Anyonic excitations of hardcore anyons

J. Wildeboer, S. Manna, A.E.B. Nielsen, arXiv:1711:00845 (2017)

J. Wildeboer, N.E. Bonesteel, Phys. Rev. B 94, 045125 (2016)

S. Manna, J. Wildeboer, G. Sierra, A.E.B. Nielsen, Phys. Rev. B 98, 165147 (2018)

S. Manna, J. Wildeboer, A.E.B. Nielsen, arXiv:1810:12288 (2019)

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Anyons in Quantum Many-Body Systems, workshop at MPIPKS

Topological Order in FQH

- **systems with topological order**

fractional Quantum Hall effect: incompressable quantum liquid with fractionalized excitations (Laughlin, 1983)

Quantum Spin Liquids:

- quantum Hall-type states on lattices:

$\Longrightarrow$ Kalmeyer-Laughlin (KL) spin liquid, see for example: N.E. Bonesteel, PRB 2000,
J. Wildeboer, N.E. Bonesteel, unpublished

$\Longrightarrow$ non-Abelian Moore-Read (MR) spin liquid, see for example:

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huge playground:

review Kalmeyer-Laughlin spin liquid

and play with it !

Kalmeyer-Laughlin chiral spin liquid

- precursor: Abelian spin-1/2 chiral spin liquid
  Kalmeyer and Laughlin, PRL 59, 2095 (1987)

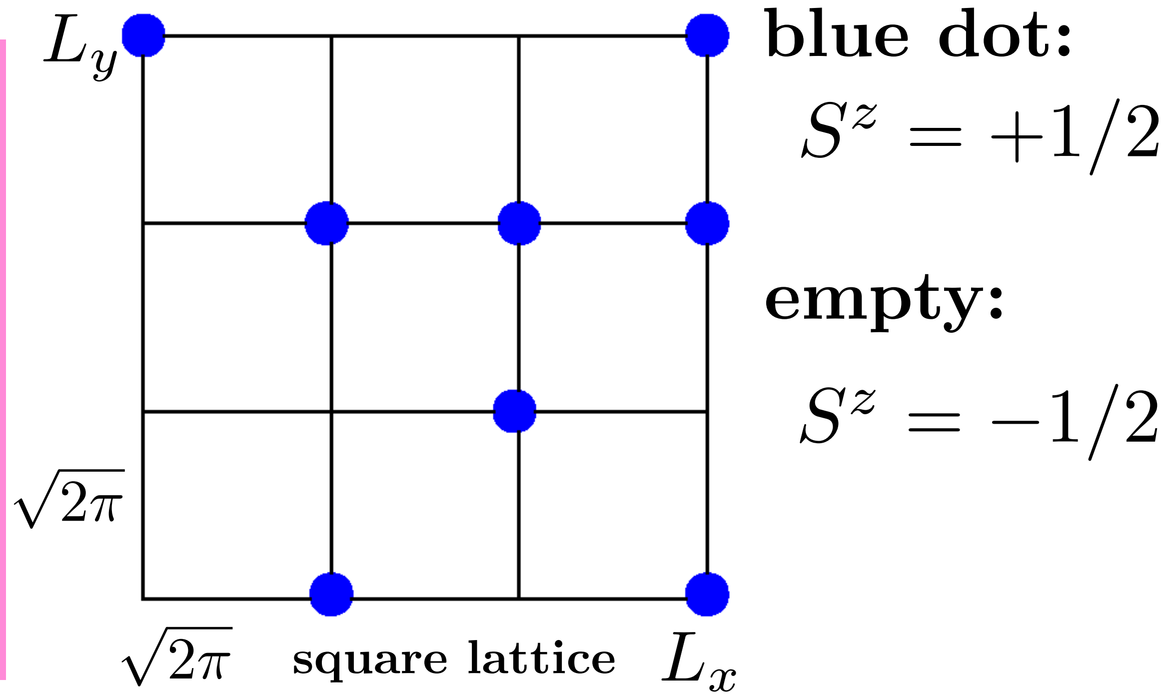
- bosonic FQHE $\nu = 1/2$:

$$\Psi[z_i] = \prod_{i<j}^{N} (z_i - z_j)^2 \prod_{i=1}^{N} e^{-\frac{1}{4}|z_i|^2} \qquad \text{Laughlin} \quad (q = \frac{1}{\nu} = 2)$$

- spin-$\frac{1}{2}$ liquid wave function:

$$|\Psi\rangle = \sum_{\{z_1,\ldots,z_N\}} \Psi(z_1,\ldots,z_N) \prod_{i=1}^{N} G(z_i)\, S^+_{z_1} \ldots S^+_{z_N} |\downarrow\downarrow \ldots \downarrow\rangle$$

- lattice system: $z = (n_x + i\, n_y)\sqrt{2\pi}$
- one flux quantum per plaquette
- singlet in the therm. limit



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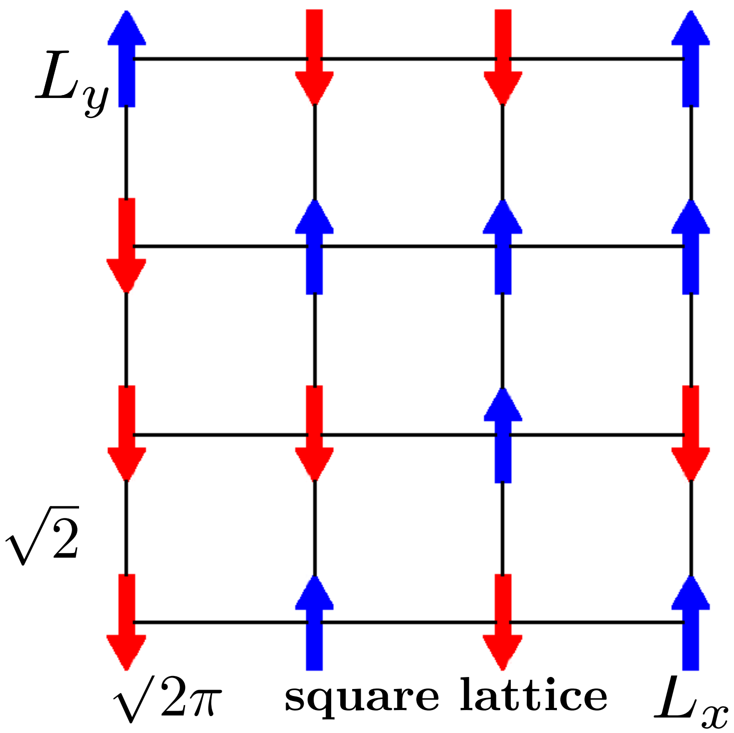


blue dot:
$S^z = +1/2$

empty:
$S^z = -1/2$

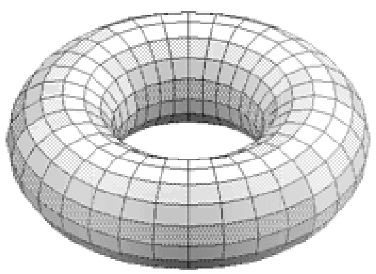
Kalmeyer-Laughlin chiral spin liquid

- Abelian CSL on the torus: 2-fold ground state degeneracy !

  - bosonic FQHE $\nu = 1/2$:

$$\Psi_n[z_i] = \vartheta_1[Z - \frac{n}{2}L_x]^2 \prod_{i<j}^{N} \vartheta_1[z_i - z_j]^2 \prod_{i=1}^{N} e^{-y_i^2/2}$$

$n = 0, 1$

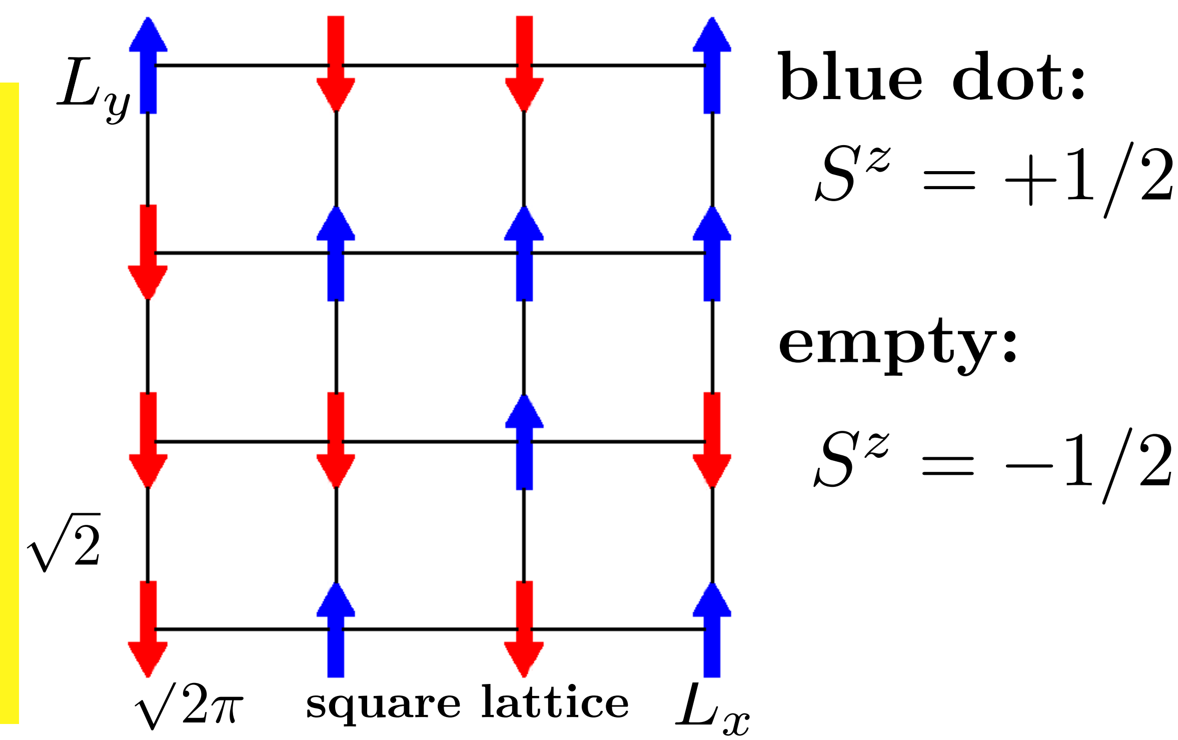


square lattice

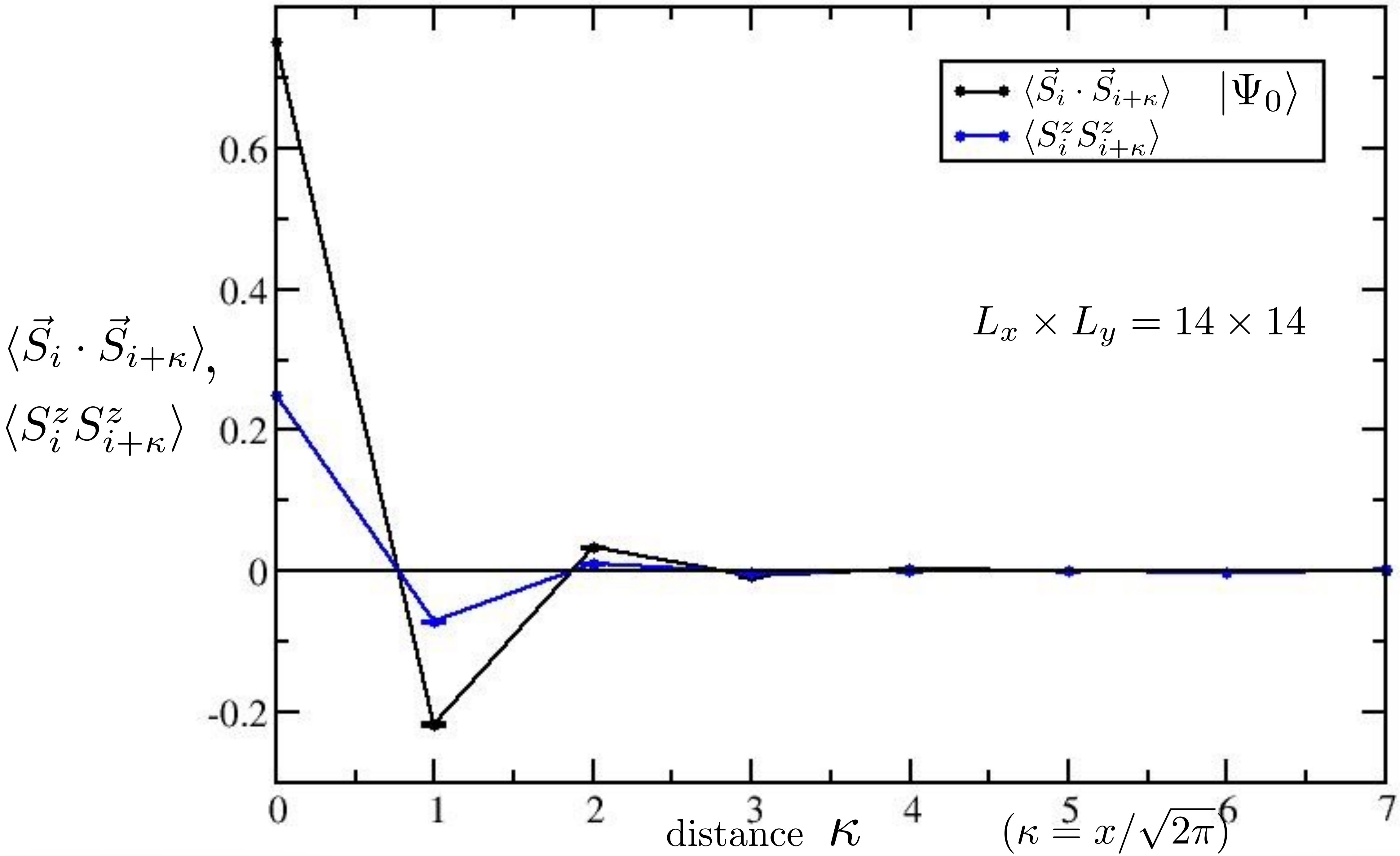
- spin-$\frac{1}{2}$ liquid wave function:

$$|\Psi_n\rangle = \sum_{\{z_1,\dots,z_N\}} \Psi_n(z_1,\dots,z_N) \prod_{i=1}^{N} G(z_i)\, S^+_{z_1} \dots S^+_{z_N} |\downarrow\downarrow \dots \downarrow\rangle$$

- lattice system: $z = (n_x + i\, n_y)\sqrt{2\pi}$
- one flux quantum per plaquette
- **exact singlet !**



Kalmeyer-Laughlin CSL: correlations (torus)



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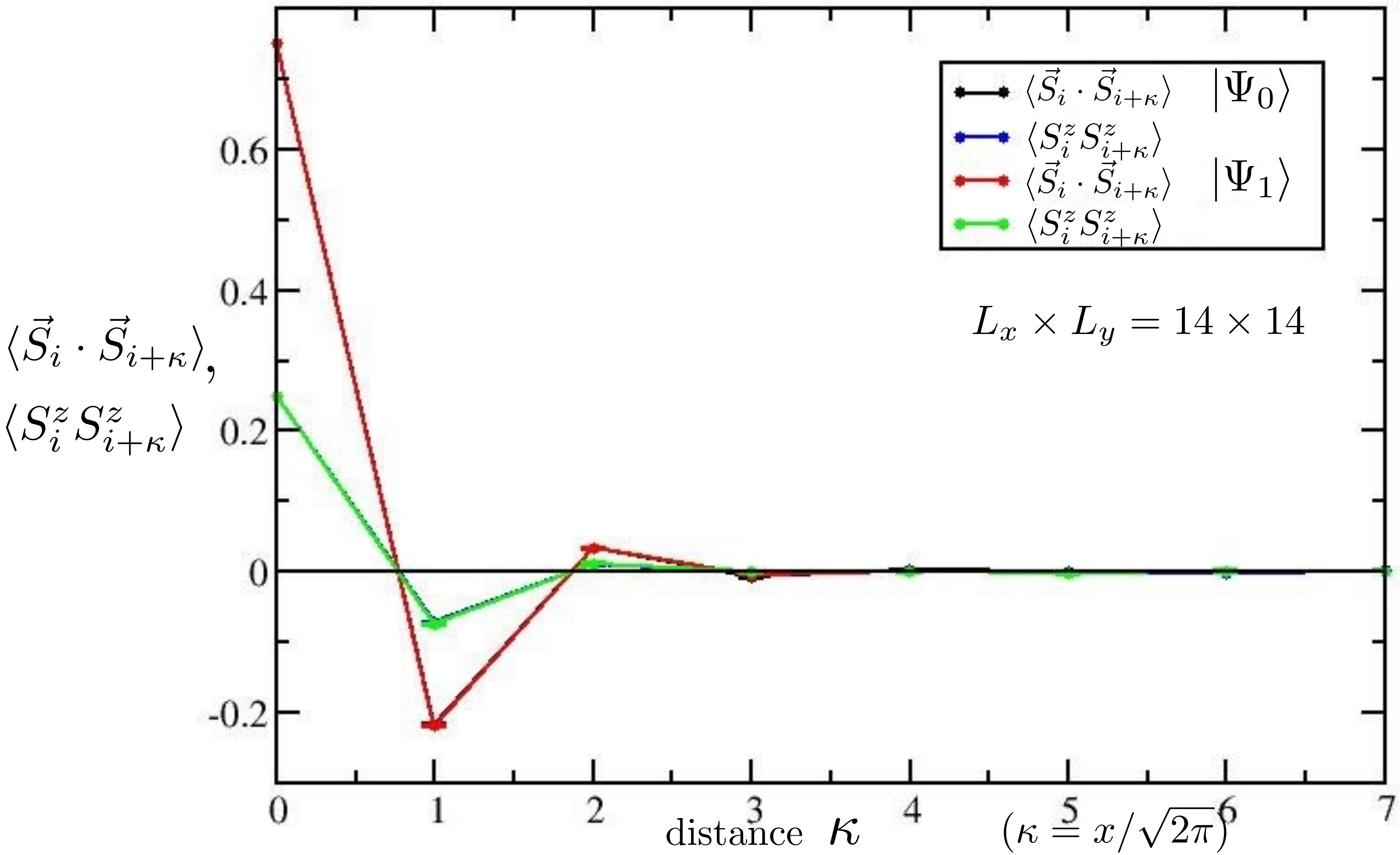

- states $|\Psi_0\rangle$ and $|\Psi_1\rangle$ are undistinguishable by their correlations
- $\langle \vec{S}_i \cdot \vec{S}_{i+x} \rangle = 3 \times \langle S_i^\alpha S_{i+x}^\alpha \rangle$, $\alpha = x, y, z$

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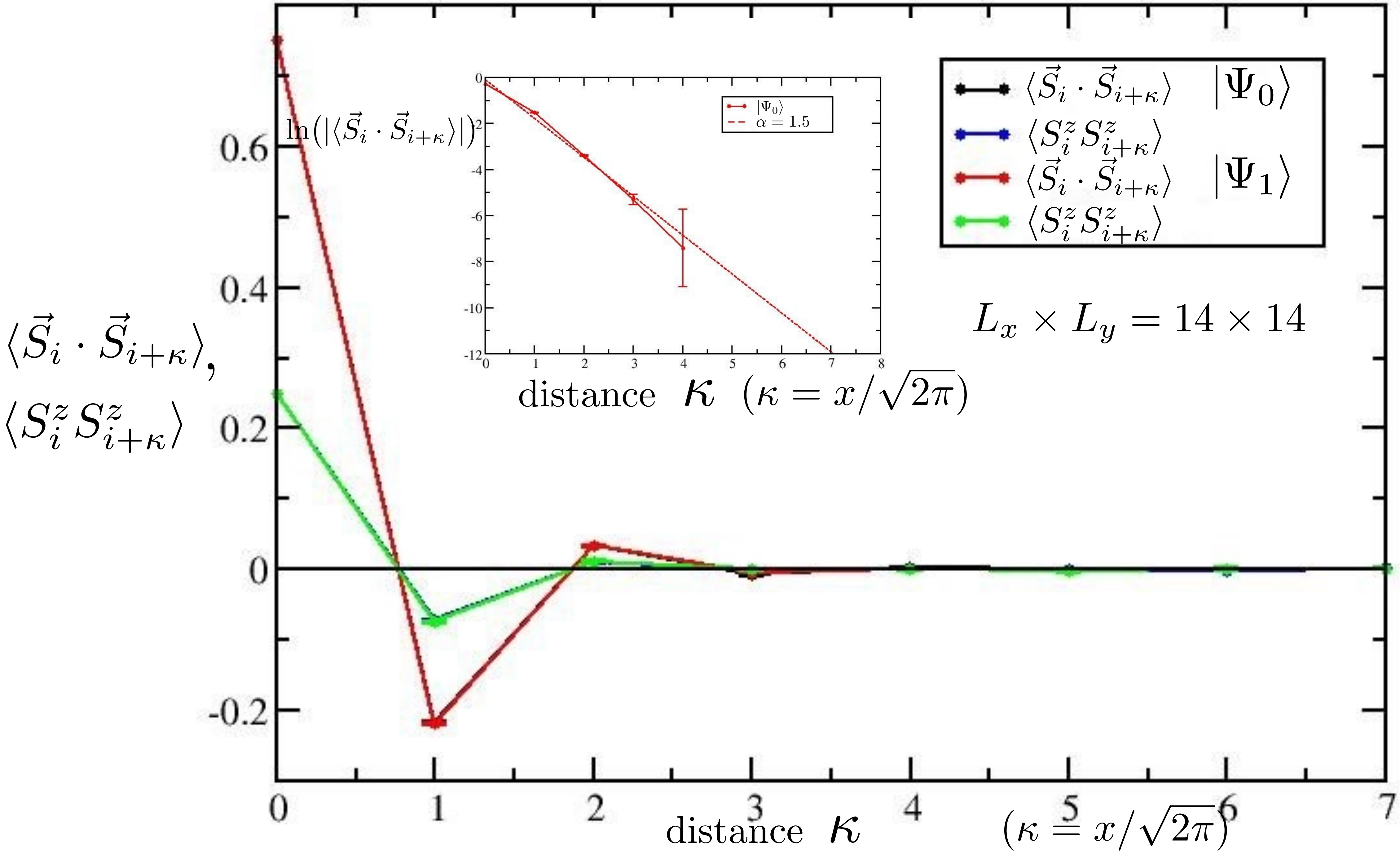

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Conformal Field Theory Wave Functions

- lattice with lattice sites at positions in complex plane: $z_j = x_j + iy_j$ with $j = 1, \ldots, N$

  local basis at site j: $|n_j\rangle$ $n_j \in \{0, 1\}$

- family of states (CFT states) from chiral correlator of vertex operators:

$$\Psi(n_1, \ldots, n_j) \propto \langle V_{n_1}(z_1) \ldots V_{n_N}(z_N) \rangle$$

$$\Rightarrow V_{n_j}(z_j) = \chi_j^{n_j} e^{i\pi \sum_{k(<j)} n_k n_j} : e^{i(qn_j - \eta_j)\Phi(z_j))/\sqrt{q}} :$$

$\Phi(z)$: chiral boson field from c = 1 free-boson CFT

$: \ldots :$ normal ordering

q : positive number

η_j are positive parameters with average $N^{-1} \sum_j \eta_j = \eta \in (0, 1]$

χ_j : phase factors

charge neutrality condition $\sum_i (qn_i - \eta_i) = 0$ fixes number of particles to $\sum_i^N n_i = \eta N / q = M$

M has to be integer $\Longrightarrow$ η / q is lattice filling fraction, interpolates between continuum and lattice $(\eta = 1)$

Anyonic CFT Wave Functions

- from Conformal Field Theory:

$$|\psi_q\rangle = \sum_{n_1,n_2,\ldots,n_N} \psi_q|n_1,n_2,\ldots,n_N\rangle$$

with amplitudes $$\psi_q = \mathcal{C}^{-1}\delta_n \prod_{i<j}(z_i - z_j)^{q n_i n_j} \prod_{i \neq j}(z_i - z_j)^{-n_i}$$

$\mathcal{C}$: real renormalization constant

$$\delta_n = 1 \text{ for } \sum_i n_i = N/q$$

For integer q the states are the normal Laughlin states with both the particles and the background charge are restricted to be on the specified lattice sites !

- re-express the states as follows:

$$\psi_q \propto \delta_n \prod_{i<j}(Z_i - Z_j)^q \prod_{\{i,j|Z_i \neq z_j\}} (Z_i - z_j)^{-1}$$

$$Z_j \in \{z_1, z_2, \ldots, z_N\}$$

Observation: wave function acquires a phase factor $\phi = e^{i\pi q}$ if one particle is moved counter clockwise another particle

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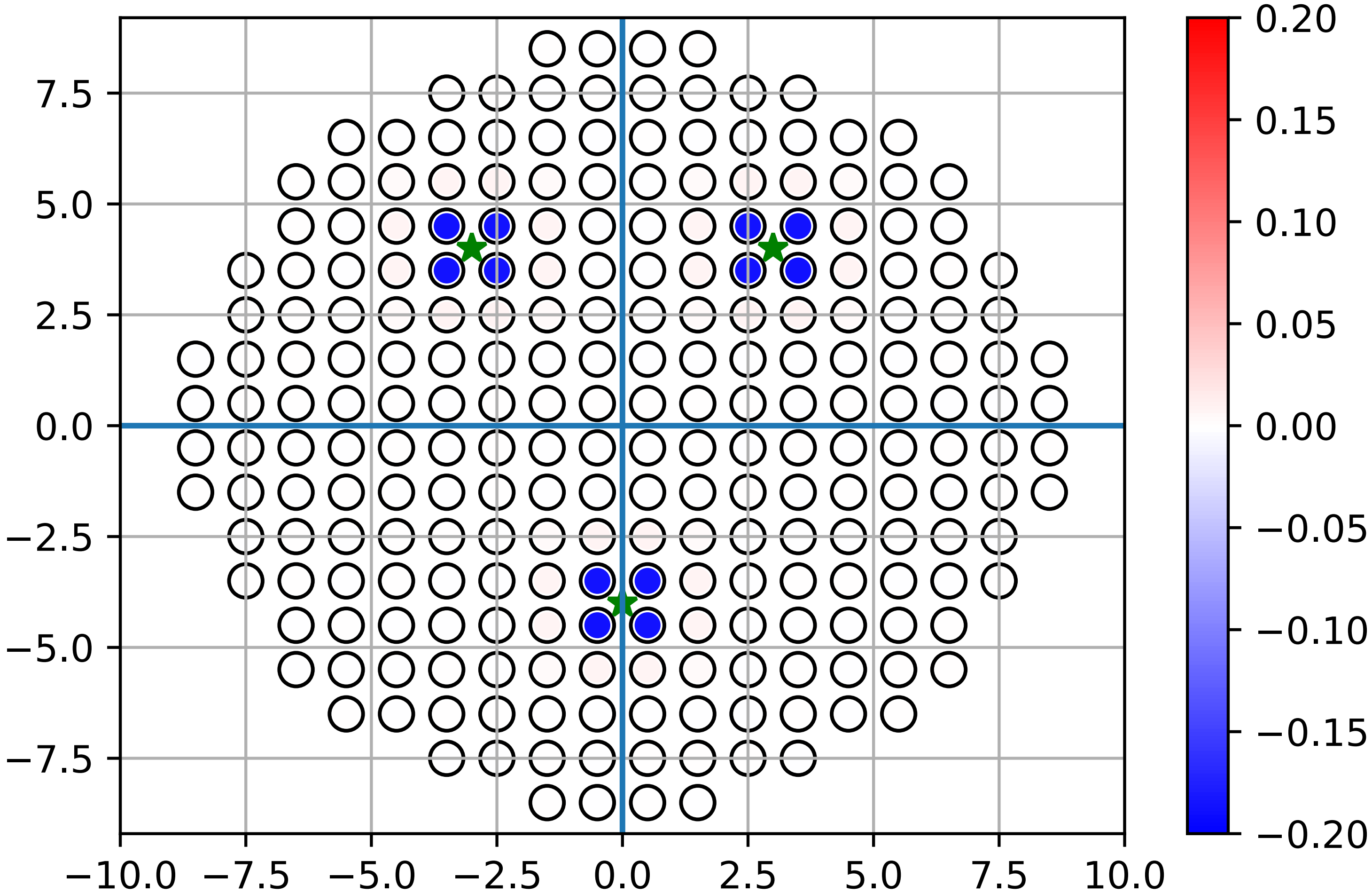
Observation: wave function acquires a phase factor $\phi = e^{i\pi q}$ if one particle is moved counter clockwise another particle

q odd $\Longrightarrow$ fermions $\phi = -1$

q even $\Longrightarrow$ bosons $\phi = +1$

q non-integer $\Longrightarrow$ **hardcore anyons** $\phi \neq \pm 1$

Charge Distribution of Anyons: q = 3/2



variational Monte Carlo study

(wave function Monte Carlo)

coloring = $\langle \psi_{q,\vec{\omega}} | n_i | \psi_{q,\vec{\omega}} \rangle - \langle \psi_q | n_i | \psi_q \rangle$

coloring = negative charge distribution of anyons by summation over all sites in small region around each anyon

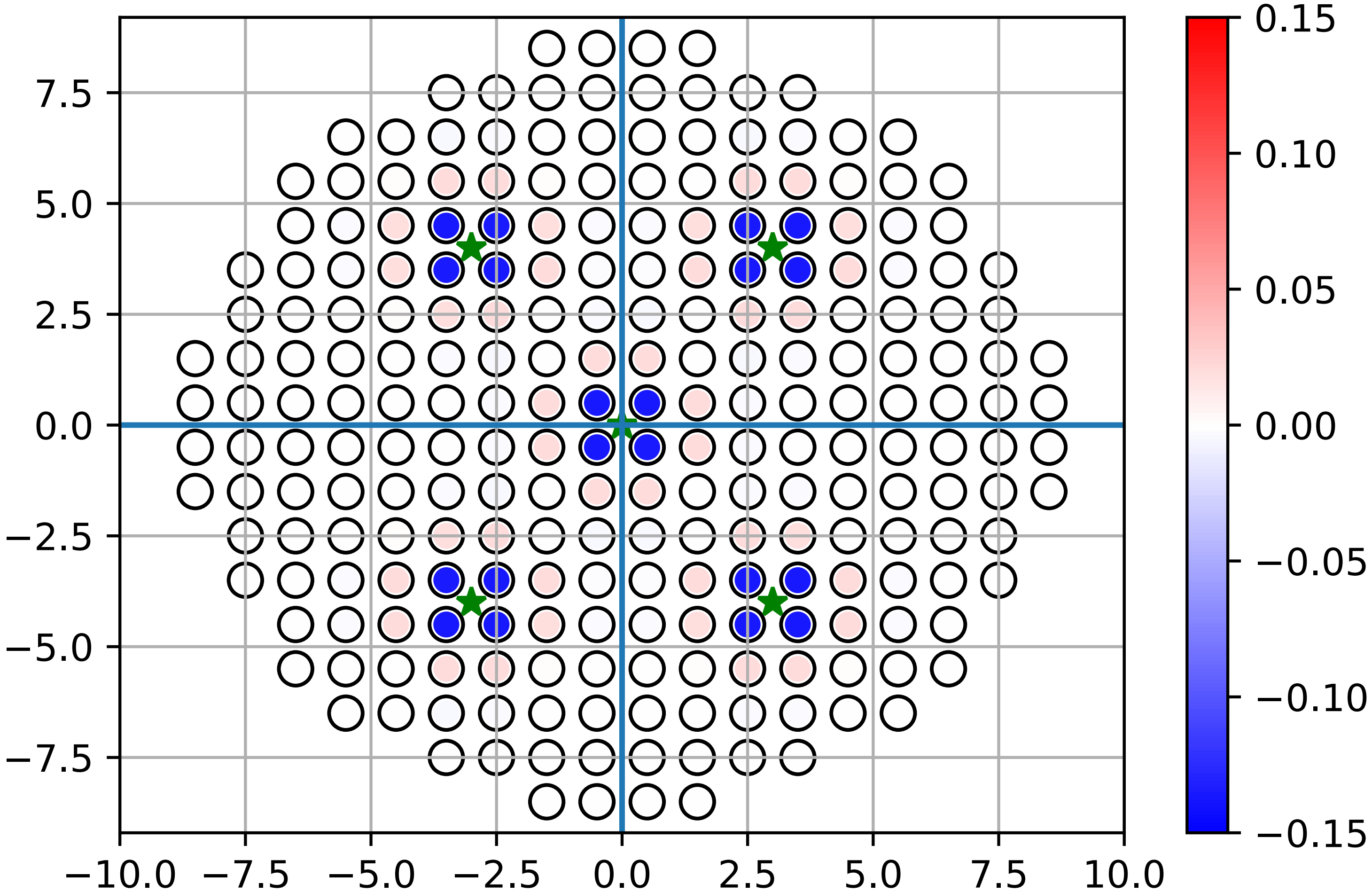
→ **3 anyons of charge 2/3 with q = 3/2**

- ❍ lattice sites
- ★ position of anyons

coloring = difference of expectation value of number of particles on a site with and without anyons present: $\langle \psi_{q,\vec{\omega}} | n_i | \psi_{q,\vec{\omega}} \rangle - \langle \psi_q | n_i | \psi_q \rangle$

⟹ **screening**

Charge Distribution of Anyons: q = 5/2



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coloring = negative charge distribution of anyons by summation over all sites in small region around each anyon

$\longrightarrow$ **5 anyons of charge 2/5 with q = 5/2**

❍ lattice sites

★ position of anyons

coloring = difference of expectation value of number of particles on a site with and without anyons present: $\langle \psi_{q,\vec{\omega}} | n_i | \psi_{q,\vec{\omega}} \rangle - \langle \psi_q | n_i | \psi_q \rangle$

$\Longrightarrow$ **screening**

Anyonic CFT Wave Functions: Berry Phase

- add Q anyons with charge p_j/q with $p_j \in \mathbb{N}$ at positions ω_j

$$\Rightarrow \psi_{q,\vec{\omega}} = \mathcal{C}_{\vec{\omega}}^{-1} \delta_n \underbrace{\prod_{i,j} (\omega_i - z_j)^{p_i n_j}}_{\text{new factor to include } Q \text{ anyons}} \prod_{i<j} (z_i - z_j)^{q n_i n_j} \prod_{i \neq j} (z_i - z_j)^{-n_i}$$

reduce number of particles: $\delta_n = \begin{cases} 1 & \text{for} \quad \sum_{j=1}^{N} n_j = (N - \sum_{j=1}^{Q} p_j)/q \\ 0 & \text{otherwise} \end{cases}$

positions of Q anyons: $\vec{\omega} = (\omega_1, \omega_2, \ldots, \omega_Q)$

real renormalization constant: $\mathcal{C}_{\vec{\omega}}$

$\Rightarrow$ **What is the result of braiding the coordinate ω_k around the coordinate ω_j ?**

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- Berry phase:

$$\theta_k = i \oint_c \langle \psi_{q,\vec{\omega}} | \frac{\partial \psi_{q,\vec{\omega}}}{\partial \omega_k} \rangle d\omega_k + c.c.$$
$$= i \frac{p_k}{2} \oint_c \sum_i \frac{\langle \psi_{q,\vec{\omega}} | n_i | \psi_{q,\vec{\omega}} \rangle}{\omega_k - z_i} d\omega_k + c.c.$$

We are looking for: $\Delta\theta_k = \theta_{k,in} - \theta_{k,out}$

$\theta_{k,in}$ $(\theta_{k,out})$ Berry phase when ω_k is well inside (outside) the closed path c

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$$= 2\pi p_k p_j / q$$

Anyonic CFT Wave Functions: Berry Phase

- add Q anyons with charge p_j/q with $p_j \in \mathbb{N}$ at positions ω_j

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- same result for Berry phase as for integer q state (Laughlin) states
- state $|\psi_q\rangle$ hosts anyons of charge p_j/q if screening occurs

Summary

- Conformal Field Theory (CFT) powerful tool to derive FQH wave functions from correlators
- Laughlin (spin liquid) state with non-integer q: state constructed from anyons

$\longrightarrow$ systems of anyons can support the formation of anyonic quasi-particles (screening)

(i) braiding one of the original anyons around another: $\phi = e^{i\pi q}$

(ii) braiding an emergent anyon of charge p_k/q around another emergent quasiparticle with charge p_j/q: $\phi = e^{2\pi i p_k p_j / q}$

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future/outlook:

- generalize to the case of a torus, find braiding statistics! Difficult problem!
- non-integer q calculation for Moore-Read state
- FQH lattice systems: huge playground, quasi holes and quasi electrons

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poster
Sourav Manna

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