MAX PLANCK INSTITUTE FOR THE PHYSICS OF COMPLEX SYSTEMS

Two-Phase Continuum Models for Geophysical Particle-Fluid Flows

Dresden, 14 March - 15 April 2016

…natural complexities and laboratory analogues

Luigi Fraccarollo Michele Larcher

UNIVERSITÀ DEGLI STUDI DI TRENTO
Dipartimento di Ingegneria Civile, Ambientale e Meccanica

March 24th, 2016

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Bed erosion in dry uniform-unsteady granular flows

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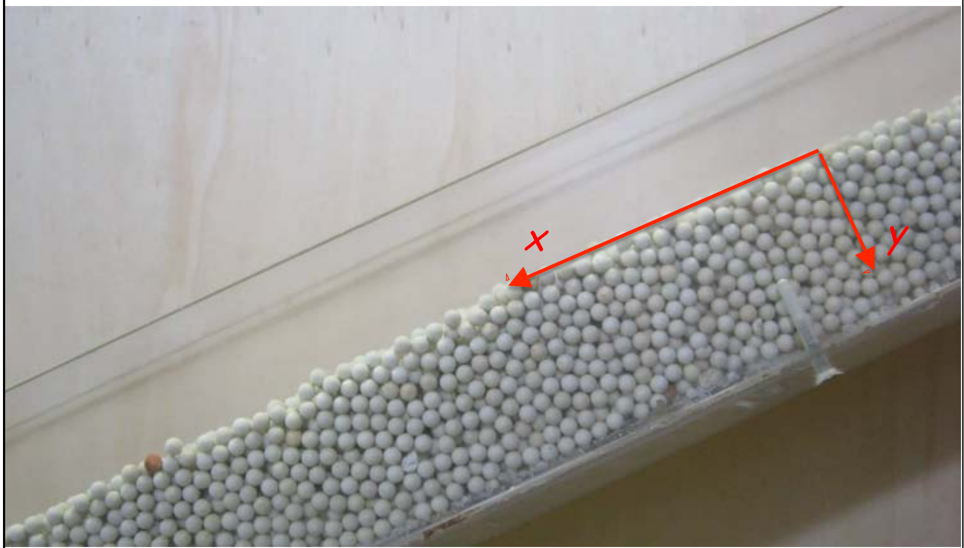
The uniform-unsteady problem




Michele Larcher
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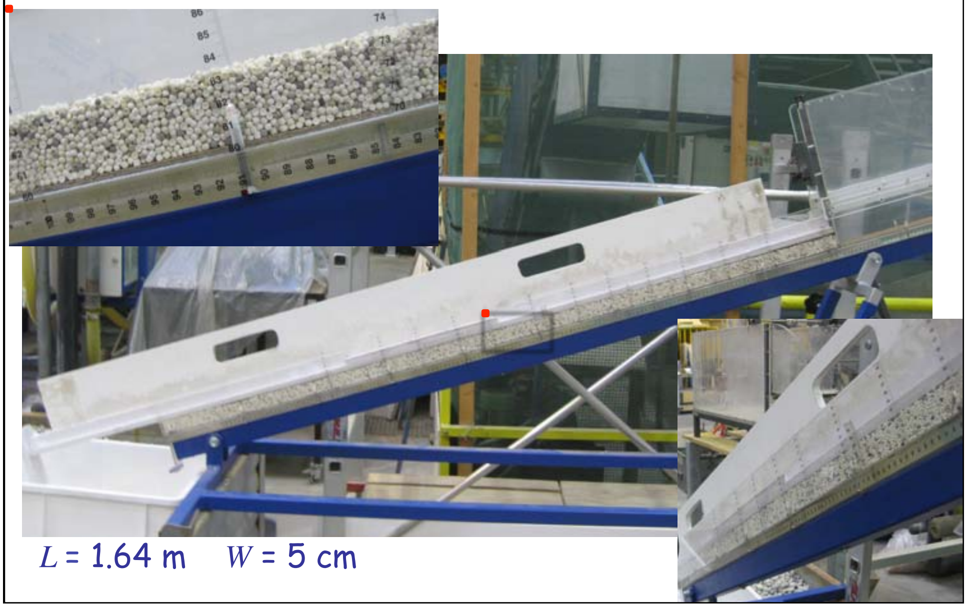
The uniform-unsteady problem



L = 1.64 m W = 5 cm

UNIVERSITÀ DEGLI STUDI DI TRENTO

Dipartimento di Ingegneria Civile Ambientale e Meccanica
Corso di Laurea Magistrale in Ingegneria per l'Ambiente ed il Territorio

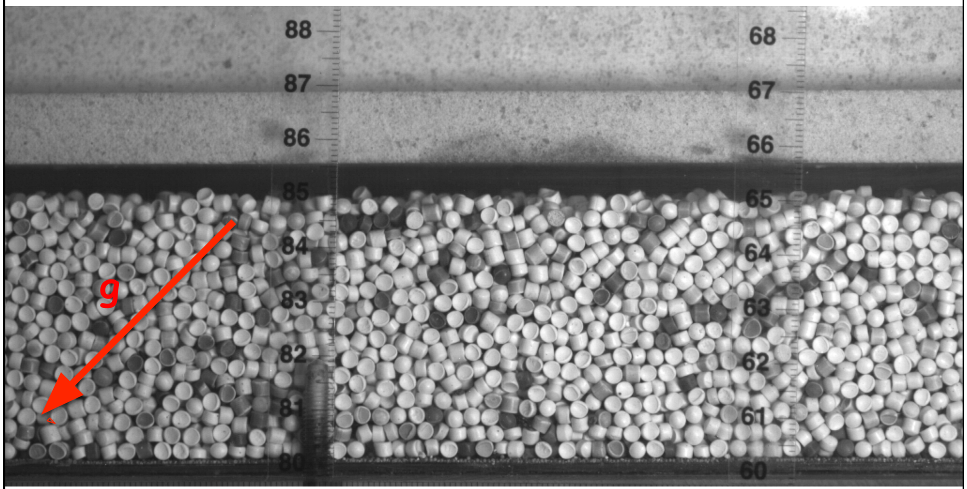
Analisi teorico-sperimentale del moto vario e uniforme di ammassi granulari secchi

Laureanda:
Anna Prati

Relatori:
Prof. Ing. Luigi Fraccarollo
Prof. Ing. Michele Larcher

The uniform-unsteady problem

Geomorphic time for dry granular flow



Dataset with PVC cylinders

N. test	*Slope[°]*	*Depth [cm]*	*Fps*	*Shutter*	*Resolution*
1	45	5	250	1/4000	1024 x 512
2	45	5.6	250	1/4000	1024 x 512
3	45	6.8	250	1/4000	1024 x 512
4	45	6.4	250	1/4000	1024 x 512
5	45	6.5	250	1/4000	1024 x 512
6	41.9	5.2	1000	1/4000	1024 x 512
7	41.3	5	2000	1/4000	1024 x 512
8	41.4	5.2	1000	1/4000	1204 x 1024
9	41.4	5	1000	1/4000	1024 x 512
10	41.4	5.5	1000	1/4000	1024 x 512
11	41.4	6.5	1000	1/4000	1024 x 512
12	41.4	6.5	1000	1/4000	1024 x 512
13	41.4	6.5	1000	1/4000	1024 x 512
14	41.4	2.5	1000	1/4000	1024 x 512
15	41.4	2.9	2000	1/4000	1024 x 512
16	41.4	2.5	2000	1/4000	1024 x 512
17	38	2.5	2000	1/4000	1024 x 512
18	45	5	1000	1/4000	1024 x 512

Dataset with PVC cylinders

N. test	*Slope[°]*	*Depth [cm]*	*Fps*	*Shutter*	*Resolution*
19	38	5	1000	1/4000	1024 x 512
20	38	5	1000	1/4000	1024 x 512
21	41.4	5	1000	1/4000	1024 x 512
22	41.4	5	1000	1/4000	1024 x 512
23	41.4	5	1000	1/4000	1024 x 512
24	41.4	5	1000	1/4000	1024 x 512
25	41.4	5	1000	1/4000	1024 x 512
26	41.4	5	1000	1/4000	1024 x 512
27	45	5	1000	1/4000	1024 x 512
28	45	5	1000	1/4000	1024 x 512
29	45	3	1000	1/4000	1024 x 512
30	45	3.2	2000	1/4000	1024 x 512
31	41.4	3	2000	1/4000	1024 x 512
32	41.4	3	2000	1/4000	1024 x 512

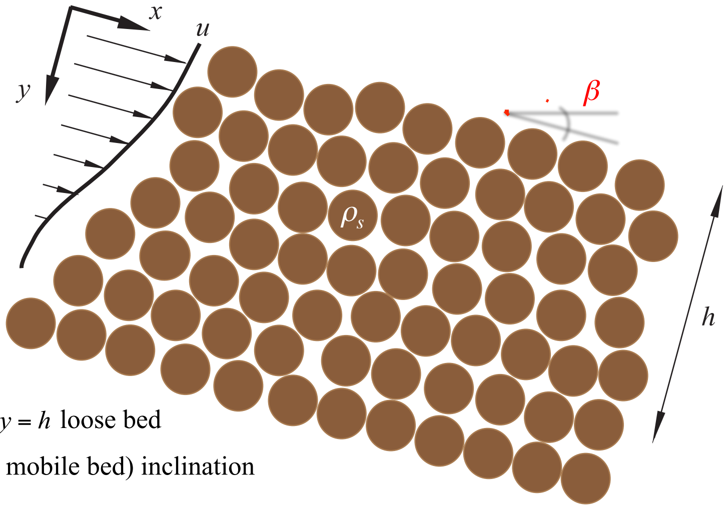
ρ_s = 1.51 kg/m^3 d = 3.50 mm ϕ = 31° e = 0.6 μ_w = 0.32

Dataset with plastic spheres

N. test	*Slope[°]*	*Depth [cm]*	*Fps*	*Shutter*	*Resolution*
41	32	5	1000	1/4000	1024 x 512
42	38	3	1000	1/frame	1024 x 512
43	35	3	1000	1/2000	1024 x 1024
44	35	3	1000	1/2000	1024 x 1024
45	32	3	1000	1/2000	1024 x 1024
46	32	3	1000	1/8000	1024 x 1024
47	33.5	3.4	1000	1/8000	1024 x 1024
48	33.5	3	1000	1/8000	1024 x 1024
49	33.5	3	1000	1/8000	1024 x 1024
50	33.5	3	1000	1/8000	1024 x 1024
51	35	3	1000	1/8000	1024 x 1024
52	35	3	1000	1/8000	1024 x 1024
53	38	3	1000	1/8000	1024 x 1024
54	38	3	1000	1/8000	1024 x 1024
55	38	3	1000	1/8000	1024 x 1024

ρ_s = 0.98 kg/m^3 d = 0.45 mm ϕ = 21° e = 0.75 μ_w = 0.31

Fully developed flow



$y = 0$ free surface, $y = h$ loose bed

β free surface (and mobile bed) inclination

ρ_s solid mass density

ν grain concentration

g gravitational acceleration

Variables

u grain velocity

h flow depth

T granular temperature

α bed yield stress ratio

Make lengths dimensionless by grain diameter d, velocities by $(gd)^{1/2}$, and stresses by $\rho_s g d$.

Goals

Given the angle of inclination, predict the **depth** of a fully-developed flow, the **velocity profile** and the **volume flux**

Grain Momentum Balance

transverse:

$$p' = \nu \cos\beta$$

flow:

$$s' = -\nu \sin\beta + 2\mu_w \frac{p}{W}$$

wall friction (μ_w)

chute width (W)

Depth of flow

$$\frac{d}{dy}(s) = -\nu \sin\beta + 2\mu_w \frac{p}{W}$$

$$p = \nu y \cos\beta$$

$$\frac{s}{p} = -\tan\beta + \frac{\mu_w}{W}\frac{p}{\nu\cos\beta}$$

obtain the dependence of s/p on y:

$$\frac{s}{p} = -\tan\beta + \frac{\mu_w}{W} y$$

…depth of flow

At the bed ($y = h$), $s/p = \alpha$, so:

$$\alpha = \tan\beta - \frac{\mu_w}{W} h$$

or:

$$h = \frac{W}{\mu_w}\left(\tan\beta - \alpha\right)$$

Particle Energy Balance

Jenkins & Berzi (2010)
Larcher & Jenkins (2014)

$$su' - \Gamma = 0$$

$$s = \mu u' \qquad p = 2(1+e)\nu G T$$

$$\mu = \frac{4J}{5\pi^{1/2}(1+e)}\frac{p}{T^{1/2}} \qquad J = \frac{(1+e)}{2} + \frac{\pi}{4}\frac{(3e-1)(1+e)^2}{\left[24-(1-e)(11-e)\right]} \qquad G = \frac{0.59c}{0.60-c}$$

collisional dissipation:
$$\Gamma = \frac{12}{\pi^{1/2}}\frac{\nu G}{L}(1-e^2)T^{3/2}$$

Jenkins (2006, 2007)

cluster size:
$$L = \frac{1}{2}\hat{c}G^{1/3}\frac{u'}{T^{1/2}}$$

Concentration

Eliminate L from the energy balance and solve for $u'/T^{1/2}$:

$$\frac{u'}{T^{1/2}} = \frac{15}{J}\frac{1-e^2}{\hat{c}G^{1/3}}$$

Use this to eliminate $u'/T^{1/2}$ from the shear stress with s/p:

$$\frac{s}{p} = \left[\frac{192}{25\pi^{3/2}}\frac{J^2(1-e)}{\hat{c}(1+e)^2}\right]^{1/3}\frac{1}{G^{1/9}} = \tan\beta - \frac{\mu_w}{W}y$$

and substitute in:
$$\nu = \frac{0.63G}{0.60+G}$$

Temperature

Solve the pressure for T and obtain its variation with depth:

$$T = \frac{p}{2(1+e)\nu G} = \frac{\cos\beta\, y}{2(1+e)G}$$

Velocity profile

Invert the shear stress and integrate:

$$u' = \frac{5\pi^{1/2}}{4J}(1+e)\frac{s}{p}T^{1/2}$$

$$\frac{s}{p} = \tan\beta - \frac{\mu_w}{W}y$$

velocity profile:

$$u(y) = \frac{5\pi^{1/2}}{4J\sqrt{2G}}\left(1+e\right)^{1/2}\left(\cos\phi\right)^{1/2}\left\{\frac{2}{3}\tan\phi\left[h^{3/2}-y^{3/2}\right]-\frac{2}{5}\frac{\mu_w}{W}\left[h^{5/2}-y^{5/2}\right]\right\}$$

Total Volume Flux

$$q = W\int_0^h \nu u(y)\,dy =$$

$$= \frac{5\pi^{1/2}W}{4\bar{J}\sqrt{2\bar{G}}}\left(1+\frac{e}{2}\right)^{1/2}\nu\left(\cos\beta\right)^{1/2}\left\{\frac{2}{5}h^{5/2}\tan\beta - \frac{2}{7}\frac{\mu_w}{W}h^{7/2}\right\}$$

Unsteady: 1 equation approach

Extended kinetic theory (algebraic integration)

Momentum balance in the x direction

$$\rho\frac{\partial u}{\partial t} = \rho g\sin\beta + \frac{\partial \tau}{\partial y} - 2\frac{\mu_W}{W}\rho g y\cos\beta$$

Similarity assumption for the variables involved: velocity, concentration

$$\frac{dh}{dt} = -\frac{dz}{dt} = f(u_s, h_s)$$

Unsteady: 2 equations approach

Extended kinetic theory (algebraic integration)

Momentum balance in the y direction

Energy balance

Similarity assumption for the variables involved: velocity, concentration

$$\begin{cases} f_1(u_s,h_s)\dfrac{du_s}{dt} + f_2(u_s,h_s)\dfrac{dh}{dt} = f_3(u_s,h_s) \\ g_1(u_s,h_s)\dfrac{du_s}{dt} + g_2(u_s,h_s)\dfrac{dh}{dt} = g_3(u_s,h_s) \end{cases}$$

1 equation approach: solution

Flow depth:

$$\frac{\partial h(t)}{\partial t} = \left\{ \rho g \sin\beta h(t) - \rho g \cos\beta h(t) R - \frac{\mu_W}{W} \rho g \cos\beta h(t)^2 \right\} /$$
$$\left\{ \left(\frac{5\sqrt{\pi}}{4J}\frac{1}{d}\left[\frac{(1+e) g\cos\beta}{2G}\right]^{1/2} \frac{1}{\left[\frac{2}{3}\tan\beta h^{3/2} - \frac{2}{5}\frac{\mu_W}{W}h^{5/2}\right]} \right.\right.$$
$$\left. h(t)^{1/2}\left(\tan\beta - \frac{\mu_W}{W}h(t)\right)\left[\frac{2}{3}\tan\beta\left[\frac{3}{5}h(t)^{5/2}\right] - \frac{2}{5}\frac{\mu_W}{W}\left[\frac{5}{7}h(t)^{7/2}\right]\right]\right) +$$
$$+ \left(\left(\frac{5\sqrt{\pi}}{4J}\frac{1}{d}\left[\frac{(1+e) g\cos\beta}{2G}\right]^{1/2} \left[\frac{2}{3}\tan\beta h(t)^{3/2} - \frac{2}{5}\frac{\mu_W}{W}h(t)^{5/2}\right]\right) \right.$$
$$\left.\left. \frac{1}{35}\frac{\left(-5\frac{2}{5}\frac{\mu_W}{W}h + 7\frac{2}{3}\tan\beta\right)\left(-5\frac{2}{5}\frac{\mu_W}{W}h + 3\frac{2}{3}\tan\beta\right)}{\left(-\frac{2}{5}\frac{\mu_W}{W}h + \frac{2}{3}\tan\beta\right)^2} \right) \right\}$$

1 equation approach

Velocity profile:

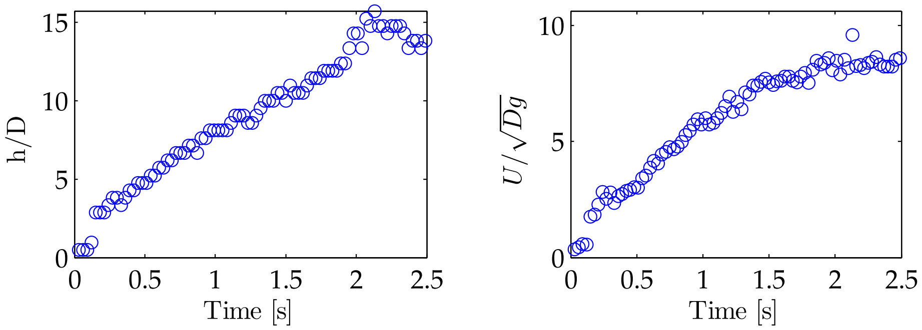
$$u(y) = u_S \frac{\left[\frac{2}{3}\tan\beta\left(h^{3/2} - y^{3/2}\right) - \frac{2}{5}\frac{\mu_W}{W}\left(h^{5/2} - y^{5/2}\right)\right]}{\left[\frac{2}{3}\tan\beta h^{3/2} - \frac{2}{5}\frac{\mu_W}{W}h^{5/2}\right]} = u_S(t) f_U(y,t)$$

Maximum velocity:

$$u_S = u|_{y=0} = \frac{5\sqrt{\pi}}{4J}\frac{1}{d}\left[\frac{(1+e)g\cos\beta}{2G}\right]^{1/2}\left[\frac{2}{3}\tan\beta h^{3/2} - \frac{2}{5}\frac{\mu_W}{W}h^{5/2}\right]$$

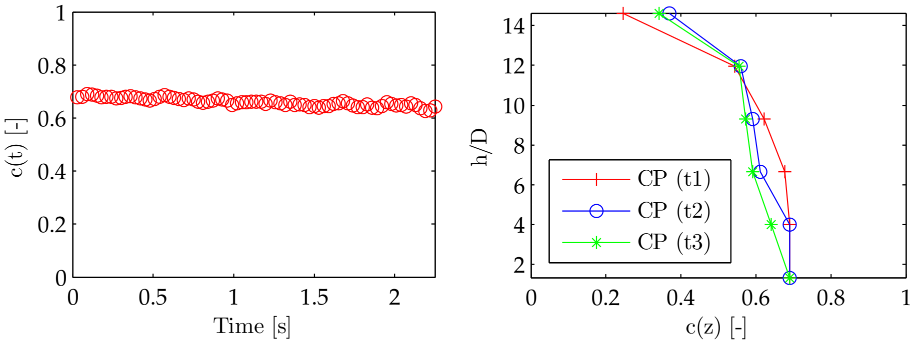
Experimental results

Run 06:



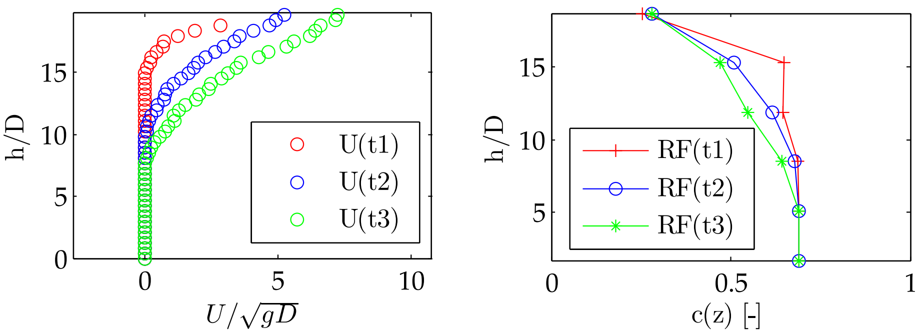
Experimental Results

Run 06:

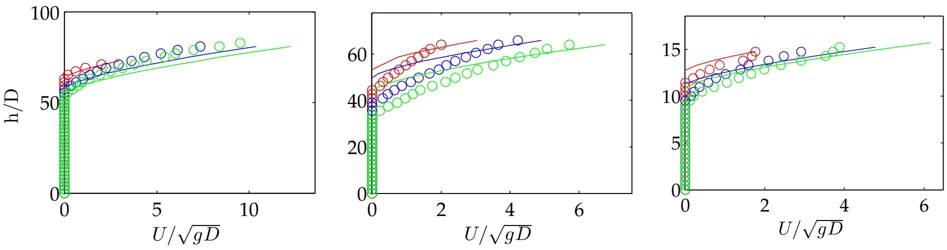


Experimental Results

Run 13:



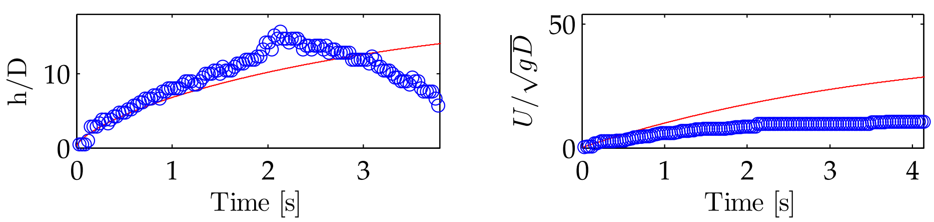
Prediction



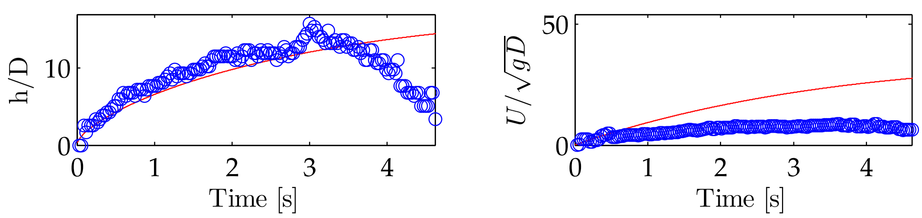
t_i [s]	*prova n. 06*	*prova n. 50*	*prova n. 53*
t_1	0.15	0.15	0.09
t_2	0.45	0.60	0.15
t_3	0.60	0.72	0.21

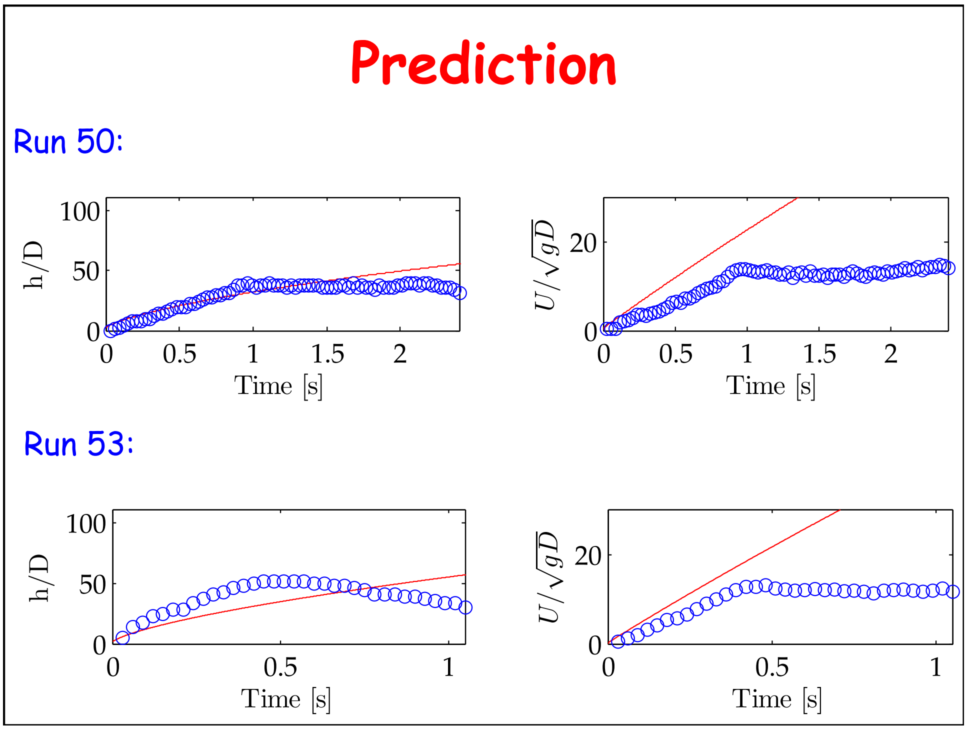
Prediction

Run 06:



Run 13:




Prediction
Run 50:
h/D
100
50
0
0 0.5 1 1.5 2
Time [s]
$U/\sqrt{gD}$
20
0
0 0.5 1 1.5 2
Time [s]
Run 53:
h/D
100
50
0
0 0.5 1
Time [s]
$U/\sqrt{gD}$
20
0
0 0.5 1
Time [s]


Snow avalanches
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