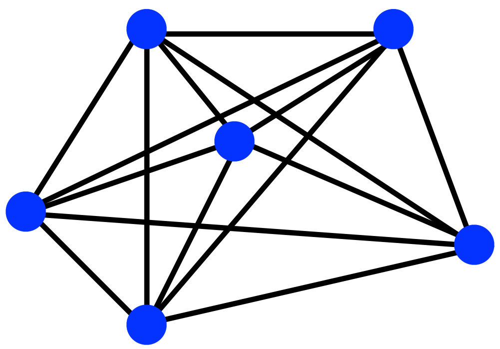
Motivation: Transport on random quantum networks

$$\boxed{H = \sum_{m \neq n} V(\mathbf{r}_m - \mathbf{r}_n) |m\rangle\langle n|}$$

$\mathbf{r}_m$, $\mathbf{r}_n$: random positions of sites m, n



$$P(\mathbf{r}', \mathbf{r}, t) = \overline{\sum_{mn} \left\langle m \left| e^{-iHt/\hbar} \right| n \right\rangle \left\langle n \left| e^{iHt/\hbar} \right| m \right\rangle \delta(\mathbf{r}' - \mathbf{r}_m) \delta(\mathbf{r} - \mathbf{r}_n)}$$

$\overline{(\dots)}$: average over random positions

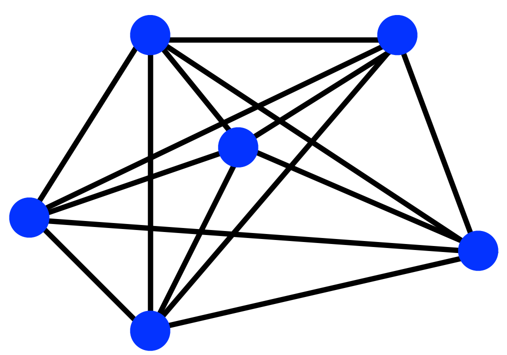
- **Character of transport: diffusion or localization?**
- **Theoretical prediction of relevant transport quantities?**

$(D,\ L_{\text{loc}}, \dots)$

Motivation: Transport on random quantum networks

$$H = \sum_{m \neq n} V(\mathbf{r}_m - \mathbf{r}_n) |m\rangle\langle n|$$

$\mathbf{r}_m$, $\mathbf{r}_n$: random positions of sites m, n

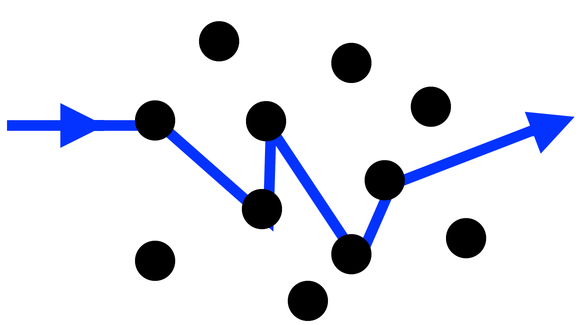


$$P(\mathbf{r}', \mathbf{r}, t) = \overline{\sum_{mn} \langle m | e^{-iHt/\hbar} | n \rangle \langle n | e^{iHt/\hbar} | m \rangle \, \delta(\mathbf{r}' - \mathbf{r}_m) \delta(\mathbf{r} - \mathbf{r}_n)}$$

$\overline{(\ldots)}$: average over random positions

- **Character of transport: diffusion or localization?**
- **Theoretical prediction of relevant transport quantities?** $(D,\ L_{\mathrm{loc}}, \ldots)$

Different model: $H = \dfrac{\hbar^2 k^2}{2m} + V(\mathbf{r})$ (particle in a random potential)



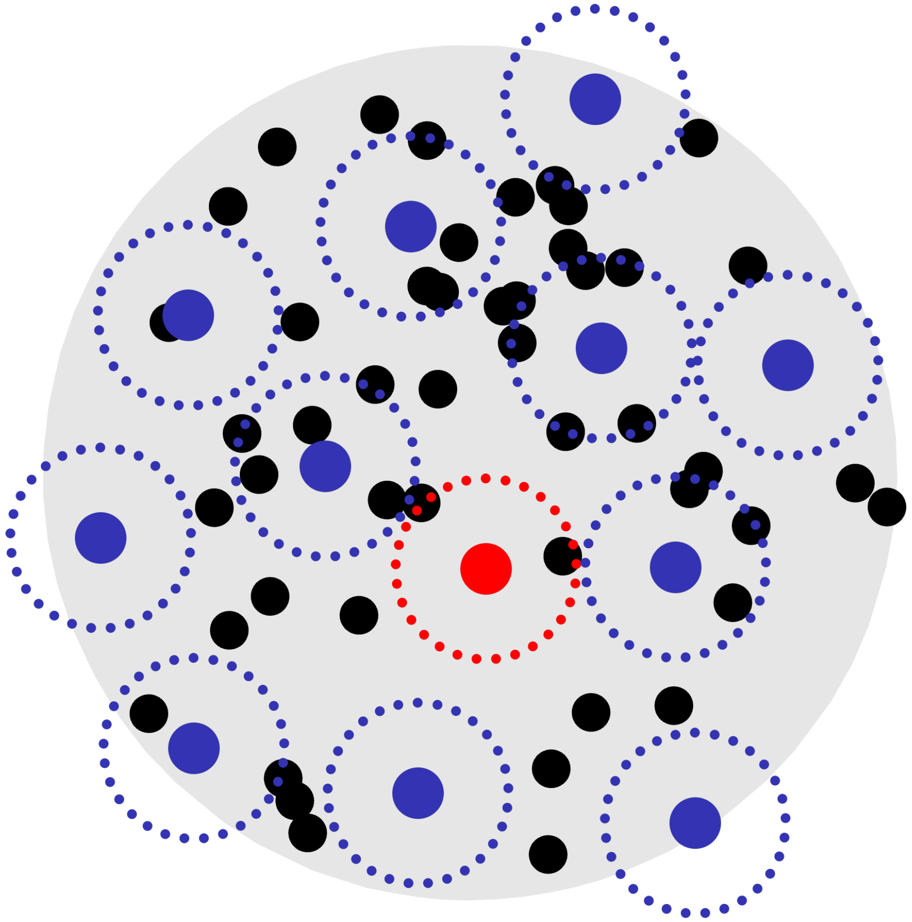
classical diffusive transport if $1/(k\ell) \ll 1$ (in 3D)

- **Analogous criterion for discrete networks?**

Excitation transfer in a disordered cloud of Rydberg atoms

- **frozen cloud** with N randomly placed Rydberg atoms
- **Rydberg blockade**: atoms are spheres with radius $r_b/2$
- **two Rydberg states**

$$|\mathrm{P}\rangle = |n\mathrm{P}_{3/2,m_j=3/2}\rangle$$
$$|\mathrm{S}\rangle = |n\mathrm{S}_{1,m_j=1/2}\rangle$$



- **a single** P in a sea of $(N-1)$S:

$$|i\rangle = |\mathrm{S}\rangle_1 \dots |\mathrm{S}\rangle_{i-1} |\mathrm{P}\rangle_i |\mathrm{S}\rangle_{i+1} \dots |\mathrm{S}\rangle_N$$

- **Hamiltonian:**

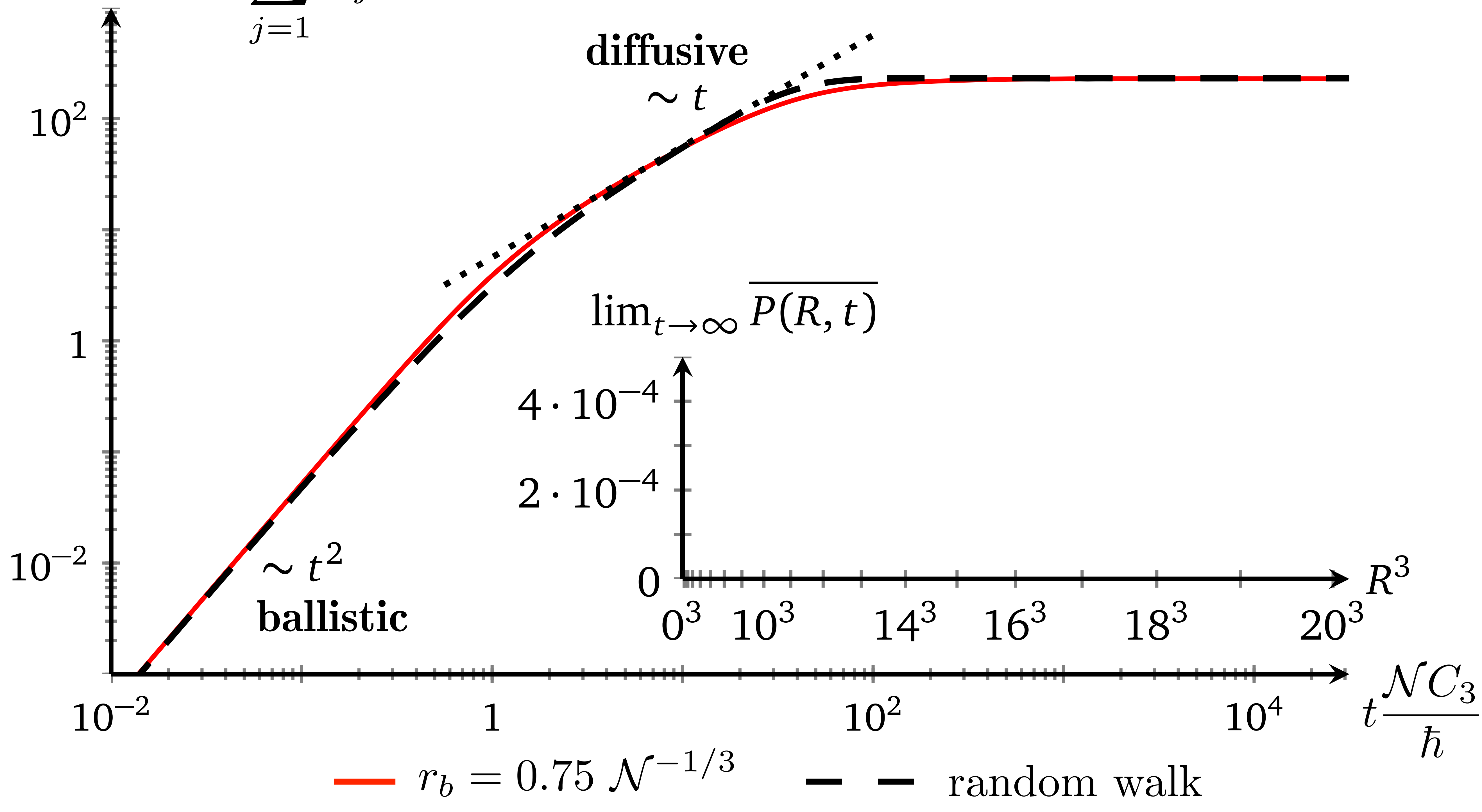
$$H = C_3 \sum_{i \neq j} \frac{3\left(\hat{\mathbf{R}}_{ij} \cdot \hat{\mathbf{Z}}\right)^2 - 1}{R_{ij}^3} |i\rangle\langle j|$$

G. Günther et. al., Science ***342****, 954 (2013)*

Excitation energy transport

- **mean squared displacement**

$$\overline{R^2(t)} = \sum_{j=1}^{N} \overline{R_{ji}^2 |\langle j| \exp(-iHt)|i\rangle|^2} \qquad N = 31623$$



Excitation energy transport

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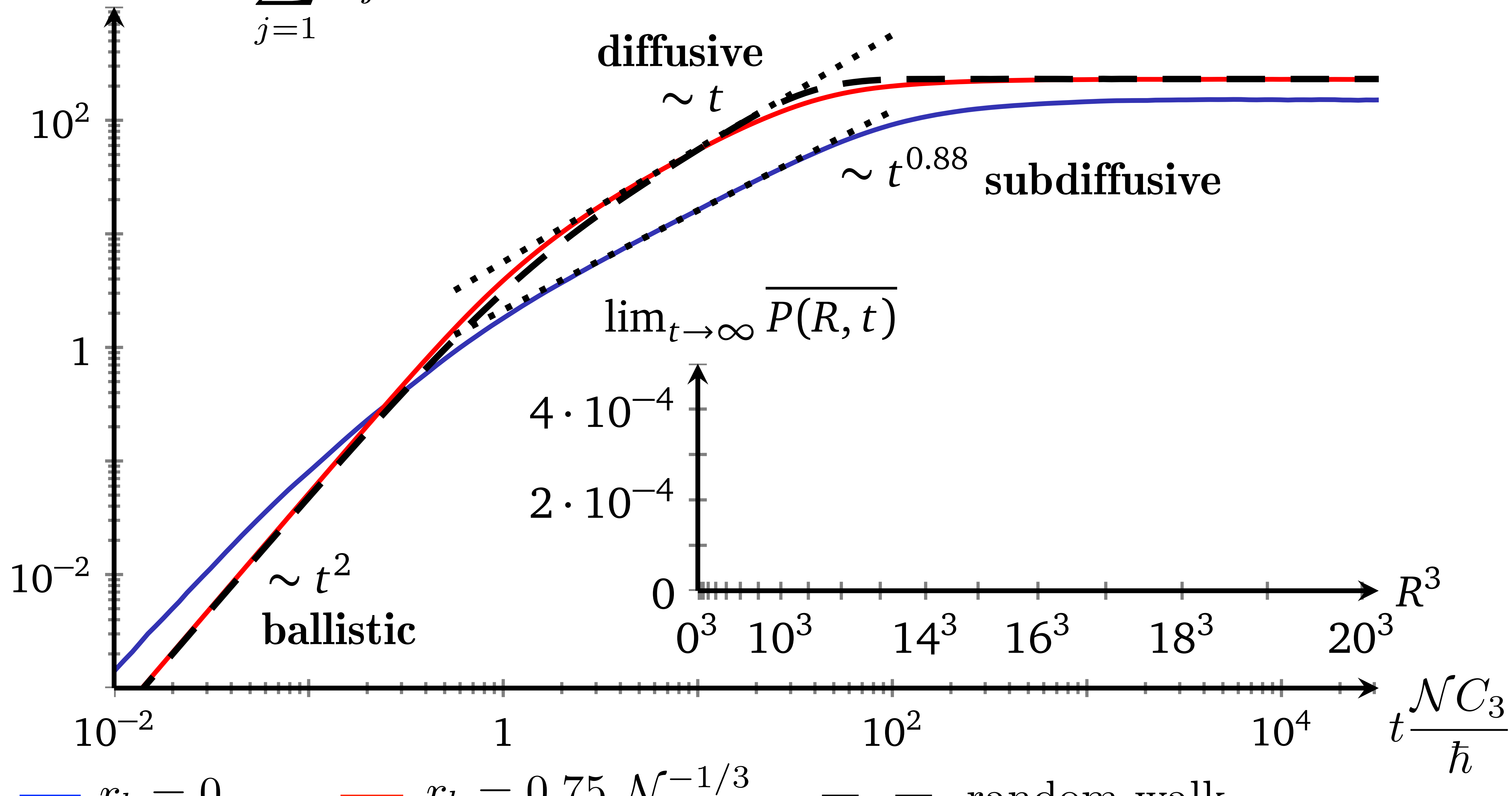


Microscopic theory of excitation transfer

Locator expansion:

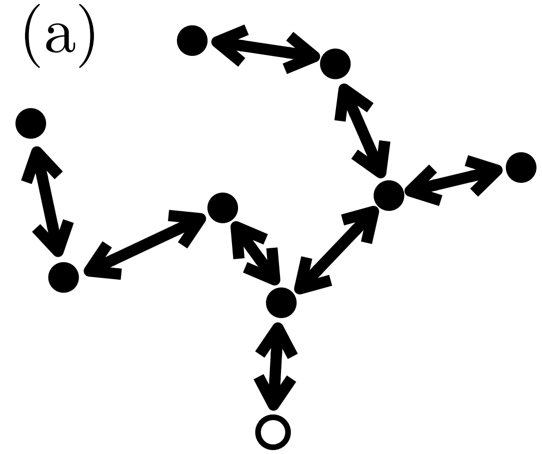
$$\frac{1}{z-\mathcal{H}} = \frac{1}{z} + \frac{1}{z}\mathcal{H}\frac{1}{z} + \frac{1}{z}\mathcal{H}\frac{1}{z}\mathcal{H}\frac{1}{z} + \ldots$$

Diagrams

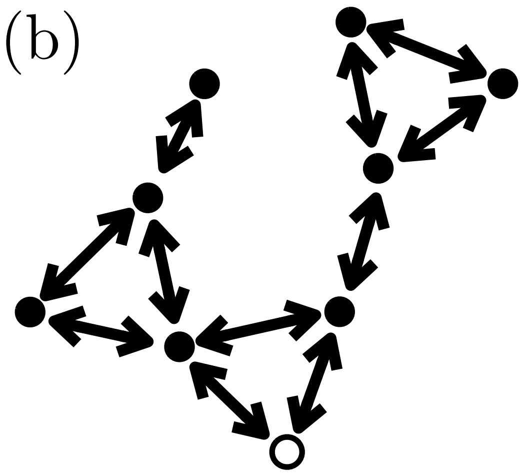
• $1/z$

→ V_{ij}

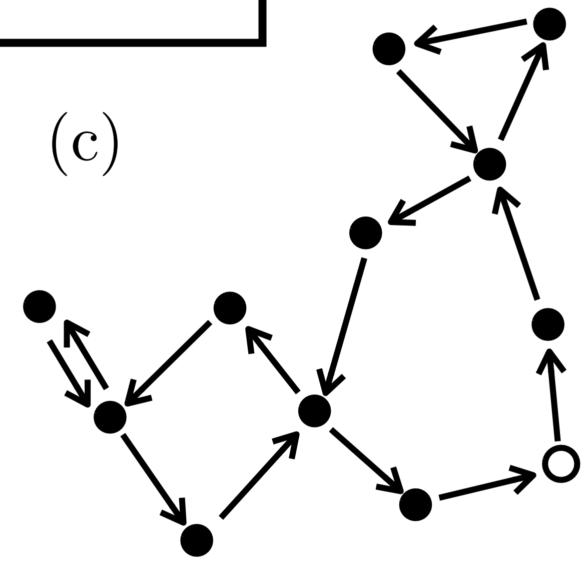
(a)



(b)



(c)

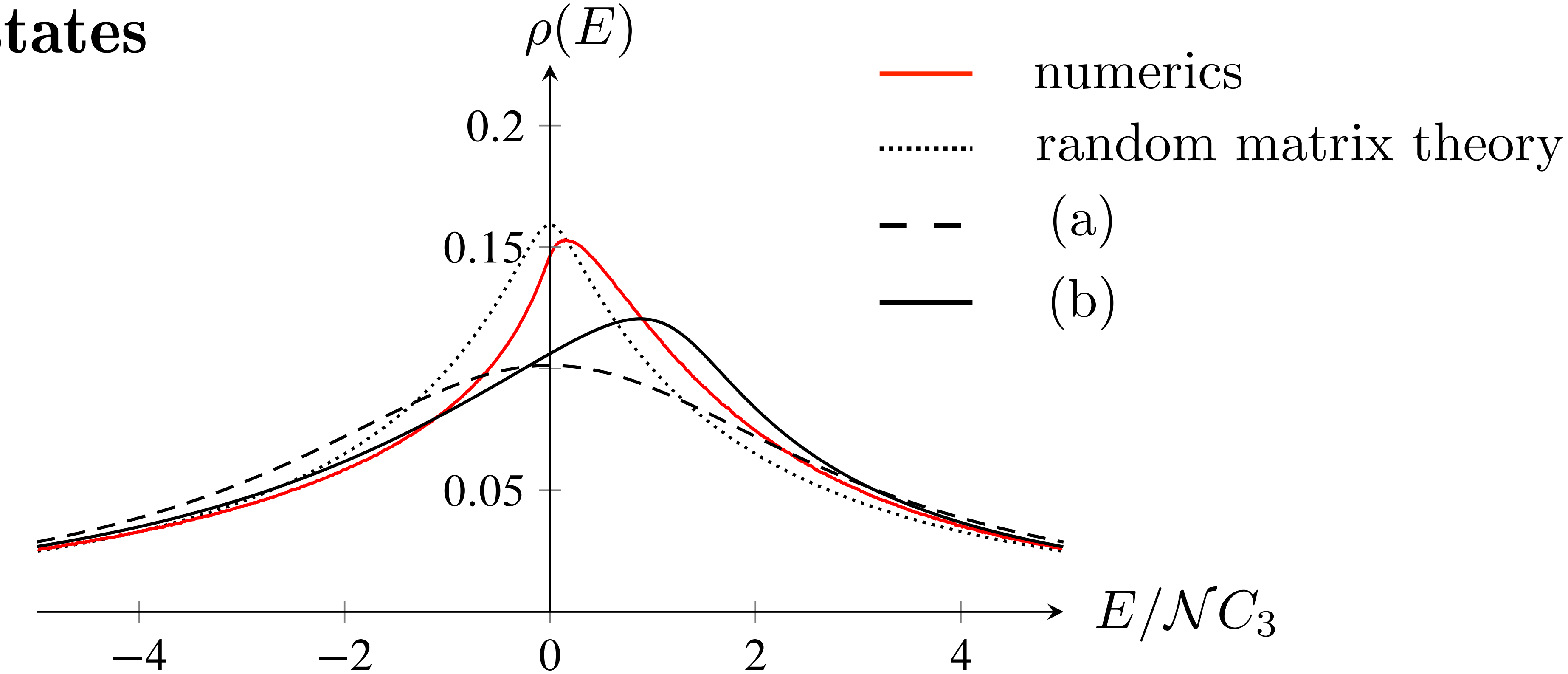


Elyutin (1981)

Matsubara/Toyozawa (1961)

Density of states

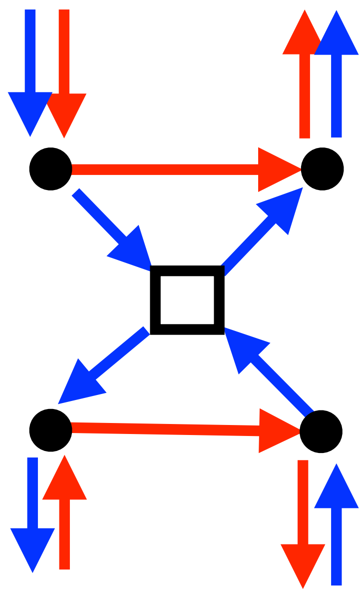
$r_b = 0$



*T. Scholak, T. Wellens, and A. Buchleitner, PRA **90**, 063415 (2014)*

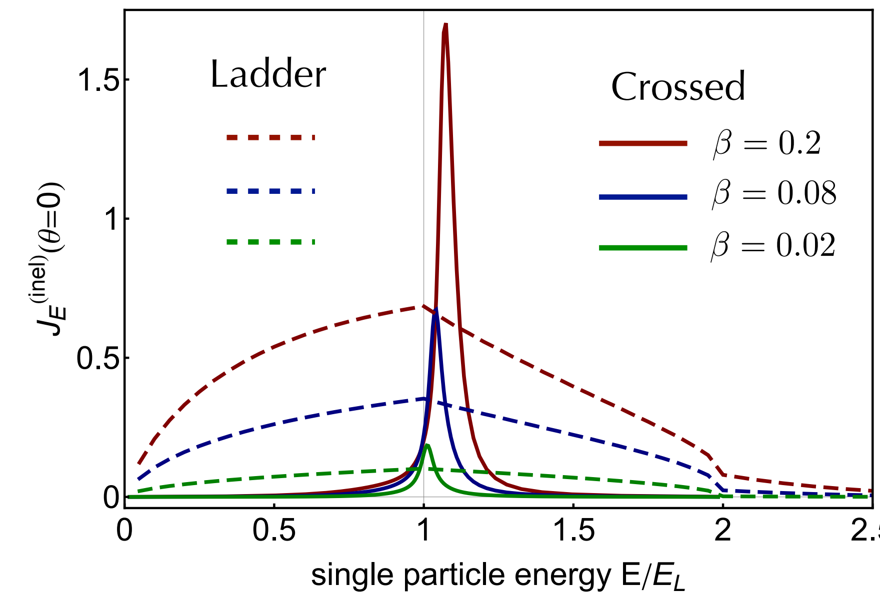
Conclusions

- Scattering theory for interacting bosons in weak random potentials



– inelastic collisions
  $\rightsquigarrow$ thermalization

– effect of interactions on coherent backscattering



T. Geiger, T. Wellens, A. Buchleitner, PRL ***109****, 030601 (2012)*

T. Geiger, A. Buchleitner, T. Wellens, New J. Phys. ***15****, 115015 (2013)*

- Character of excitation transport in disordered ultra-cold Rydberg gases controllable via blockade radius r_b and density $\mathcal{N}$:

  – $r_b \sim \mathcal{N}^{-1/3}$: diffusive (limited by finite size)

  – $r_b \ll \mathcal{N}^{-1/3}$: subdiffusive (coherent transport)

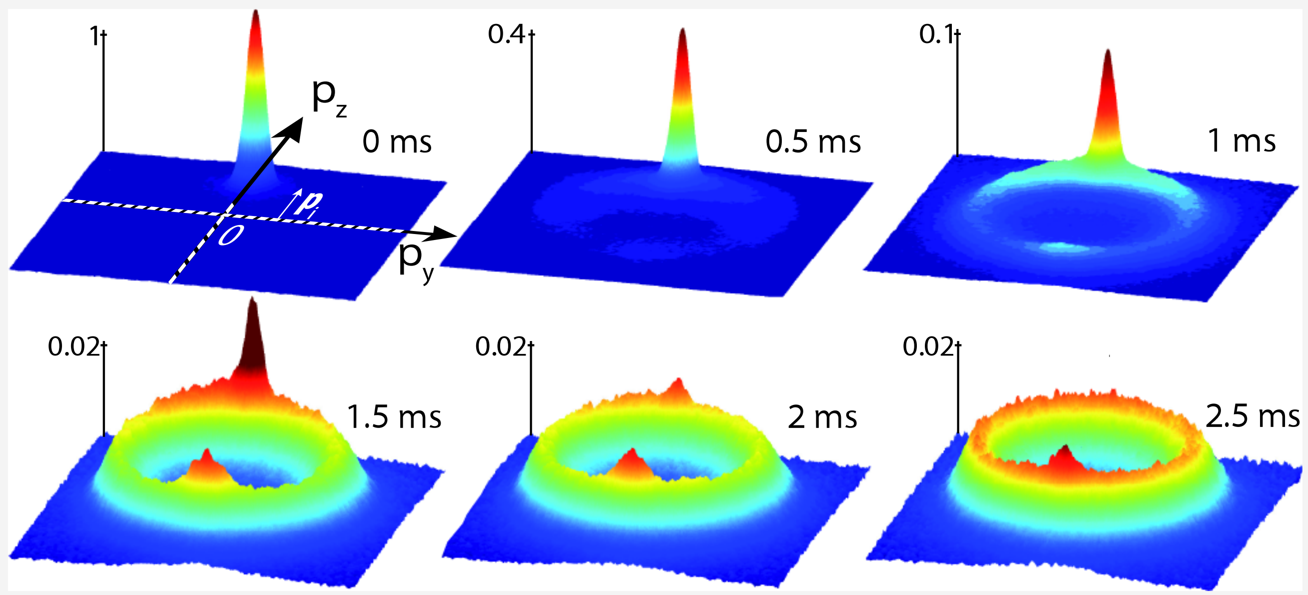
T. Scholak, T. Wellens, and A. Buchleitner, PRA ***90****, 063415 (2014)*

Conclusions

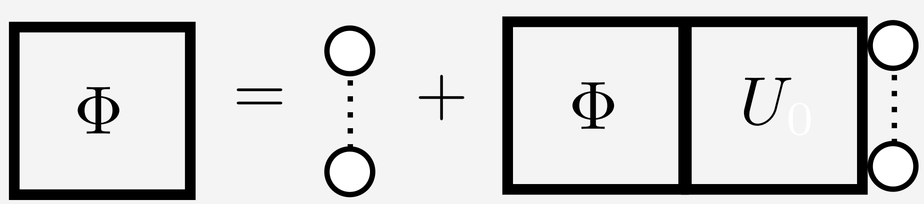
- Scattering theory for interacting bosons in weak random potentials

Outlook

- Interacting bosons: time-dependent scenario



- Excitation transport in Rydberg gases: diagrammatic theory



*T. Wellens and R. A. Jalabert, PRA **94**, 144209 (2016)*

*T. Scholak, T. Wellens, and A. Buchleitner, PRA **90**, 063415 (2014)*