Two-Phase Continuum Models for Geophysical Particle-Fluid Flows

Time-dependent measurements for incipient bed load discharge on shallow open channel flows

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**General framework**

\[ \nabla \]

\[ \bar{u}(z) \]

\[ \rho, \mu \]

\[ \Delta \rho_* = \frac{\rho_s}{\rho} - 1 \]

\[ \tau_b \leftrightarrow \tau_c \]

wash load

\[ w_s \]

suspension

bedload

\[ q_0 ; u_0 ; h_0 ; \]

\[ \rightarrow \tau_b ; u_* ; \]

\[ u_* > u_{*c} : \]

Transport

\[ q_* = A(Sh - Sh_c)^B. \]

**Shields number**

\[ Sh = \frac{u_*^2}{(\Delta \rho_* g D)}; \]

(Shields, 1936; Paphitis, 2001; Beheshti and Ataie-Ashtiani, 2008)

(Meyer-Peter and Müller, 1948; Wong and Parker, 2006)
Dimensionless analysis

Reynolds: \[ \text{Re} = 4 \frac{u_0 h_0}{\nu} \] flow dynamics;

Froude: \[ \text{Fr} = \frac{u_0}{\sqrt{g h_0 \cos \theta}} \] flow hydraulic regimen;

Shields: \[ \text{Sh} = \frac{u^2}{\Delta \rho \cdot g D} \] particles initiation of motion;

Particle Reynolds: \[ \text{Re}_p = \frac{u_0 D}{\nu} \] particle-flow interaction;

Stokes: \[ \text{St} = \frac{T_p}{T_f} \] particles-flow interaction;

Particle Froude: \[ \text{Fr}_p = \frac{u_0}{\sqrt{g D}} \] flow capacity of transport;

Rouse: \[ \frac{w_s}{\kappa u_*} ; \frac{w_s}{u_*} ; \frac{u_*}{w_s} \] dominant mode of sediment transport;

(suspension or movability)
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(suspension or movability)
Non-stationary transport - Turbulence

\[ u_* \propto u_0 \]

Figure 1 – Fluctuations of the instantaneous velocity around the mean value. Particle entrainment occurs sporadically. (Ancey et al., 2008).
Non-stationary transport - Grain-size distribution

Heterogeneous

Homogeneous

Figure 2 – Solid discharges of materials with different granulometry (Frey et al., 2003).
Non-stationary transport - Flow instabilities

Free surface instabilities & Bedforms

Figure 3 – Evolution of roll waves and sediment transport (Davies, 1990).

Figure 4 – Comparison of stability lines for roll waves and bedforms (Colombini and Stocchino, 2005).
Main objective

Non-stationary sediment transport

Experimentally study time-dependent effects on sediment transport for runoff flows.
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Non-stationary sediment transport

Experimentally study time-dependent effects on sediment transport for runoff flows.

Close to threshold cases:

Shields number

$Sh < 2.5Sh_c$;

(Recking et al., 2009)

Turbulence effects:
Experimental project

Figure 5 – Sketch of experimental setup.

- shallow water flow $h_0/l \ll 1$;
- PIV method for local flow dynamics measurements;
- contact needle for global measurements;
- combined shadowgraph and PTV methods for local sediment transport measurements;
Test sand

Figure 6 – Histogram for particles used in experiments and MEV images.

- non-cohesive;
- regular shape;
- $0.15 < D < 0.25$ mm;
- $\text{Sh}_c = 0.062$;
- favorable to bedload (given the exp. limitations, $w_s \approx 2\text{cm/s}$);
# Dimensionless parameters

<table>
<thead>
<tr>
<th>Dimensionless</th>
<th>Range</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>$3000 &lt; \text{Re} &lt; 4000$</td>
<td>Turbulent (close to transition)</td>
</tr>
<tr>
<td>Fr</td>
<td>$1.1 &lt; \text{Fr} &lt; 1.5$</td>
<td>Torrential flows</td>
</tr>
<tr>
<td>Sh</td>
<td>$0.068 &lt; \text{Sh} &lt; 0.165$</td>
<td>particles initiation of motion</td>
</tr>
<tr>
<td>Rouse</td>
<td>$2.4 &lt; \frac{W_s}{u_*} &lt; 3.6$</td>
<td>bed-load transport</td>
</tr>
</tbody>
</table>

$\text{Sh}_c = 0.062 \text{ (Paphitis, 2001)}$
Methodology debrief

For flow dynamics

- determination of friction velocity $u_*$ based on theoretical turbulent profile; assumption $u_*(t) = F\{\langle u \rangle (y_0, t)\}$; correction of results based on global values (contact needle);
- Obtaining of time-dependent friction velocity $u_*(t)$.

For sediment transport

- Image acquisition from fast-recording camera; images processing on Matlab; PTV algorithm; correction of particles size/velocities; time dependent discharge computation; comparison to global values from mean weighted discharge;
- Obtaining of time-variable transport rate $q(t)$. 
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Statistically...

$$q(t) \equiv f(u_*(t))$$
Friction velocity obtaining

In wall coordinates:
\[ u^+ = u(y)/u_* \]
and
\[ y^+ = u_*y/\nu \]

For viscous sublayer:
\[ u^+ = y^+; \]
For log region:
\[ u^+ = \kappa^{-1} \log y^+ + 5; \]

Figure 7 – Turbulent characteristics of average profile of mean flow velocity. Lines indicate theoretical values: solid line is \( u^+ = \kappa^{-1} \log y^+ + 5 \) for log region; dashed line is \( u^+ = y^+ \) for viscous layer.
Turbulent intensities - $u_{RMS}^+$

(Antonia and Krogstad, 2001)

$$u_{RMS}^+ = A \exp \left(-\frac{y^+}{Re^*}\right) \left[1 - \exp \left(-\frac{y^+}{B}\right)\right] + Cy^+ \exp \left(-\frac{y^+}{B}\right)$$

$A = 2$, $B = 8$, and $C = 0.34$;

$Re^* = u^*_\text{PIV} h_0/\nu$ is the Reynolds number using friction velocity as reference.

Figure 8 – Turbulent intensities for runs gb1 to gb8. Lines indicate computed values following empirical results from Antonia and Krogstad (2001). Longitudinal turbulent intensities $u_{RMS}^+$; dark line represent run computed value for gb1, and gray line, for gb8.
Friction velocity $u_*(t)$

Bottom shear stress:

$\tau_b \sim \rho u_*^2 \sim \rho gh_0 \sin \theta$

For our experiments:

- $u_* \sim 0.01 \text{ ms}^{-1}$;
  $\rightarrow \tau_b \sim (10^3)(10^{-2})^2 \sim 0.1 \text{ Pa}$;

- $h_0 \sim 0.01 \text{ m}$;
  $\rightarrow \tau_b \sim (10^3)(10^1)(10^{-2})(10^{-2}) \sim 1 \text{ Pa}$;

The precision required to measure fluctuations would be $\sim 0.01 \text{ Pa}$

(Detert et al., 2010; Amir et al., 2014)

So, an indirect measure is pursued...
Friction velocity correlation

$\tau_b(t)$?

- $u_*^2 \propto \tau_b \propto u_0^2$
- $u(y_0) \propto u_0 \rightarrow u_* \propto u(y_0)$

Dimensionally and statistically we can assume that:

- $\bar{u}_*^2(t) \propto \bar{\tau}_b(t) \propto \bar{u}^2(y_0, t)$
- so there is a function $F$ that:
  $\bar{u}_*(t) = F\{\bar{u}(y_0, t)\}$
  
  (Ould Ahmedou et al., 2007)

- the hypothesis: $F$ is also valid for instantaneous variables, so that:
  $u_*(t) = F\{u(y_0, t)\}$

Figure 9 – Correlation between $\langle \bar{u} \rangle(y_0)$ and $u_{*\text{PIV}}$. First-order polynomial approximation and 95% confidence boundaries.
Friction velocity correlation

\[ \tau_b(t) \]?

Figure 9 – Correlation between \( \langle \bar{u} \rangle(y_0) \) and \( u_{*_{PIV}} \). First-order polynomial approximation and 95% confidence boundaries.
Friction velocity signal

Figure 10 – Signals of $u_*(t)$ and $Sh(t)$ for a close to threshold experiment run. The dotted dark line represent $Sh_c$. 
Experimental project

Top view of the trap.

Side view, camera perspective. (red arrow)
Experimental project

(Ho et al., 2014)
Particles identification method

Original image

Linear histogram normalization

Binary Image

- Based on maximum and minimum images from series, each image gray level is adjusted;

- Canny filter is applied, using Otsu’s threshold method, to identify particle edges;

- Morphological operations (closing, filling, opening, watershed) are performed.

(Frey et al., 2003)
Particle Tracking Velocimetry

Particle mass:
\[ m_i^k = \frac{\pi \rho_s D_i^k}{6} \]

Particle displacement:
\[ \Delta r_i^k \]

Particle velocity:
\[ \vec{v}_i^k = \frac{\Delta r_i^k}{\Delta t} \]

Total mass at time \( t^k \):
\[ M(t^k) = \sum_{i}^{N_p} m_i^k ; \]

Time dependent discharge:
\[ q(t^k) = \frac{M(t^k)}{\Delta t_{esc}^k} , \]

Time scale of particles permanence:
\[ \Delta t_{esc}^k = \frac{\Delta y_{ROI}^k}{\bar{v}_{y_i}^k} ; \]

Mean:
\[ \bar{v}_{y_i}^k = \frac{1}{N_p} \sum_{i}^{N_p} v_{y_i}^k ; \]

Weighted-mean:
\[ \bar{v}_{y_i}^k = \frac{1}{M(t^k)} \sum_{i}^{N_p} m_i^k v_{y_i}^k ; \]
Instantaneous discharge measurement

\[ q_{s_{\text{PTV}}} = 2.35 q_{s_{\text{Wg}}} \]

Figure 11 – Correlation between sediment discharge computation based on images and weighted mass.

<table>
<thead>
<tr>
<th>Measure</th>
<th>(u_\ast)</th>
<th>(\text{Sh})</th>
<th>(\text{Fr})</th>
<th>(\bar{\bar{q}}_W)</th>
<th>(\bar{\bar{q}}_{\text{PTV}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>gb1-2</td>
<td>0.014</td>
<td>0.068</td>
<td>1.16</td>
<td>0.065</td>
<td>0.064</td>
</tr>
<tr>
<td>gb1-3</td>
<td>0.014</td>
<td>0.075</td>
<td>1.20</td>
<td>0.179</td>
<td>0.386</td>
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<tr>
<td>gb1-4</td>
<td>0.015</td>
<td>0.079</td>
<td>1.10</td>
<td>0.204</td>
<td>0.336</td>
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<tr>
<td>gb2-1</td>
<td>0.021</td>
<td>0.158</td>
<td>1.55</td>
<td>1.187</td>
<td>2.967</td>
</tr>
<tr>
<td>gb2-2</td>
<td>0.021</td>
<td>0.159</td>
<td>1.54</td>
<td>0.896</td>
<td>2.107</td>
</tr>
</tbody>
</table>

Top view of the trap.
Sh = 0.079, Fr = 1.10
Sh = 0.079, Fr = 1.10
Results

$Sh = 0.159, Fr = 1.54$

- First wave of sediments
- Initiation stage
- Steady solid transport
- Disturbed transport

$q_s (gm^{-1}s^{-1})$

Time (s)

Sample image of sediment movement and stability phases.
$Sh = 0.159, Fr = 1.54$
Comparison to classical formulas

Meyer-Peter & Muller (1948):

\[ q_s = 8(Sh - Sh_c)^{3/2} \]

Wong & Parker (2006):

\[ q_s = 4.93(Sh - Sh_c)^{8/5} \]
PSD

Results

Steady transport

$u(t)$

$-\frac{5}{3}$

Frequency (Hz)

PSD

Steady transport

$u(t)$

$-\frac{5}{3}$
Discussions

- $q_{s\text{RMS}}$ and $u_{\text{*RMS}}$;
- characteristic time scales;
- additional effect: pulsating flows;
Thank you!

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Further analysis...

**Mean flow velocity and turbulent intensities**

Figure 12 – Results for average profiles of mean flow velocity $\langle \bar{u} \rangle (y)$ and standard deviation $\langle u_{\text{RMS}} \rangle (y)$ for runs gb1 to gb8.
Turbulent intensities - $v_{\text{RMS}}^+$

(Antonia and Krogstad, 2001)

$$v_{\text{RMS}}^+ = 1.14 \exp \left( \frac{-0.76y}{h_0} \right)$$

Figure 13 – Turbulent intensities for runs gb1 to gb8. Lines indicate computed values following empirical results from Antonia and Krogstad (2001). Vertical turbulent intensities $v_{\text{RMS}}^+$; dark line represent computed values.
Friction velocity correction

**Figure 14** – Correlation between friction velocity computed through both methods $u_\text{CN}$ and $u_\text{PIV}$. Solid line represents linear relation between both methods for friction velocity calculation. Dashed line represents equality $u_\text{PIV} = u_\text{CN}$.

$$u_* = u_\text{CN} = 1.08u_\text{PIV} - 0.006$$

with a correlation coefficient $R^2 = 0.96$. 

Further analysis...
Grain-size distribution

Figure 15 – Probability distribution function from captured images in comparison to calibrated grain-size distribution.

→ Correction of particles diameter.
Further analysis...

$\Delta y_{ROI}$ influence

Figure 16 – Influence of $\Delta y_{ROI}$ on $q_s(t)$.

Filtering of data, reducing discretization noise.

$\rightarrow$ Imposes time window of signal acquisition: $\Delta t_{esc}$.

In order to be comparable to PIV signal: $\Delta y_{ROI} = 6.5 \text{mm}$

$(\Delta t_{esc} = 0.13 - 0.14 \text{s})$
Further analysis...

Δy_{ROI} influence

Figure 16 – Influence of Δy_{ROI} on q_s(t).

→ Imposes time window of signal acquisition: Δt_{esc}.
In order to be comparable to PIV signal: Δy_{ROI} = 6.5mm
(Δt_{esc} = 0.13 – 0.14s)
Experimental run

1. arrange the experiment;
2. partially block the outlet;
3. set flow discharge and slope;
4. start data acquisition (PIV and PTV);
5. release the outlet;
6. after 1’30”, data acquisition stops;
7. block outlet;
Further analysis...

PSD

Frequency (Hz)

Steady transport

Non-steady transport (bedforms)

\[ u_\text{a}(t) \]

\[ \frac{5}{3} \]