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The evolution of segregation in dense inclined flows of binary mixtures of spheres

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The evolution of segregation in dense inclined flows of binary mixtures of spheres

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Physical observation

Heavier particles tend to sink in a medium of lower density due to gravitational effects

Smaller particles tend to percolate downwards through the interstices due to geometrical effects, inducing larger particles to float

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Physical observation

Confirmed in

- Free-surface flows (Drahun & Bridgewater 1983, …)
- Rotating tumblers (Alonso, Satoh & Miyanami 1991, Felix & Thomas 2004, Jain, Ottino & Lueptow 2005, Hill et al. 2010, …)
- Rotating tubes (Metcalfe & Shattuck 1996, …)

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Context

Industry:

- Segregation is usually undesired
- Mixing is encouraged

Nature:

- Debris flow: reverse grading
- Snow and rock avalanches


Snow avalanches
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Large, light human "sphere"…

Assumptions

- Binary mixtures of spheres
- Rigid, bumpy, inclined channel
- Dense, collisional flows
- Absence of sidewalls
- Steady, uniform flow of the mixture
- Time and space evolution of segregation
- Small differences in size and/or mass

Goal

- Predict the concentration of the mixture
- Predict the profiles of mixture velocity and granular temperature
- Predict the evolution of concentration fractions of the two types of spheres
- Comparison with DEM simulations and physical experiments

Kinetic theory

- A theory based on measured particle properties
- Governing equations based on fundamental physical principles: *i)* the balances of mass, *ii)* momentum and *iii)* energy for the two species and the mixture

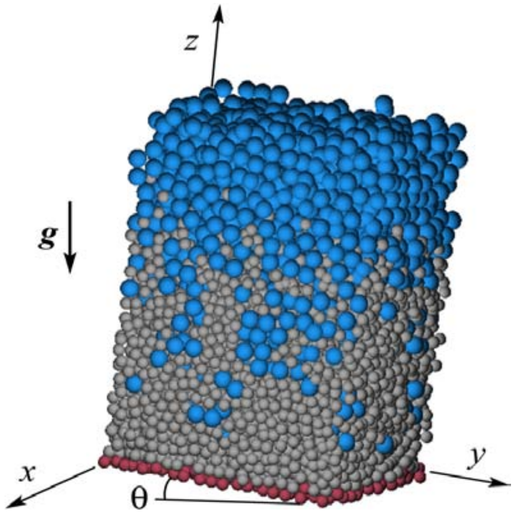
Steady, fully developed, dense flows

Larcher & Jenkins, Phys. Fluids 25: 113301, 2013

- Binary mixture of inelastic spheres (Arnarson & Jenkins 2004)
- Extension of the kinetic theory of Garzo & Dufty (1999) for identical inelastic spheres by Jenkins (2007)
- Correlated collisions and particle clusters accounted for through an additional length-scale in the rate of collisional energy dissipation (Jenkins & Berzi 2010)

Steady, fully developed, dense flows

Tripathi and Khakhar, Phys. Fluids 23, 113302 (2011)



Segregation evolution: Dense flows

- Rate at which segregation takes place ∝ average distance between the edges of spheres
- The rate at which momentum is transferred in the mixture is inversely proportional to that distance
- The flow of the mixture reaches a fully developed state much rapidly than do C_A and C_B

Fully developed flows

Therefore, we assume that the flow of the mixture reaches a fully developed state much more rapidly than do the concentrations of the two species

Variables

Radii: r_A, r_B $d = r_A + r_B$ Masses: m_A, m_B

$\delta r = r_A / r_B - 1$ $\delta m = (m_A - m_B)/(m_A + m_B)$

Number densities: n_A, n_B $n = n_A + n_B$

Number fraction, species A: $f_A = N_A / N$

Mass densities: $\rho_i = m_i n_i$ $\rho = \rho_A + \rho_B$

Volume fractions: $c_i = (4/3) n_i \pi r_i^3$ $c = c_A + c_B$

Variables

Mixture velocity: $u = (\rho_A u_A + \rho_B u_B)/\rho$

$v = (\rho_A v_A + \rho_B v_B)/\rho$

Granular temperature: $T_i \equiv \frac{m_i \langle C_i^2 \rangle}{3} \quad i = A, B$

Mixture temperature: $T = \frac{n_A T_A + n_B T_B}{n}$

Balance equations: segregation

Difference of mass balance of the two species

$$\frac{\partial}{\partial t}(\rho_A - \rho_B) + \frac{\partial}{\partial x}(\rho_A u_A - \rho_B u_B) + \frac{\partial}{\partial y}(\rho_A v_A - \rho_B v_B) = 0$$

Using the components of the diffusion velocity

$$\tilde{u}_A = u_A - u \text{ and } \tilde{v}_A = v_A - v$$

$$\rho \frac{\partial}{\partial t}\left(\frac{\rho_A - \rho_B}{\rho}\right) + \rho u \frac{\partial}{\partial x}\left(\frac{\rho_A - \rho_B}{\rho}\right) + \rho v \frac{\partial}{\partial y}\left(\frac{\rho_A - \rho_B}{\rho}\right)$$
$$+ 2\frac{\partial}{\partial x}\left[\frac{\rho_A \rho_B}{\rho}(\tilde{u}_A - \tilde{u}_B)\right] + 2\frac{\partial}{\partial y}\left[\frac{\rho_A \rho_B}{\rho}(\tilde{v}_A - \tilde{v}_B)\right] = 0.$$

Balance equations: segregation

where the difference in the vector diffusion velocities is driven by gradients in the mixtures pressure, P, the kinetic energy of the velocity fluctuations, T, the gradient of the chemical potentials, μ_A, and the number densities of the two species (Arnarson & Jenkins 2004)

$$\tilde{\boldsymbol{v}}_A - \tilde{\boldsymbol{v}}_B = -\frac{n^2}{n_A n_B} D_{AB} \left\{ -\frac{\rho_A}{n\rho T} \nabla P + \frac{1}{nT} \left(n_A + 2\frac{m_A}{m_{AB}} K_{AB} + K_{AA} \right) \nabla T + \frac{n_A}{nT} \left(\frac{\partial \mu_A}{\partial n_A} \nabla n_A + \frac{\partial \mu_A}{\partial n_B} \nabla n_B \right) + \frac{1}{T} K_T^{(A)} \nabla T \right\}.$$

Balance equations: segregation

and the diffusion coefficient is (Arnarson & Jenkins 2004):

$$D_{AB} \equiv \frac{3}{2n g_{AB}} \left(\frac{2T m_{AB}}{\pi m_A m_B} \right)^{1/2} \frac{1}{8 r_{AB}^2}$$

where the radial distribution function at collision is (Mansoori et al. 1971):

$$g_{ij} = \frac{1}{1-c} + \frac{3 r_i r_j}{r_{ij}} \frac{\xi_2}{(1-c)^2} + 2 \left(\frac{r_i r_j}{r_{ij}} \right)^2 \frac{\xi_2^2}{(1-c)^3}$$

$$\xi_2 \equiv 4\pi (n_A r_A^2 + n_B r_B^2)/3 \qquad K_{ij} = 2\pi r_{ij}^3 n_i n_j g_{ij}/3.$$

Balance equations: segregation

The approximate form of the difference of mass balance of the two species becomes:

$$\rho\frac{\partial X}{\partial t}+\rho u\frac{\partial X}{\partial x}+\rho v\frac{\partial X}{\partial y}+\frac{\partial}{\partial x}\left[\frac{m_A n}{4}\left(1-4X^2\right)\left(\tilde{u}_A-\tilde{u}_B\right)\right]$$
$$+\frac{\partial}{\partial y}\left[\frac{m_A n}{4}\left(1-4X^2\right)\left(\tilde{v}_A-\tilde{v}_B\right)\right]=0, \qquad X\equiv(n_A-n_B)/(2n)$$

where:

$$\tilde{\boldsymbol{v}}_A-\tilde{\boldsymbol{v}}_B=-D_{AB}\left[(\Gamma_1\delta m+R_1\delta r)\frac{\nabla T}{T}-(\Gamma_2\delta m+R_2\delta r)\frac{m_{AB}\boldsymbol{g}\cos\phi}{2T}+\frac{\nabla X}{0.25-X^2}\right]$$

$$R_1=\frac{5}{58}\left[2+\frac{c(3-c)}{2-c}-\frac{12}{5}G\right]+2G\left[3+\frac{c(3-c)}{2-c}\right]-\frac{12cH(1+4G)}{1+4G+4cH}\doteq-4.35G.$$

$$\Gamma_1=\frac{179}{29}G+\frac{105}{116}\doteq 6.17G,$$

$$D_{AB}=\frac{\pi^{1/2}}{16}\frac{r_{AB}}{G}\left(\frac{2T}{m_{AB}}\right)^{1/2}$$

$$G=5.69c\frac{c_M-0.49}{c_M-c}\quad H\equiv\frac{dG}{dc}=G\frac{c_M}{c(c_M-c)},\qquad \Gamma_2=2,\quad R_2=-\frac{12cH}{1+4G+4cH}\doteq-3$$

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Balance equations: segregation

$$\tilde{\boldsymbol{v}}_A-\tilde{\boldsymbol{v}}_B=-D_{AB}\left[(\Gamma_1\delta m+R_1\delta r)\frac{\nabla T}{T}-(\Gamma_2\delta m+R_2\delta r)\frac{m_{AB}\boldsymbol{g}\cos\phi}{2T}+\frac{\nabla X}{0.25-X^2}\right]$$

- The inhomogeneity in the specie's concentration results from an imbalance between the gradient of the difference in the species' number fractions, *X*, and a segregation flux that contains contributions from both the gradient of the mixture temperature and gravity
- Each contribution to the segregation flux is linear in δr and δm.

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Balance equations: mixture flow

- Extension of kinetic theory that incorporates an additional lengthscale in the rate of collisional dissipation associated with chains or clusters
- Transport coefficients given by Garzo & Dufty (1999) for identical, very dissipative spheres, modified by Arnarson & Jenkins (2004) for binary mixtures of nearly elastic spheres.
- Uniform concentration across the flow (Silbert et al. 2001; Tripathi & Khakhar 2011)

Balance equations: mixture flow

The component of the mixture momentum balance across the flow gives

$$T = \frac{m_{AB}(h-y)}{4(1+e)G} g \cos\phi \, (1 + 2X\delta m).$$

From the mixture energy balance, $Su' - \Gamma = 0$:

$$G = \left\{ \frac{4J}{5\pi^{1/2}} \frac{1}{1+e} \left[\frac{15}{J} \frac{(1-e^2)}{\alpha} \right]^{1/3} \frac{1}{\tan\phi} \right\}^9 [1 + 3X(\delta r + \delta m)]$$

$$J = \frac{(1+e)}{2} + \frac{\pi}{4} \frac{(3e-1)(1+e)^2}{[24-(1-e)(11-e)]}$$

and

$$c = \frac{c_M G}{G + 5.69\,(c_M - 0.49)}$$

Balance equations: mixture flow

The velocity of the mixture, u, follows from the balance of mixture momentum along the flow and the expression for the mixture shear stress:

$$u = u_0 + \frac{5\pi^{1/2}}{6J}\frac{1}{r_{AB}}\left(\frac{1+e}{2G} g\cos\phi\right)^{1/2}\left[h^{3/2} - (h-y)^{3/2}\right]\tan\phi(1 - X\delta r)$$

We approximate the slip velocity at the bottom u_0 = 0

Evolution in time

For **uniform, time-dependent segregation** the mass balance reduces to

$$\rho\frac{\partial X}{\partial t} + \frac{\partial}{\partial y}\left[\frac{m_A n}{4}\left(1 - 4X^2\right)\left(\tilde{v}_A - \tilde{v}_B\right)\right] = 0.$$

or, equivalently, using the expressions for the diffusion velocity, D_{AB} and T:

$$\frac{\partial X}{\partial \tau} = \left(\frac{r_{AB}}{h}\right)^{3/2}\frac{(\pi\cos\phi)^{1/2}}{128G^{3/2}}\left(\frac{2}{1+e}\right)^{1/2}\frac{\partial}{\partial z}\left[(1-z)^{1/2}\left\{[(2(1+e)G\Gamma_2 - \Gamma_1)\,\delta m + (2(1+e)GR_2 - R_1)\,\delta r]\frac{1-4X^2}{1-z} + 4\frac{\partial X}{\partial z}\right\}\right].$$

Evolution in time

In order to conserve the total number of particles of the two species, a new variable is introduced:

$$\zeta \equiv Xn/\bar{n}, \qquad \bar{\zeta} \equiv \frac{1}{2}\left(\frac{\bar{n}_A}{\bar{n}} - \frac{\bar{n}_B}{\bar{n}}\right) = \frac{1}{2}\left(\frac{N_A}{N} - \frac{N_B}{N}\right)$$

or:

$$X = \frac{\left(\hat{c}_A + \hat{c}_B\right)\zeta}{2\left[c - \left(\hat{c}_A - \hat{c}_B\right)\zeta\right]} \doteq \frac{\left(\hat{c}_A + \hat{c}_B\right)}{2c}\zeta$$

$$\hat{c}_A \equiv \frac{\bar{n}}{\bar{n}_A}\bar{c}_A \quad \text{and} \quad \hat{c}_B \equiv \frac{\bar{n}}{\bar{n}_B}\bar{c}_B.$$

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Evolution in time

If lengths and time are normalized by:

$$z \equiv y/h \ \text{ and } \ \tau \equiv t/(r_{AB}/g)^{1/2}$$

then:

$$\frac{\partial\zeta}{\partial\tau} = \left(\frac{r_{AB}}{h}\right)^{3/2}\frac{(\pi\cos\phi)^{1/2}}{128G^{3/2}}\left(\frac{2}{1+e}\right)^{1/2}\frac{2c}{\left(\hat{c}_A+\hat{c}_B\right)}$$
$$\times\frac{\partial}{\partial z}\left\{\frac{\left[(2(1+e)G\Gamma_2-\Gamma_1)\,\delta m+(2(1+e)GR_2-R_1)\,\delta r\right]}{(1-z)^{1/2}}\right.$$
$$\left.\times\left[1-\frac{\left(\hat{c}_A+\hat{c}_B\right)^2}{c^2}\zeta^2\right]+2(1-z)^{1/2}\frac{\left(\hat{c}_A+\hat{c}_B\right)}{c}\frac{\partial\zeta}{\partial z}\right\}.$$

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Steady longitudinal segregation

Lengths associated with the evolution of segregation are significantly larger than the flow depth

Therefore we assume that in the mass balance the streamwise derivatives are neglegible compared with the cross-stream derivatives

$$\rho u \frac{\partial}{\partial x}\left(\frac{\rho_A - \rho_B}{\rho}\right) + 2\frac{\partial}{\partial y}\left[\frac{\rho_A \rho_B}{\rho}\left(\tilde{v}_A - \tilde{v}_B\right)\right] = 0.$$

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Steady longitudinal segregation

At lowest order in δr and δm:

$$\left[1-(1-z)^{3/2}\right]\tan\phi \frac{\left(\hat{c}_A+\hat{c}_B\right)}{2\bar{c}}\frac{\partial\zeta}{\partial\ell} = \frac{3J}{160(1+e)}\left(\frac{r_{AB}}{h}\right)^2\frac{1}{G}$$
$$\times\frac{\partial}{\partial z}\left\{\frac{\left[2(1+e)G(2\delta m+R_2\delta r)-(\Gamma_1\delta m+R_1\delta r)\right]}{(1-z)^{1/2}}\right.$$
$$\left.\times\left[1-\frac{\left(\hat{c}_A+\hat{c}_B\right)^2}{\bar{c}^2}\zeta^2\right]+2(1-z)^{1/2}\frac{\left(\hat{c}_A+\hat{c}_B\right)}{\bar{c}}\frac{\partial\zeta}{\partial z}\right\}.$$

with $z \equiv y/h$ and $\ell \equiv x/h$

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Comparison with experiments and simulations

The evolution of segregation was obtained by solving the segregation equation with the Matlab embedded solver *pdepe*, suitable for initial BVP for parabolic-elliptic PDE in one space-like and one time-like variable.

The initial boundary sets the concentration profile (through ζ).

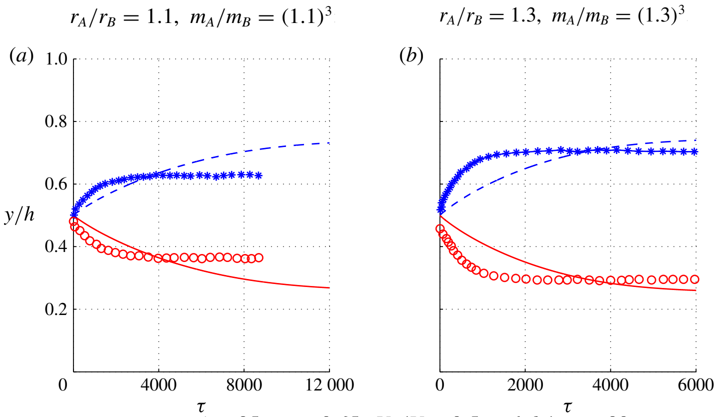
The two boundary conditions require the flux to vanish at the base, $z = 0$, and the top, $z = 1$.

Thornton et al. (2012) simulations

- Chute flow over a rigid bumpy base
- Periodic box at angle 25° - 0°
- 5000 small particles
- Total volume of large and small are equal

Thornton et al. (2012) simulations

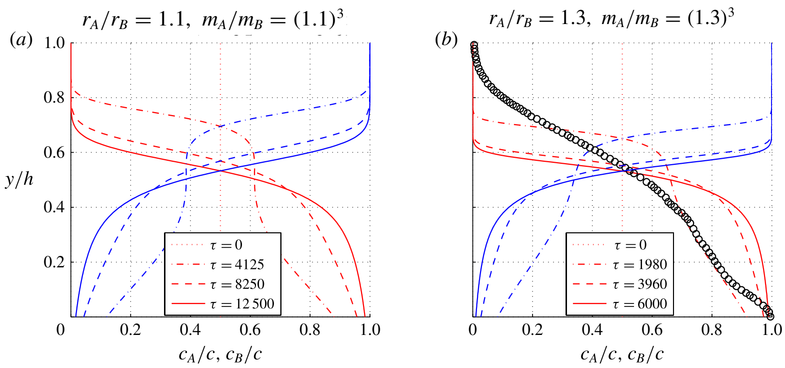
COM of large and small spheres



$\phi = 25°,\ e = 0.65,\ V_A/V = 0.5$ and $h/r_{AB} = 20.$

Thornton et al. (2012) simulations

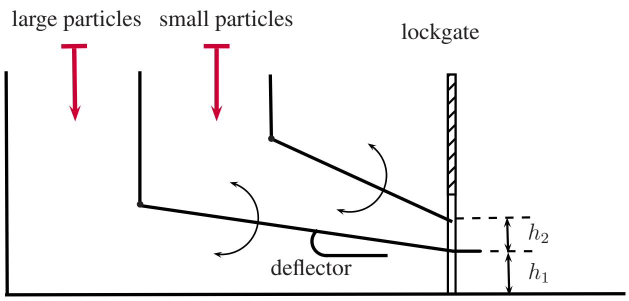
Relative concentration profiles of large and small spheres



$\phi = 25°,\ e = 0.65,\ V_A/V = 0.5$ and $h/r_{AB} = 20.$

Wiederseiner et al. (2011) EXP.

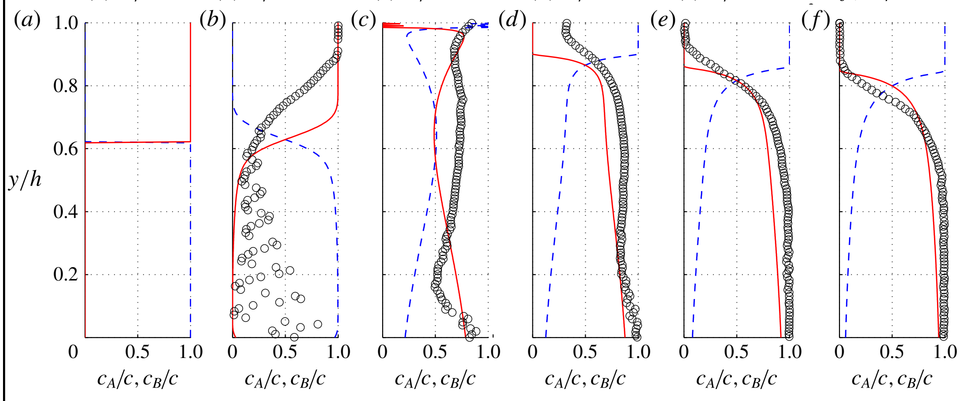
- Flume: 3 m long, 2 cm wide
- Inclination: 0 - 45°, rigid bumpy bed
- Spherical glass beads
- r_A = 2 mm; r_B = 1 mm; ρ_s = 2500 kg/m^3



Wiederseiner et al. (2011) EXP.

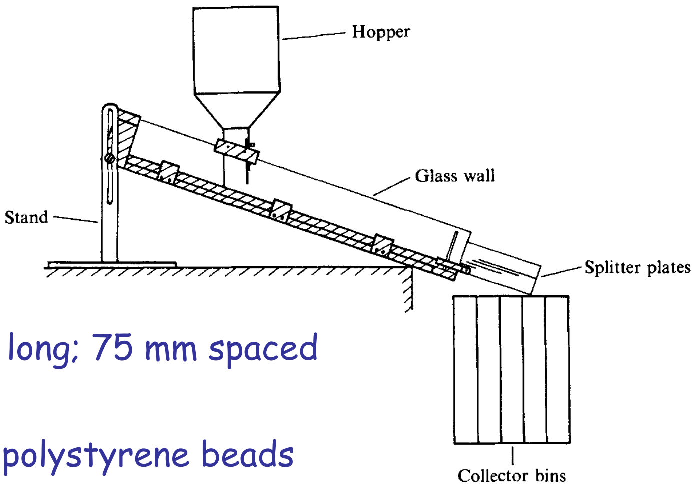
Large and small glass beads

(a) $x/L = 0$, (b) $x/L = 0.01$, (c) $x/L = 0.25$, (d) $x/L = 0.5$, (e) $x/L = 0.75$, (f) $x/L = 1$



$\phi = 29°$, $e = 0.65$, $r_A/r_B = 2$, $m_A/m_B = (r_A/r_B)^3$, $V_A/V = 0.62$ and $h/r_{AB} = 16$

Savage & Lun (1988) EXP.

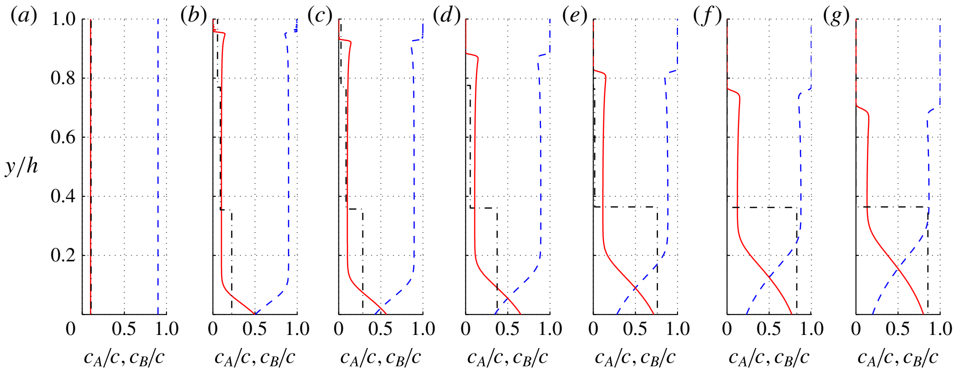


- Flume 1 m long; 75 mm spaced sidewalls
- Spherical polystyrene beads
- r_A = 0.8 mm; r_B = 0.47 mm

Savage & Lun (1988) EXP.

ϕ = 26°, h/r_{AB} = 11.8; large (blue), small (red)

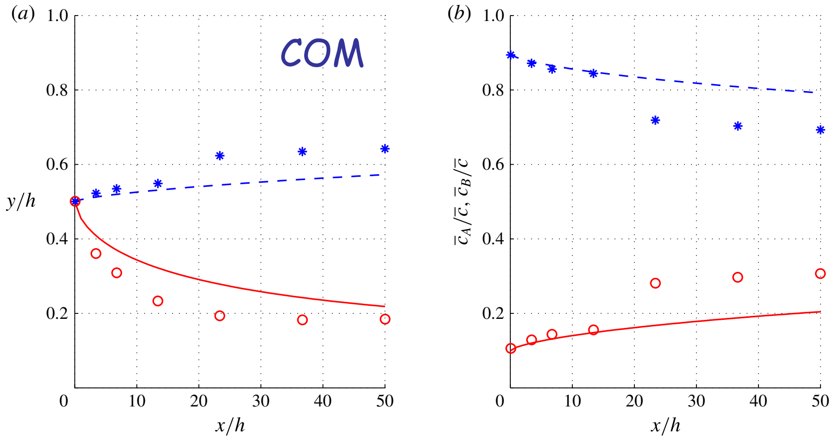
(*a*) $x/h=0$, (*b*) $x/h=3$, (*c*) $x/h=7$, (*d*) $x/h=13$, (*e*) $x/h=23$, (*f*) $x/h=37$, (*g*) $x/h=50$.



$\phi=26°$, $e=0.7$, $r_A/r_B=1.7$, $m_A/m_B=(1.7)^3$, $V_A/V=0.9$ and $h/r_{AB}=11.8$

Savage & Lun (1988) EXP.

ϕ = 26°, h/r_{AB} = 11.8; large (blue), small (red)



$\phi = 26°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $V_A/V = 0.9$ and $h/r_{AB} = 11.8$

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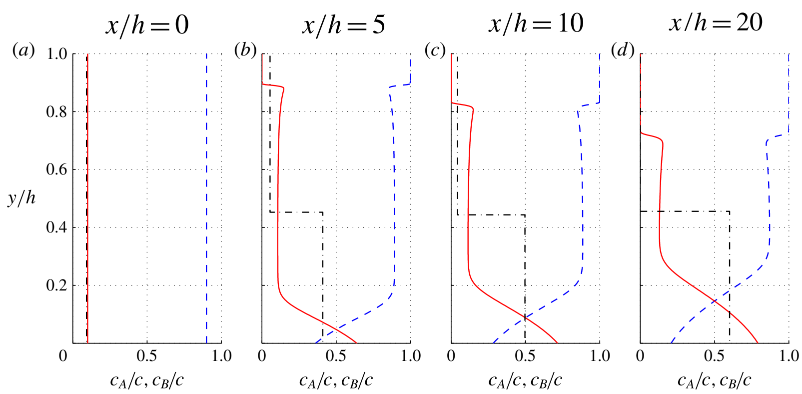
Savage & Lun (1988) EXP.

ϕ = 26°, h/r_{AB} = 7.8; large (blue), small (red)



$\phi = 26°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $V_A/V = 0.9$ and $h/r_{AB} = 7.8$

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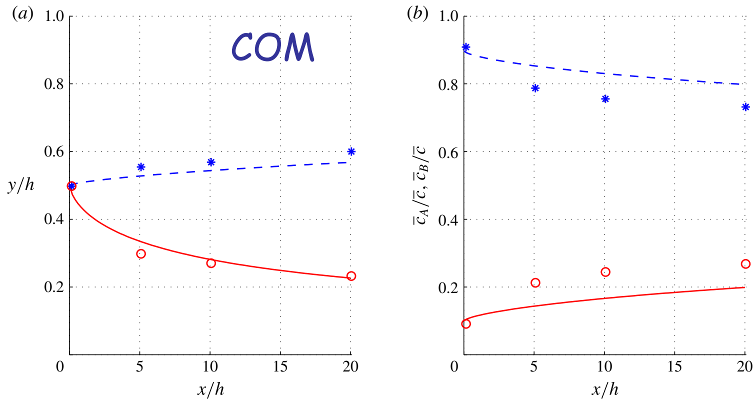
Savage & Lun (1988) EXP.

ϕ = 26°, h/r_{AB} = 7.8; large (blue), small (red)



$\phi = 26°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $\dot{V}_A/\dot{V} = 0.9$ and $h/r_{AB} = 7.8$

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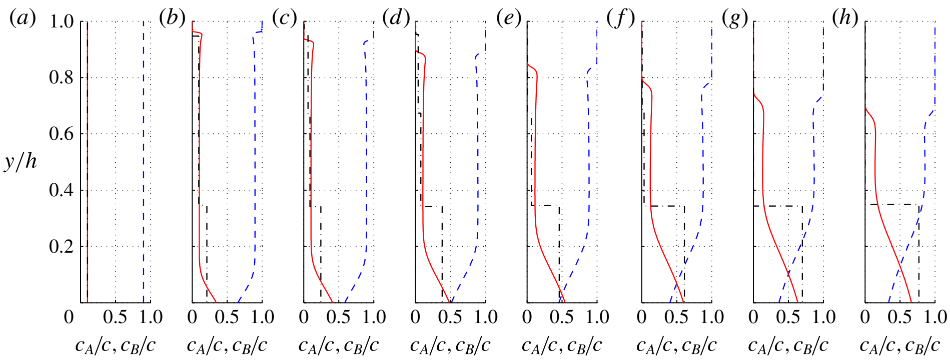
Savage & Lun (1988) EXP.

ϕ = 28°, h/r_{AB} = 11.8; large (blue), small (red)

(a) $x/h = 0$, (b) $x/h = 3$, (c) $x/h = 7$, (d) $x/h = 13$, (e) $x/h = 23$, (f) $x/h = 37$, (g) $x/h = 50$, (h) $x/h = 63$



$\phi = 28°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $V_A/V = 0.9$ and $h/r_{AB} = 11.8$

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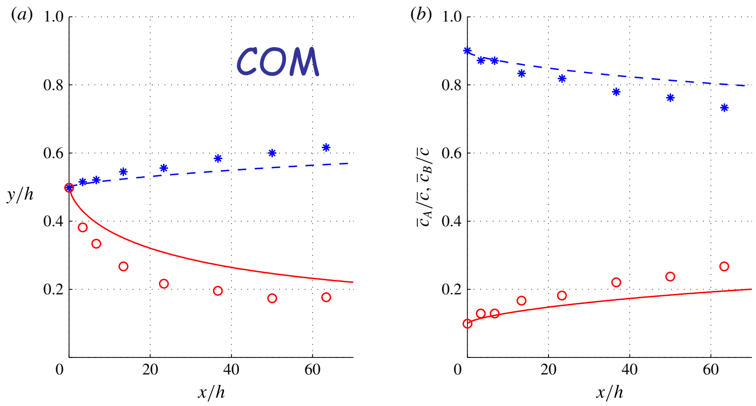
Savage & Lun (1988) EXP.

ϕ = 28°, h/r_{AB} = 11.8; large (blue), small (red)

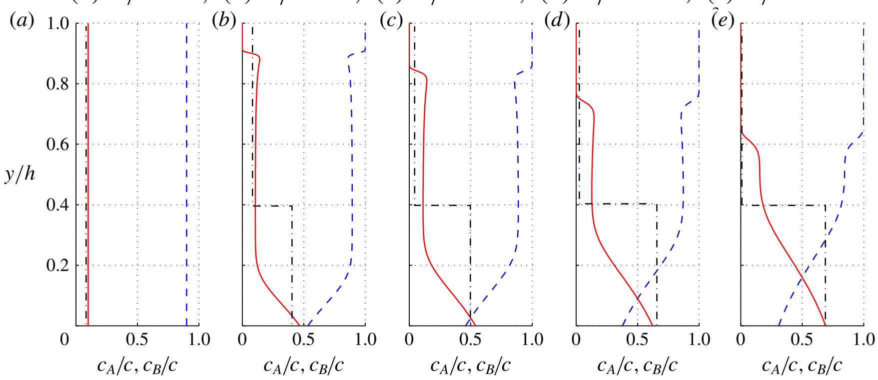


$\phi = 28°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $V_A/V = 0.9$ and $h/r_{AB} = 11.8$

Savage & Lun (1988) EXP.

ϕ = 28°, h/r_{AB} = 7.8; large (blue), small (red)

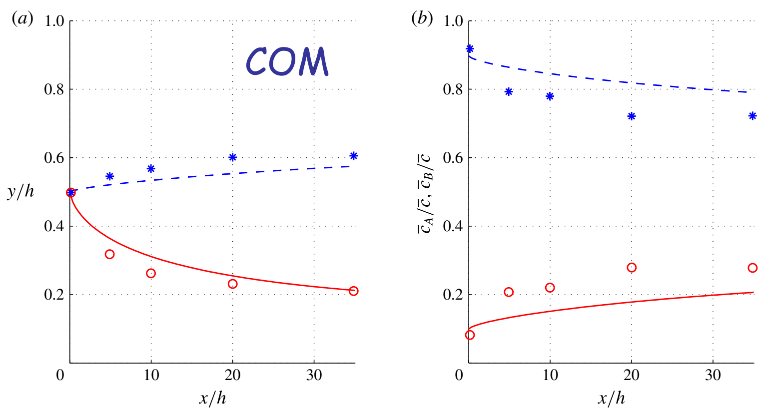
(a) $x/h = 0$, (b) $x/h = 5$, (c) $x/h = 10$, (d) $x/h = 20$, (e) $x/h = 35$



$\phi = 28°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $V_A/\dot{V} = 0.9$ and $h/r_{AB} = 7.8$

Savage & Lun (1988) EXP.

ϕ = 28°, h/r_{AB} = 7.8; large (blue), small (red)



$\phi = 28°$, $e = 0.7$, $r_A/r_B = 1.7$, $m_A/m_B = (1.7)^3$, $V_A/V = 0.9$ and $h/r_{AB} = 7.8$

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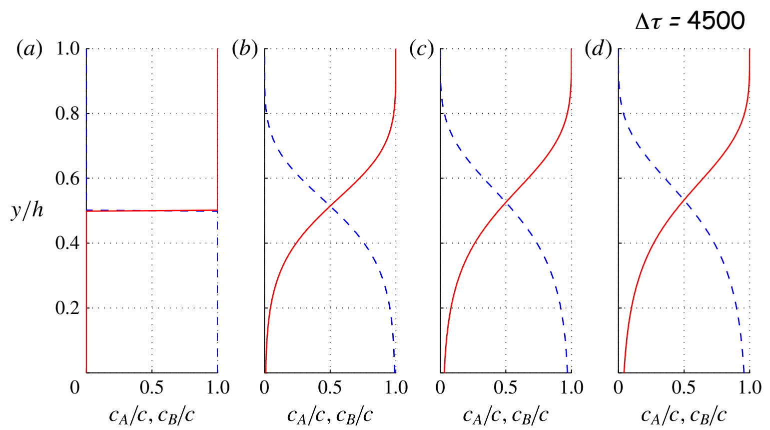
Equal size, different masses

r_A/r_B = 1, m_A/m_B = 2; heavy (blue), light (red)



$\phi = 25°$, $e = 0.7$, $V_A/V = 0.5$ and $h/r_{AB} = 20$.

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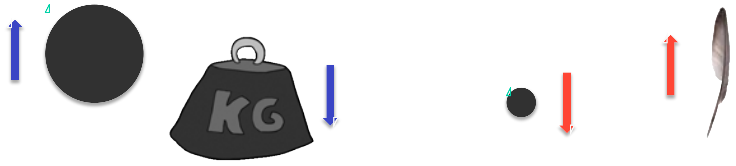
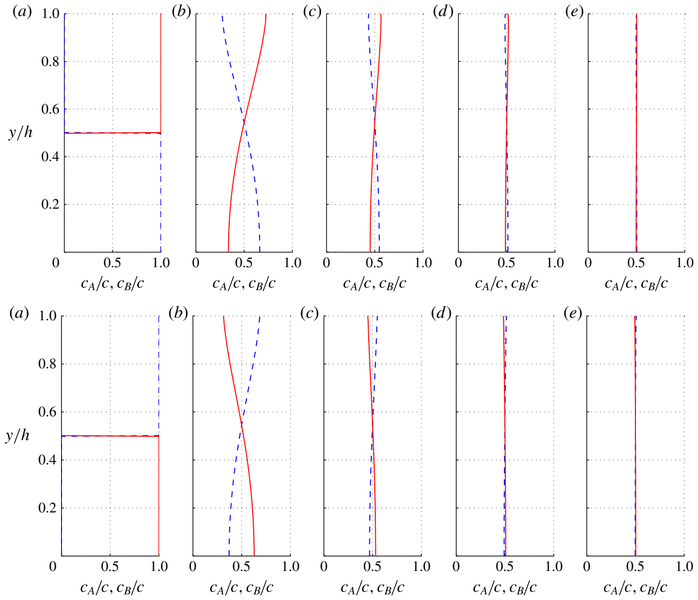
Different sizes, different masses

Large, heavy

Small, light



r_A/r_B = 1.04, m_A/m_B = 2; heavy (blue), light (red)



$\Delta\tau$ = 22500

$\Delta\tau$ = 27000

Conclusion

- Governing equations based on the balances of mass, momentum and energy
- The flow reaches uniformity before segregation ($D_{AB} \propto 1/G$, $\mu \propto G$)
- Larger and lighter particles rise
- Slower segregation for thicker flows
- Slower evolution for steeper slopes
- Faster evolution if one species is diluted in the other
- No segregation for particular radii and mass combinations

Thank you

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