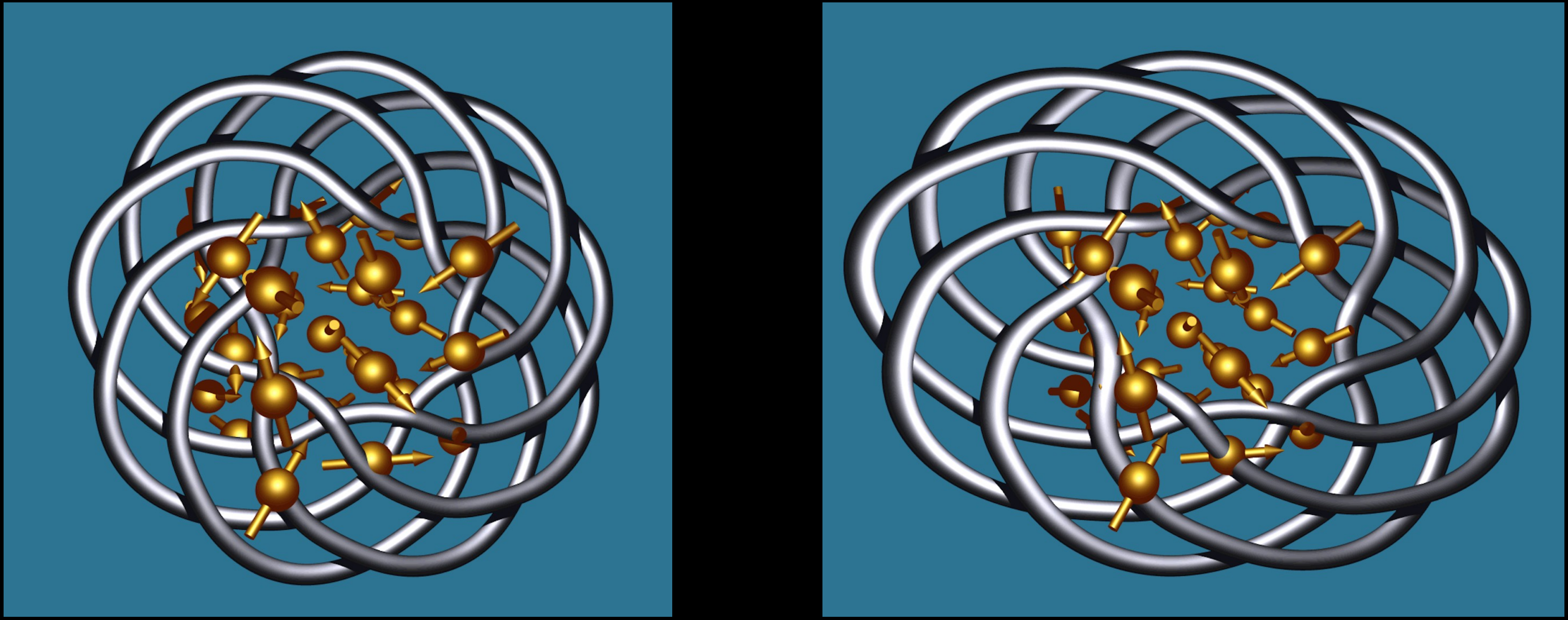
Geometry and dynamics in the fractional quantum Hall effect: from graviton oscillation to quantum many-body scars

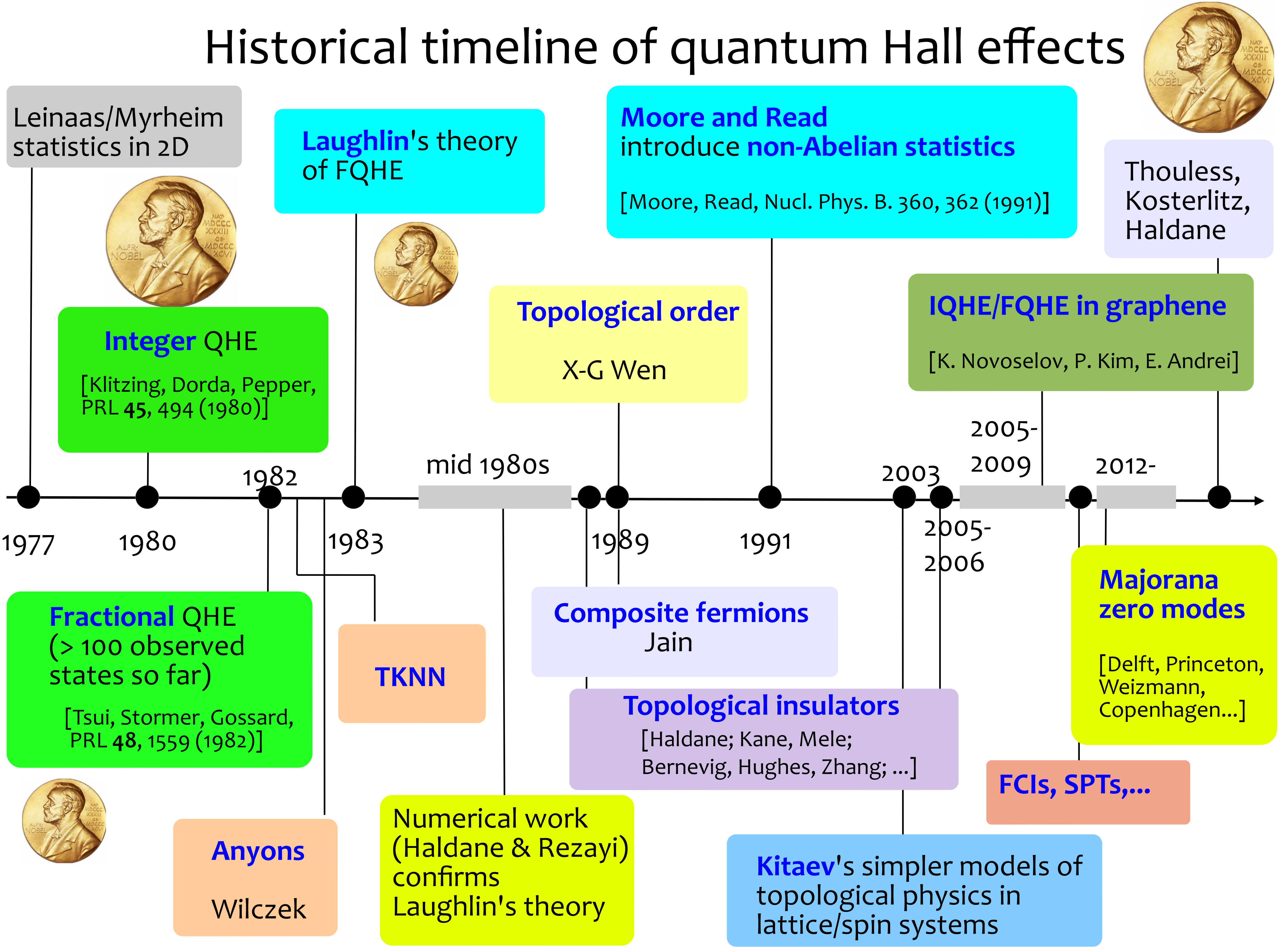
Zlatko Papic



Anyons in Quantum Many-Body Systems, MPI PKS, Dresden, 21/01/2019

UNIVERSITY OF LEEDS

Historical timeline of quantum Hall effects



1977

Leinaas/Myrheim statistics in 2D

1980

Integer QHE

[Klitzing, Dorda, Pepper, PRL **45**, 494 (1980)]

1982

Fractional QHE (> 100 observed states so far)

[Tsui, Stormer, Gossard, PRL **48**, 1559 (1982)]

TKNN

Anyons

Wilczek

1983

Laughlin's theory of FQHE

mid 1980s

Numerical work (Haldane & Rezayi) confirms Laughlin's theory

1989

Topological order

X-G Wen

Composite fermions

Jain

1991

Moore and Read introduce **non-Abelian statistics**

[Moore, Read, Nucl. Phys. B. 360, 362 (1991)]

2003

Kitaev's simpler models of topological physics in lattice/spin systems

2005-2006

Topological insulators

[Haldane; Kane, Mele; Bernevig, Hughes, Zhang; ...]

2005-2009

IQHE/FQHE in graphene

[K. Novoselov, P. Kim, E. Andrei]

FCIs, SPTs,...

2012-

Majorana zero modes

[Delft, Princeton, Weizmann, Copenhagen...]

Thouless, Kosterlitz, Haldane

Historical timeline of quantum Hall effects

Leinaas/Myrheim
statistics in 2D

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Laughlin's theory
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Topological order

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[Moore, Read, Nucl. Phys. B. 360, 362 (1991)]

IQHE/FQHE in graphene

Thouless,
Kosterlitz,
Haldane

Q: FQHE is a mature subject (nearing 40 years).
What is there left to understand about FQHE?

197

2006

Fractional QHE
(> 100 observed
states so far)

[Tsui, Stormer, Gossard,
PRL **48**, 1559 (1982)]

Anyons

Wilczek

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Topological insulators
[Haldane; Kane, Mele;
Bernevig, Hughes, Zhang; ...]

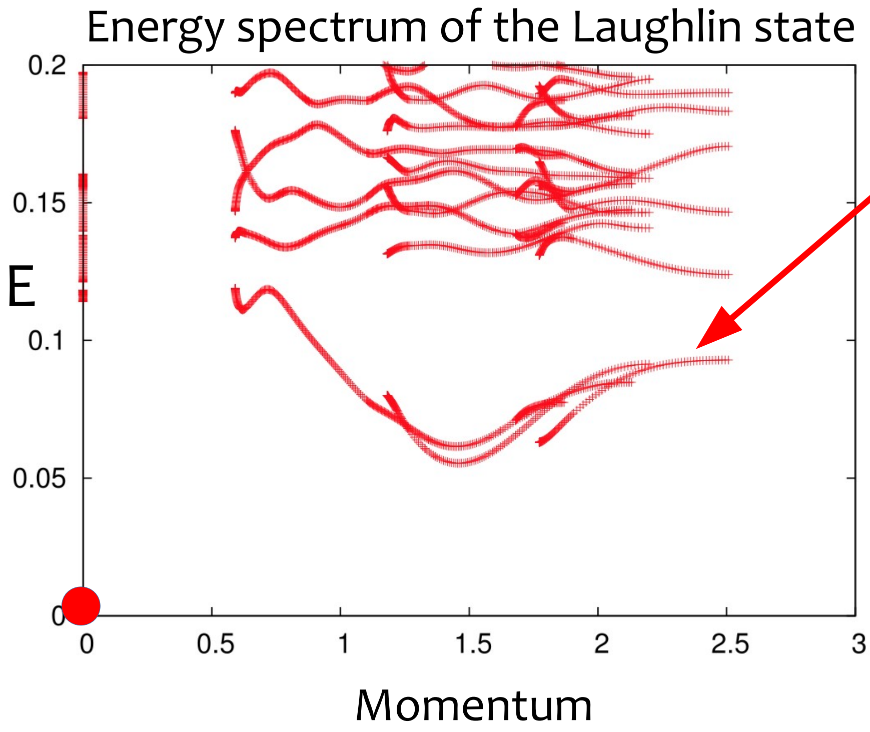
Kitaev's simpler models of
topological physics in
lattice/spin systems

**Majorana
zero modes**

[Delft, Princeton,
Weizmann,
Copenhagen...]

FCIs, SPTs,...

A: Pretty much anything that extends beyond TQFT or strictly ground state properties:



- Why is there a (neutral) gap?

- What is the Lagrangian of a FQH system?

- Bulk dynamics of FQH systems?

Outline

- Introduction to the fractional quantum Hall effect

- Geometric quench of fractional quantum Hall states

[Zhao Liu, Andrey Gromov, ZP, PRB **98**, 155140 (2018)]

- Constrained dynamics and quantum many-body scars

[C. Turner, A. Michailidis, D. Abanin, M. Serbyn, ZP, Nature Physics **14**, 745 (2018);
PRB **98**, 155134 (2018)]
(see also viewpoints by V. Dunjko & M. Olshanii, *Escape the thermal fate*
& N. Robinson, *Cold Atoms Bear a Quantum Scar*)

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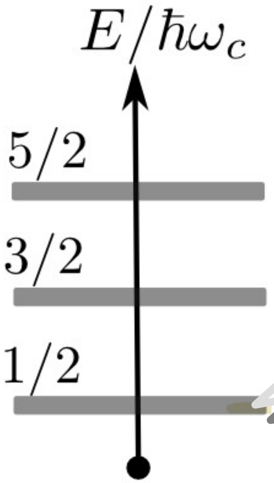
Electrons in a Landau level

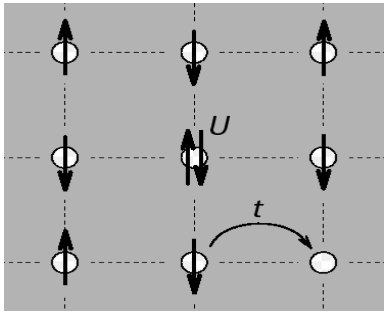


$$\mathcal{H} = \int \frac{d^2q\,\ell_B^2}{(2\pi)^2} V_{\mathbf{q}} \bar{\rho}_{\mathbf{q}} \bar{\rho}_{-\mathbf{q}}$$

$$\bar{\rho}_{\mathbf{q}} = \mathcal{P}\rho_{\mathbf{q}}\mathcal{P} \qquad \ell_B = \sqrt{h/eB} = 1$$

non-commuting!

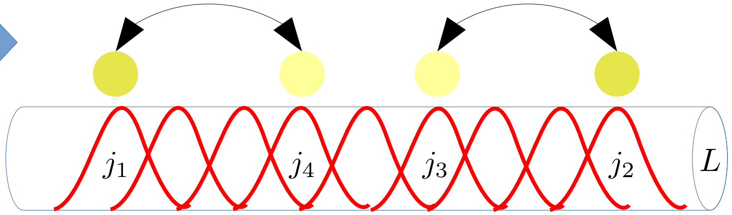
Hubbard model



$$H = -t \sum_{<i,j>,\sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c.$$
$$+U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- Lattice system
- Competition between kinetic and potential energy

FQHE



$$\mathcal{H} = \sum_{\{j_i\}} V_{j_1 j_2 j_3 j_4} c^\dagger_{j_1} c^\dagger_{j_2} c_{j_3} c_{j_4}$$

Like a Coulomb gas except:

(1) Matrix elements $V_{j_1 \ldots j_4} \sim e^{-(2\pi j_i)^2/L^2}$

(2) Only pairs hop: $j_1 + j_2 = j_3 + j_4$

Continuum system, no “kinetic” energy

2D interacting electron gas

Effective theory (Chern-Simons)

Theoretical approach to FQHE

[Laughlin '83]

Variational wavefunctions

Laughlin states, Composite Fermion states, states constructed by conformal field theory (Moore-Read, Read-Rezayi)

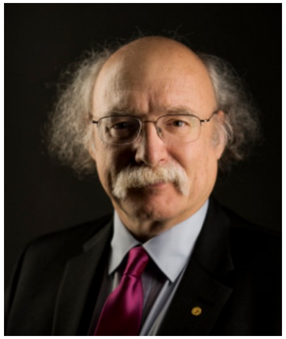
$$\Psi = \prod_{i<j} (z_i - z_j)^3 e^{-\frac{1}{4\ell_B^2} \sum_k |z_k|^2}$$

$$z \equiv x + iy$$

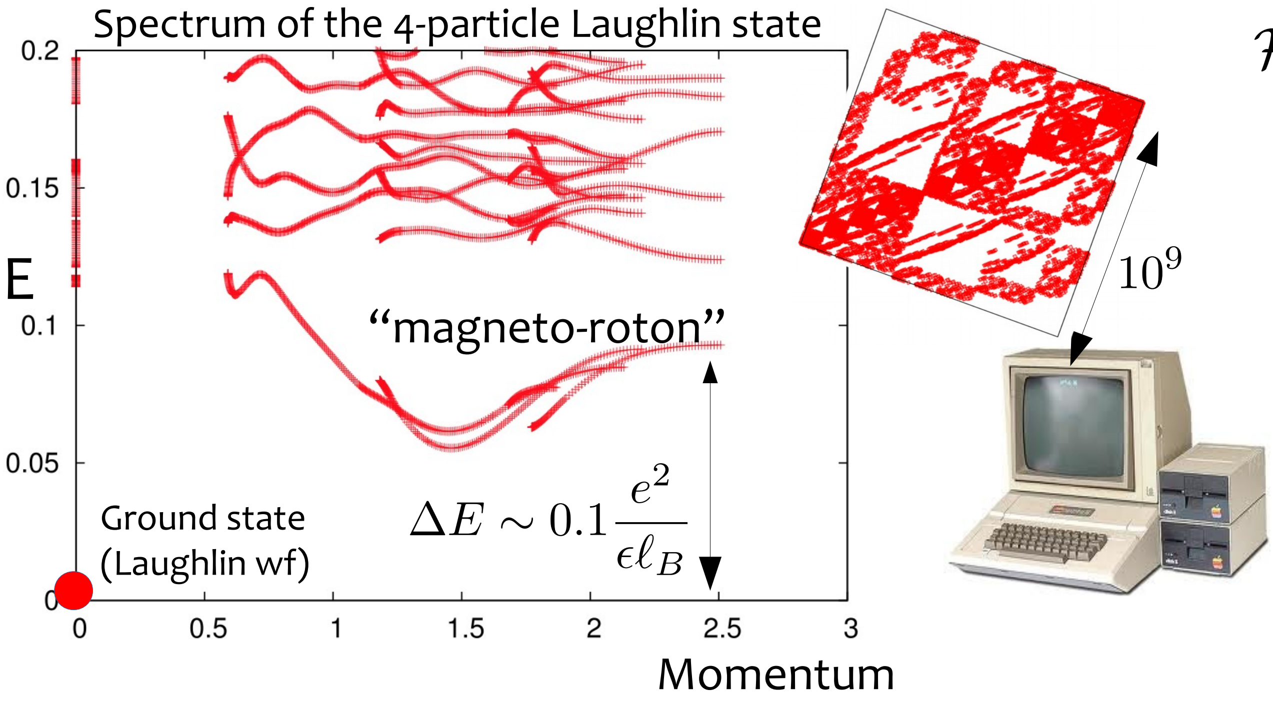
Microscopic simulations

(exact diagonalization)

Haldane pseudopotentials



[Haldane, '83]



Spectrum of the 4-particle Laughlin state

$$\mathcal{H} = \int \frac{d^2 q \ell_B^2}{(2\pi)^2} V_1(\mathbf{q}) \bar{\rho}_{\mathbf{q}} \bar{\rho}_{-\mathbf{q}}$$

$$V_1(\mathbf{q}) = L_1(q^2) e^{-q^2/2}$$

$$q^2 \equiv q_x^2 + q_y^2$$

Projector onto particle pairs with given angular momentum

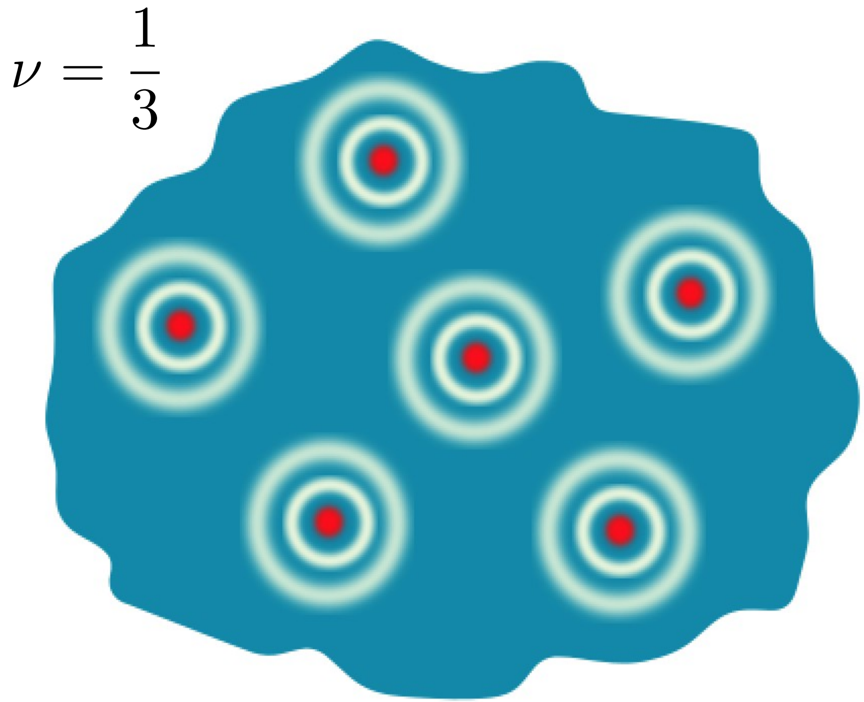
$$\int \frac{d^2 q \ell_B^2}{(2\pi)^2} V_1(\mathbf{q}) \bar{\rho}_{\mathbf{q}} \bar{\rho}_{-\mathbf{q}} |\Psi\rangle = 0$$

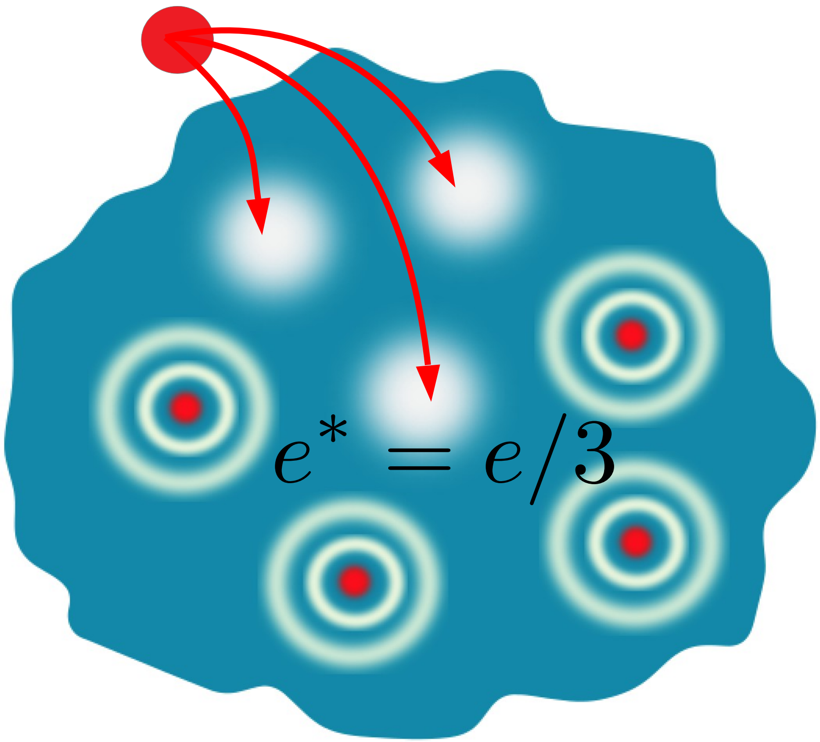
[Haldane '83; Trugman, Kivelson '85]

Incompressible electron fluids with fractionalization of charge and statistics

Electrons “bind” magnetic fluxes , i.e., surround themselves with correlation holes.
The composite objects condense into **incompressible quantum fluids.**

[Laughlin ‘83, Jain ‘89, Read/Moore ‘91,...]

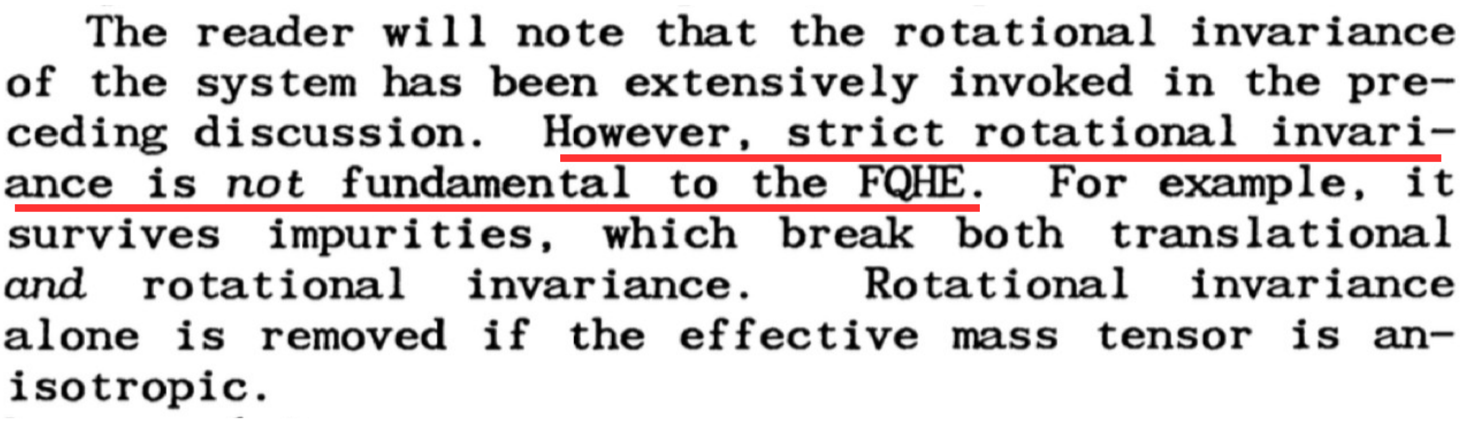




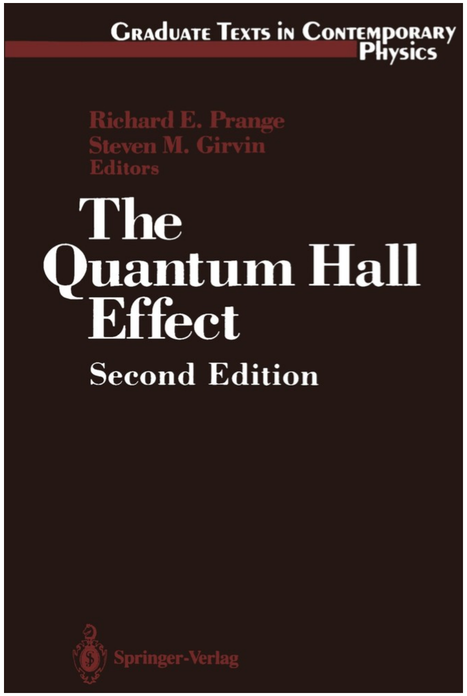
These defects have “topological” interactions
that leave an imprint on the spectrum around an impurity,
which could be seen in STM
[see Ali Yazdani’s talk on Wednesday]

What is the role of rotational invariance?

[F D. M. Haldane in *Prange/Girvin*;
PRL **107**, 116801 (2011)]

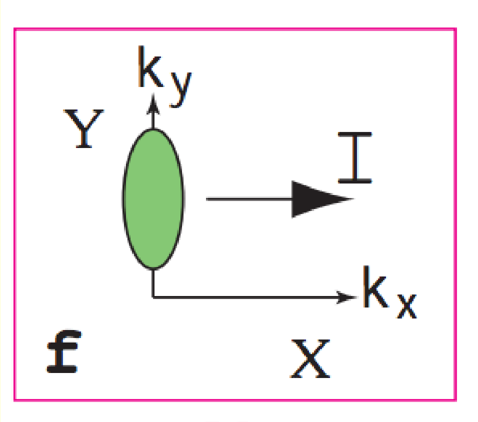


The reader will note that the rotational invariance of the system has been extensively invoked in the preceding discussion. However, strict rotational invariance is *not* fundamental to the FQHE. For example, it survives impurities, which break both translational *and* rotational invariance. Rotational invariance alone is removed if the effective mass tensor is anisotropic.



Why care?

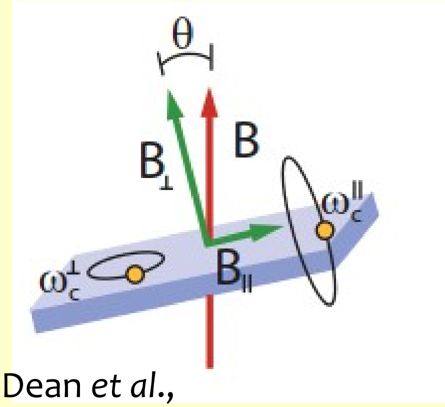
Anisotropic bands



AlAs under strain

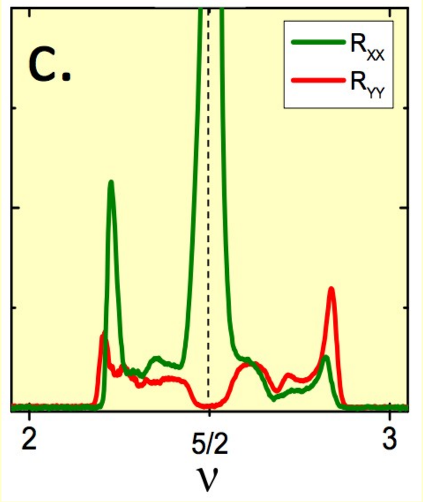
[Gokmen et al.,
Nat. Phys. **6**, 621 (2010)]

Tilted magnetic field



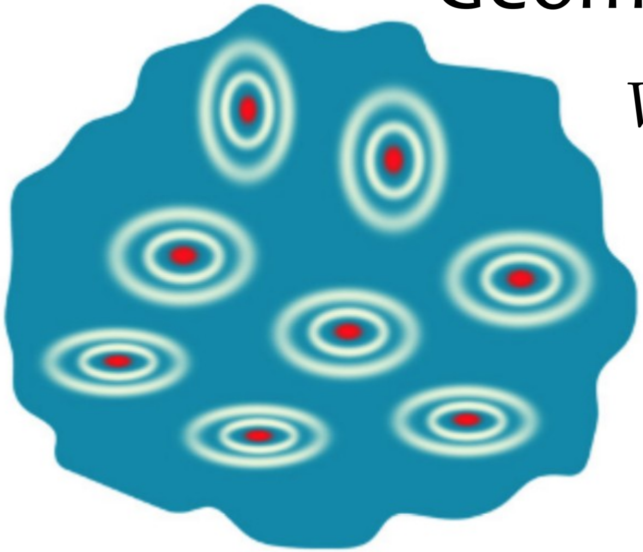
[C.R. Dean *et al.*,
PRL **101**, 186806 (2008);
J. Xia *et al*, PRL **105**, 176807 (2010);
Liu *et al.*, PRB **88**, 035307 (2013)]

Nematic quantum Hall



[N. Samkharadze *et al.*, Nat. Phys. **12**, 191 (2016)]

Geometry of the Laughlin state



$$V_1(q,g) = L_1(q_g^2)e^{-q_g^2/2} \qquad q_g^2 \equiv g^{ab}q_a q_b$$

$$\det g = 1$$

$$\hat{V}_1(q,g)|\Psi_L(g)\rangle = 0$$

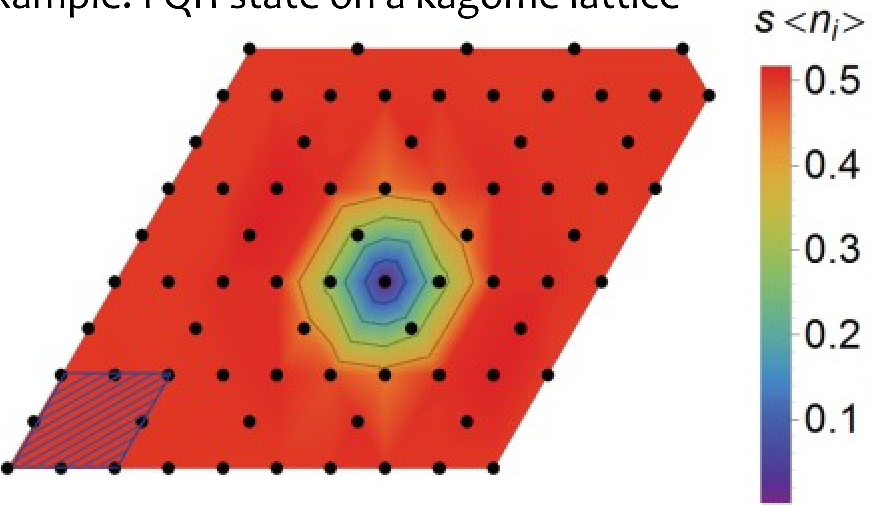
$$\Psi_L(\{\mathbf{r}_i\}) \propto \prod_{i<j}\left[z_i - z_j + f_g^2(\partial_{z_i} - \partial_{z_j})\right]^3 \hat{1}$$

Splits the zeros
$z_1 - z_2 = 0, \pm i f_g$

[Haldane ‘09; Read ‘09; Haldane ‘11; Can, Laskin & Wiegmann ‘14; Ferrari & Klevtsov ‘14; Bradlyn & Read ‘15;...]

The metric describes the “shape” of the anyon

Example: FQH state on a kagome lattice



[Zhao Liu, R. N. Bhatt, and Nicolas Regnault, PRB 91, 045126 (2015)]

Q: What are the dynamical consequences of this metric?

[Zhao Liu’s talk on Thursday]

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[C. Turner, A. Michailidis, D. Abanin, M. Serbyn, ZP, Nature Physics **14**, 745 (2018);
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(see also viewpoints by V. Dunjko & M. Olshanii, *Escape the thermal fate*
& N. Robinson, *Cold Atoms Bear a Quantum Scar*)

Geometric quench of fractional quantum Hall states

Geometric quench protocol:

- Instantaneous tilt amounts to sudden change

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow g' = \begin{pmatrix} e^{A} & 0 \\ 0 & e^{-A} \end{pmatrix}$$

Tilt angle (or mass anisotropy)

- Assume closed system i.e., evolving according to Schroedinger equation:

$$|\psi(t)\rangle = \exp(-\frac{i}{\hbar} H(g')t)|\psi(0)\rangle$$

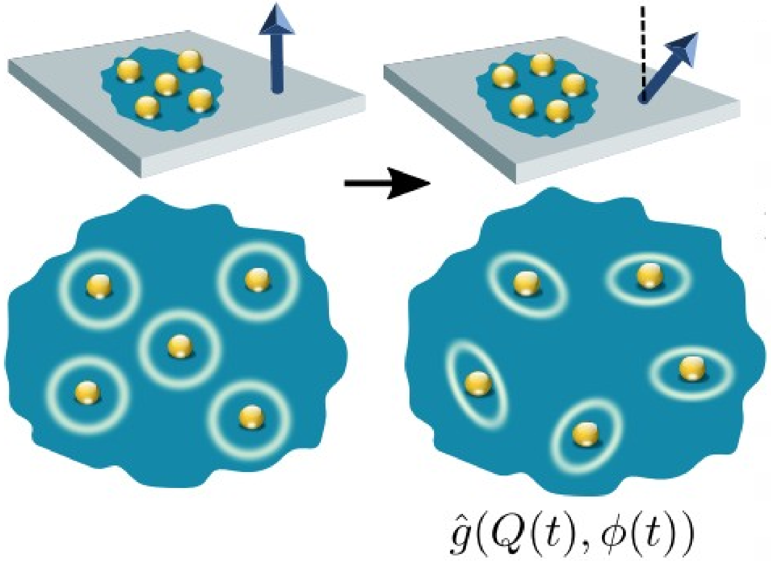
- Determine the metric by maximizing overlap:

$$\max_g |\langle \Psi_{\mathrm{Laughlin}}(g)|\Psi_{\mathrm{exact}}\rangle|^2$$

(Assume adiabatic protocol, i.e.,
the ground state before AND after quench is in the Laughlin phase.
Then, the maximum overlap should be close to 1.)

- Metric is parametrized by Q (stretch) and ϕ (rotation).

Q: what is EOM for $Q(t)$ and $\phi(t)$?



Idea: to probe metric, suddenly tilt the magnetic field (global quench)

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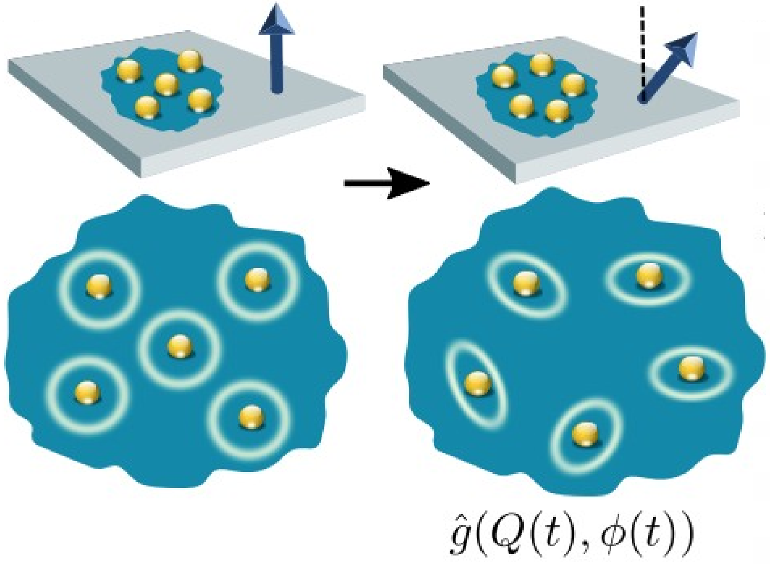


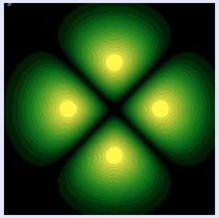
Idea: to probe metric, suddenly tilt the magnetic field (global quench)

Why do we want to do this?

Motivation: Study “graviton” dynamics

- Quadrupole (2qp+2qh)

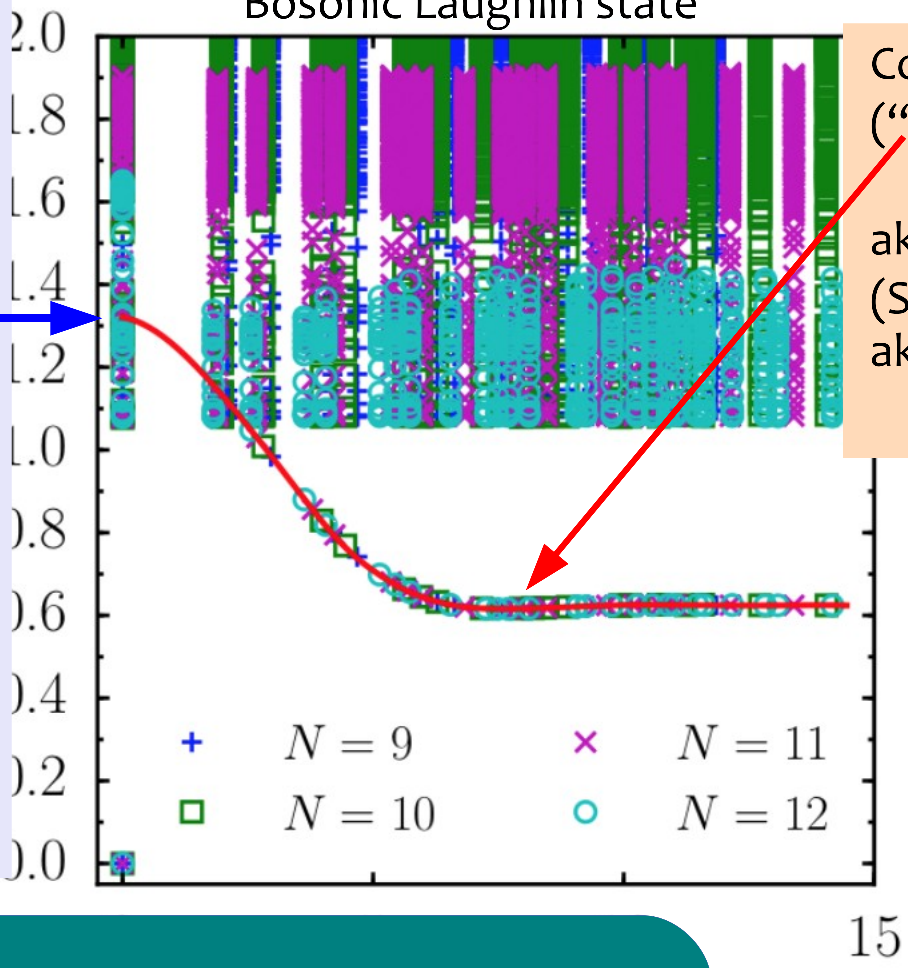
  [Zhang, Hansson, Kivelson ‘89]



- Angular momentum L=2
- Described by a metric (geometrical DOF)

  = “graviton”

  [Haldane ‘11;
  Golkar, Nguyen, Son ‘16]



Bosonic Laughlin state

Collective mode (“magnetoroton”)

aka SMA mode (Single Mode Approximation)
aka GMP mode

[Girvin, MacDonald, Platzman, ‘85]

Observed in experiment:

Inelastic light scattering
[Kang et al, ‘01]

Surface acoustic wave
[Kukushkin et al, ‘09]

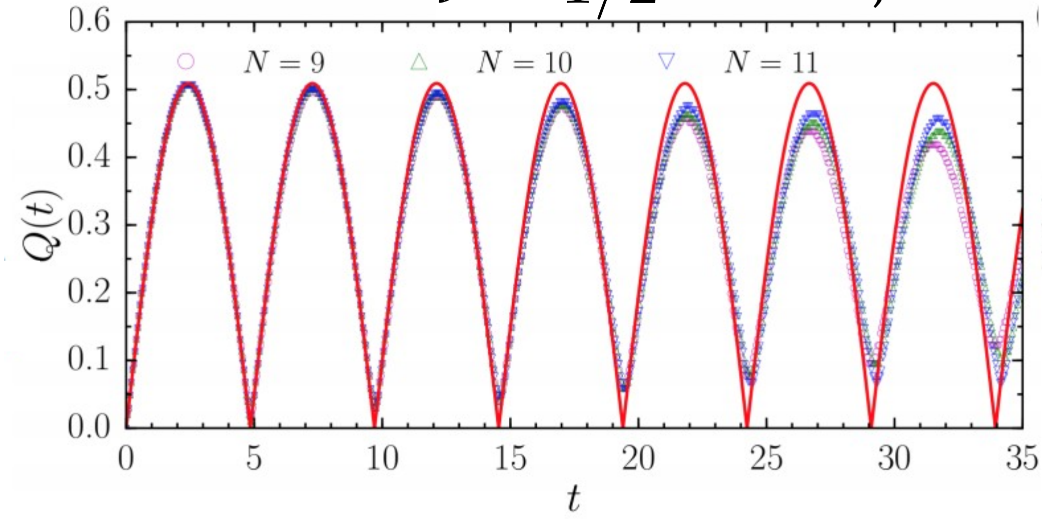
Can we excite the graviton by quench?
How will it move?
Not obvious as it is in the continuum...

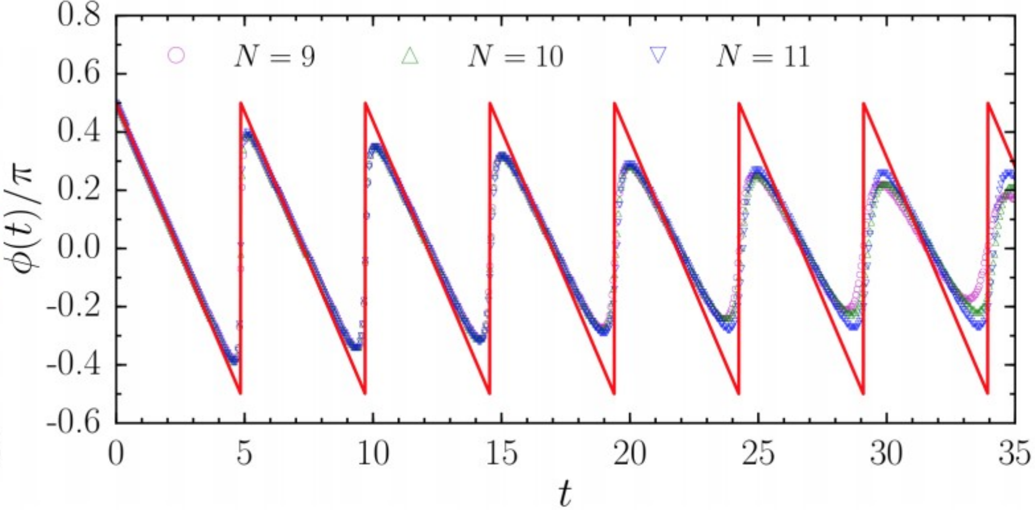
Accurate microscopic wavefunctions exist for the entire excitation branch

[Bo Yang, ZP, Hu, Haldane, ‘12]

Main result: Harmonic motion of the graviton

$\nu = 1/2$ bosons, contact interaction $A_0 = 0, A_1 = \ln 1.3 \approx 0.262$





[Zhao Liu, Andrey Gromov, ZP, PRB **98**, 155140 (2018)]

Physical picture: harmonic motion of the graviton

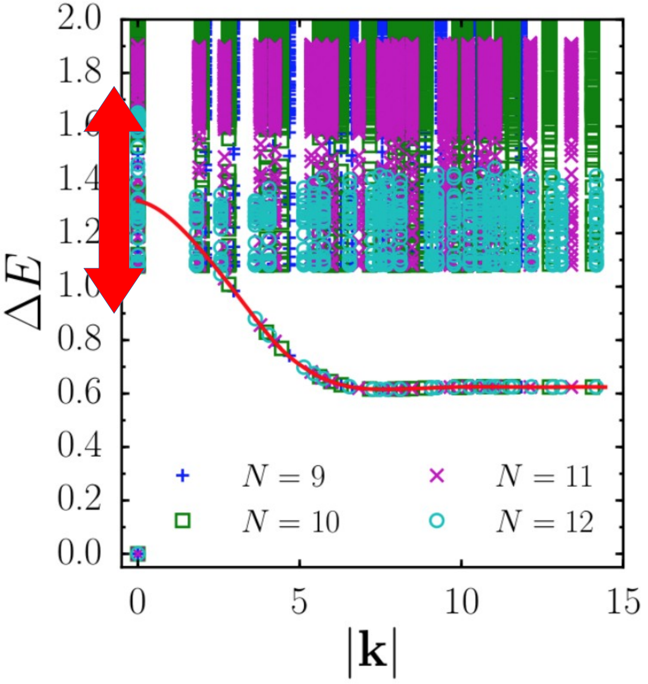
Red lines

= fit to a harmonic solution:

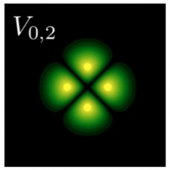
$$Q(t) = \pm 2A \sin \frac{E_\gamma t}{2}$$

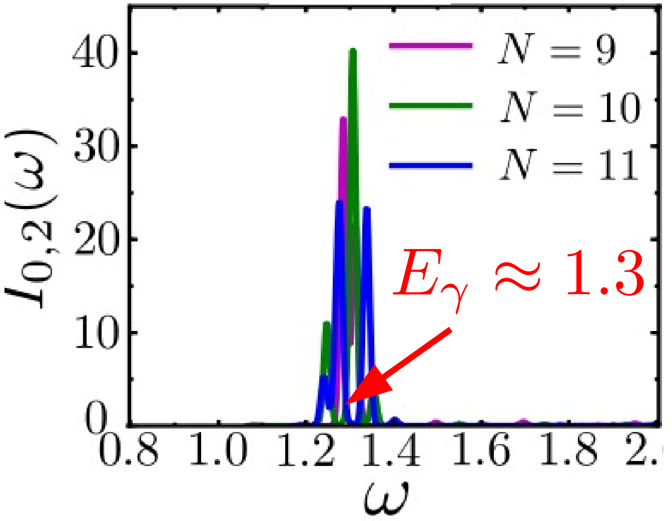
$$\phi(t) = (\pi \mp \frac{\pi}{2}) - \frac{E_\gamma t}{2}$$

$E_\gamma \approx 1.296, A \approx 0.255$



Spectral function for quadrupolar pseudopotential





[B. Yang, Z. Hu, CH Lee, ZP, PRL **118**, 146403 (2017)]

Bimetric theory interpretation and open questions

Bimetric theory = gapped dynamics of a spin-2 DOF interacting with ambient geometry
[Gromov, Son ‘17]

Gap of spin-2 mode

Tune the theory to nematic transition

$$\mathcal{L} = \frac{\nu\varsigma}{2\pi\ell_B^2}\hat{\omega}_0 - \frac{m}{2}\left(\frac{1}{2}g_{ij}\hat{g}^{ij} - \gamma\right)^2$$

Intrinsic metric
Q, ϕ

Temporal component of the Levi-Civita spin connection

Extrinsic metric in quench Hamiltonian

$$g = \begin{pmatrix} e^{A} & 0 \\ 0 & e^{-A} \end{pmatrix}$$

- Linearized prediction of bimetric theory is the *same* as fitting equations

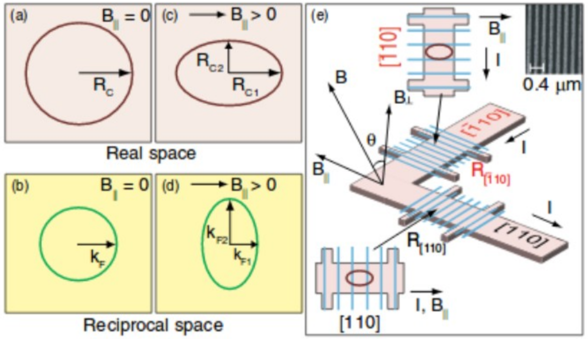
$$\dot{\phi}Q = E_\gamma(A\cos\phi - Q)$$
$$\dot{Q} = E_\gamma A\sin\phi$$

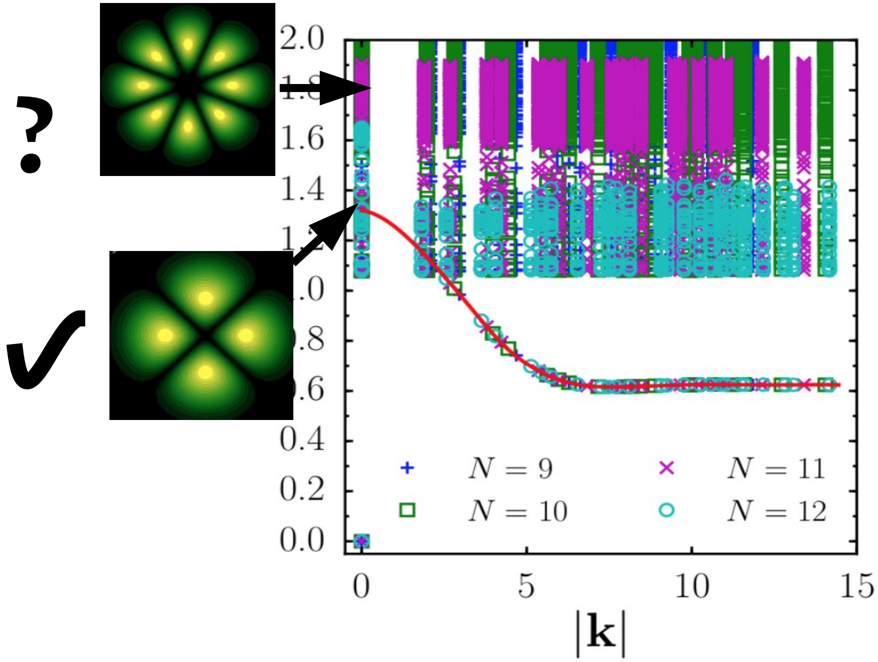
- Geometric quench = an **accurate microscopic test of bimetric theory**

Open questions:

- Dynamics of **higher spin excitations**
- Dynamics of **multicomponent/non-Abelian** states with a richer spectrum of collective modes?
- **Experimental realizations**

[Kamburov et al, ‘13]





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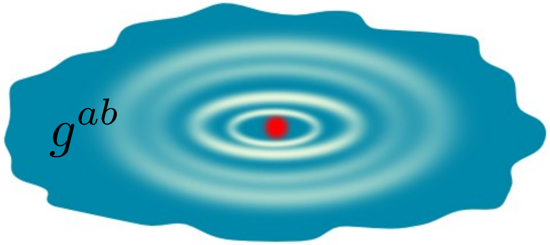
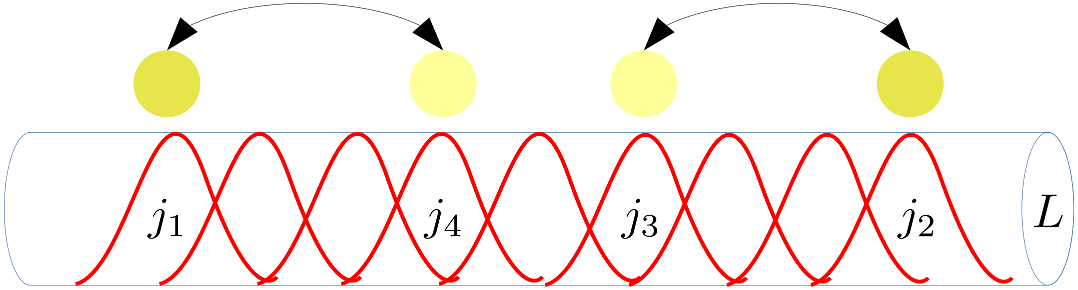
Quantum Hall state in a “thin cylinder” geometry





$$H = \sum_{j_1,\ldots,j_4} V_{j_1 j_2 j_3 j_4} c^\dagger_{j_1} c^\dagger_{j_2} c_{j_3} c_{j_4}$$

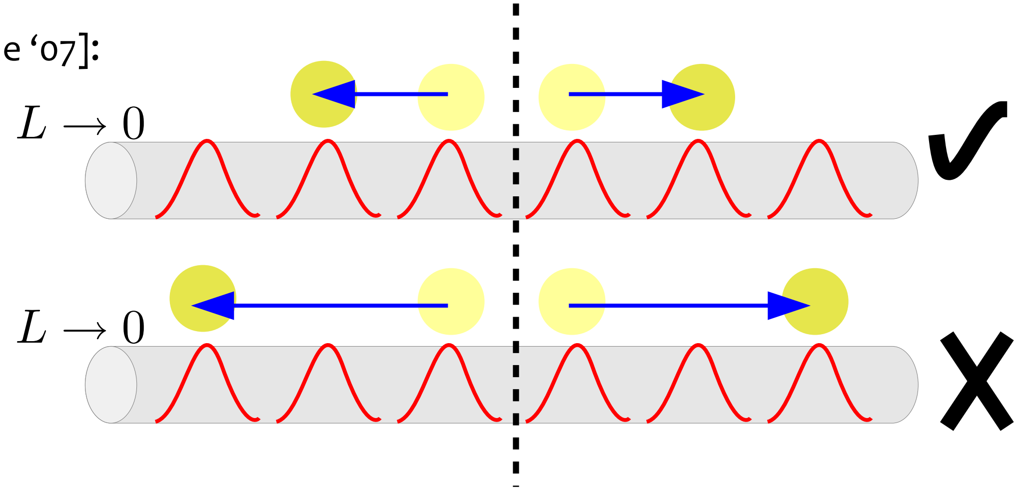
Like a Coulomb gas except:

(1) Gaussian amplitudes $V_{j_1 \ldots j_4} \sim \exp(-(2\pi j_i)^2/L^2)$

(2) Particles only hop in pairs: $j_1 + j_2 = j_3 + j_4$

On a thin cylinder, most matrix elements are suppressed.
The dominant process is NN “squeezing” [Bernevig, Haldane ‘07]:

$$H = \sum_j c^\dagger_j c^\dagger_{j+3} c_{j+1} c_{j+2} + \text{h.c.} + \text{other}$$



[Seidel and Lee, Bergholtz and Karlhede '05;
Nakamura *et al.*, ‘10; ZP, ‘13]

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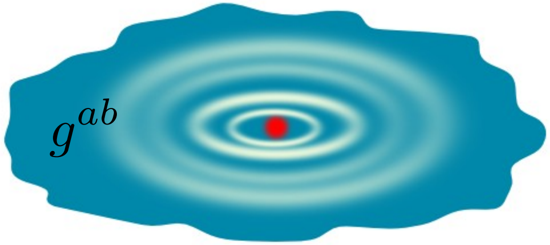

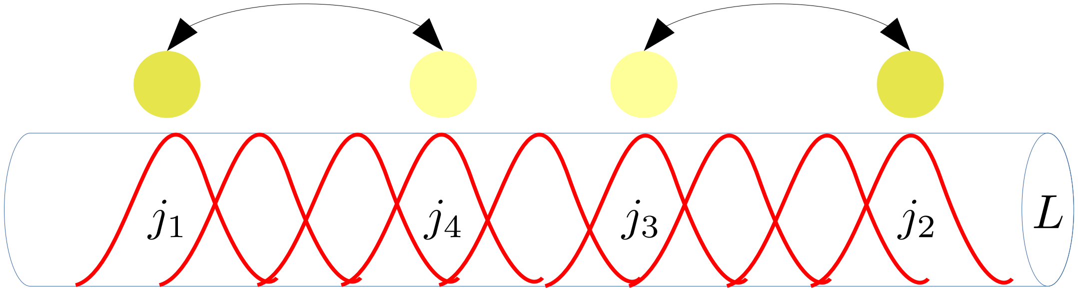

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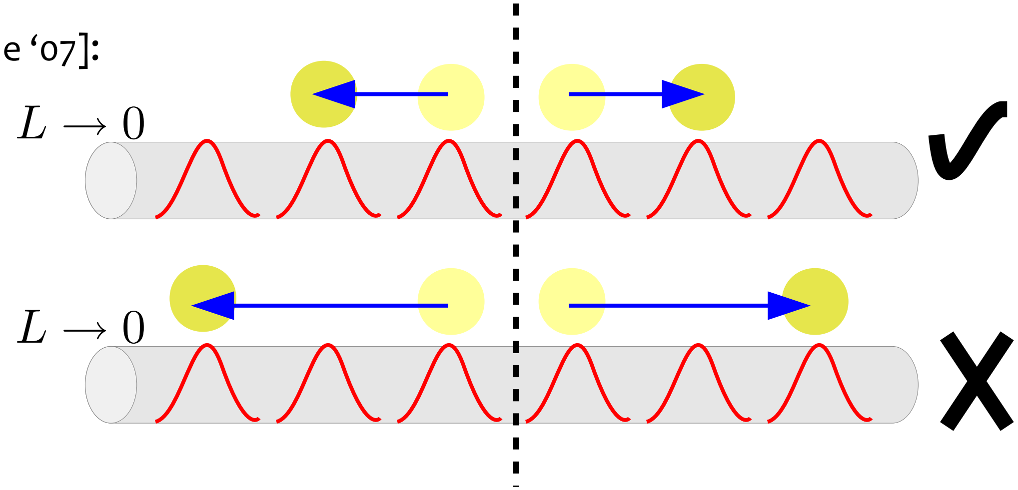
$$H = \sum_j c^\dagger_j c^\dagger_{j+3} c_{j+1} c_{j+2} + \text{h.c.} + \text{other}$$

Why is this interesting?



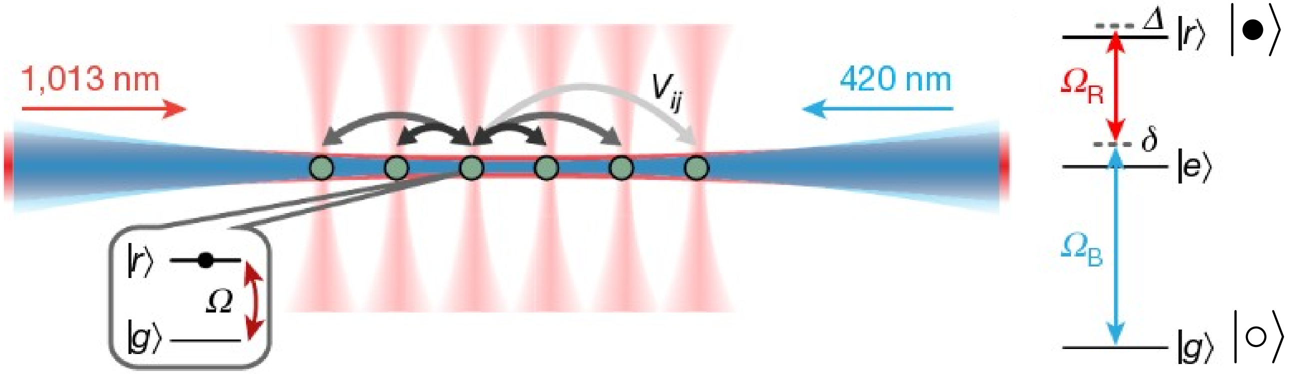
[Seidel and Lee, Bergholtz and Karlhede '05; Nakamura *et al.*, ‘10; ZP, ‘13]

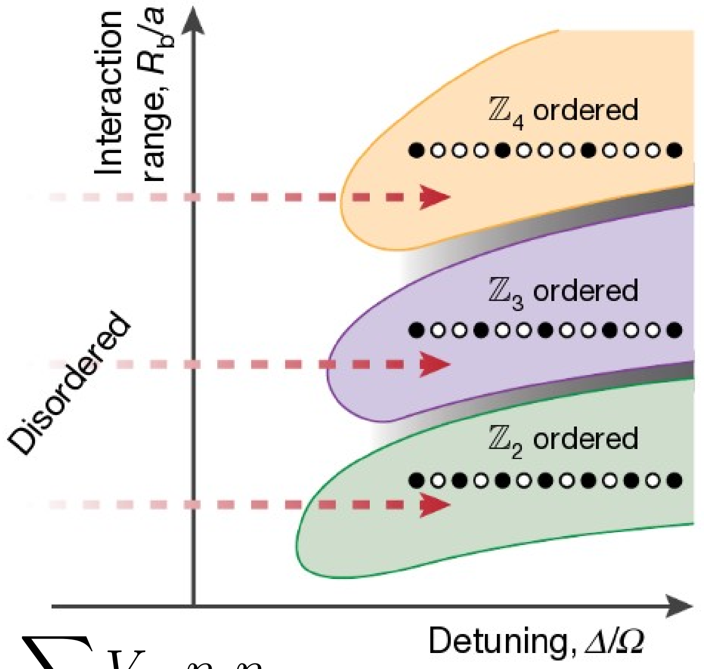
Constrained Many-Body Dynamics in a Quantum Simulator

doi:10.1038/nature24622

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien[1], Sylvain Schwartz[1,2], Alexander Keesling[1], Harry Levine[1], Ahmed Omran[1], Hannes Pichler[1,3], Soonwon Choi[1], Alexander S. Zibrov[1], Manuel Endres[4], Markus Greiner[1], Vladan Vuletić[2] & Mikhail D. Lukin[1]





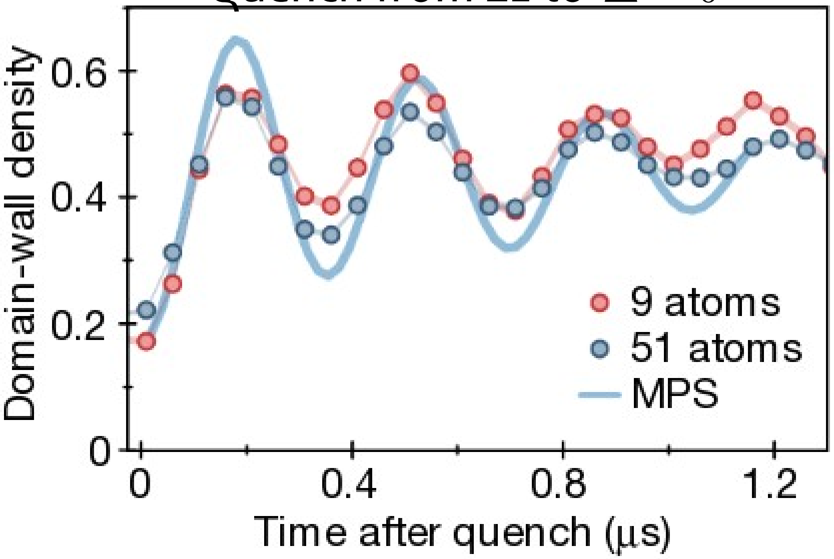
[also, the 53-qubit simulator with trapped ions: J. Zhang *et al.*, Nature **551**, 601 (2017)]

$$H = \sum_j \left(\frac{\Omega}{2} X_j - \Delta \cdot n_i\right) + \sum_{i<j} V_{i,j} n_i n_j$$

$$V_{i,j} \sim 1/r_{i,j}^6$$

$$|\circ_i\rangle \rightarrow n_i = 0$$
$$|\bullet_i\rangle \rightarrow n_i = 1$$

Quench from Z2 to $\Delta = 0$



Even more striking is the coherent and persistent oscillation of the crystalline order after the quantum quench. With respect to the quenched Hamiltonian ($\Delta = 0$), the energy density of our $\mathbb{Z}_2$-ordered state corresponds to that of an infinite-temperature ensemble within the manifold constrained by Rydberg blockade. Also, our Hamiltonian does not have any explicitly conserved quantities other than total energy. Nevertheless, the oscillations persist well beyond the natural timescale of local relaxation ($1/\Omega$) and the fastest timescale ($1/V_{i,i+1}$).

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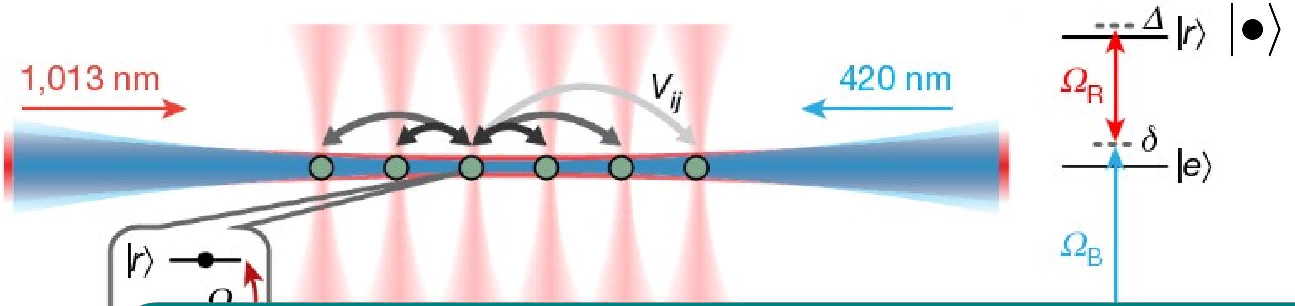
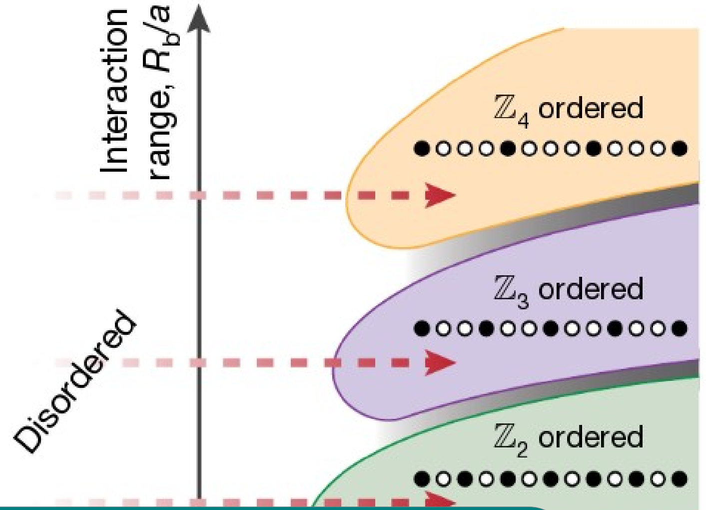
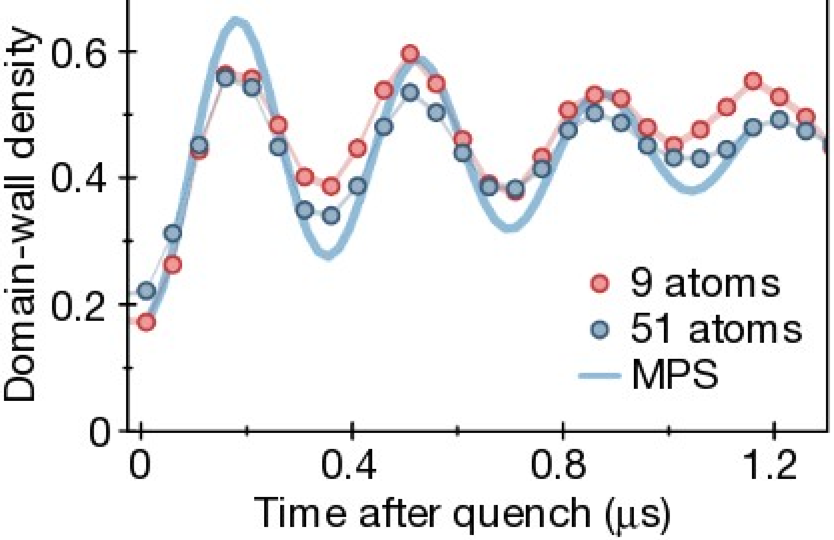




What causes coherent oscillations in a thermalizing system?
How is this related to FQHE?

[also, t
J. Zhan

$n_i = 0$
$n_i = 1$



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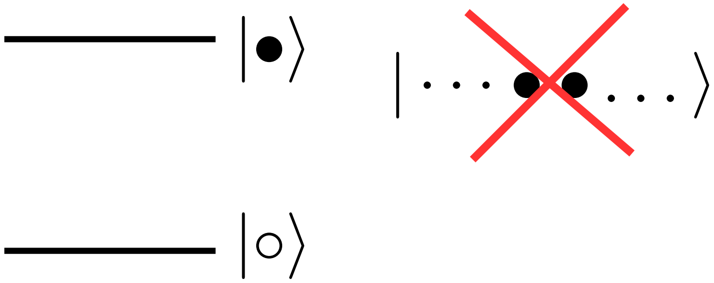
Effective model: Fibonacci chain

For homogeneous couplings
in the limit $V_{i,i+1} \gg \Omega \gg \Delta$:

Hilbert space
=
tensor product of spins ½
+ a kinematic constraint



[Lesanovsky & Katsura, PRA **86**, 041601 (2012)]

$$H = \sum_j P_{j-1} X_j P_{j+1}$$

Projector
$P = |\circ\rangle\langle\circ|$

$X = |\circ\rangle\langle\bullet| + |\bullet\rangle\langle\circ|$

$$|\cdots\circ\circ\circ\ldots\rangle \leftrightarrow |\cdots\circ\bullet\circ\ldots\rangle$$

Rydberg chain

All atoms in ground state:
$|\circ\circ\circ\circ\ldots\rangle$

Z2 state:
$|\circ\bullet\circ\bullet\ldots\rangle$

Hilbert space
= no two NN atoms are both excited
$\dim \sim \phi^N$
"Fibonacci chain"

PXP Hamiltonian

1/3 FQHE thin cylinder

Laughlin root state:
$|100100100100\ldots\rangle$

Maximally* squeezed:
$|011000011000\ldots\rangle$
(only NN squeezings!)

Hilbert space
= only configurations reachable by NN squeeze operations from the root
$\dim \sim \phi^N$

$$H = \sum_j c_j^\dagger c_{j+3}^\dagger c_{j+1} c_{j+2} + \text{h.c.}$$

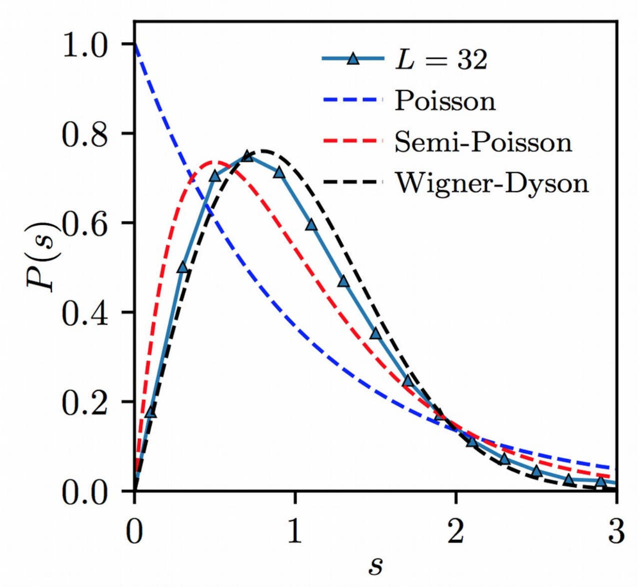
Rich ground state phase diagram
[Fendley, Sengupta, Sachdev, PRB **69**, 075106 (2004)]

"The Golden Chain" integrable model
[Feiguin *et al.*, PRL **98**, 160409 (2007)]

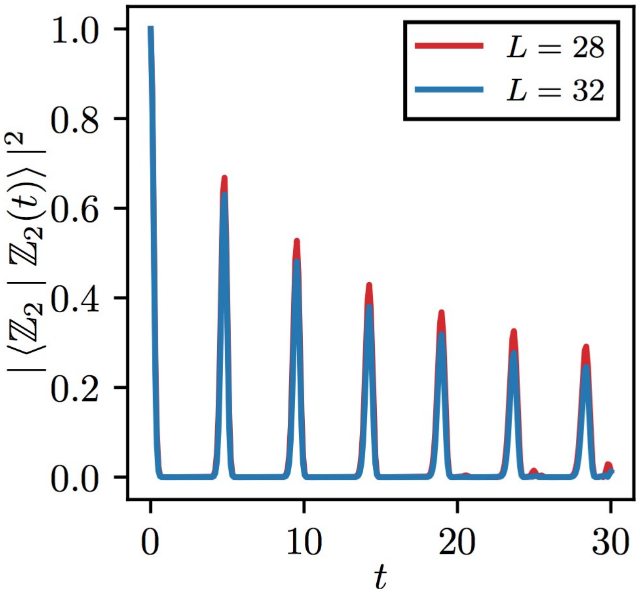
A puzzle: persistent oscillations in a thermalizing system

The model is **not integrable**:

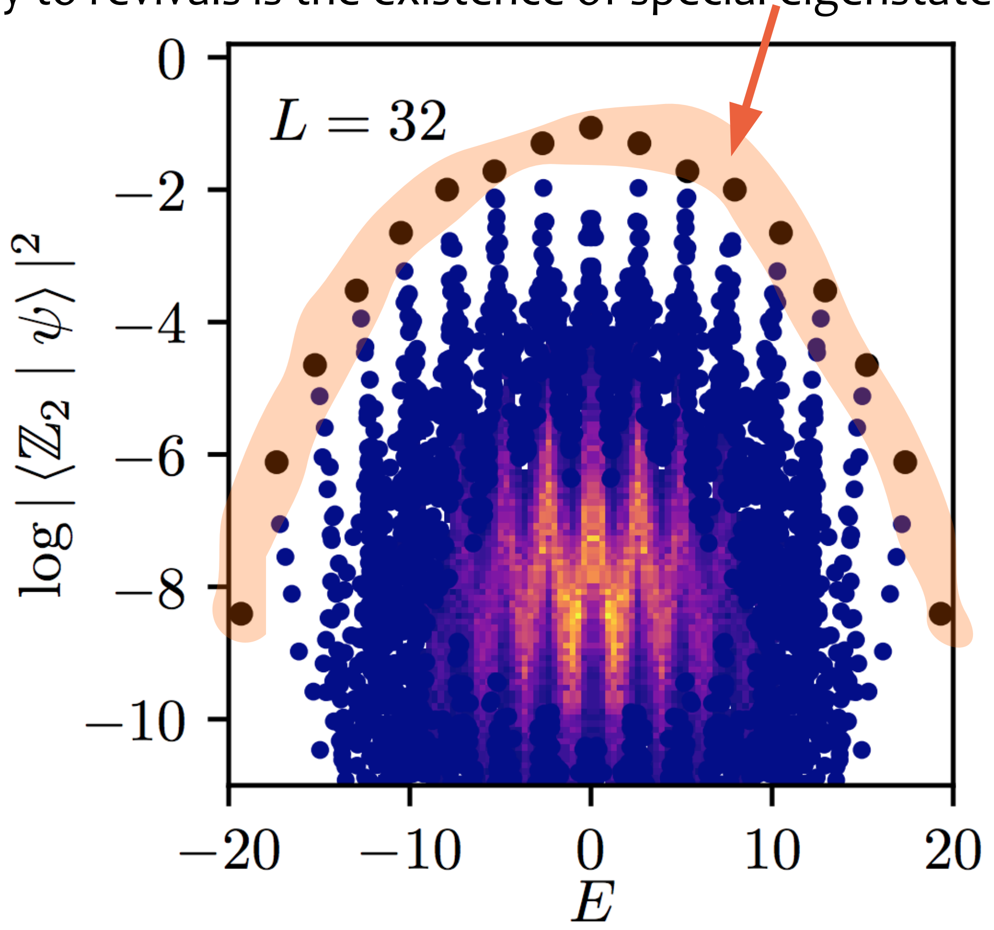
Statistics of energy level spacings



Yet the wavefunction **revives in a quench**:



The key to revivals is the existence of special eigenstates

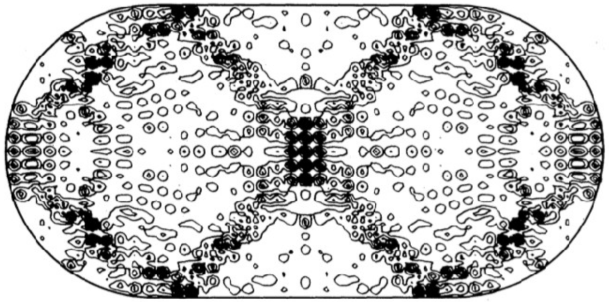


- Special states are atypical: low-entangled, and equally spaced throughout the spectrum
- Weak violation of Eigenstate Thermalization Hypothesis
- Can be understood using “forward scattering” approximation

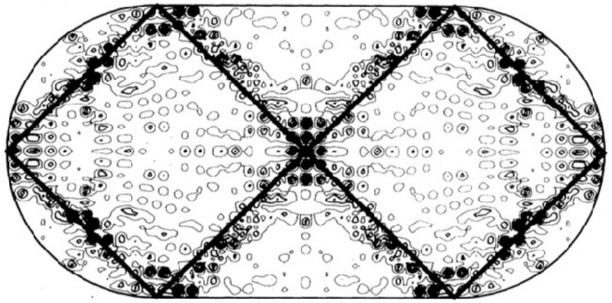
[C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, Z. Papic, Nature Physics **14**, 745 (2018)]

Quantum scars

Ernst Chladni

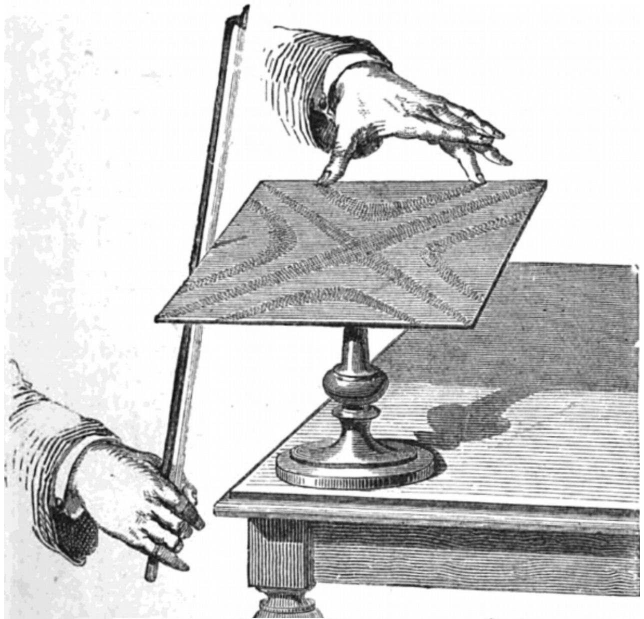


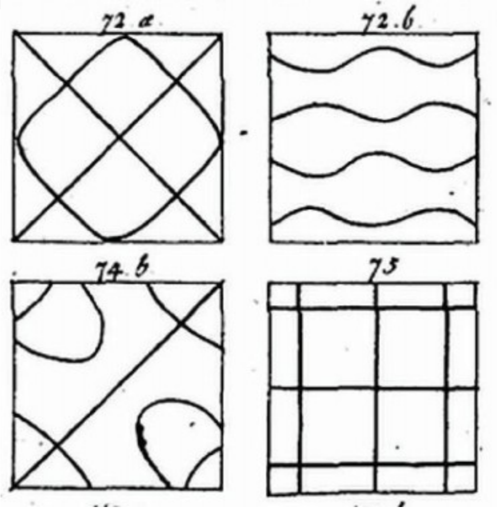
Scarred wavefunction



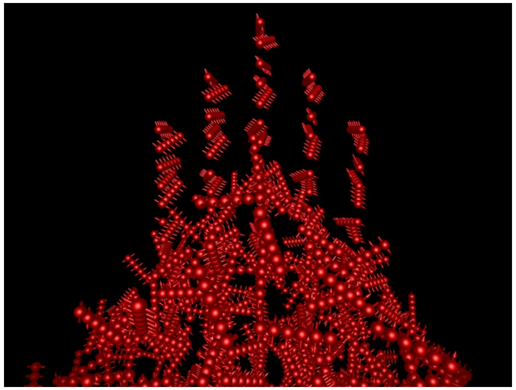
Unstable classical periodic orbit

Eric Heller

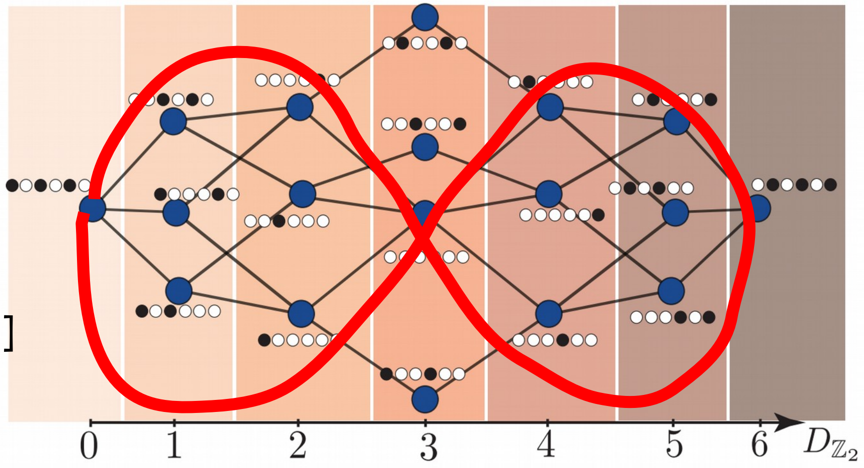




Many-body scars



- Concentration of many-body eigenstates in parts of Hilbert space



- Periodic “orbits” and revivals from certain simple initial states

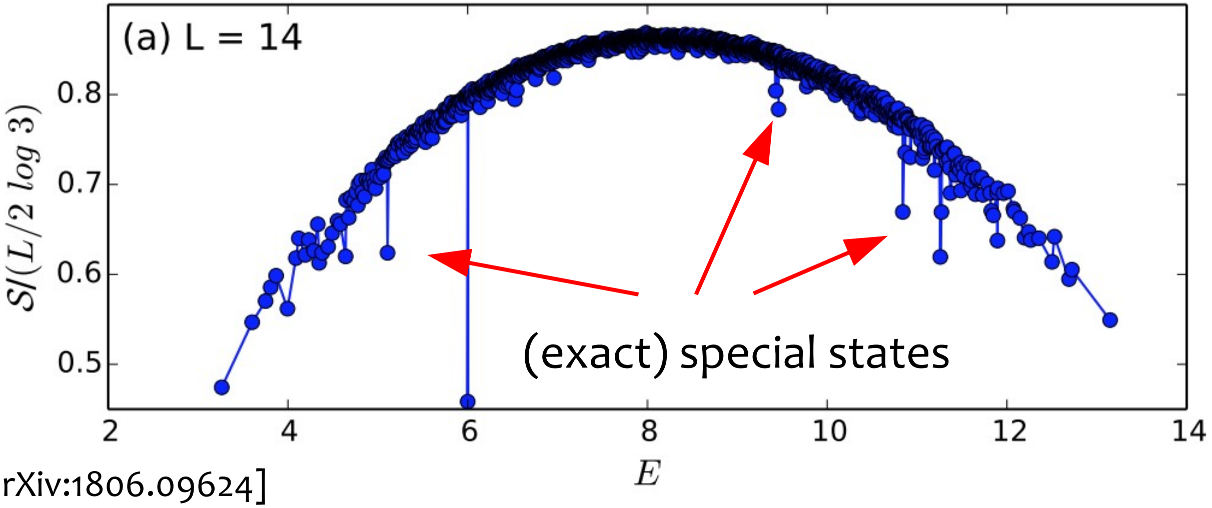
[W. W. Ho *et al*, arXiv:1807.01815]

Where do scars come from?

- Constrained Hilbert space?

Analytical construction of exact non-ergodic eigenstates in AKLT model

$$H = \sum_j \vec{S}_j \vec{S}_{j+1} + \frac{1}{3}(\vec{S}_j \vec{S}_{j+1})^2$$

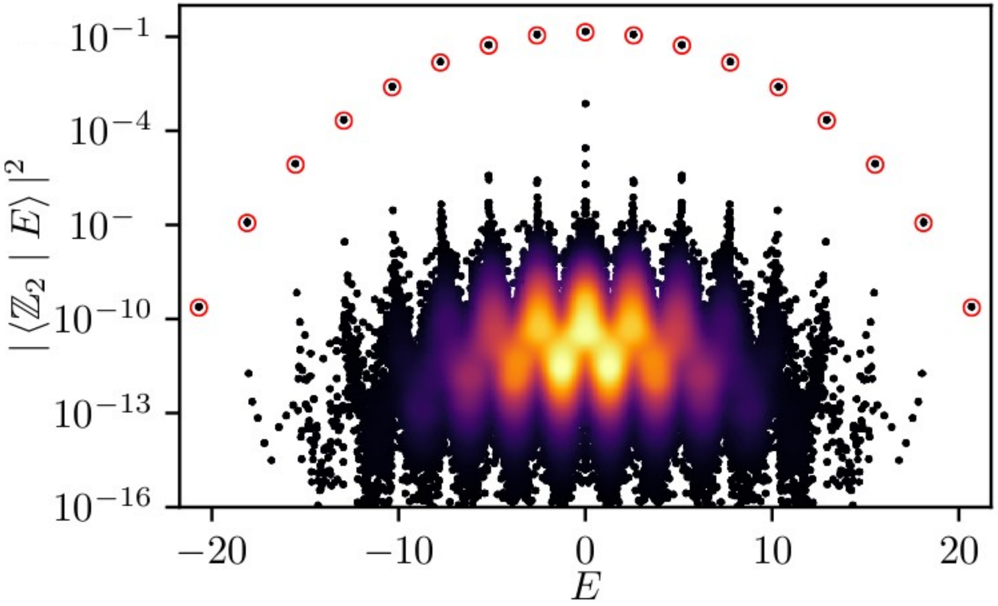


(exact) special states

[Arovas, Physics Letters A 137, 431 (1989); Moudgalya, Regnault, Bernevig, arXiv:1708.05021; arXiv:1806.09624]

- Integrability and “perfect” scars?

Adding certain perturbations enhances scars:



[Choi *et al.*, arXiv:1812.05561; C.-J. Lin and O. I. Motrunich, arXiv:1810.00888 (2018); see also Khemani, Laumann, Chandran, arXiv:1807.02108]

- Supersymmetry? [Fendley, Schoutens ‘05]

$$H = \{Q, Q^\dagger\} \qquad Q^2 = (Q^\dagger)^2 = 0$$

$$H_{\rm SUSY} = \sum_j P_{j-1} c_j^\dagger c_{j+1} P_{j+2} + h.c. + \text{diag}$$

$$PXP \simeq \sqrt{H_{\rm SUSY}} \quad ?$$

- Eigenstate “embedding”?

$$H = \sum_k \alpha_k P_k \qquad P_k|\psi\rangle = 0$$

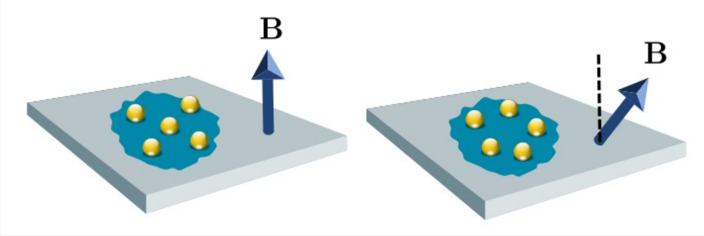
[N. Shiraishi and T. Mori, PRL **119**, 030601 (2017); Seulgi Ok *et al.*, arXiv:1901.01260 (2019)]

[Titus Neupert’s talk on Friday]

Conclusions

- Two examples of coherent many-body dynamics inspired by FQH systems

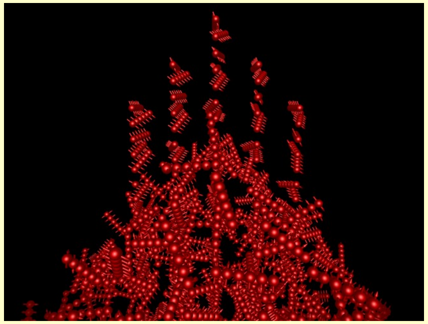
- Geometric quench: a tool to study “graviton” oscillations in gapped FQH states



- Allows to test predictions of bimetric theory and reveals new phenomena beyond that theory

- **Dynamical response of non-Abelian states, nematic phases, CFL, FCIs ?**

- Constrained Hilbert space facilitates a formation of quantum many-body scars



- Analog of periodic orbits in single-particle chaotic systems, manifested also by the presence of special non-ergodic eigenstates throughout the spectrum

- **Other scarred models? Full classification of periodic orbits? Applications?**

Acknowledgments


Zhao Liu

ZHEJIANG UNIVERSITY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BROWN


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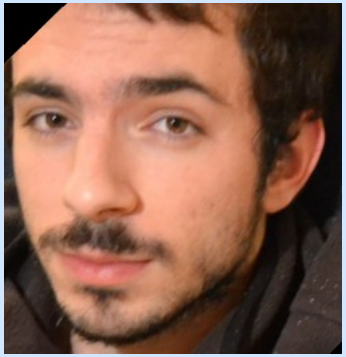
UNIVERSITY OF LEEDS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