Double-Diffusive Sedimentation

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• Motivation
• Governing equations
• Results: buoyant river outflows:
  - double-diffusive sedimentation
  - ‘fingering’ vs. ‘leaking’ modes
• Scaling analysis and physical interpretation
• Summary and outlook
Coastal margin processes
Sedimentation from river plumes: Motivation

- $10^{10}$ tons of sediment are transported by rivers into the world’s oceans every year → important to understand sedimentation in river plumes

Mississippi river plume
- drainage basin size: $3.3 \times 10^6$ km$^2$
- annual sediment yield: $1.2 \times 10^2$ t/km$^2$

Santa Clara river plume
- drainage basin size: $4.2 \times 10^3$ km$^2$
- annual sediment yield: $1.4 \times 10^3$ t/km$^2$

→ a large fraction of the sediment supply into the oceans is due to small, mountainous streams
Sedimentation from river plumes: Configuration

Hypopycnal river plumes:

density of the river (fresh water + sediment) < density of ocean (water + salinity)

→ river outflow propagates along the ocean surface

• focus on the downstream density stratification
Sedimentation from river plumes: Double-diffusion

Base density profile:

consider local downward perturbation of fluid element across opposing gradients
Sedimentation from river plumes: Double-diffusion

Base density profile:

salinity diffuses inward more rapidly than particles diffuse outward
Sedimentation from river plumes: Double-diffusion

Base density profile:

• potential for double-diffusive instability

→ fluid element will continue to sink
Traditional case: Salt fingers

- warm, salty water above cold, fresh water:

  

  ![Image of salt fingers](image)

  *Huppert and Turner (1981)*

- dominant process for the vertical flux of salt in the ocean
- robust against shear
- believed to be responsible for the formation of the thermohaline staircase

→ for salt/sediment system, how does double-diffusion affect sedimentation?
Sedimentation from river plumes: Experiments

- previous experimental work by Parsons et al. (2001):

  - convective ‘fingering’ mode
    - space filling
  - ‘leaking’ mode
    - localized, structures move along interface

  → goal: understand mechanisms driving these modes, and their influence on the effective particle settling velocity
Sedimentation from river plumes

Effect of settling velocity:

- settling process creates potential for Rayleigh-Taylor instability

\[ z \]

\[ \text{Sediment} \]

\[ \text{Salt} \]

\[ \text{nose height } H \]

\[ \text{density profile} \]
Framework: Dilute flows

Assumptions:
- volume fraction of particles $< O(10^{-3})$
- particle radius $\ll$ particle separation
- small particles with negligible inertia

Dynamics:
- effects of particles on fluid continuity equation negligible
- coupling of fluid and particle motion primarily through momentum exchange, not through volumetric effects
- particle loading modifies effective fluid density
- particles follow fluid motion, with superimposed settling velocity
Moderately dilute flows: Two-way coupling (cont’d)

Governing dimensionless eqns:

\[ \rho - 1 = \alpha S + \gamma C \]
\[ \nabla \cdot \mathbf{u} = 0 \]
\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \nabla^2 \mathbf{u} - \frac{\rho'}{\rho} \mathbf{g'} \]
\[ \frac{\partial S}{\partial t} + \mathbf{u} \cdot \nabla S = \frac{1}{S_c} \nabla^2 S \]
\[ \frac{\partial C}{\partial t} - \frac{V_p}{\tau S_c} \frac{\partial C}{\partial z} + \mathbf{u} \cdot \nabla C = \frac{1}{\tau S_c} \nabla^2 C \]

Characteristic quantities:

\[ L^c = \left( \frac{\nu^2}{g'} \right)^{1/3} , \quad T^c = \left( \frac{L^c \nu^2}{\nu} \right) , \]
\[ U^c = \left( \nu g' \right)^{1/3} , \quad g' = \frac{\Delta \rho_c}{\rho_0} g , \]
\[ V_{st} = \frac{g \frac{d_p^2}{\rho_p} (\rho_p - \rho_f)}{18 \mu_f} \]

Dimensionless parameters:

settling velocity \[ V_p = \frac{V_{st}}{(\nu g')^{1/3}} \]

stability ratio \[ R_s = \frac{\alpha}{\gamma} \]

Schmidt number \[ Sc = \frac{\nu}{\kappa_s} \]

diffusivity ratio \[ \tau = \frac{\kappa_s}{\kappa_c} \]
Sedimentation from river plumes: Numerical simulations

- Two dimensions:
  - streamfunction, vorticity-formulation of Navier-Stokes equations
  - Boussinesq approximation
  - spectral/compact finite differences

- Three dimensions:
  - IMPACT code (Henniger and Kleiser 2011)
  - primitive variable formulation of Navier-Stokes equations
  - Boussinesq approximation
  - staggered grid
  - 6th order compact finite differences
  - massively parallel
Sedimentation from river plumes: Numerical simulations

\[
V_p = 0.04, \quad S_c = 0.7, \quad R_s = 2, \quad \tau = 25
\]
Sedimentation from river plumes: Numerical simulations
Mammatus clouds
Volcanic ash plume
Sedimentation from river plumes: Mean fields

- thickening of the plume-dominated region \( \sim \) time \( \rightarrow \) convectively dominated
- vigorous convective motion
- ‘streaks’ due to the release of buoyant plumes
Sedimentation from river plumes: Mean fields

fit concentration profiles with erf → determine interface location, thickness

sediment concentration

salinity

time = 300, 400, 500
Sedimentation from river plumes: Mean fields

- both interface thicknesses grow diffusively
- sediment interface thickness grows faster, in spite of smaller molecular diffusivity!
- sediment interface moves downward, but more slowly than Stokes settling velocity
- salinity interface moves upward
Sedimentation from river plumes: Mean fields

Why does the salinity interface move upward?

• the instability is centered around the unstable sediment interface, which moves downward into the region of high salinity
• the region of high salinity gets mixed more strongly → the $s=0.5$ contour is displaced upwards
Sedimentation from river plumes: Mean fields

Turbulent diffusivities:

- turbulent sediment diffusivity is about twice as high as turbulent salinity diffusivity, even though the molecular salinity diffusivity is 25 times larger than ‘molecular’ sediment diffusivity → consistent with numerical observations
Sedimentation from river plumes: Mean fields

Quasisteady measures of sedimentation dynamics

- ratio of turbulent diffusivities, ratio of interface thicknesses and ratio of turbulent fluxes all approach quasisteady values → will be important for scaling analysis

\[ \tau_{turb} = \frac{K_c}{K_s} \]

\[ \frac{l_c}{l_s} \]

\[ \frac{F_s}{F_c} \]
Sedimentation from river plumes: Mean fields

Ratio of nose height to salinity interface thickness:

- ratio of nose height to salinity interface thickness approaches quasisteady state, and remains ≪1
  - sediment interface remains embedded in the region of strong salinity gradient
  - double diffusion remains important
in the region  \( z < 0 \), the effective settling velocity is \( O(1) \), rather than \( V_{st} = 0.04 \), i.e., it scales with the buoyancy velocity of the system, not the Stokes velocity.
Sedimentation from river plumes: Leaking mode (higher Sc)
**Sedimentation from river plumes: Leaking mode**

Horizontal cross-cuts through sediment concentration field:

- Nonlinear evolution of initial, localized plumes results in web-like structure.
- Characterized by sheets, rather than plumes.

→ time increases
Sedimentation from river plumes: fingering vs. leaking

\( x,t \)-diagrams of sediment concentration at fixed vertical location:

- **fingering mode**
  - weak horizontal motion

- **leaking mode**
  - strong horizontal motion and merging

- explains different modes observed by Parsons et al. (2001)
Scaling of nose height with in-/outflow ratio:

\[ \frac{H}{l_s} \propto V_p S_c^{0.5} R_s \]

\[ R_s = 2.0 \hspace{1cm} 7.0 \hspace{1cm} 70.0 \]

\[ S_c = 0.7 \hspace{1cm} 0.04 = V_p \]

→ quasisteady ratio of nose height to salinity interface thickness scales with ratio of sediment inflow into nose region to sediment outflow from nose region
**Sedimentation from river plumes: Parametric study**

Physical interpretation:

- **for small settling velocity, the rate of sediment inflow from above is low →** this low rate of sediment inflow can be balanced by conventional double-diffusive outflow of sediment below → there is little accumulation of sediment in the nose region → height of nose region remains small

- **for large settling velocity, the rate of sediment inflow from above is high →** this high rate of sediment inflow cannot be balanced by traditional double-diffusive sediment outflow below → sediment accumulates in the nose region → height of nose region increases until it is thick enough for Rayleigh-Taylor instability to form, which leads to increased sediment outflow below → new balance between in- and outflow into the nose region is established
Double-diffusive sedimentation: Open questions

Currently under investigation:

• linear concentration gradients vs. initial step profiles

• influence of shear:
  - Kelvin-Helmholtz vs. double-diffusive instabilities
  - does Holmboe instability form?

• based on recent findings for thermohaline double-diffusive instabilities:
  - diffusive vs. convective mode
  - do collective instability modes form?
  - do horizontal intrusions form?
  - do “gamma-instability” and “staircases” form?
Summary

- double-diffusive sedimentation in river outflows dramatically enhances the effective settling velocity
- settling velocity scales with buoyancy velocity, not with Stokes velocity
- two mechanisms drive the process:
  - double-diffusive instability of salt vs. sediment
  - settling of sediment creates ‘nose region,’ Rayleigh-Taylor instability
- ratio of nose height/salinity interface thickness $H/l_s$ determines regime
- for low Schmidt numbers, low stability ratios and small Stokes settling velocities, traditional double-diffusive instability causes convective ‘fingering’ mode
- for high Schmidt numbers, large stability ratios and large Stokes settling velocities, settling of sediment causes ‘leaking’ mode, via interaction of Rayleigh-Taylor and double-diffusive instability modes through ‘phase-locking’
- overall dynamics is governed by the in-/outflow of sediment into/from the nose region