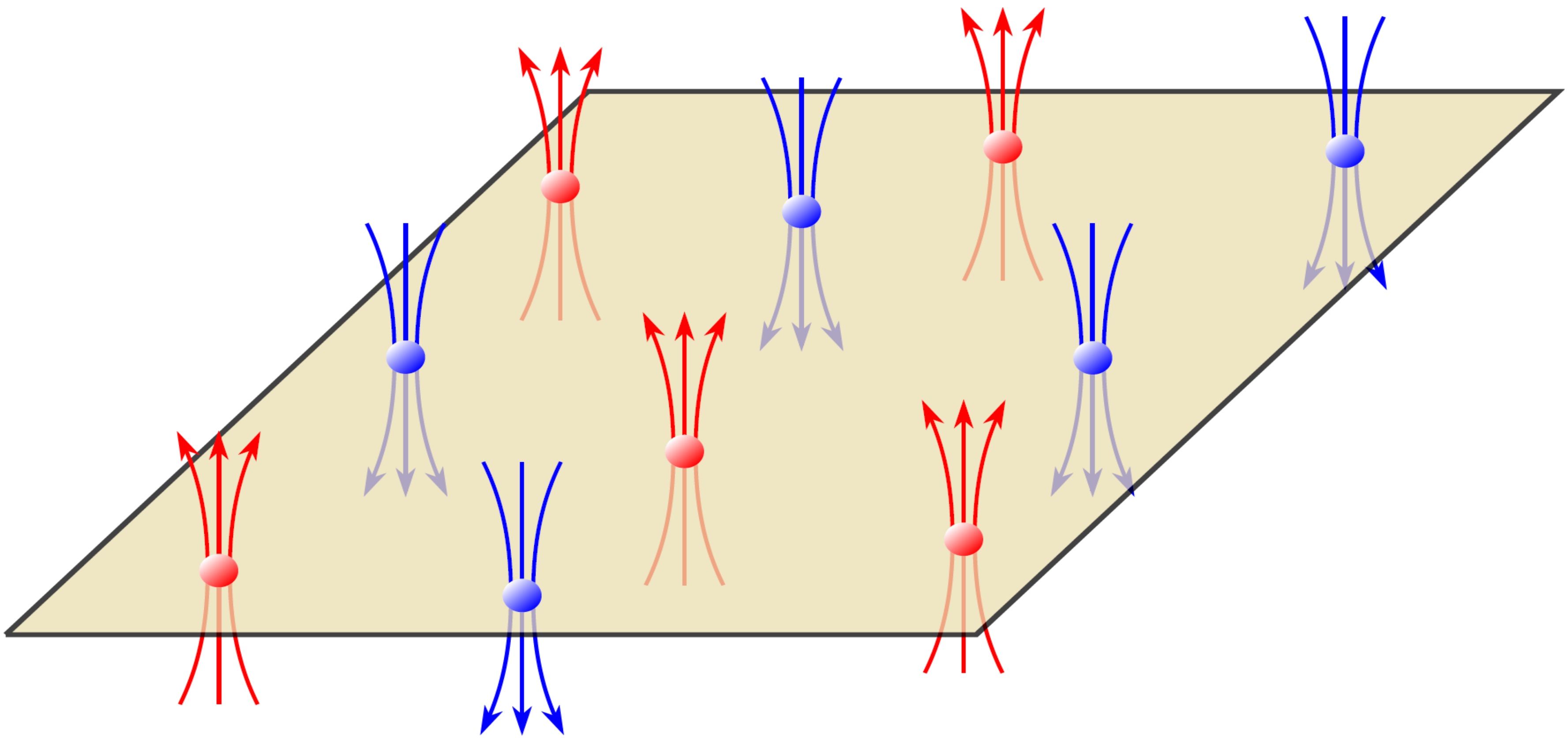
Fractional Excitonic Insulator

Charles Kane
University of Pennsylvania



Fractional Excitonic Insulator

I. Introduction : Quantum Hall Effect

- Fractional Chern Insulator
- Band Inversion paradigm

II. Fractional Excitonic Insulator:
A correlated fluid of electrons and holes

- Variant on Laughlin wavefunction
- Composite fermion theory
- Exact Hamiltonian

III. Higher angular momentum band inversion

- A route to FEI ?
- A target for band structure engineering
- excitonic phases and mean field theory

Yichen Hu

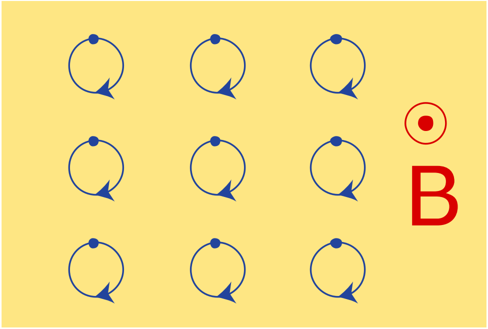
Jörn Venderbos

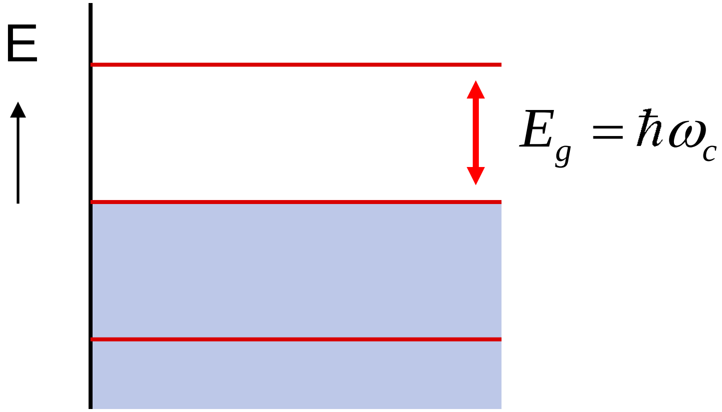
Hu, Venderbos and Kane, PRL **121**, 126601 (2018).

Venderbos, Hu and Kane, PRB **98**, 235160 (2018).

Integer Quantum Hall Effect

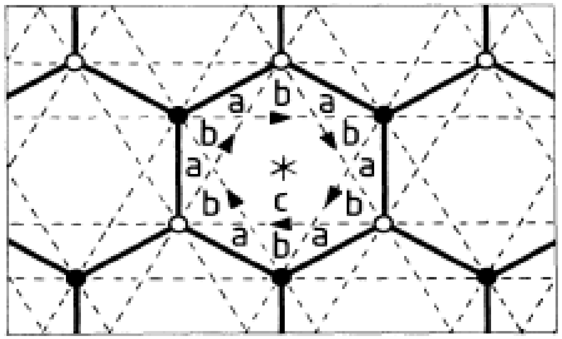
Landau levels

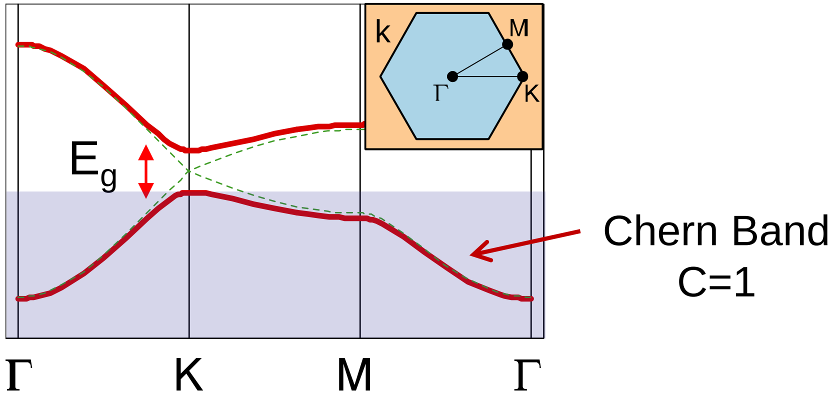




Chern Insulator: B=0

e.g. Haldane 1988





Fractional Quantum Hall Effect

Laughlin State

Fractionally filled Landau level: $\nu = 1/m$

$$\Psi(\{z_i\}) = \prod_{i<j} (z_i - z_j)^m e^{-\sum_i |z_i|^2/4\ell_B^2}$$

- Strongly correlated incompressible quantum fluid with bulk energy gap
- $\sigma_{xy} = (1/m)e^2/h$
- Fractional charge/statistics

Fractional Chern Insulator

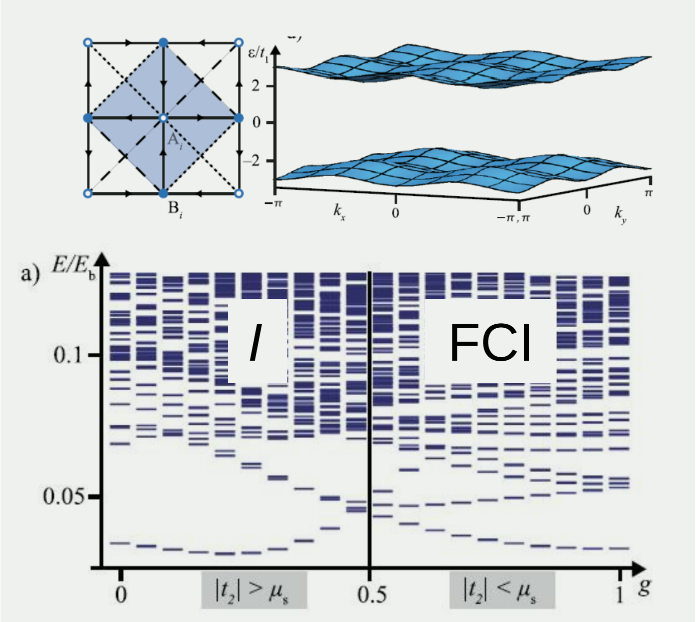
Tang, et al. '11 ; Neupert et al. '11
Sun et al. '11 ; Regnault, Bernevig '11

Fractionally filled Chern band at B=0

Requires:

- Nearly flat band:
  bandwidth < interaction energy
- Fractional band filling: $\nu = 1/m$

Numerical evidence in model systems

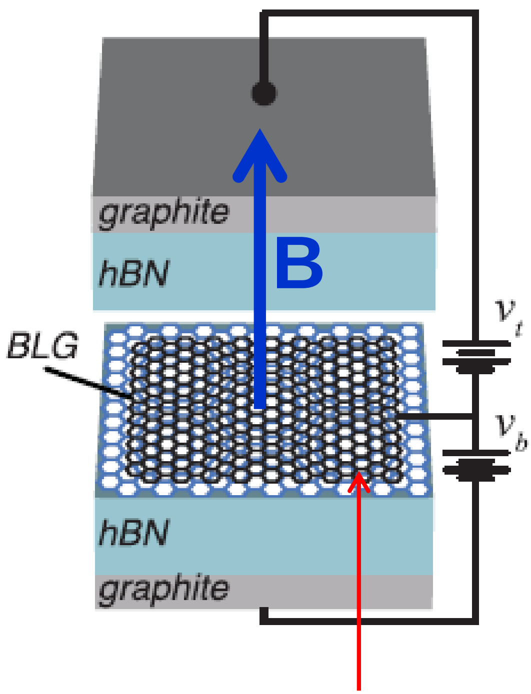


Neupert et al. '11

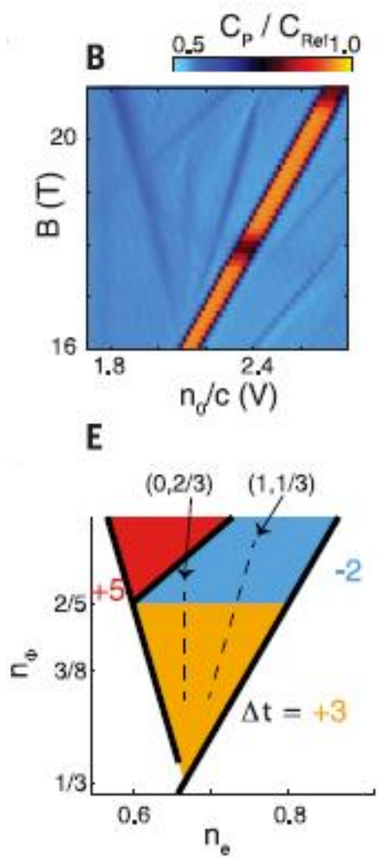
Fractional quantum Hall states observed in moiré superlattices of bilayer graphene

Spanton, …, Young, Science 2018

“Hofstadter Chern Insulator”: Finite B, periodic potential



periodic potential
due to moiré pattern

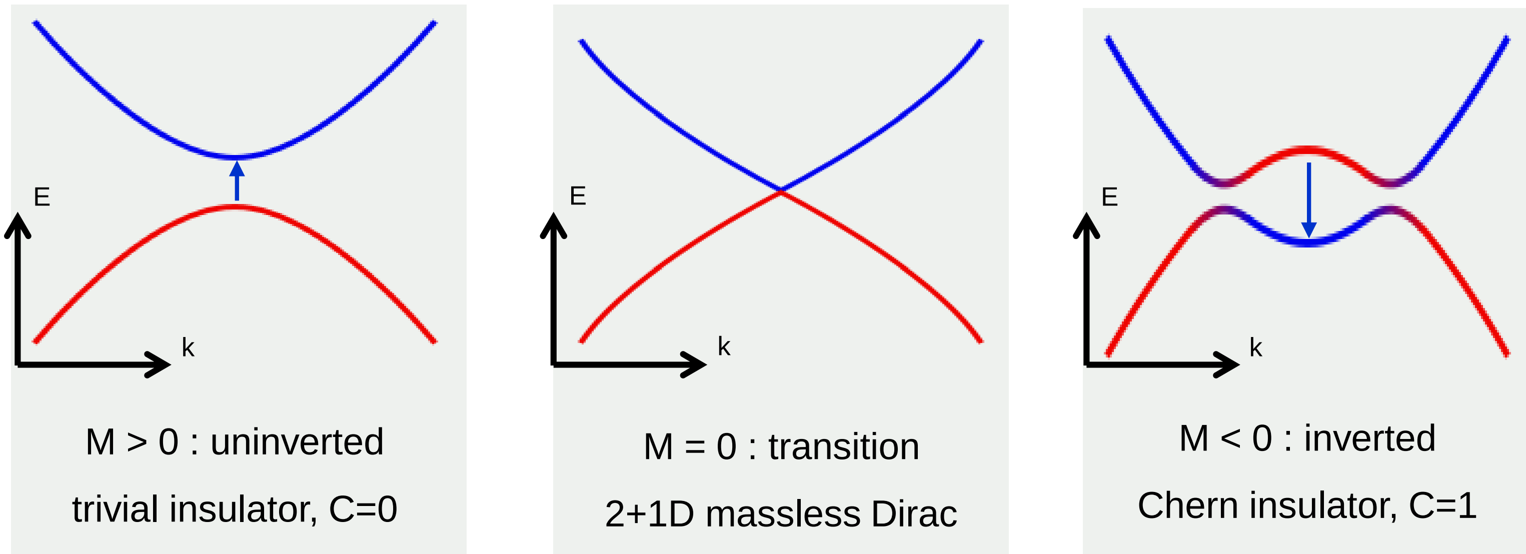


penetration field capacitance C_p:

probe of bulk gap

Identify incompressible quantum Hall states in fractionally filled Hofstadter-Chern bands.

Band Inversion Paradigm



- "opposite" of the flat Landau level limit
- Ground state topology determined by low energy states near Fermi energy
- Two band k·p model: simplest theory of topological transition

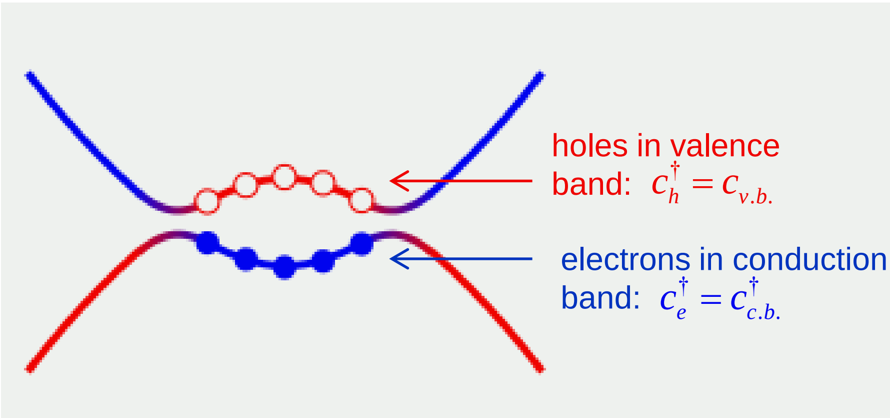
$$H = \left(M + k^2\right)\sigma_z + \mathrm{v}\left(k_x\sigma_x + k_y\sigma_y\right)$$

Dirac mass

large k regularization

Read, Green '01;
Bernevig, Hughes, Zhang '07;
… many others

Electron – Hole Fluid



$$H = \sum_k \varepsilon_k \left(c_{ke}^\dagger c_{ke} + c_{kh}^\dagger c_{kh} \right) + \Delta_k c_{-ke}^\dagger c_{kh}^\dagger + h.c.$$

$$\varepsilon_k = k^2 + M \qquad \Delta_k = \mathrm{v}(k_x + ik_y)$$

p+ip "excitonic pairing"

"BCS" wavefunction $\quad |\Phi_0\rangle = \prod_k \left(u_k + v_k c_{-ke}^\dagger c_{kh}^\dagger \right) |0\rangle \quad \longleftarrow$ vacuum = filled valence band

p+ip excitonic condensate * $\quad |\Phi_0\rangle \propto e^{\int dz dw\, g(z-w) \psi_e^\dagger(z) \psi_h^\dagger(w)} |0\rangle \quad \left(g_k = \frac{v_k}{u_k} = \frac{\Delta_k}{\varepsilon_k + \sqrt{\varepsilon_k^2 + |\Delta_k|^2}} \right)$

(* no spontaneously broken symmetry)

pair wavefunction

$$g(z = x + iy \to \infty) \sim \begin{cases} M > 0: & e^{-|z|/\xi} \quad \text{"strong paired"} \quad \left(g_{k\to 0} \sim \frac{\mathrm{v}(k_x + ik_y)}{2M} \right) \\ M < 0: & \frac{1}{z} \quad \text{"weak paired"} \quad \left(g_{k\to 0} \sim \frac{2M}{\mathrm{v}(k_x + ik_y)} \right) \end{cases}$$

Fractional Excitonic Insulator

A strongly correlated fluid of electrons and holes
leading to B=0 "anomalous FQHE" at ν=1/m

Proposed ground state: (see also Dubail and Read PRB 2015)

$$\left|\Psi_m\right\rangle = \sum_{N=1}^{\infty} \frac{f^N}{N!}\left|\Psi_m^N\right\rangle$$

$\left|\Psi_m^N\right\rangle =$ wave function for N electrons at z_i
N holes at w_j

$$\Psi_m^N\left(\left\{z_i, w_j\right\}\right) = \frac{\prod_{i<i'}(z_i - z_{i'})^m \prod_{j<j'}(w_j - w_{j'})^m}{\prod_{i,j}(z_i - w_j)^m}$$

Similar to Laughlin (or Halperin bilayer) wavefunction, except :

- No Gaussian factor
- Singular denominator

m=1 : Non interacting Chern insulator

$|\Psi_1\rangle$ is the exact ground state of a simple free fermion model.

$$H = \sum_k \varepsilon_k \left(c^\dagger_{ke} c_{ke} + c^\dagger_{kh} c_{kh} \right) + \Delta_k c^\dagger_{-ke} c^\dagger_{kh} + h.c.$$

$$\varepsilon_k = \frac{1}{2}\left(k^2 - \mathrm{v}^2\right) \qquad \Delta_k = \mathrm{v}(k_x + ik_y)$$

$$E_k = \sqrt{\varepsilon_k^2 + |\Delta_k|^2} = \frac{1}{2}\left(k^2 + \mathrm{v}^2\right)$$

$$g_k = \frac{v_k}{u_k} = \frac{\mathrm{v}}{k_x + ik_y}$$

$$g(z = x + iy) = \frac{\mathrm{v}}{2\pi z}$$

$$|\Phi_0\rangle \propto e^{\int dz dw g(z-w) \psi_e^\dagger(z) \psi_h^\dagger(w)} |0\rangle = \sum_{N=1}^{\infty} \frac{f^N}{N!} \det\left[\frac{1}{z_i - w_j}\right] \left| \{z_i, w_j\} \right\rangle \qquad \left(f = \frac{\mathrm{v}}{2\pi} \right)$$

Cauchy Determinant Identity:

$$\det\left[\frac{1}{z_i - w_j}\right] = \frac{\prod_{i<i'}(z_i - z_{i'}) \prod_{j<j'}(w_j - w_{j'})}{\prod_{i,j}(z_i - w_j)}$$

Conclude $|\Phi_0\rangle = |\Psi^0_{m=1}\rangle$ with f = v/2π.

Ground state wavefunction as a CFT correlator

Laughlin State

$$\Psi_m^N\left(\{z_i\}\right)=\left\langle\left(\prod_{i=1}^{N}e^{im\varphi(z_i)}\right)O_{background}\right\rangle=\prod_{i<i'}(z_i-z_{i'})^m e^{-\sum_{i=1}^{N}|z_i|^2/4}$$

Fractional Excitonic Insulator

$$\Psi_m^N\left(\{z_i,w_i\}\right)=\left\langle\left(\prod_{i=1}^{N}e^{im\varphi(z_i)}\right)\left(\prod_{j=1}^{N}e^{-im\varphi(w_j)}\right)\right\rangle=\frac{\prod_{i<i'}(z_i-z_{i'})^m\prod_{j<j'}(w_j-w_{j'})^m}{\prod_{i,j}(z_i-w_j)^m}$$

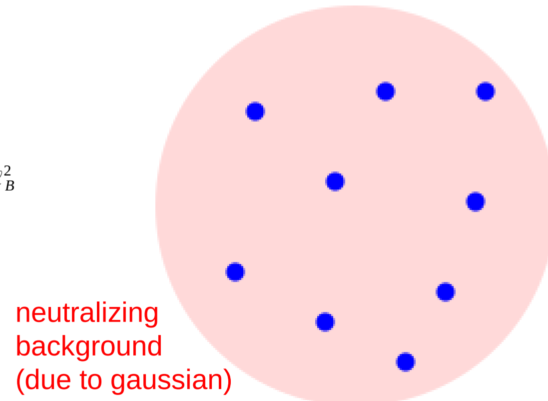
Plasma Analogy

(Dubail, Read 2015)

$\langle\Psi_m|\Psi_m\rangle$ ~ partition function for classical plasma $\quad \beta V = \sum_{i<j} 2m\, q_i q_j \log\frac{\xi}{|z_i - z_j|}$

Laughlin plasma

$$\prod_{i<j}(z_i - z_j)^m e^{-\sum_i |z_i|^2/4\ell_B^2}$$

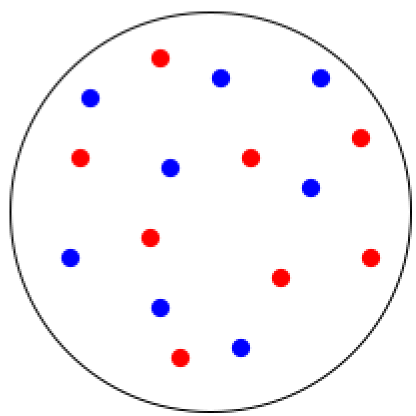


m <~ 11: fluid (plasma screening)

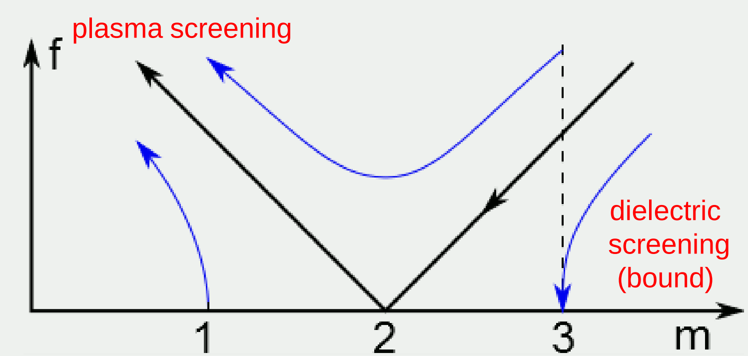
m >~ 11: crystal (dielectric screening)

Electron – hole plasma

$$\frac{\prod_{i<i'}(z_i - z_{i'})^m \prod_{j<j'}(w_j - w_{j'})^m}{\prod_{i.j}(z_i - w_j)^m}$$

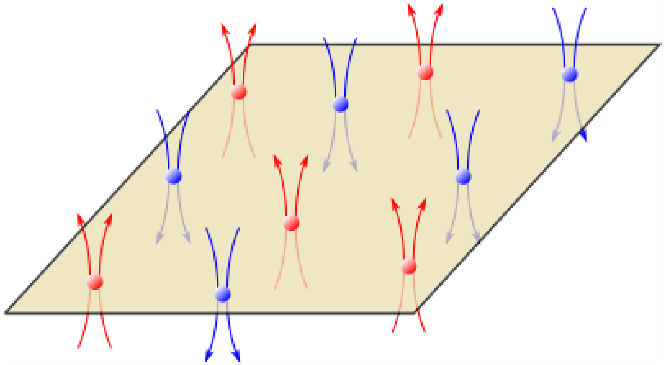


Kosterlitz Thouless Problem



For sufficiently large fugacity f, $|\Psi_m\rangle$ describes a fractional quantum Hall fluid.

Composite Fermion Model



Singular gauge transformation*:

- attach flux $+ (m-1)\, h/e$ to each electron
- attach flux $- (m-1)\, h/e$ to each hole
- $\psi_{e(h)}(\mathbf{r}) = \psi^{CF}_{e(h)}(\mathbf{r}) e^{\pm i(m-1)\Theta(\mathbf{r})}$ $\qquad \nabla \times \nabla\Theta = 2\pi(\psi_e^\dagger \psi_e - \psi_h^\dagger \psi_h)$

Mean field theory:

- If $\langle\rho_e\rangle = \langle\rho_h\rangle$, then $B_{av} = 0$.
- If composite fermions form Chern insulator (C=1), then the original fermions form a fractional quantum Hall fluid

Chern Simons theory:

- $L = L_0[\psi, a+A] + \dfrac{\varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda}{4\pi(m-1)} \quad \rightarrow \quad L = \dfrac{\varepsilon_{\mu\nu\lambda}}{4\pi}\left[(a+A)_\mu \partial_\nu (a+A)_\lambda + \dfrac{a_\mu \partial_\nu a_\lambda}{m-1} \right]$
- $\rightarrow \quad L = \dfrac{\varepsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}{4\pi m} \quad \rightarrow \quad \boxed{\sigma_{xy} = \dfrac{1}{m}\dfrac{e^2}{h}}$

Towards a Hamiltonian

Seek the *exact question to the answer*

The Laughlin state is the exact zero energy eigenstate of a model Hamiltonian with short range repulsive interactions: Haldane 1983, Trugman and Kivelson 1985

Alternative approach for Jastrow type wavefunctions:
C.L. Kane, S.A. Kivelson, D.H. Lee and S.C. Zhang, PRB 1991

- Construct annihilation operators: $Q_{e(h)}(z)\left|\Psi_m\right\rangle = 0$

- $\left|\Psi_m\right\rangle$ is an exact ground state of

$$H_m = \int d^2z \left[Q_e^\dagger Q_e + Q_h^\dagger Q_h \right]$$

- m=1: H_1 is the exact non interacting Chern insulator Hamiltonian.

- m>1: H_m involves (2m-1) body interactions.

Compute $\partial_{z^*}\Psi(\{z_i, w_j\})$,use fact:

$$\partial_{z^*}\frac{1}{(z-w)^m} = \frac{\pi}{(m-1)!}\partial_w^{m-1}\delta^2(z-w)$$

$$Q_{e(h)}(z) = \partial_{z^*}\psi_{e(h)} - \mathrm{v}\left(\partial_z - ia\right)^{m-1}\psi^\dagger_{h(e)}$$

$$a(z) = m\int d^2u \frac{\psi_e^\dagger\psi_e - \psi_h^\dagger\psi_h}{z-u}$$

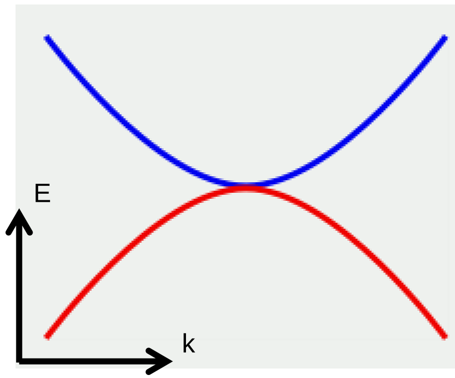
Possible route to Fractional Excitonic Insulator

Turn off interactions in exact Hamiltonian:

$$H = \sum_k k^2 \left(c_{ke}^\dagger c_{ke} + c_{kh}^\dagger c_{kh} \right) + \mathrm{v}(k_x + ik_y)^m c_{-ke}^\dagger c_{kh}^\dagger + h.c.$$

quadratic
band touching

angular momentum
m excitonic pairing



Non interacting ground state :

$$g(z \to \infty) \sim \frac{1}{z^m}$$

$$\left|\Phi_0\right\rangle \propto \sum_{N=1}^{\infty} \frac{1}{N!} \det\left[g(z_i - w_j) \right] \left| \{z_i, w_j\} \right\rangle$$

$$\det\left[\frac{1}{(z_i - w_j)^m} \right] = \frac{P\left(\{z_i, w_j\}\right)}{\prod_{i,j} (z_i - w_j)^m}$$

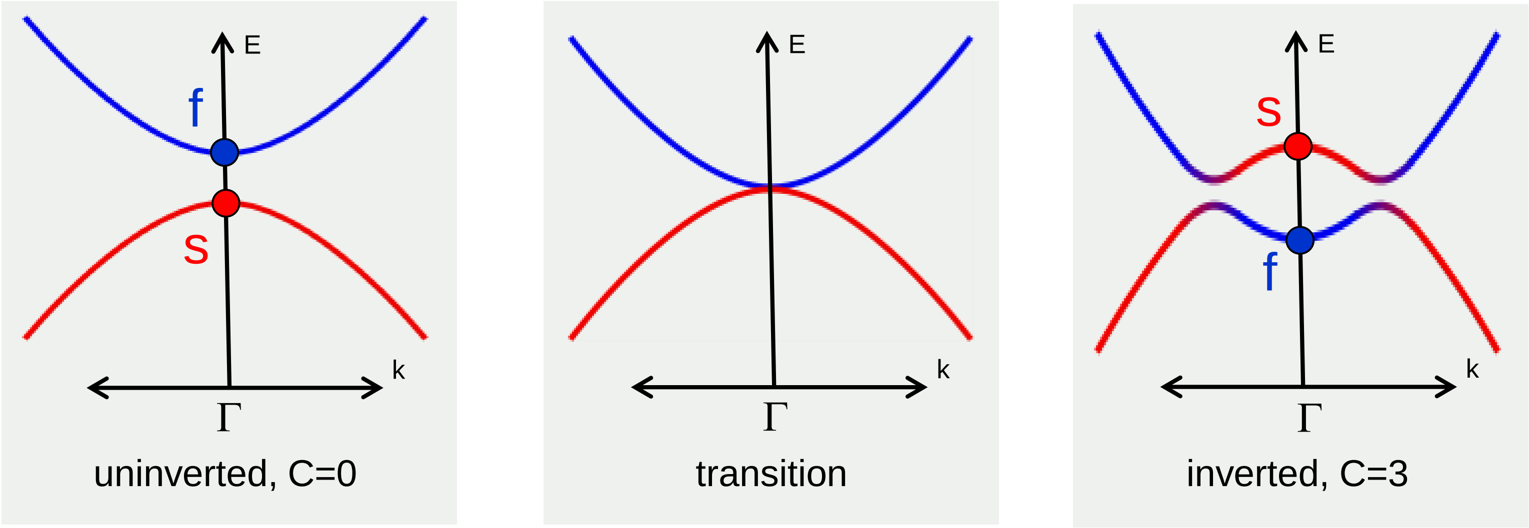
Short range repulsive interactions :

Put all the required zeros of the wave function on top of the particles

$$\frac{P\left(\{z_i, w_j\}\right)}{\prod_{i,j}(z_i - w_j)^m} \to \frac{\prod_{i<i'}(z_i - z_{i'})^m \prod_{j<j'}(w_j - w_{j'})^m}{\prod_{i,j}(z_i - w_j)^m}$$

Higher angular momentum band inversion

In crystal with C_6 rotational symmetry, invert bands that differ in angular momentum by 3 at Γ: e.g. s and f states.



Unconventional topological transition

- Chern number changes by 3 at transition
- Quadratic band touching (with l=3 coupling) at transition
- Expect interactions to be important
- Target for band structure engineering

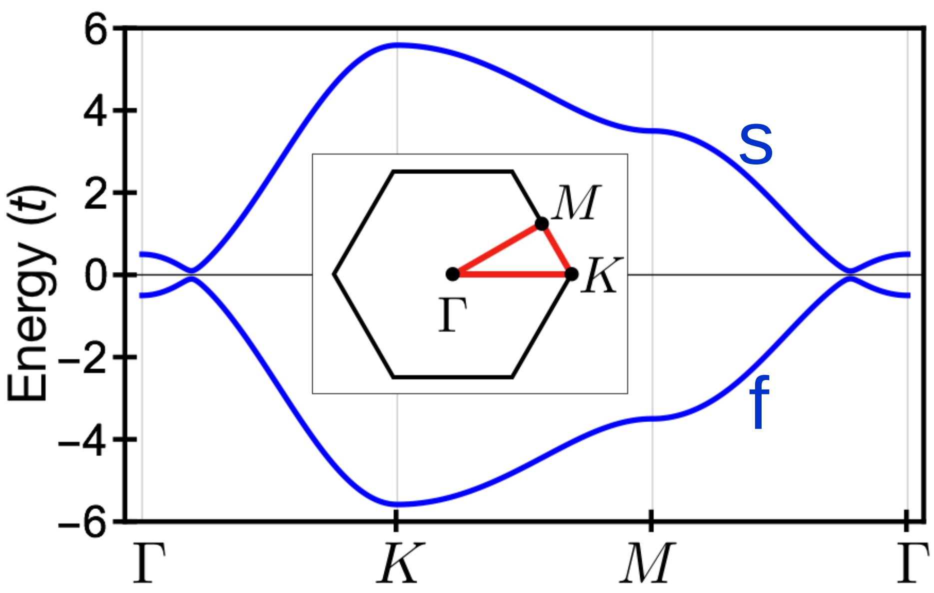
Tight Binding Model

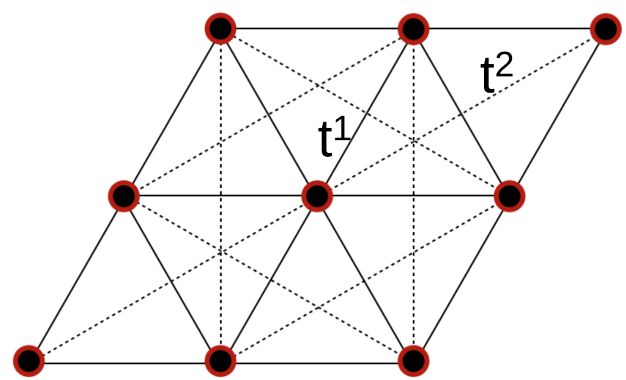
Two band model: triangular lattice with s (m=0) and f (m=3) orbitals

$$H = \sum_{i,j} \begin{pmatrix} s_i^\dagger & f_i^\dagger \end{pmatrix} \begin{pmatrix} h_{ij}^s & \Delta_{ij} \\ \Delta_{ji}^* & h_{ij}^f \end{pmatrix} \begin{pmatrix} s_j \\ f_j \end{pmatrix}$$

$$h_{ij}^{s(f)} = \varepsilon^{s(f)} \delta_{ij} + t_{ij}^{1\, ss(ff)}$$

$$\Delta_{ij} = \left(t_{ij}^{1\, sf} + t_{ij}^{2\, sf} \right) e^{3i\theta_{ij}} \qquad \text{(p+ip)}^3$$





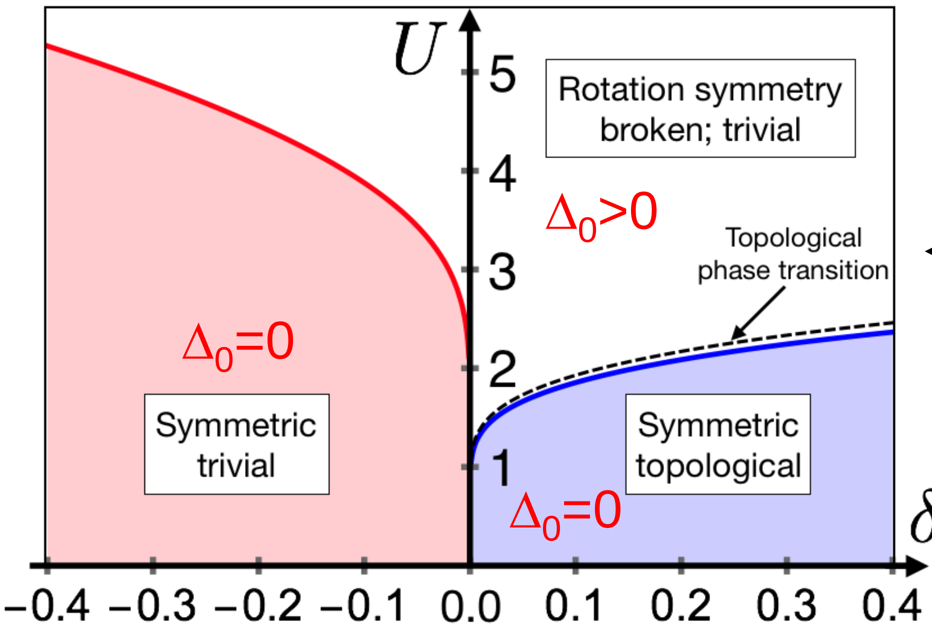
Mean field excitonic instability

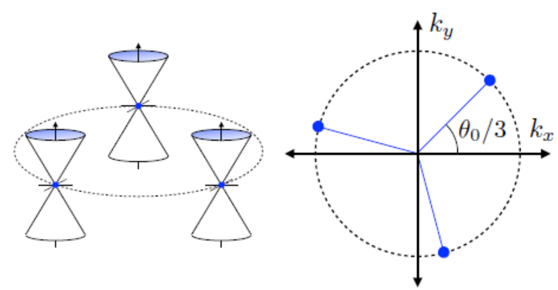
With interactions, energy can be lowered by spontaneously breaking symmetry

- s-wave excitonic order parameter: $\langle \psi_e^\dagger \psi_h^\dagger \rangle = \Delta_0 e^{i\theta_0}$
- spontaneously lowers rotational symmetry to C_3

$$H = \sum_k \varepsilon_k \left(c_{ke}^\dagger c_{ke} + c_{kh}^\dagger c_{kh} \right) + \Delta_k c_{-ke}^\dagger c_{kh}^\dagger + h.c. \qquad \varepsilon_k = k^2 + m \qquad \Delta_k = \Delta_0 e^{i\theta_0} + \mathrm{v}(k_x + ik_y)^3$$

broken symmetry
order parameter





- Interplay between topological and symmetry breaking transitions.

Conclusion

Fractional excitonic insulator

- A correlated fluid of electrons and holes can exhibit a fractional quantum Hall state at zero magnetic field with a stoichiometric band filling.
- Described by variant of Laughlin wavefunction
- Target for numerics on strongly interacting model systems

Higher angular momentum band inversion

- Unconventional topological transition: $\Delta C=3$.
- Promising venue for FEI in presence of strong repulsive short range interactions.
- Target for band structure engineering, real materials prediction