PARTICLE-INDUCED FINGERING

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Max Planck Institute for the Physics of Complex Systems
BUT FIRST....
A typical sequence of side-view images from the initially sessile drop to the runback threshold. Test conditions were $V = 50 \mu L$ and $\theta = 0$. The critical flow speed was $U_{\text{crit}} = 17.7 \text{m/s}$.

(b) Final drop profiles, split by volume and inclination angle. Plots show inclination angles of $\theta = 0, 10, 20, 30, \ldots$ beginning at the top. Drops smaller than the $Bo$ threshold in figure 9 are shown with solid lines; larger drops are shown with dashed lines.

Figure 23: Sideview drop profiles.

The profile as the flow speed increases. The pressure inside a sessile drop is higher than that of the surrounding fluid due to the surface tension along the interface. Flow over the drop and gravity combine to alter the pressure field both inside and outside the drop. The effects of these forces can be seen in the evolution of the drop profile.

Splitting reconstructed profiles into groups based on both inclination angle and drop size relative to the Bond number threshold, figure 23(b) shows the final sub-critical reconstructed sideview profile. The profile and band displayed are the mean and standard deviation of a collection of drop profiles. Drops smaller than the $Bo$ threshold in figure 9 possess a region of negative curvature on the upwind side of the drop. Larger drops whose stability limits have been shown to be dominated by the force of gravity (see figure 10) maintain configurations with curvature similar to the sessile drop case. In the hydrostatic configuration, the force exerted by surface tension due to the interface curvature resists the essentially uniform pressure field around the drop. For smaller drops where the stability limit is dominated by airflow effects, the complex pressure field surrounding the drop is creates the negative interface curvature on the upwind drop side.

Reconstructed volumes begin at about 70% to 90% of the applied drop volume and decrease to about 60% to 85% at runback. This is thought to be measurement error. Although evaporation could contribute to the problem, the initially low values suggest it is mainly a measurement issue. As the experiment proceeds, added complexity of reconstructing deformed drop shapes decreases further the reconstructed volume.

4.2.3. Contact Angle Evolution

The measurement of profile sequences enables further exploration of the details of drop evolution and stability. Of particular interest is the evolution of the contact angle distribution. Figure 24 shows the mean and standard deviation of contact angle for all...
We also like particles...
VISCOUS FINGERING

"invading" phase \( \mu_i \)

\[ \Rightarrow \]

"defending" phase \( \mu_d \)

Interface is **unstable** when \( \mu_i < \mu_d \).

Saffman & Taylor 1958

Wooding 1969

Paterson 1981

Praud & Swinney 2005

Pihler-Puzovic et al 2012
Within the packing, as the granular packing is unable to dilate, the capillary forces at the interface is transmitted through force chains with the reservoir of air of volume at high injection rate. Finally, we consider displacement dynamics uncovered and presented in phase diagrams illustrating the variability we focus on the dynamic response of the frictional tonian.

A yield stress well below the close packing limit, can be attributed to observed for dense granular suspensions, such as shear banding and Bingham or yield stress.

Oils are governed by threshold-limited dynamics. Of interest in engineering applications is the role of frictional granular the rheology to jump rapidly from solid- to liquid-like behaviour. Of example, they have a key role in landslides, where inter-particle friction has a central role, have received less attention.

Recently, several authors have considered displacement patterning typically found in consumer products and industrial materials. I

In their landmark paper from 1958, Sa et al. found that many puzzling phenomena are ubiquitous in nature and engineering. For fluids, in which non-Newtonian behaviour

VISCOUS FINGERING + PARTICLES

(viscous fingering)
Within the packing. As the granular packing is unable to dilate, the initial frictional forces create force chains that stabilize the system. Earthquakes and granular avalanches are thus observed for dense granular suspensions, such as shear banding and sedimentation or creaming induced by gravity.

Recently, several authors have considered displacement patterning in granular suspensions, in which care is taken to match the density filling fraction $\phi$. The low rate means that viscous forces are negligible. Capillary forces influence the dynamics of the mixture (where fluid has a profound effect), and decreases with increasing loading fraction $\phi$. Only the slightest difference in behaviour is perhaps not surprising as both yield stress and friction forces act locally and thereby extensively influence the origin of jamming is closely related to Newtonian fluid-like behaviour. Of all dynamic interactions are less dominant, as the air is compressed by the constant, slow driving of the piston, there is simple setup has since finally yields, and the pressurized air invades the Hele-Shaw cell, two parallel plates separated by a small gap, as an idealized model system for the study of viscous fingering in most Newtonian and non-Newtonian fluids.

The invading air/fluid interface $R(t)$ radially depends on channel width and velocity, and instead in $h$, $d$, $\phi$, and the injection rate $Q$. The dependence on channel width, contrary to what is generally expected, is independent of channel width, and instead in $\phi$. The existence of a characteristic labyrinthine structure as seen in granular avalanches and granular avalanches in most Newtonian and non-Newtonian fluids is characteristic of the length scale of the problem.

Recently Fall et al. (2000) have considered displacement patterning in settling granular mixtures, in which care is taken to match the density filling fraction $\phi$. The low rate means that viscous forces are negligible. Capillary forces influence the dynamics of the mixture (where fluid has a profound effect), and decreases with increasing loading fraction $\phi$. Only the slightest difference in behaviour is perhaps not surprising as both yield stress and friction forces act locally and thereby extensively influence the origin of jamming is closely related to Newtonian fluid-like behaviour. Of all dynamic interactions are less dominant, as the air is compressed by the constant, slow driving of the piston, there is simple setup has since finally yields, and the pressurized air invades the Hele-Shaw cell, two parallel plates separated by a small gap, as an idealized model system for the study of viscous fingering in most Newtonian and non-Newtonian fluids.

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**PARTICLES + VISCOUS FINGERING**

Particles **destabilize** the interface!

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Tang et al. 2000; Ramachandran & Leighton 2010
accumulates a front of compacted granular material of thickness with the viscous regime observed as hydrodynamic interactions dominate tent behaviour familiar from processes such as stick-slip sliding. At low rate, we focus on the dynamic response of the frictional forces, which create force chains that stabilize the system. As the mobilized, leading to destabilization and buckling of the load, unjamming, or yielding occurs as grain–grain contacts become linked to the presence of force chains within the packing, increasing the applied stress on the macro-level.

Frictional forces in granular suspensions, in which care is taken to match the density to CO$_2$, have recently been considered displacement patterning in their landmark paper from 1958, Sa and Taylor introduced an approach 1, where the system takes on the properties of a metastable for an extended period, followed by a sudden displacement progress bubble by bubble in a stick-slip manner. The origin of jamming is closely associated with the yield stress and the static friction, respectively. Similarity in behaviour is perhaps not surprising as both yield stress and friction forces act locally and thereby effectively stabilize the system. Where the air/fluid interface advances and is simple setup has since been found that many puzzling phenomena are strongly related to such complex materials as fluids and granular materials. Furthermore, the capillary forces and friction at the interface.

When we increase the filling fraction the injected air advances in a finger-like manner. Pressurized air is injected into a conical Hele-Shaw cell loaded with polydisperse glass beads in an experiment on viscous fingering in most Newtonian and non-Newtonian fluids. The geometry of the channel is fixed, and we measure the gas pressure and gas volume for the stress in the packing, such that the Janssen proportionality constant giving the ratio of transverse normal force on the conical walls to the static friction experienced by the front of the air.

During the static periods, the gas pressure increases linearly with the time, and into the viscoelastic regime is observed as hydrodynamic interactions dominate tent behaviour familiar from processes such as stick-slip sliding and the transitions between them. At low rate, forces create force chains that stabilize the system. The dynamic response of the frictional forces, which create force chains that stabilize the system. As the mobilized, leading to destabilization and buckling of the load, unjamming, or yielding occurs as grain–grain contacts become linked to the presence of force chains within the packing, increasing the applied stress on the macro-level.

Particles destabilize the interface!
PARTICLE-INDUCED FINGERING

\[

d/h = 0.3 \quad h/d = 3.5
\]

\[

\phi_0 = 20\% \\
Q = 150\text{mL/min}
\]

\[

h/d = 11
\]

\[

0\% \quad 5\% \quad 10\% \quad 15\% \quad 20\% \quad 25\% \quad 30\%
\]
Particle-induced fingering

\[ h/d = 11 \]

\[ h/d = 3.5 \]

\[ \phi_0 = 20\% \]

\[ Q = 150 \text{ mL/min} \]

"continuum"
What is the critical particle concentration in the continuum limit?
What is the critical particle concentration in the continuum limit?

In this talk, we will focus on:
1. What is the mechanism of fingering?
2. Can we model the particle-laden flow upstream of the interface?
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

"shear-induced migration"
Leighton & Acrivos 1987
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

\[ \vec{v}_r^p > \vec{v}_r \]
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

\[ \bar{v}_r > \bar{v}_r \]
**MECHANISM OF FINGERING**

Particle accumulation on the fluid-fluid interface.
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

“fountain flow”  Coyle et al 1987; Karnis & Mason 1967
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

Tang et al 2000; Ramachandran & Leighton 2010, 2007 (particle accumulation on the meniscus in a tube)
MECHANISM OF FINGERING

particle accumulation on the fluid-fluid interface.

Particle accumulation near the interface

Miscible viscous fingering

Interfacial deformations

“μ_d”

"μ_i"

\( \phi_0 = 35\% \)

3 cm
**MECHANISM OF FINGERING**

Particle accumulation on the fluid-fluid interface.

Particle accumulation due to shear-induced migration.

velocity ratio \( \beta \equiv \frac{\bar{v}_r^p(r)}{\bar{v}_r(r)} > 1 \)

necessary condition for accumulation

where \( \bar{v}_r^p \equiv \int_{-h/2}^{h/2} v_r^p(r, z) \phi(r, z) dz \)

& \( \bar{v}_r = \frac{Q}{2\pi rh} \)
Particle accumulation on the fluid-fluid interface.

**Particle accumulation due to shear-induced migration**

**Velocity ratio**

\[ \beta \equiv \frac{v_r^p(r)}{\bar{v}_r(r)} > 1 \]

necessary condition for fingering

Calculate \( \beta(\phi_0, h/d) \) by resolving the upstream flow field!
CONTINUUM MODEL

**Suspension balance approach**  Nott & Brady 1994

- **Mixture mass conservation**
  \[ \nabla \cdot \mathbf{v} = 0 \]

- **Mixture momentum**
  \[ \nabla \cdot \mathbf{P} = 0 \]

- **Particulate mass conservation**
  \[ \frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}^p) = 0 \]

- **Particulate momentum**
  \[ \nabla \cdot \mathbf{P}^p - \frac{18 \mu_1}{d^2} \frac{\phi}{f(\phi)} (\mathbf{v}^p - \mathbf{v}) = 0 \]

- **Kinematic relationship**
  \[ \mathbf{v} = \phi \mathbf{v}^p + (1 - \phi) \mathbf{v}^f \]

- **Constitutive relationships**
  \[ \mathbf{P} = -p \mathbf{I} + \mu_1 (\nabla \mathbf{v} + \nabla \mathbf{v}^T) + \mathbf{P}^p \]

  \[ \mathbf{P}^p = \mathbb{P}^p(\dot{\gamma}, \mu_n(\phi)) + (\mu_s(\phi) - \mu_1) (\nabla \mathbf{v} + \nabla \mathbf{v}^T) \]

- **Empirical relationships**
  \[ \mu_s \approx \mu_1 \frac{e^{-2.34\phi}}{(1 - \phi/\phi_m)^3} \]

**Lubrication approximations:**
\[ h/\bar{R} \ll 1 \]
\[ \mathbf{v} = \frac{F(z)}{r} \mathbf{e}_r, \quad \frac{\partial}{\partial \theta} = 0 \]

(axisymmetric)

\[ \nabla \cdot \mathbf{P}^p = \mu_1 \nabla \dot{\gamma} \]

\[ \mathbf{P}^p \approx \mu_1 \nabla \dot{\gamma} \]

Zarraga et al 2000
CONTINUUM MODEL

suspension balance approach + lubrication approximations

mixture momentum

particulate mass + momentum

constant mixture flux

constant particulate flux

$$r^* \frac{dp^*}{dr^*}(z^* - 1/2) = \frac{\mu_s(\phi)}{\mu_1} \frac{dF^*}{dz^*}$$

$$\text{const} = \frac{\mu_n(\phi)}{\mu_1} \frac{dF^*}{dz^*}$$

$$\frac{1}{2} = \int_0^{1/2} F^*(z^*) dz^*$$

$$\frac{\phi_0}{2} = r^* \int_0^{1/2} \phi(z)v_r^{p*} dz^*$$

$$\phi = \phi(z) \text{ only}$$

"upstream" quasi-1D regime
CONTINUUM MODEL

**suspension balance approach + lubrication approximations**

- **mixture momentum**
  \[
  r^* \frac{dp^*}{dr^*}(z^* - 1/2) = \frac{\mu_s(\phi)}{\mu_1} \frac{dF^*}{dz^*}
  \]

- **particulate mass + momentum**
  \[
  \text{const} = \frac{\mu_n(\phi)}{\mu_1} \frac{dF^*}{dz^*}
  \]

- **constant mixture flux**
  \[
  \frac{1}{2} = \int_0^{1/2} F^*(z^*) dz^*
  \]

- **constant particulate flux**
  \[
  \frac{\phi_0}{2} = r^* \int_0^{1/2} \phi(z) v_r^{P*} dz^*
  \]

**velocity ratio**
\[
\beta = \frac{\int_{-h/2}^{h/2} v_r^p(r, z) \phi(r, z) dz}{\phi(r) \bar{v}_r(r) h} > 1
\]

necessary condition for fingering

\[
\phi = \phi(z) \text{ only}
\]
CONTINUUM MODEL

**suspension balance** approach + **lubrication** approximations

- mixture momentum
- particulate mass + momentum
- constant mixture flux
- constant particulate flux

\[ r^* \frac{dp^*}{dr^*}(z^* - 1/2) = \frac{\mu_s(\phi)}{\mu_1} \frac{dF^*}{dz^*} \]

\[ \text{const} = \frac{\mu_n(\phi)}{\mu_1} \frac{dF^*}{dz^*} \]

\[ \frac{1}{2} = \int_0^{1/2} F^*(z^*)dz^* \]

\[ \frac{\phi_0}{2} = r^* \int_0^{1/2} \phi(z) v_{r^*}^p dz^* \]

**velocity ratio**

\[ \beta = \frac{\int_{-h/2}^{h/2} v_{r^*}^p(r, z)\phi(r, z)dz}{\bar{\phi}(r)\bar{v}_r(r)h} > 1 \]

Can we validate the velocity ratio experimentally?

"upstream" quasi-1D regime

\[ \phi = \phi(z) \text{ only} \]
MODEL VALIDATION: from accumulation to fingering

mass conservation in upstream regime

\[ \phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v_r^P(z)\phi(z)dz \]

\[ \beta = \frac{\phi_0}{\bar{\phi}_{up}} \]

measure experimentally

“upstream” quasi-1D regime

\[ \bar{\phi}_{up} = \frac{1}{h} \int_{-h/2}^{h/2} \phi(z)dz \]
MODEL VALIDATION: from accumulation to fingering

mass conservation in upstream regime

\[ \phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v_r^p(z)\phi(z)dz \]

\[ \beta = \phi_0 / \bar{\phi}_{up} \]

measure experimentally

"upstream" quasi-1D regime

\( 1.39 \text{ mm} \)

\( 1.27 \text{ mm} \)

\( 1.15 \text{ mm} \)

\( \beta^{1D} \)

\( \beta^{exp} \)

\( 0 \)

\( 0.05 \)

\( 0.1 \)

\( 0.15 \)

\( 0.2 \)

\( 0.25 \)

\( 0 \)

\( 0.2 \)

\( 0.4 \)

\( 0.6 \)

\( 0.8 \)

\( 1 \)

\( \bar{\phi}(r) \)

\( \bar{\phi}_{up} \)

\( r / R(t) \)

\( \phi_0 = 17\% \)
MODEL VALIDATION: from accumulation to fingering

mass conservation in upstream regime

\[ \phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v^p_r(z) \phi(z) dz \]

\[ \beta = \frac{\phi_0}{\bar{\phi}_{\text{up}}} \]

Rate of particle accumulation decreases with increasing \( \phi_0 \).

Rate of particle accumulation does not directly determine likelihood of fingering.
MODEL VALIDATION: from accumulation to fingering

mass conservation in upstream regime

\[ \phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v_r^p(z) \phi(z) dz \]

\[ \beta = \frac{\phi_0}{\bar{\phi}_{up}} \]

\[ V^{*p}_{\text{accum}} = \frac{V^p - V^p_{up}}{V^p} \sim \phi_0 \left(1 - \frac{1}{\beta}\right) \]

where \( V^p = \phi_0 \pi R^2 h \) & \( V^p_{up} = \frac{\phi_0}{\beta} \pi (R - w)^2 h \)
MODEL VALIDATION: from accumulation to fingering

Mass conservation in upstream regime

\[ \phi_0 Q = 2\pi r \int_{-h/2}^{h/2} v^p_r(z) \phi(z) \, dz \]

\[ \beta = \frac{\phi_0}{\phi_{up}} \]

\[ V^*_{\text{accum}} = \frac{V^p - V^p_{\text{up}}}{V^p} \sim \phi_0 \left(1 - \frac{1}{\beta}\right) \]

More particles collect near the interface with increasing \( \phi_0 \).

More likely to finger with increasing \( \phi_0 \).

Xu, Kim & Lee [in review]
particle-induced fingering

**Summary:** characterization of particle-induced fingering; continuum model formulation

**On-going:** prediction of the onset of fingering based on stability analysis

**Big picture:** coupled dynamics of particles & interface dynamics

effect of particles on **draining**

**pattern formation** with analogy to elastic instability

formation & breakage of **particle band**

Kim, Xu & Lee [ICTAM; in preparation]

Brau et al 2011
**Summary:** characterization of particle-induced fingering; continuum model formulation

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Kim, Xu & Lee [ICTAM; in preparation]

Brau et al 2011
Thank you.