Tensor Network (PEPS) approach to

Abelian and/or non-Abelian Chiral Spin Liquids

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Goals:

- Classify/construct chiral spin liquids, analogs of the Fractional Quantum Hall states
- Identify simple (local) quantum spin models hosting these CSL

Relevant framework :

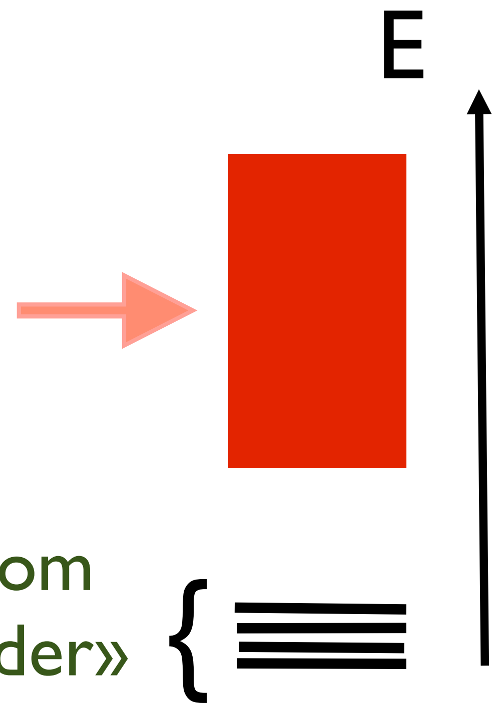
"Projected Entangled Pair States" (PEPS) !

Exotic «topological liquids» beyond the «order parameter» paradigm

* no spontaneous broken symmetry
* no local order but...
* Topological order X. G. Wen

Excitations are fractionalized anyons

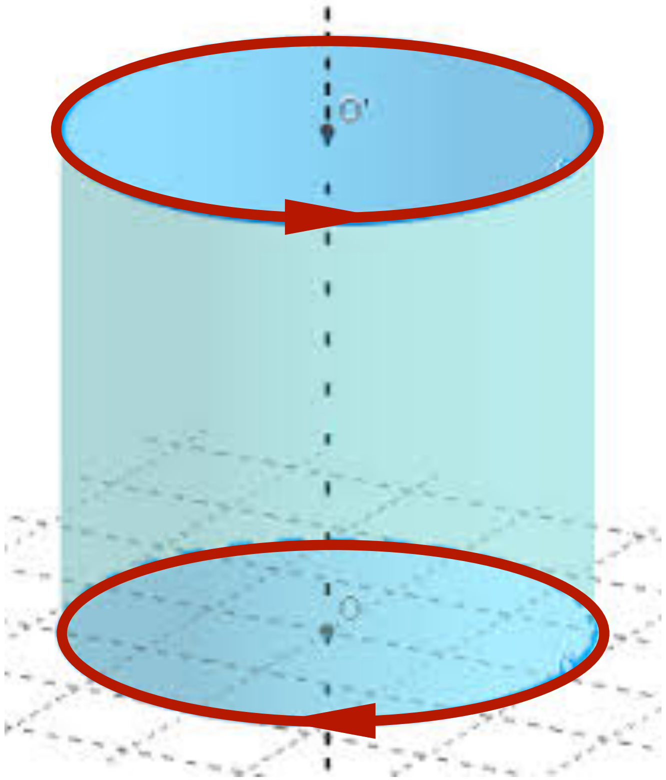
Degeneracy from «topological order»



if T & P are broken :

chiral spin liquids

analogs of FQH states



Protected edge modes described by $SU(2)_k$ CFT

↓

"Long range entanglement"

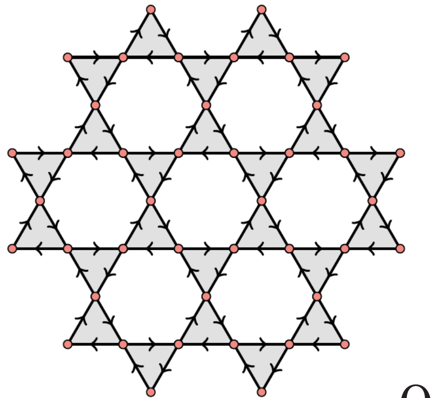
New formalism needed :

Tensor networks

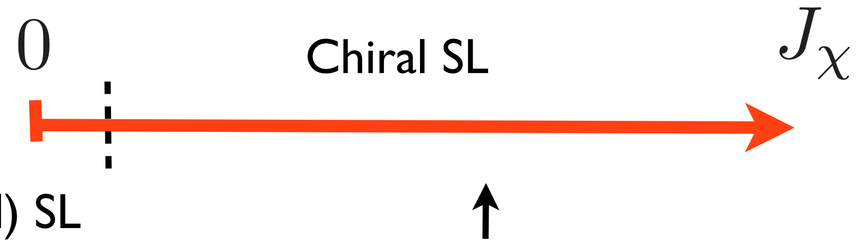
Abelian chiral SL in spin-1/2 chiral AFM (I)

B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig
Nature Communications 5, 5137 (2014)

Kagome lattice



$$H = J_{\mathrm{HB}} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



Edge modes : chiral c=1 CFT ($SU(2)_1$ WZW)

Parent Hamiltonian of Abelian Kalmeyer-Laughlin CSL

$$\Psi_{\text{Laughlin}}(z_1, z_2, ..., z_M) = \prod_{i<j}(z_i - z_j)^q \exp\left(-\frac{1}{4}\sum_i |z_i|^2\right)$$

(z_i are position of up spins on 2D square lattice)

q=2 : bosons at $\nu = 1/2 \rightarrow \text{spins } \ 1/2$

Can be rewritten as CFT correlator of $SU(2)_1$ CFT primary fields :

$$\psi_{\text{P0}}^{\text{CFT}}(s_1, s_2, ..., s_N) \propto \left\langle \phi_{s_1}(z_1)\phi_{s_2}(z_2)...\phi_{s_N}(z_N) \right\rangle$$ (Moore-Read 1991)

↓

Parent Hamiltonien (long-range) → truncation →

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle k,l \rangle\rangle} \mathbf{S}_k \cdot \mathbf{S}_l + \lambda_c \sum_{\square(ijkl)} i(P_{ijkl} - P_{ijkl}{}^{-1}),$$

A. Nielsen, G. Sierra, J.I. Cirac, Nature Com. 4, 2864 (2013)

Parent Hamiltonian of non-Abelian Moore-Read CSL

Bosonic Moore-Read Pfaffian state at $\nu = 1\ (q = 1)$

$$\psi(w_1, \ldots, w_M) \propto \prod_{i<j} (w_i - w_j)^q \mathrm{Pf}\left[\frac{1}{w_i - w_j}\right] e^{-\frac{1}{4}\sum_i |w_i|^2}$$

non-Abelian anyons : $\sigma \times \sigma = 1 + \Psi$

like Kitaev's honeycomb non-Abelian phase

Can be written as CFT correlator of primary fields of $SU(2)_2$ CFT

↓

Parent Hamiltonien (long-range)

truncation →

Spin-1 chiral HAFM

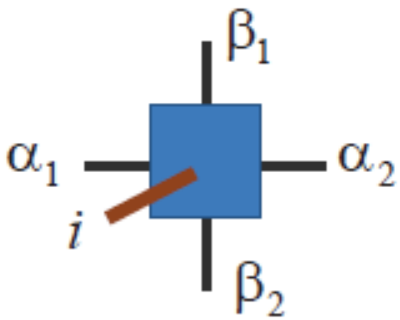
$$\begin{aligned} H = {} & J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle k,l \rangle\rangle} \mathbf{S}_k \cdot \mathbf{S}_l \\ & + K_1 \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + K_2 \sum_{\langle\langle k,l \rangle\rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \\ & + K_c \sum_{\square} [\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) + \mathbf{S}_j \cdot (\mathbf{S}_k \times \mathbf{S}_m) \\ & + \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_m) + \mathbf{S}_i \cdot (\mathbf{S}_k \times \mathbf{S}_m)] \end{aligned}$$

I. Glasser, I. Cirac, G. Sierra & A. Nielsen (2015)

PEPS tensor networks as variational ansatz

$$|\Psi\rangle = \sum C_{i_1,i_2,\ldots,i_N} |i_1, i_2, \ldots, i_N\rangle$$

$$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$$



$$i = \{1, \cdots, d_{\text{phys}}\}$$

$$\alpha, \beta = \{1, \ldots, D\}$$

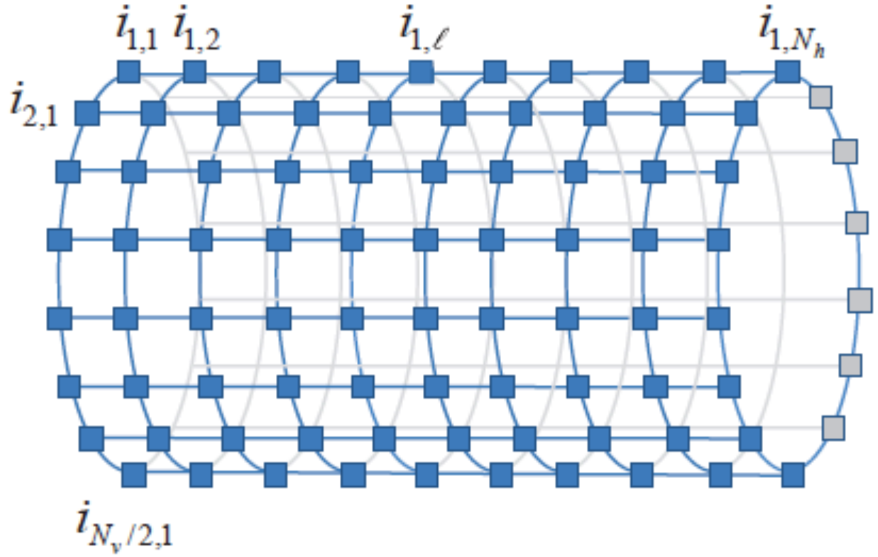
dimension of auxilary (or virtual) space

I. Cirac
F. Verstraete
G. Vidal

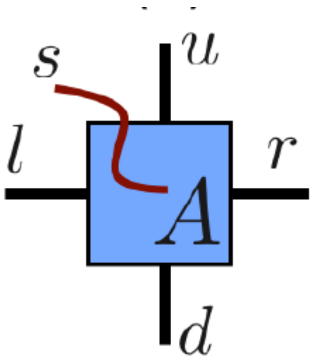
$$N = N_v N_h$$

Coefficients $C_{\{i_{1,1},\ldots,i_{N_v,N_h}\}}$ of the wavefunction

"contract" product of tensors



Guessing the PEPS wavefuntion !



Using a classification of SU(2)-invariant PEPS

M. Mambrini, R. Orus & DP, Phys. Rev. B 94, 205124 (2016)

* virtual space : $V = S_1 \oplus S_2 \oplus \cdots S_p$
* Irreps of point group
  (C4v for square lattice)

Chiral PEPS ansatz: $A = A_R + iA_I$

$$A_R = \sum_\alpha \lambda_\alpha A_\alpha^{(A_1)} \qquad A_I = \sum_\beta \gamma_\beta A_\beta^{(A_2)}$$

Different irreps !

No-go theorem for chiral PEPS ?

J. Dubail, N. Read
Phys. Rev. B 92, 205307 (2015)

Chiral TNS of free fermions (Gaussian PEPS) have no gapped local parent Hamiltonians

Exemple by

T.B. Wahl, H.-H. Tu, N. Schuch, J.I. Cirac
Phys. Rev. Lett. 111, 236805 (2013)

«They are ground states of two different kinds of free-fermion Hamiltonians: (i) local and gapless;
(ii) gapped, but with hopping amplitudes that decay according to a power law.»

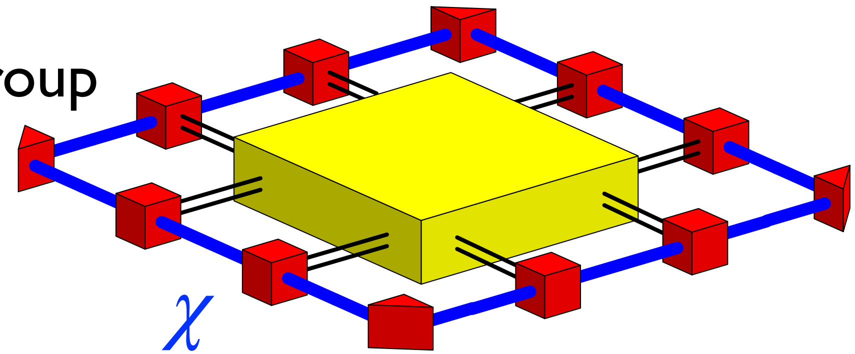
For interacting spins is there any obstruction to construct "gapped" chiral topological PEPS ?

iPEPS method

$$\langle \Psi_{\text{PEPS}} | H_{\square} | \Psi_{\text{PEPS}} \rangle$$

$$||$$

CTM Renormalization Group algorithm



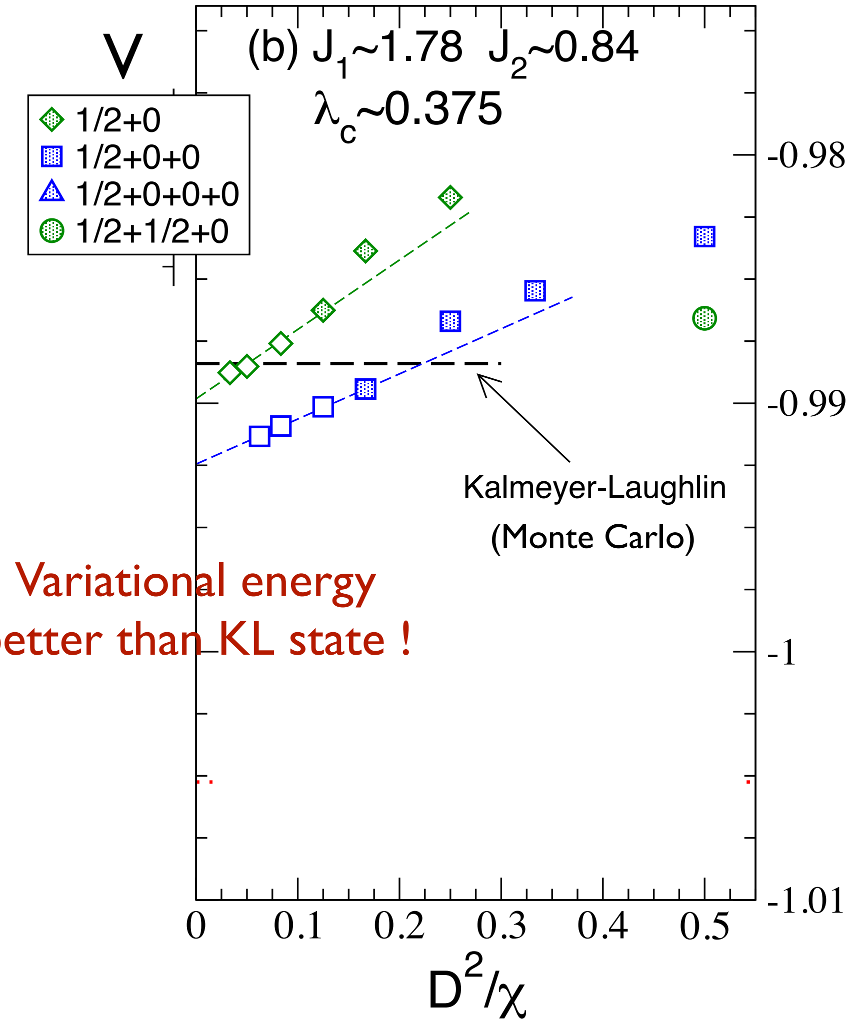
- Environment constructed by renormalization of the corner transfer matrix (CTM)

T. Nishino & K. Okunichi, J. Phys. Soc. J. **65**, 891 (1996)
R. Orus & G. Vidal, Phys. Rev. B **80**, 094403 (2009)

- Variational optimisation scheme based on a conjugate gradient method

Variational energy

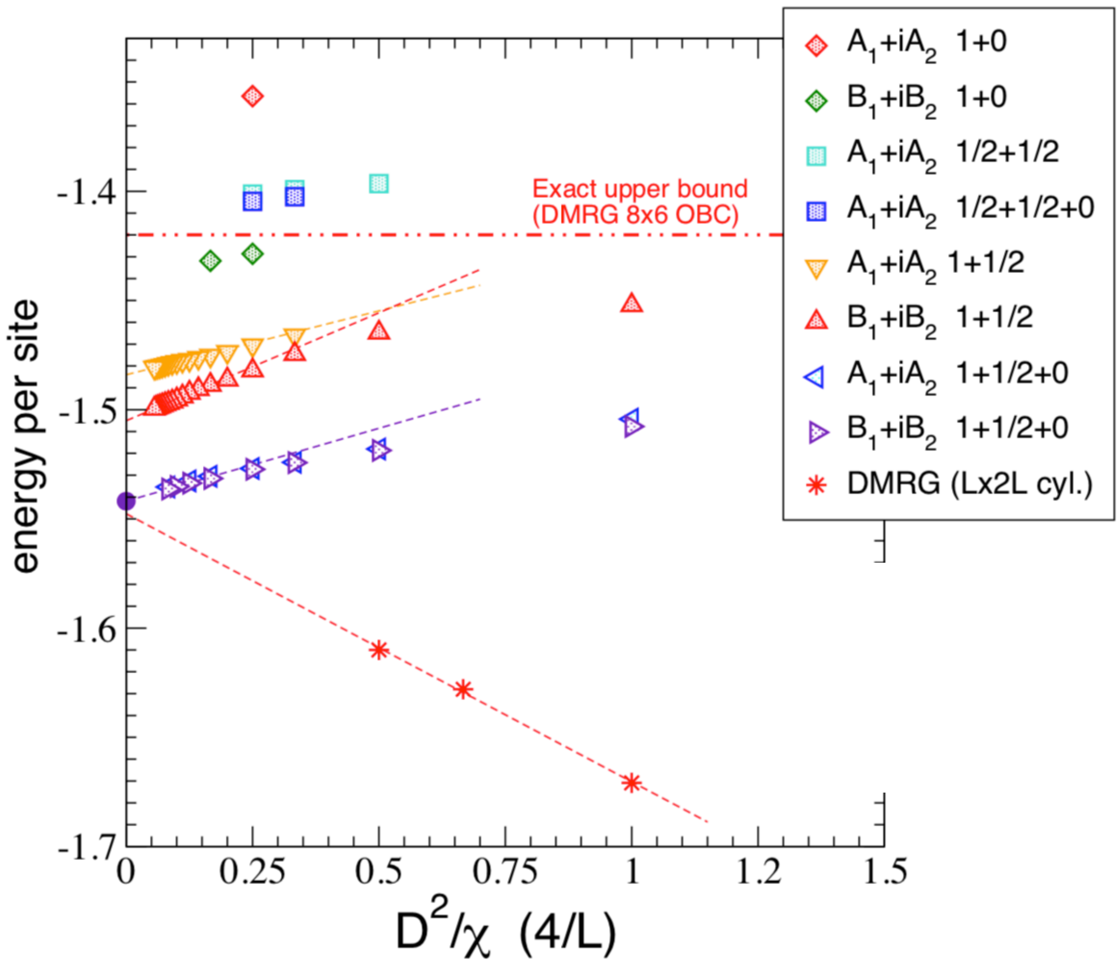
Spin-1/2



Variational energy better than KL state !

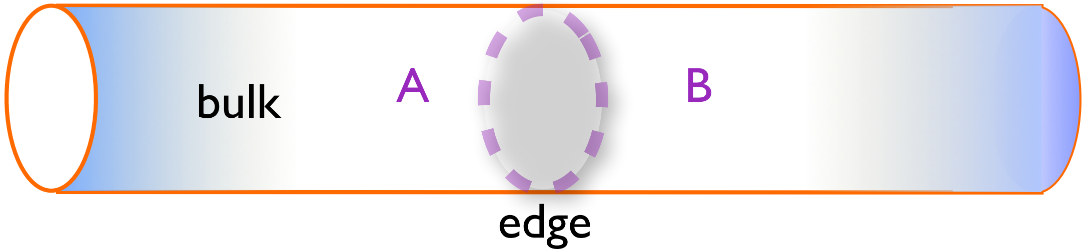
Spin-1

V



χ = environment dimension

Entanglement spectrum



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

Li & Haldane conjecture : $\rho_A = \exp(-H_b)$

One-to-one correspondence between ES and edge mode spectrum

Use PEPS bulk-edge correspondence

$$\rho_A = U \sigma_b^2 U^\dagger$$

$$\sigma_b^2 = \exp(-H_b^{\mathrm{edge}})$$

→

Compare spectrum of H_b^{edge}

to predictions of TQFT:

Bulk CFT ↔ edge CFT

Conformal tower of $SU(2)_1$ CFT

= chiral Luttinger Liquid
Central charge c=1

Even

Table 15.1. States in the lowest grades of the $\widehat{su}(2)_1$ module $L_{[1,0]}$.

L_0	S_z −2	−1	0	1	2	$su(2)$ decomposition
0			1			(0)
1		1	1	1		(2)
2		1	2	1		(2)+(0)
3		2	3	2		2(2)+(0)
4	1	3	5	3	1	(4)+2(2)+2(0)
5	1	5	7	5	1	(4)+4(2)+2(0)
6	2	7	11	7	2	2(4)+5(2)+4(0)

Odd

Table 15.2. States in the lowest grades of the $\widehat{su}(2)_1$ module $L_{[0,1]}$.

L_0	$2S_z$ −2	−1	0	1	2	3	$su(2)$ decomposition
$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$		1	2	2	1		(3)+(1)
$\frac{13}{4}$		1	3	3	1		(3)+2(1)
$\frac{17}{4}$		2	5	5	2		2(3)+3(1)
$\frac{21}{4}$		3	7	7	3		3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)

Philippe Di Francesco
Pierre Mathieu
David Sénéchal

Conformal Field Theory

“Conformal tower of states” with very precise content !

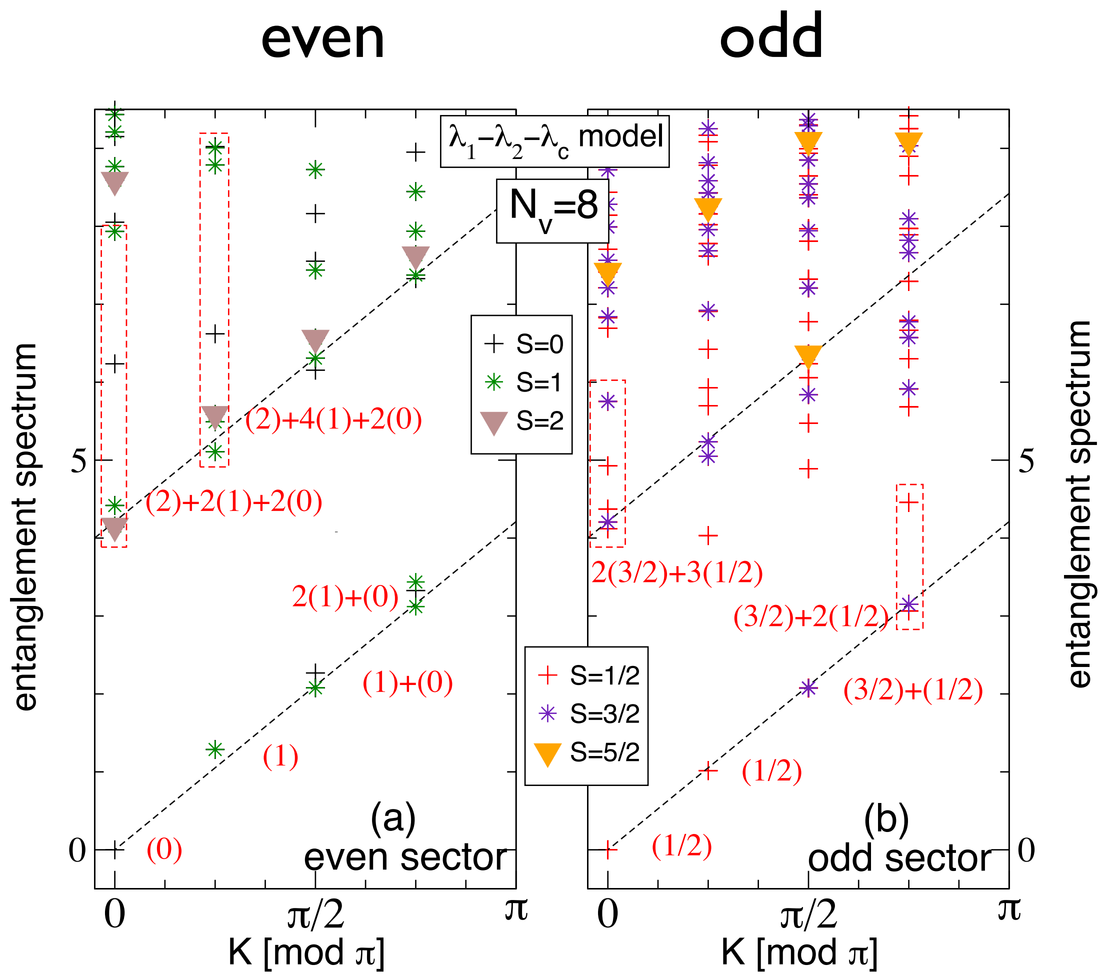
Conformal tower of $SU(2)_2$ CFT

Central charge c=3/2
Majorana (Ising) + boson

Conformal tower:

$n\backslash j$	0	$\frac{1}{2}$	1
0	(0)	$(\frac{1}{2})$	(1)
1	(1)	$(\frac{1}{2})+(\frac{3}{2})$	$(0)+(1)$
2	$(0)+(1)+(2)$	$2(\frac{1}{2})+2(\frac{3}{2})$	$(0)+2(1)+(2)$
3	$(0)+3(1)+(2)$	$4(\frac{1}{2})+3(\frac{3}{2})+(\frac{5}{2})$	$2(0)+3(1)+2(2)$
4	$3(0)+4(1)+3(2)$	$6(\frac{1}{2})+6(\frac{3}{2})+2(\frac{5}{2})$	-
5	$3(0)+8(1)+4(2)+(3)$	$10(\frac{1}{2})+10(\frac{3}{2})+4(\frac{5}{2})$	-

Entanglement spectrum for spin-1/2 CSL

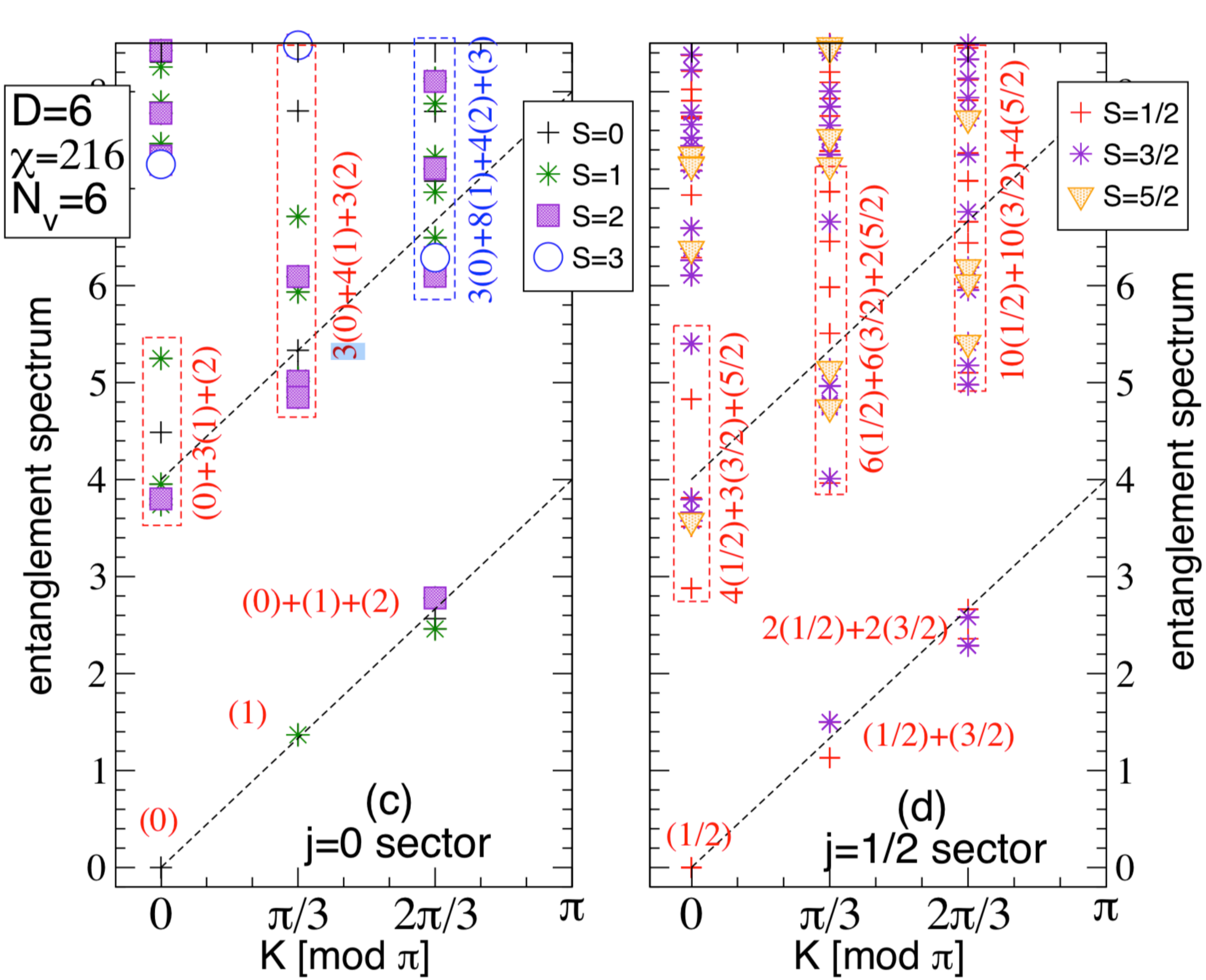


"tower of states" of chiral SU(2)$_1$ CFT

Abelian Laughlin Chiral SL

DP, Phys. Rev. B **93**, 121118 (2017)

Entanglement spectrum for spin-1 CSL



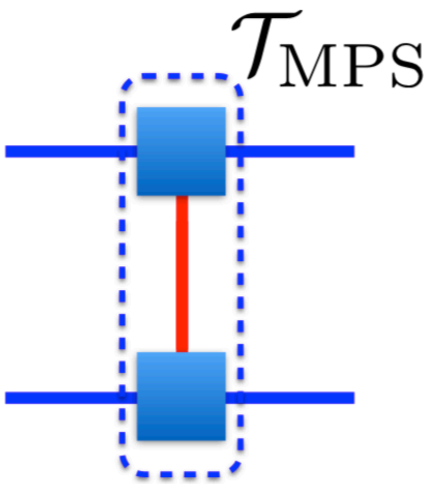
"tower of states" of chiral $SU(2)_2$ CFT & c~1.5

non-Abelian Moore-Read Chiral SL

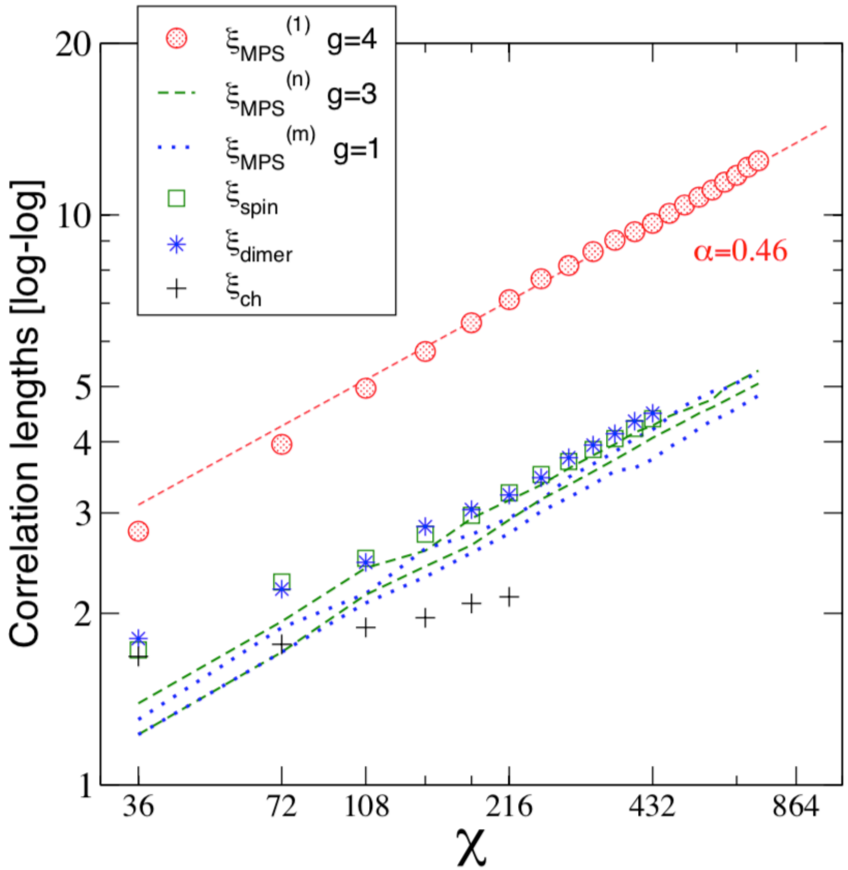
Correlation lengths

(S=1)

Transfer matrix



$$\xi_n = -1/\ln(\lambda_n/\lambda_{\max})$$

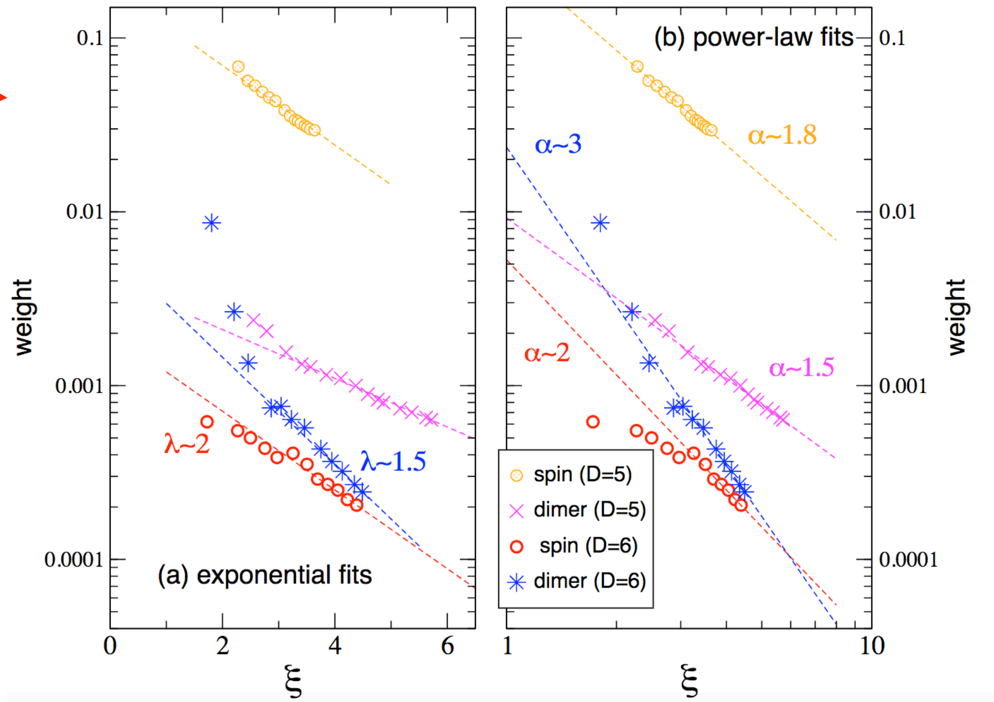


Diverging correlation length !

Gossamer long-range correlations

$$C(d) = C_{\text{bulk}}(d) + C_{\text{tail}}(d)$$

$$C_{\text{tail}}(d) = \sum_{i>i_{\text{tail}}} w(\xi_i)\, \exp(-d/\xi_i)$$



(Laplace transform)

stretched exponential

power law

Discussion & conjecture

- "Critical" bulk behavior needed to obtain gapless chiral modes (due to "bulk-edge" correspondence) : essence of no-go theorem…
- The long-range correlation "tail" is an artifact of chiral PEPS !
- NOT a practical limitation in PEPS descriptions of chiral SL

“PEPS collaborators”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