Analytical Mesoscale Modeling of Aeolian Sand Transport

Marc Lämmel, Anne Meiwald, Klaus Kroy
Aeolian Sand Transport
Mesoscale Process

potentially amenable to analytical modeling
Mean-Field approach

movie @ A. Valance & BBC
Two-species approach

saltation

reptation

movie @ A. Valance & BBC
Aeolian Structure Formation

\[ \ell_s \frac{\partial q}{\partial x} = q(1 - q/q_s) \]
Mesoscale Phenomena

Megaripple

Saltation

Dust

Meganripple
A

two saltation trajectories

BED

VTRAP compartments

V6
V5
V4
V3
V2
V1

trapped sediment contributed by each trajectory

contribution from larger trajectory to mass-flux distribution

contribution from smaller trajectory to mass-flux distribution

B

BED

HTRAP compartments

H1 H2 H3 H4 H5 H6

Saltation of uniform grains in air

By P. R. OWEN
Department of Aeronautics, Imperial College, London

(Received 14 April 1964)

Printed in Great Britain

HEIGHT RESOLVED

• particle concentration
• particle velocity
• particle flux
• hop length & height
• wind speed

DISTRIBUTIONS of grain trajectories
Hierarchical Approach

- Single Trajectory
- Distribution of Trajectories
- 2 Species
- Compare to Experiments & Simulation

A two-species model of aeolian sand transport

By BRUNO ANDREOTTI

New Journal of Physics
An open-access journal for physics

A two-species continuum model for aeolian sand transport

M Lämmel, D Rings and K Kroy
particle distribution

\[ P(z, h) \]

Prob to observe particle on trajectory of height \( h \) at \( z \)
particle distribution

$$P_\bar{h}(z, h) = P(z|h)P_\bar{h}(h)$$

Prob for particle at $z$ if on this trajectory

wind 2-spec

Prob for trajectory of height $h$
\[ P_\bar{h}(h) \propto e^{-h/\bar{h}} \]

**Reptation/Splash**

A theory for the flow of identical, smooth, nearly elastic, spherical particles

By J. F. Jenkins

Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York

And S. B. Savage

**Saltation**

\[ h \propto N ; \quad P(\text{survival}) \propto e^{-N} \]
particle distribution

$$P_{\bar{h}}(z, h) = P(z|h)P_{\bar{h}}(h)$$

Prob for particle on this trajectory at $z$

$$\frac{e^{-h/\bar{h}}}{\bar{h}}$$
particle distribution

\[ P_{\bar{h}}(z, h) = P(z|h) P_{\bar{h}}(h) \]

\[ \frac{1}{2h} \cdot \frac{1}{\sqrt{1 - z/h}} \]

\[ e^{-h/\bar{h}} \]

\[ \frac{\tilde{h}}{\bar{h}} \]
particle distribution

\[ P_{\overline{h}}(z, h) = P(z | h) P_{\overline{h}}(h) \]
height-resolved observables

- grain density: $\rho(z)$
- horizontal flux: $j(z)$
- vertical flux: $\phi(z)$
- grain-borne stress: $\tau_g(z)$
- hop length distribution: $P(z, l)$
- hop length distribution: $P(\ell) \propto -\partial_\ell \phi_\ell(0)$
particle distribution

\[ \rho_{\bar{h}}(z) = \int dh \, P_{\bar{h}}(z, h) \]

Prob for particle at \( z \) for any trajectory

\[ \sim \frac{\ln(4\bar{h}/z)}{2} \]

\[ \sim \frac{\sqrt{\pi}}{2} \frac{\exp(-z/\bar{h})}{(z/\bar{h})^{1/2}} \]

\( \bar{h} \): wind, 2 spec
particle velocity

\[ v_x(z, l, h) \approx \sqrt{2gh/4\epsilon} \quad \text{hop-length/flight-time} \]

\[ \sim \text{free-fall/const. } - \epsilon \text{ approx} \]

\[ \epsilon \equiv h/l \quad \sim \text{constant} \]
**horizontal sand flux**

\[ j_h(z) = \int dh \, v_x(h) \, P_h(z, h) = q \frac{e^{-z/\bar{h}}}{\bar{h}} \]

**vertical sand flux**

\[ \phi_\ell(z) = \int_{l(h) > \ell} dh \, v_z(z, h) \, P_h(z, h) \]

\[ v_z(z, h) = \sqrt{2g(h - z)} \quad \phi_\ell(z = 0) = q \frac{\text{erfc}\sqrt{\ell \epsilon / \bar{h}}}{\bar{h}/\epsilon} \]
horizontal sand flux

vertical sand flux

\[ \phi_\ell(z) = \int_{l(h) > \ell} dh \, v_z(z, h) \, P_h(z, h) \]

\[ v_z(z, h) = \sqrt{2g(h - z)} \]

\[ \phi_\ell(z = 0) = q \frac{\text{erfc}\sqrt{\frac{l \epsilon}{h}}}{\frac{h}{\epsilon}} \]
grain-scale experiments

Rasmussen, Mikkelsen, Sedimentology (1998)

Namikas, Sedimentology (2003)

Rasmussen, Sørensen, J. Geophys. Res. (2008)

Ho, Valance, Dupont, Moctar, Aeolian Research (2014)

Durand, Claudin, Andreotti, PNAS (2014)
horizontal sand flux

$$j_{\text{sal}}(z) + j_{\text{rep}}(z)$$

$$\frac{q_{\text{rep}}}{h_{\text{rep}}} e^{-\frac{z}{h_{\text{rep}}}}$$

Two-species approach

saltation

reptation

Two-species approach

horizontal sand flux
vertical sand flux

\[ \phi_{\ell}^{\text{sal}}(0) + \phi_{\ell}^{\text{rep}}(0) \]

\[ \ell / l_{\text{sal}} \]
Direct numerical simulations of aeolian sand ripples

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Aeolian sand beds exhibit regular patterns of ripples resulting from the interaction between topography and sediment transport. Their characteristics have been so far related to reptation transport caused by the impacts on the ground of grains entrained by the wind flow that are, for the first time to our knowledge, able to reproduce the spontaneous growth of ripples with an initial wavelength and a propagation speed increasing linearly with the wind velocity. We propose a new formation mechanism, involving resonant grain trajectories tuned with the ripple wavelength, that allows us to perform runs over long periods of time using a large 2D continuum Reynolds averaged description of hydrodynamics, presented in ref. 26, we explicitly implement a two-way coupling between a discrete element method for the particles and a continuum Reynolds averaged description of hydrodynamics, coarse-grained at a scale larger than the grain size. This coupling...
particle velocity

\[ v_x(z) = \frac{j(z)}{\rho(z)} \]
particle velocity

(a) 
(b) 
(c) 
(d) 

\[ \bar{v}(z) \text{ [m/s]} \]

\[ 2gz/v^2 \]

\[ 10^{-1} \quad 10^{0} \]

\[ 0 \quad 2 \quad 4 \quad 6 \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \]

\[ 4.9 \quad 4.0 \quad 2.8 \quad 1.9 \]

\[ 5.7 \quad 5.1 \quad 4.2 \]

\[ 6.3 \quad 5.6 \quad 4.9 \quad 4.1 \quad 3.4 \]

\[ 2.3 \]
Summary

- analytical mesoscale model of aeolian transport
  - based on grain scale physics
  - ensemble of trajectories
  - & two-species
  - height-resolved observables
  - applications to turbulent closure
    & data analysis & various mesoscale phenomena

Thank you!