Analytical Mesoscale Modeling of Aeolian Sand Transport

Marc Lämmel, Anne Meiwald, Klaus Kroy
Analytical Mesoscale Modeling of Aeolian Megaripples

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UNIVERSITÄT LEIPZIG  GeoFlo16 — Dresden
Aeolian Sand Transport

turbulent streaks
  = sand or dust?
  = important or averaged out?

mesoscale phenomenon
  potentially amendable to analytical modeling
mean-field model

saltation
Continuum saltation model for sand dunes

Gerd Sauermann,1,2 Klaus Kroy,1,* and Hans J. Herrmann1,2

depth-averaged
saturated flux/\(u^3\)

saturation transients
(mesoscale)

drag
rebound/splash

2 Fit Parameters

mean-field model
Aeolian Structure Formation
Mesoscale Phenomena

![Diagram of saltation](image)

- **saltation**
- **megaripple**
- **dust**
A two-species model of aeolian sand transport

By BRUNO ANDREOTTI

A two-species continuum model for aeolian sand transport

M Lämmel, D Rings and K Kroy
Saltation of uniform grains in air

By P. R. OWEN
Department of Aeronautics, Imperial College, London

(Received 14 April 1964)

Printed in Great Britain

HEIGHT RESOLVED
• particle concentration
• particle velocity
• particle flux
• hop length & height
• wind speed
particle distribution

$P(z, h)$  
Prob to observe particle on trajectory of height $h$ at $z$
particle distribution

\[ P_{\bar{h}}(z, h) = P(z|h) \overline{P}_{\bar{h}}(h) \]

Prob for trajectory of height \( h \)

wind 2-spec

\[ z \]

\[ h \]
particle distribution

\[ P_{\bar{h}}(z, h) = P(z| h) P_{\bar{h}}(h) \]

Prob for particle at \( z \) if on this trajectory

\[ e^{-\frac{h}{\bar{h}}} \]

\( \frac{\bar{h}}{h} \)
particle distribution

\[ P_{\bar{h}}(z, h) = P(z| h) P_{\bar{h}}(h) \]
particle distribution

\[ P_{\bar{h}}(z, h) = P(z|h) P_{\bar{h}}(h) \]
$P_h(h) \propto e^{-h/\bar{h}}$

Reptation/Splash

A theory for the motion of identical, smooth, nearly elastic, spherical particles

By J. T. JENKINS

Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, New York

AND S. B. SAVAGE


Printed in Great Britain
granular temperature

1. molecular gas with (turbulent) thermostat

\[ p = nk_B T \quad -m g n(z) = \partial_z p = k_B T \partial_z n \]
\[ n(z) \propto \exp(-mgz/k_B T) \]

2. splash: free granular gas released at z=0

\[ P(E) dE = P(v) m v d v = P(h) d h \]
\[ P(h) \propto \exp(-mgh/k_B T) \]
\[ P_{\bar{h}}(h) \propto e^{-h/\bar{h}} \]

Reptation/Splash

\[ \sim \text{barometer formula with granular temperature} \]

Saltation

\[ P_{\bar{h}}(h) \propto e^{-h/\bar{h}} \]

\[ P(\# \text{ hops} \gtrsim N) \propto e^{-N} ; \quad h \propto N ? \]
deterministic numeric solution
turbulent diffusion — how important is it?

\[ \langle \Delta z^2 \rangle \simeq D t \]

\[ D \simeq u_* h \]

\[ t_\infty \approx \sqrt{2h/g} \]

diffusion coefficient

time of flight
assume

\[ P_{\bar{h}}(h) \propto e^{-h/\bar{h}} \]

for reptons & saltons
height-resolved observables

- grain density \( \rho(z) \)
- horizontal flux \( j(z) \)
- vertical flux \( \phi(z) \)
- grain-borne stress \( \tau_g(z) \)
- hop length distribution \( P(z, l) \)
- hop length distribution \( P(l) \propto -\partial_l \phi_l(0) \)
particle distribution

\[ \rho_{\bar{h}}(z) = \int \text{d}h \, P_{\bar{h}}(z, h) \]

Prob for particle at \( z \) for any trajectory

\[ \sim \frac{\ln(4\bar{h}/z)}{2} \]
\[ \sim \frac{\sqrt{\pi}}{2} \frac{\exp(-z/\bar{h})}{(z/\bar{h})^{1/2}} \]

\( z/\bar{h} \) wind, 2 spec
horizontal sand flux

\[
j_{h}(z) = \int dh \, v_x(h) \, P_h(z, h) = \frac{q}{\bar{h}} e^{-z/\bar{h}}
\]

\[
v_x(z, l, h) \approx \sqrt{2gh}/4\epsilon
\]

vertical sand flux

\[
\phi_{\ell}(z) = \int dh \, v_z(z, h) \, P_h(z, h)
\]

\[
v_z(z, h) = \sqrt{2g(h - z)}
\]

\[
\phi_{\ell}(z = 0) = q \frac{\text{erfc} \sqrt{\ell \epsilon/\bar{h}}}{\bar{h}/\epsilon}
\]
particle velocity

\[ v_x(z) = \frac{j(z)}{\rho(z)} \]

\[ \frac{v_x(z)}{v_0} = \frac{2}{\ln 4\bar{h}/z} + \sqrt{\frac{4z}{\pi \bar{h}}} \]
grain-scale experiments

Rasmussen, Mikkelsen, Sedimentology (1998)

Namikas, Sedimentology (2003)

Rasmussen, Sørensen, J. Geophys. Res. (2008)

Ho, Valance, Dupont, Moctar, Aeolian Research (2014)

Durand, Claudin, Andreotti, PNAS (2014)
horizontal sand flux

Two-species approach

saltation

reptation

\[ j_{\text{sal}}(z) + j_{\text{rep}}(z) \]

\[ \frac{q_{\text{rep}}}{h_{\text{rep}}} e^{-z/h_{\text{rep}}} \]

\[ z/h_{\text{sal}} \]
vertical sand flux

\[ \phi_{\ell}^{\text{sal}}(0) + \phi_{\ell}^{\text{rep}}(0) \]

vs \[ \ell / \ell^{\text{sal}} \]
hop length distribution

\[ P(l|z) \propto P[z, h(l)] \partial_l h(l) \propto \frac{e^{-\ell/\bar{h}}}{\ell} \]

(b)  

Direct numerical simulations of aeolian sand ripples

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Edited by Harry L. Swinney, The University of Texas at Austin, Austin, TX, and approved September 17, 2014 (received for review July 10, 2014)

Aeolian sand beds exhibit regular patterns of ripples resulting from the interaction between topography and sediment transport. Their characteristics have been so far related to reptation transport caused by the impacts on the ground of grains entrained. As shown in ref. 26, we explicitly implement a two-way coupling between a discrete element method for the particles and a continuum Reynolds averaged description of hydrodynamics, coarse-grained at a scale larger than the grain size. This coupling...
particle velocity

(a) $v(z)$ vs. $2gz/v^2$

(b) $v(z)$ vs. $2gz/v^2$

(c) $v(z)$ vs. $2gz/v^2$

(d) $v(z)$ vs. $2gz/v^2$
Summary

- analytical mesoscale model of aeolian transport
  - based on grain scale physics
  - ensemble of trajectories
  - & two-species
  - height-resolved observables
  - applications to turbulent closure
  & data analysis & various mesoscale phenomena
Analytical Mesoscale Modeling of Aeolian Megaripples

Marc Lämmel, Anne Meiwald, Klaus Kroy
normal ripples

Direct numerical simulations of aeolian sand ripples

Orencio Durán*†, Philippe Claudin*, and Bruno Andreotti*

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Aeolian sand beds exhibit regular patterns of ripples resulting from the interaction between topography and sediment transport. Their characteristics have been so far related to reptation transport caused by the impacts on the ground of grains entrained from the interaction of the particles with their neighbors in contact. Presented in ref. 26, we explicitly implement a two-way coupling between a discrete element method for the particles and a continuum Reynolds averaged description of hydrodynamics, coarse-grained at a scale larger than the grain size. This coupling...
normal ripples

Andreotti et al., PRL 96, 028001 (2006): field 180\(\mu\)m, wind tunnel 120\(\mu\)m
Bagnold (1941): 250\(\mu\)m
Walker, MA thesis (1981): 200, 250, 320, 400, 780\(\mu\)m
Seppälä and Lindé, Geografiska Annaler. A (1978): 150\(\mu\)m
megaripple morphology

wind strength (+ saltation)

megaripple = repton dune

transport (reptation)
megaripple morphology

megaripple = repton dune

WIND Ripples

ROBERT P. SHARP
California Institute of Technology

ABSTRACT

Two types of wind ripples are distinguished; sand ripples composed of median diameter roughly between 0.30 and 0.35 mm., and granule ripples composed of approaching granule size 2–4 mm. The planimetric patterns and facing directions...
aspect ratio

reptation length: $l_r \propto u_* \sqrt{d_2 / g}$

\[
\frac{H/L}{\epsilon} = \frac{\sqrt{HL}}{\sqrt{HL} + \alpha l_r}
\]
migration speed

\[ v = -\frac{\partial_t h}{\partial_x h} \propto \frac{\partial_x q}{\partial_x h} \propto \frac{1}{L} \]

\[ \partial_x q \sim q'(\tau)\partial_x \tau \]

\[ \tau \propto H/L \]

“\[ \partial_x \sim \frac{1}{L} \]”

Lorenz and Valdez, Geomorphology 133 (2011)
Summary

- analytical mesoscale model of aeolian ripples
  - ripples vs megaripples
  - two-species ~ two particle sizes
  - megaripples = repton dunes
  - disintegration of wave structures
  - velocity scaling with length (not separation)
  - aspect ratio vs mass as predicted for dunes

thank you!