Anomalous critical and supercritical connectivity transitions

Jan Nagler
ETH Zurich & ETH Risk Center

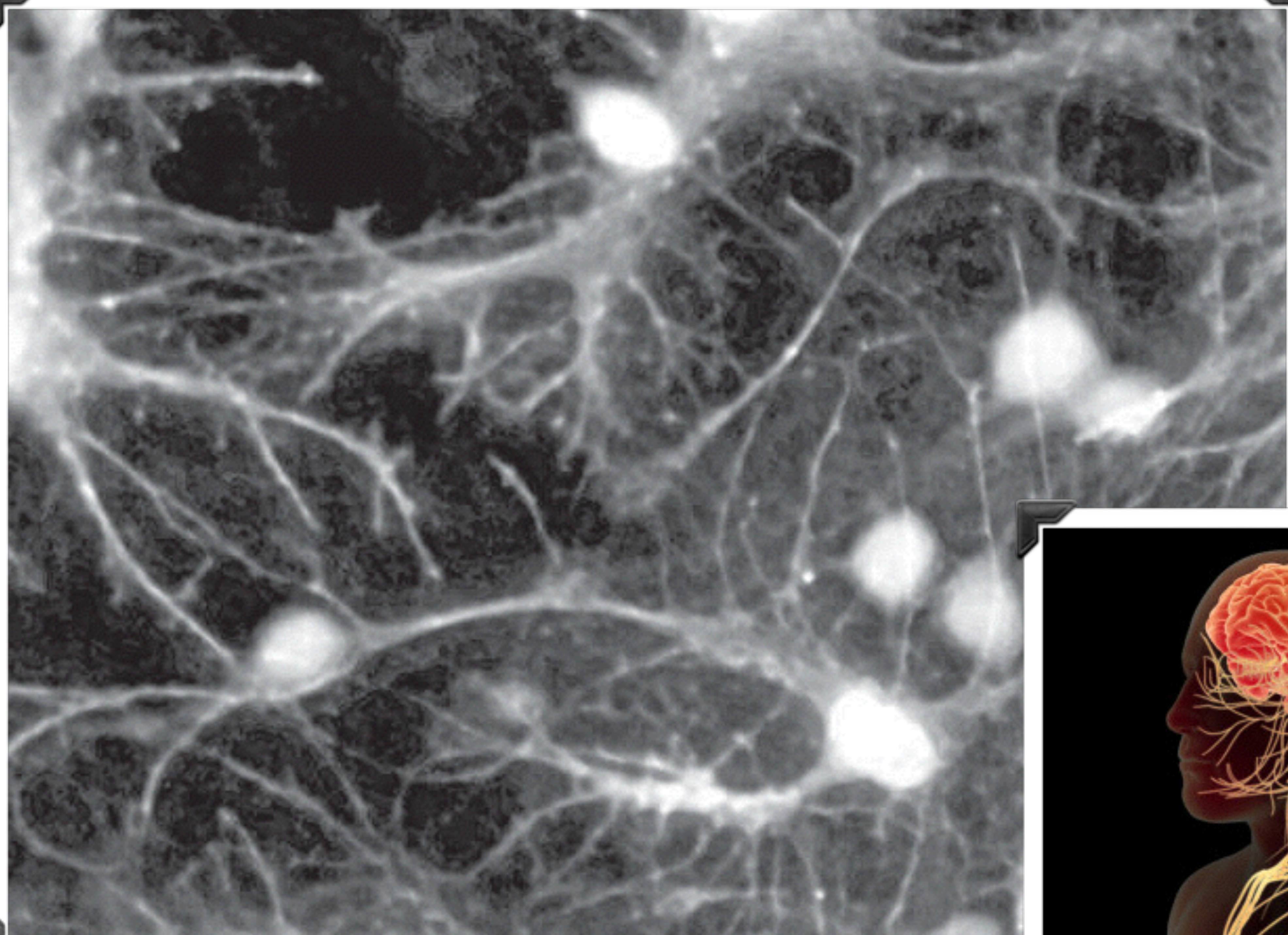
MPI PKS, very advanced group meeting, 2016

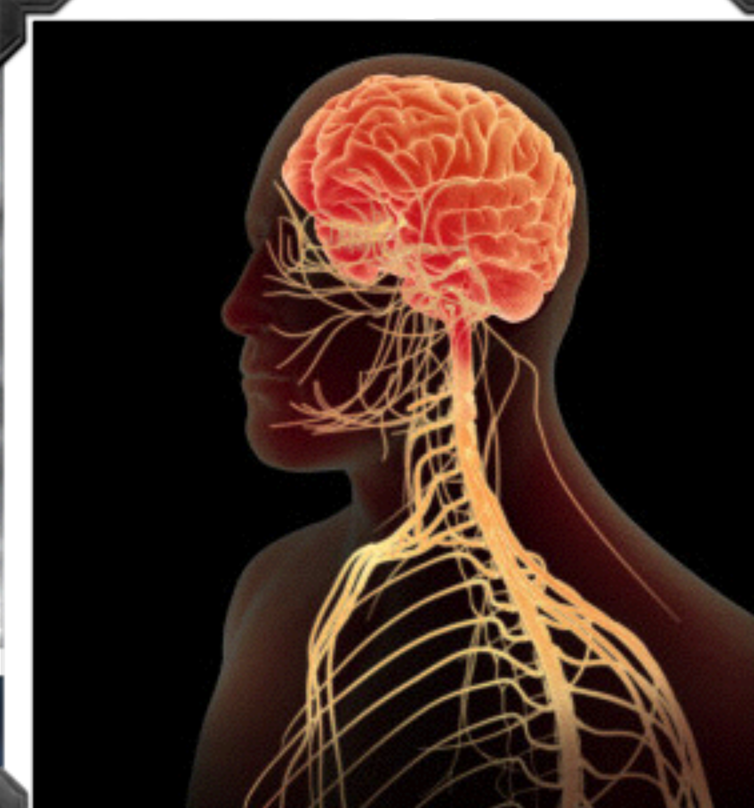
Connectivity matters

Desired connectivity
(internet, www, social/transportation networks, ...)

Verma, Russmann, Araujo, Nagler, Herrmann, Emergence of core-peripheries in networks, Nature Commun. 7:10441 (2016)

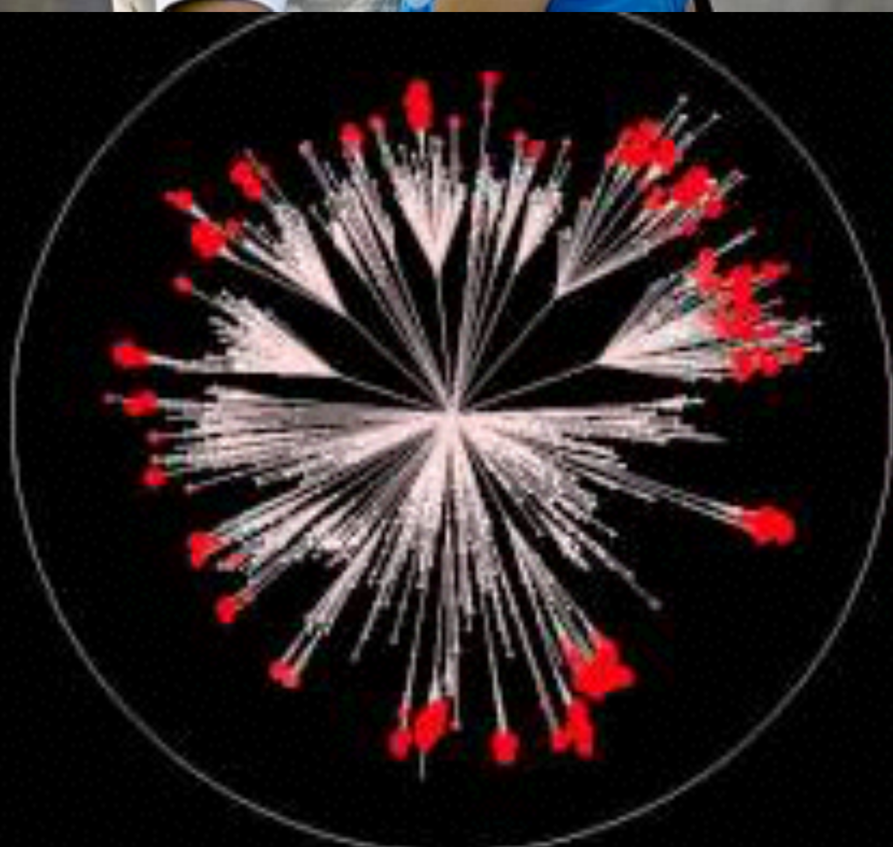
Necessary connectivity: our nervous system





Breskin et al., Percolation in living neural networks, Phys. Rev. Lett. 2006
Soriano et al., Development of input connections in neural cultures, PNAS 2008

Connectivity as a liability: spread of infectious disease or virus



Hufnagel, Brockmann, Geisel, PNAS 2004

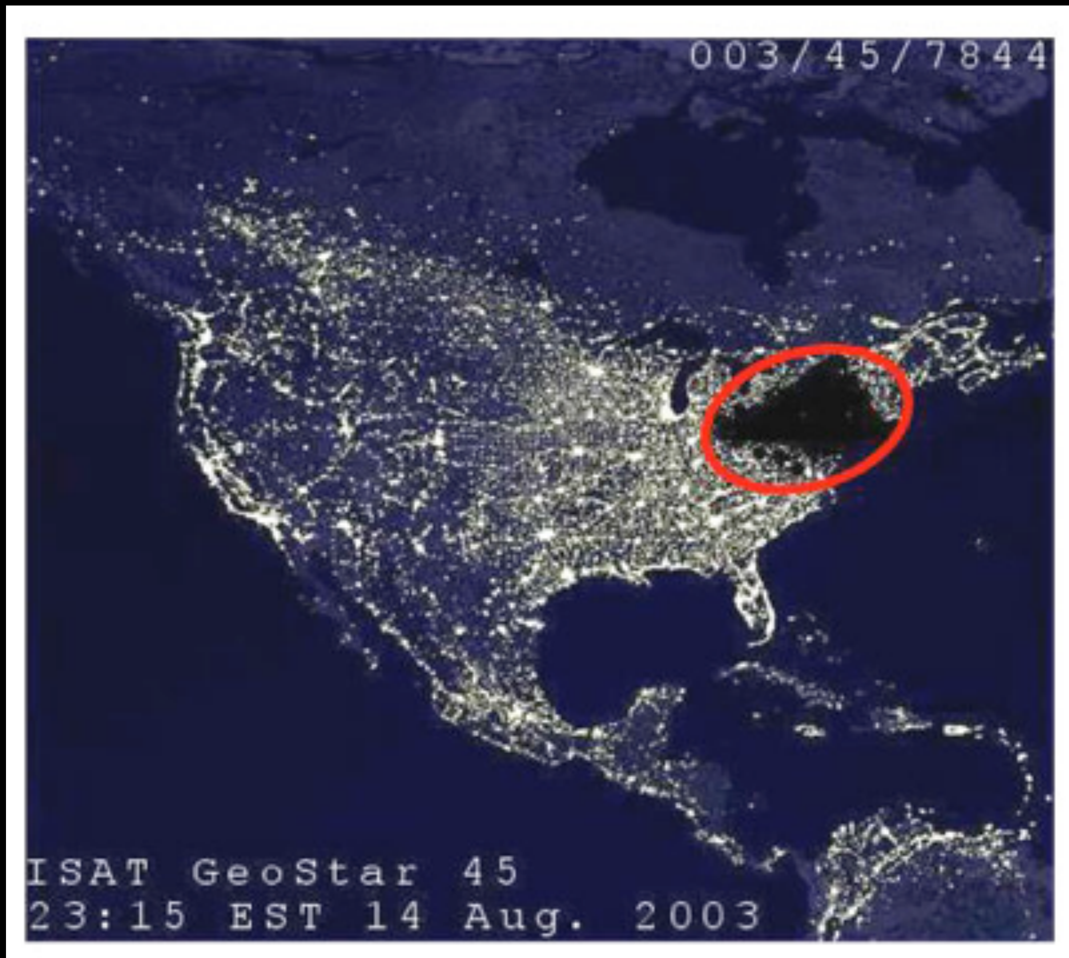
Brockmann, Hufnagel, Geisel, Nature 2006

Brockmann & Helbing, Science 2013

Challenging connectivity and cascades: finance

May, Levin & Sugihara, Complex systems: ecology for bankers, Nature 451, 893–895 2009
Haldane & May, Systemic risk in banking ecosystems, Nature 469, 351–355 2011

Broken connectivity and cascades: power outages



Italy & CH, 09/2003, 56 Mio people affected

Catastrophic cascade of failures in interdependent networks,
Buldyrev, Parshani, Paul, Stanley, Havlin,
Nature (2010)

Connectivity and cascades: Evolution
Stagnation and variability of Earth's biodiversity through species dependencies

Physics
spotlighting exceptional research

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Synopsis: Modeling Biodiversity

iStockphoto.com/johnandersonphoto

Possible Origin of Stagnation and Variability of Earth's Biodiversity

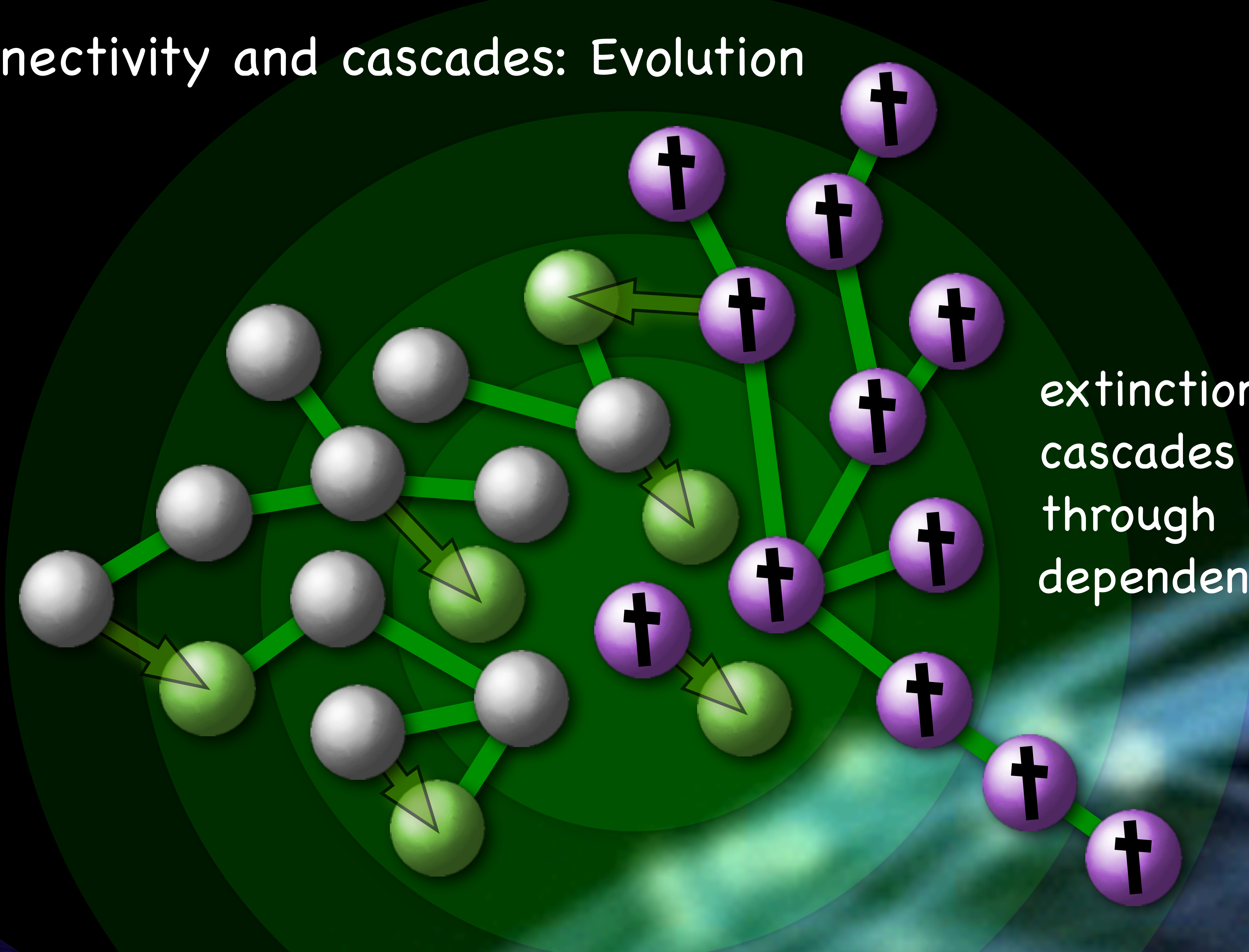
Frank Stollmeier, Theo Geisel, and Jan Nagler

Phys. Rev. Lett. **112**, 228101 (2014)

Published June 5, 2014

According to the fossil record, about 500 million years ago the number of marine species began growing exponentially and leveled off for 200 million years before exploding again. To understand how and why these changes occurred, researchers formulated a variety of models built upon different assumptions. Some models support the idea that biodiversity increased stagnation, others that the increase is an artifact of how the fossils have been collected and sampled. In a paper in *Physica*

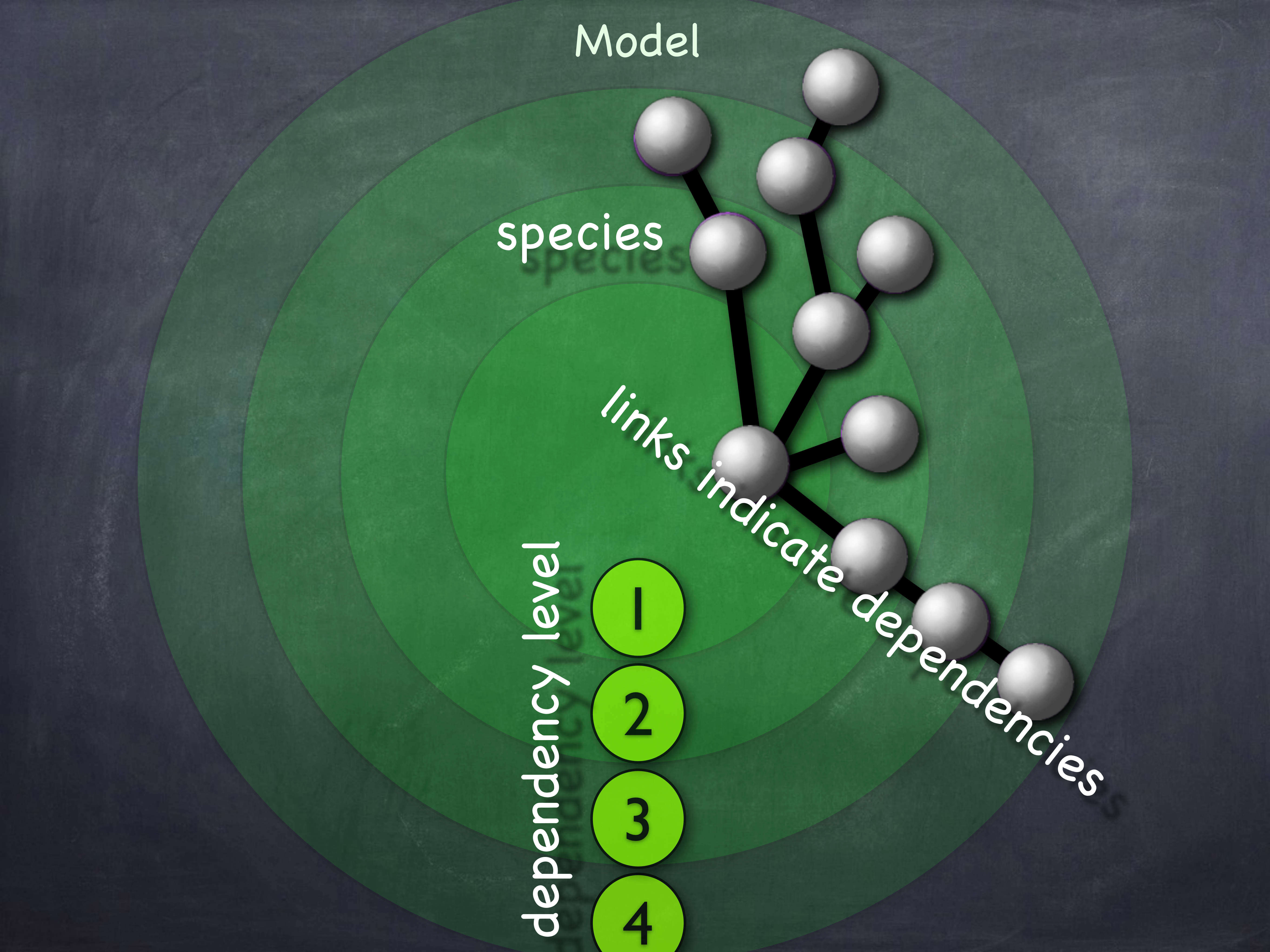
Connectivity and cascades: Evolution



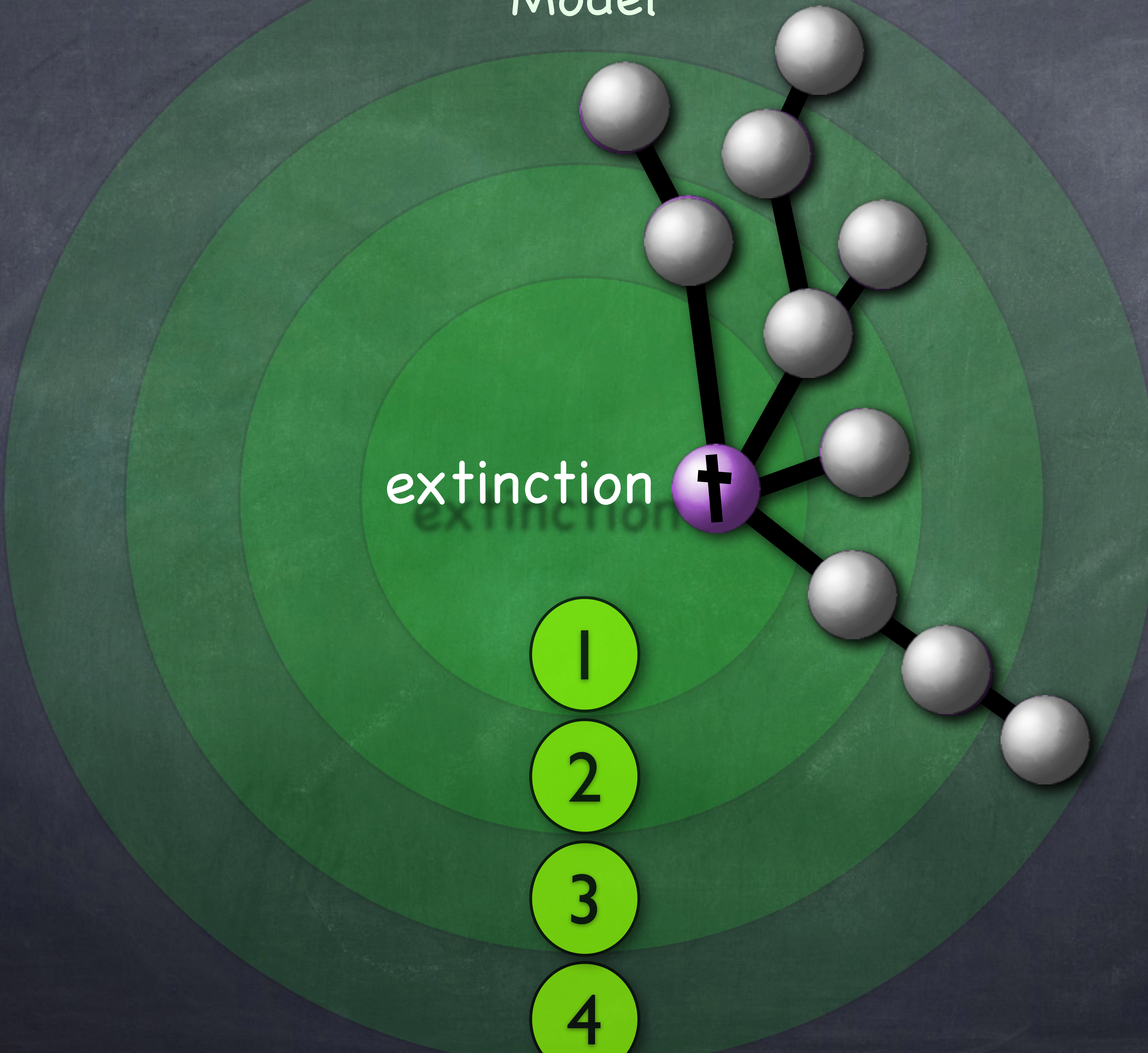
extinctions
cascades
through
dependencies

Stollmeier, Geisel and Nagler,
Possible Origin of Stagnation and Variability of Earth's Biodiversity,
Phys. Rev. Lett. 112: 2281011 (2014)

Model
1
2
3
4
∞
dependency level

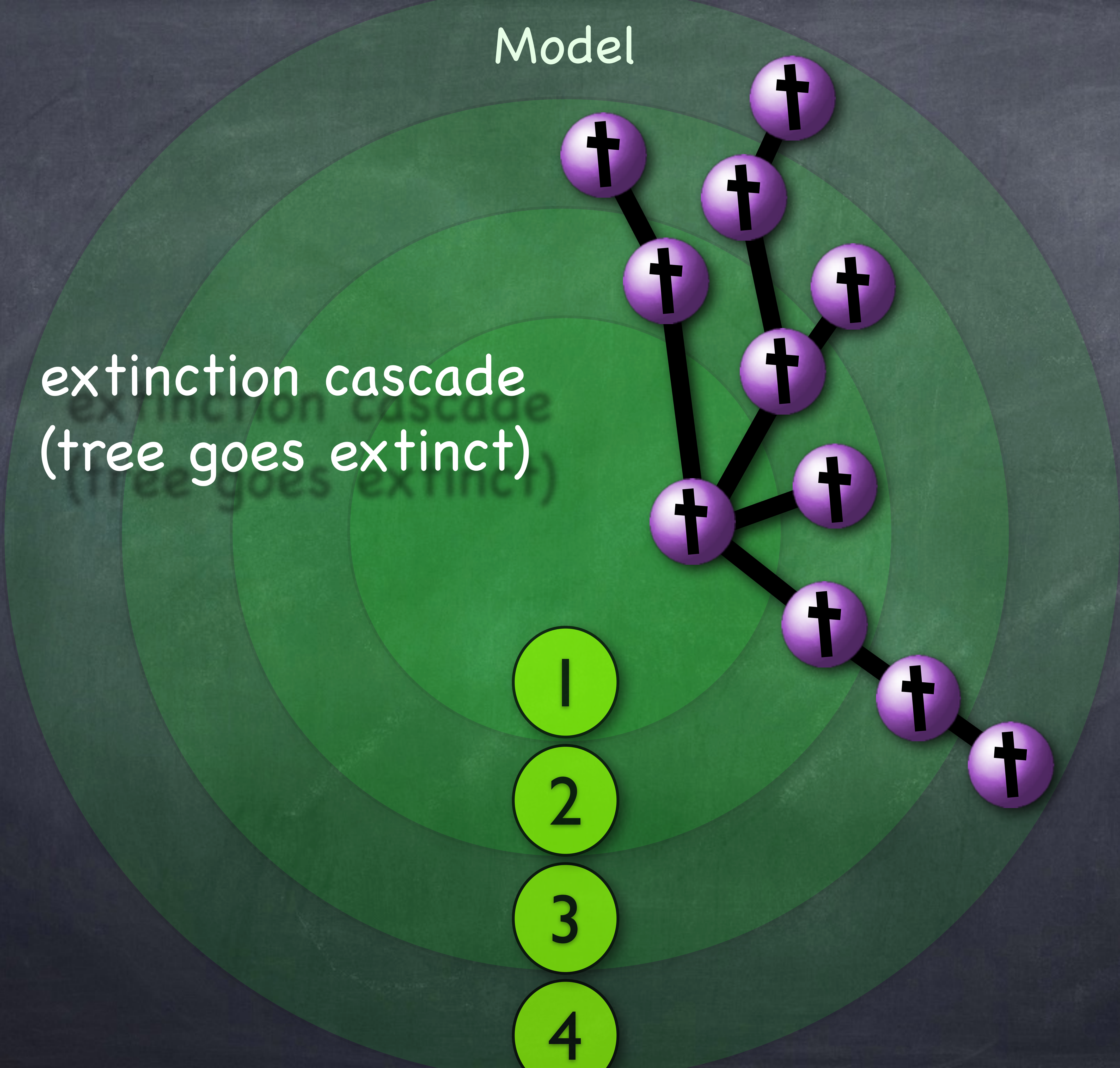

Model
species
links indicate dependencies
dependency level
1
2
3
4

Model

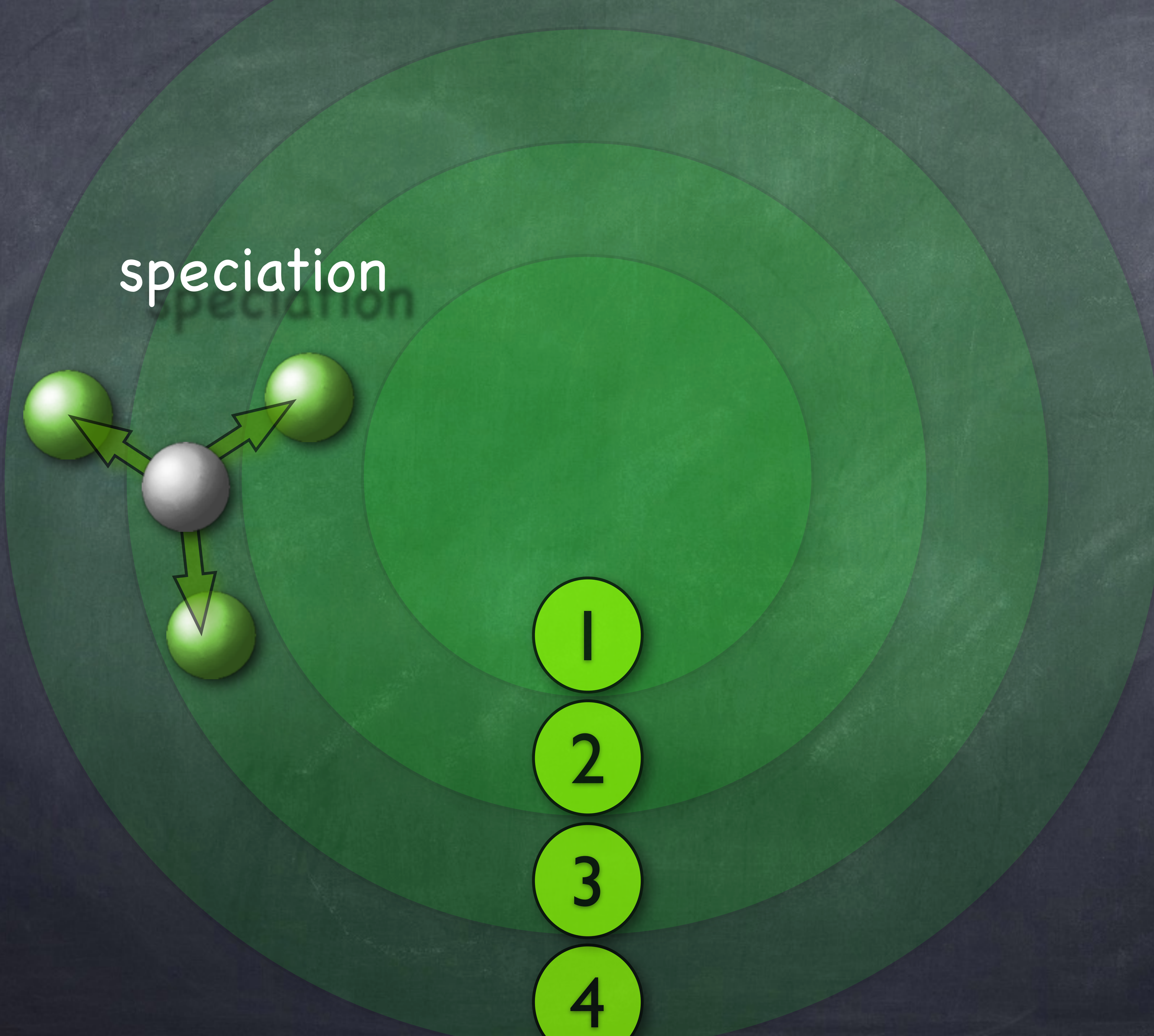


Model

extinction cascade
(tree goes extinct)



Model



Crucial ratio: lambda

extinction probability

relative extinction probability

$$\lambda = \epsilon / \mu$$

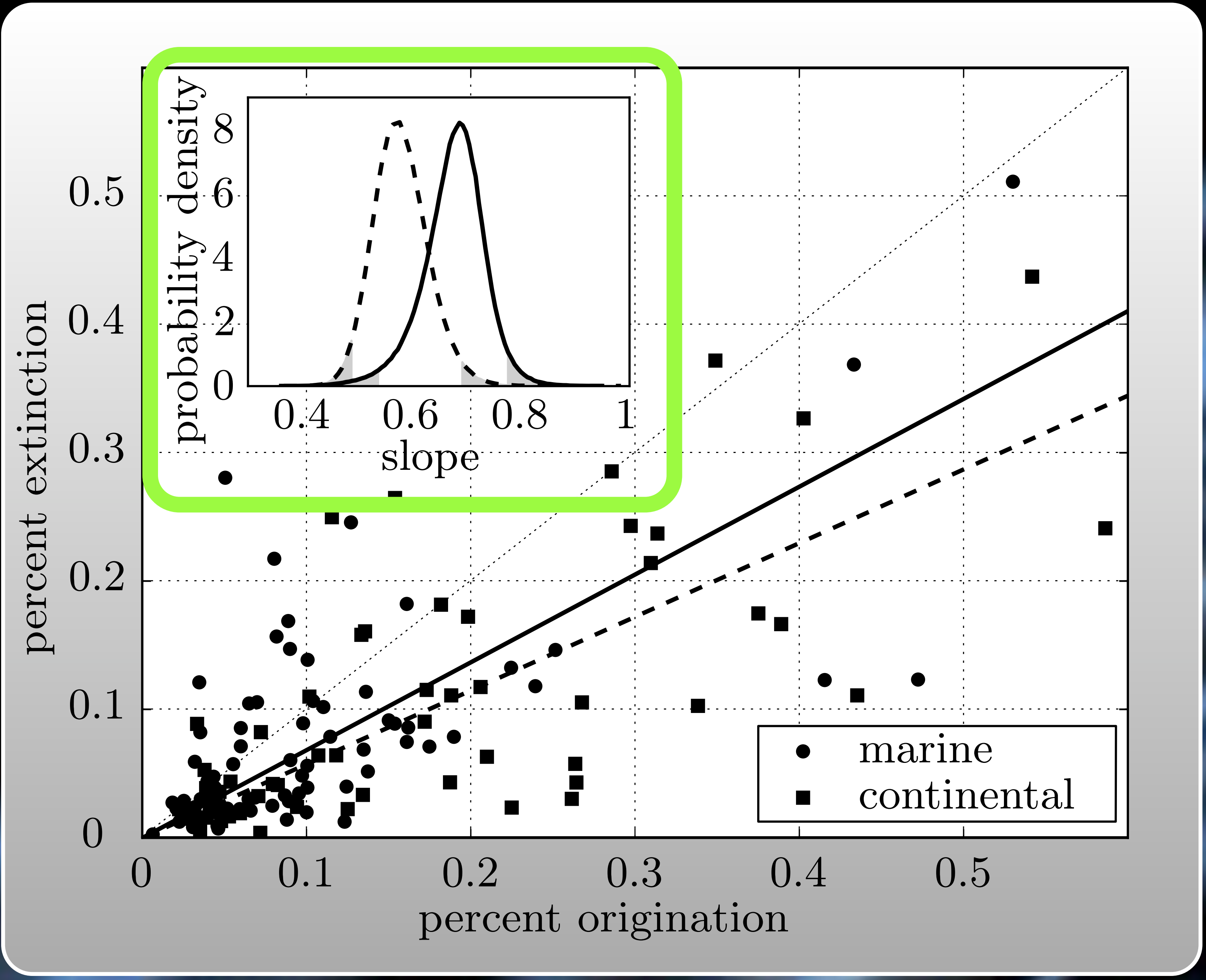
speciation probability

Stollmeier, Geisel and Nagler,
Possible Origin of Stagnation and Variability of Earth's Biodiversity,
Phys. Rev. Lett. 112: 2281011 (2014)

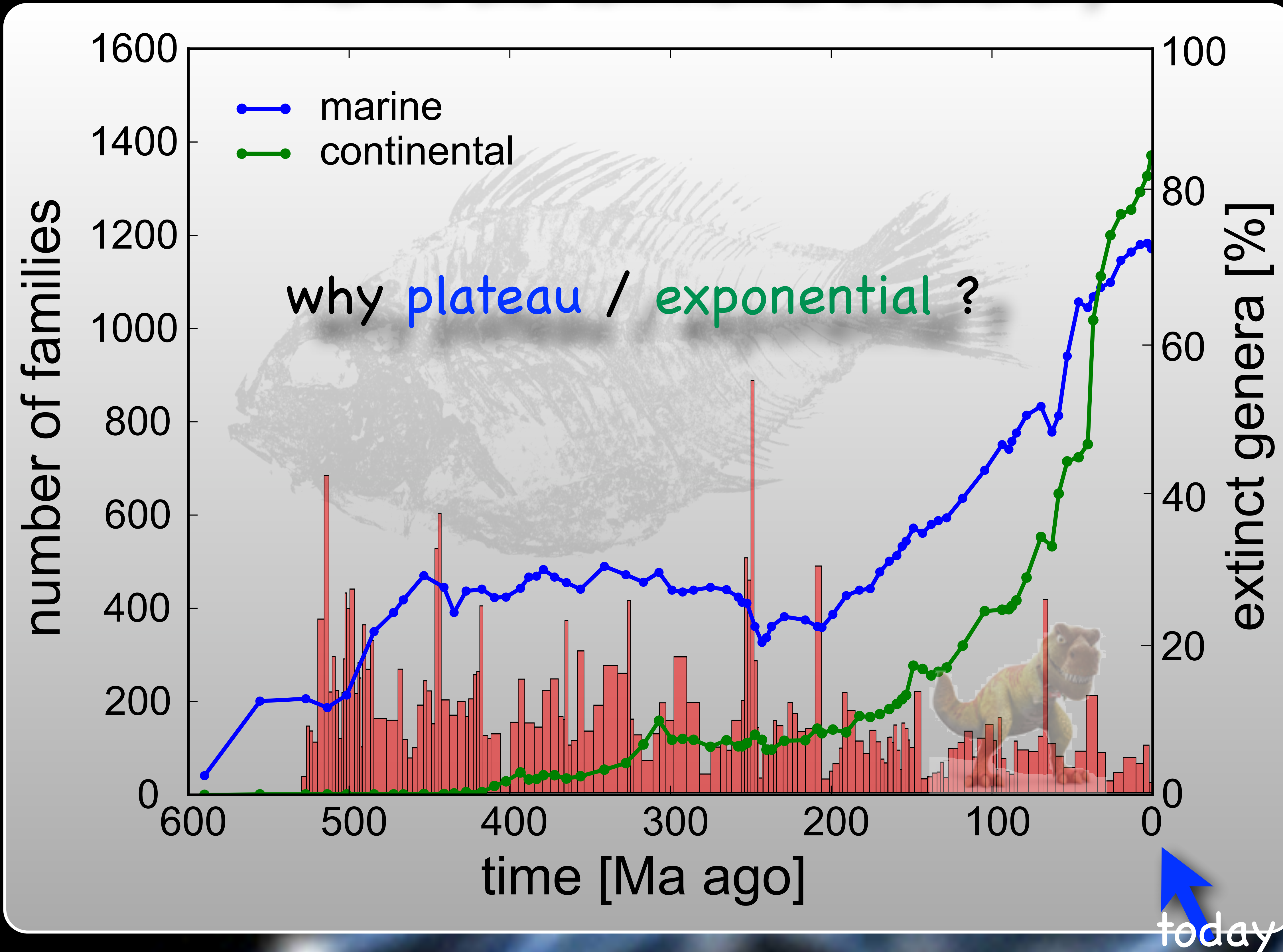
Evidence for different lambdas in data, marine & continental biodiversity

$$\lambda_{\text{cont}} = 0.57$$

$$\lambda_{\text{mar}} = 0.68$$



Marine and continental biodiversity



Sepkoski, Bull. Am. Paleo. 363 (2002); Benton, The Fossil Record 2 (Chapman & Hall London, 1993)

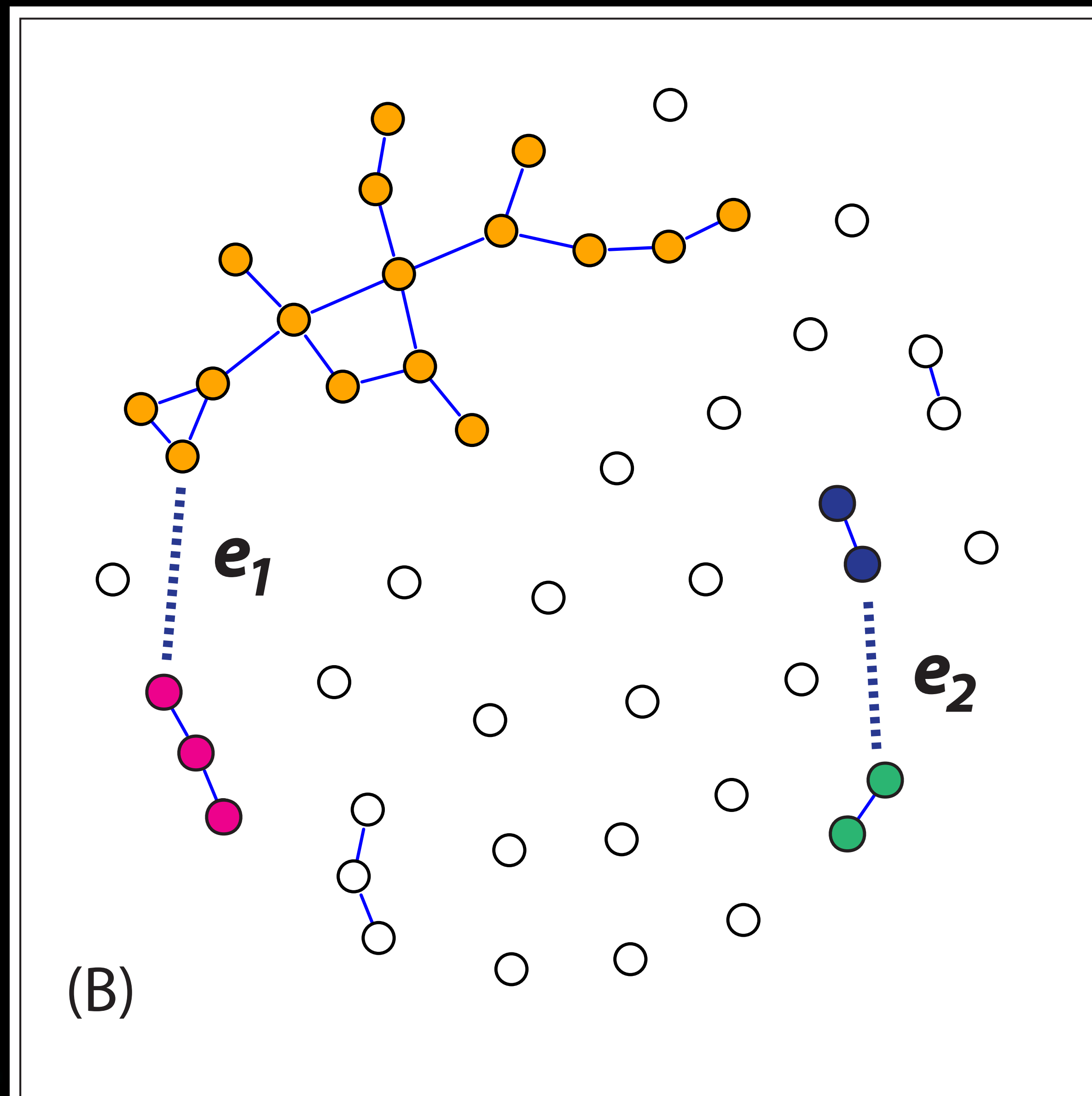
PART II

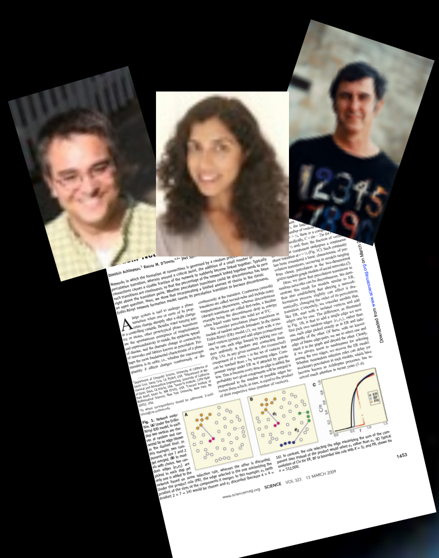
Anomalous percolation

Nagler, Levina & Timme, Nat. Phys. 2011
Nagler, Tiessen & Gutch, Phys. Rev. X 2012
Schröder, Rahbari, Nagler, Nat. Commun. 2013
Chen, Schröder, D'Souza, Sornette & Nagler, Phys. Rev. Lett. 2014
D'Souza & Nagler, Nature Physics 2015

Concept of explosive percolation

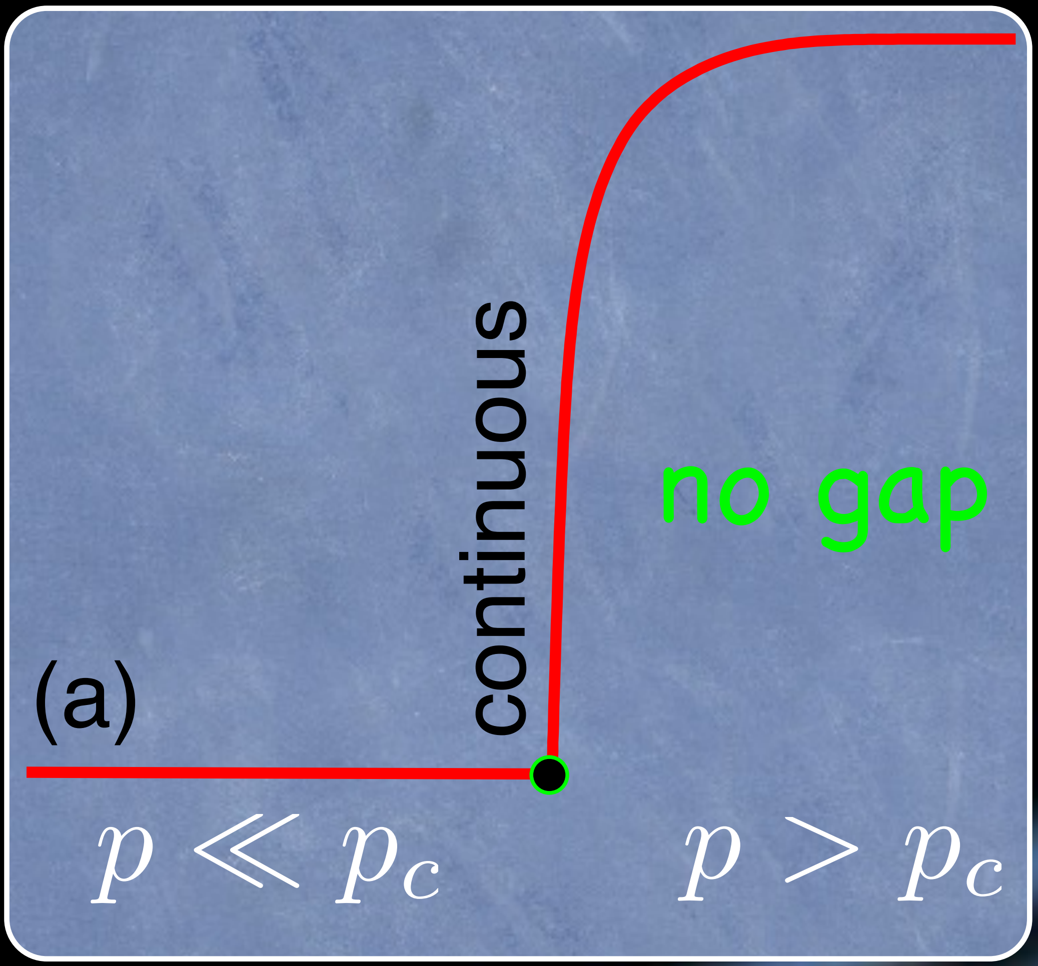
Delay connectivity by link-addition competition
(avoid emergence of large clusters)



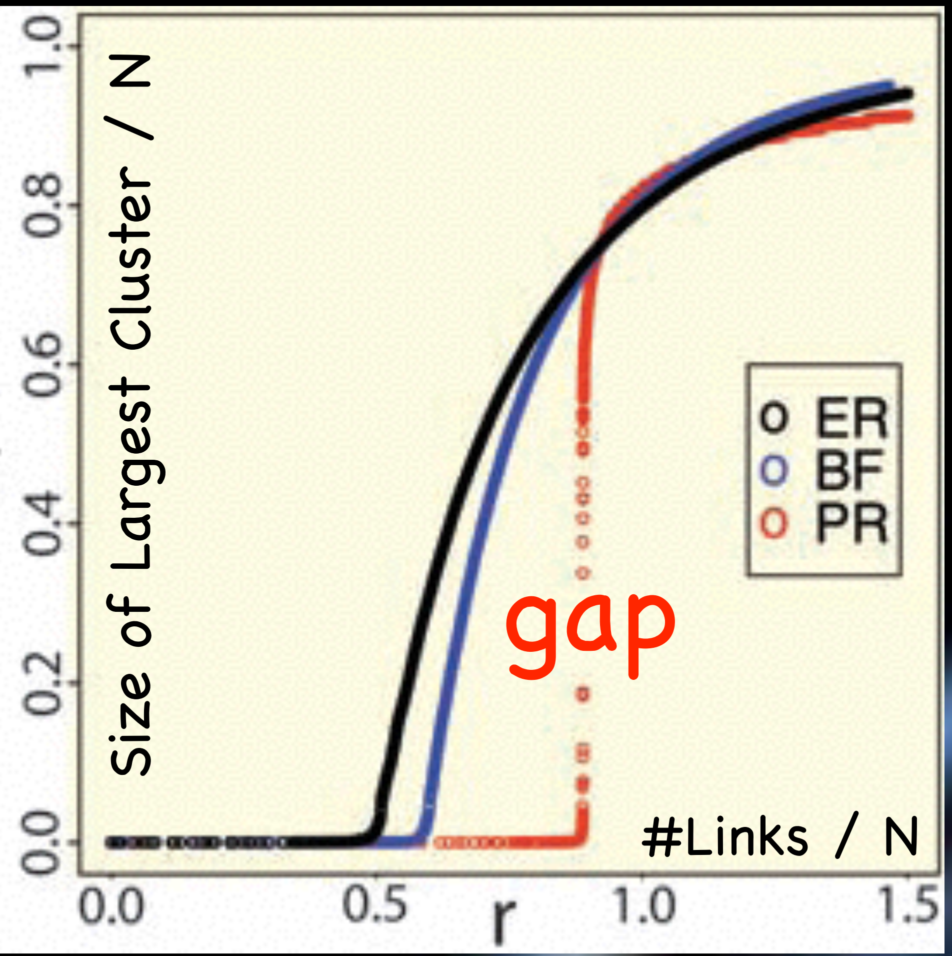


D Achlioptas, RM D'Souza, J Spencer, Science (2009)
„Explosive Percolation in Random Networks"

Explosive percolation in random networks



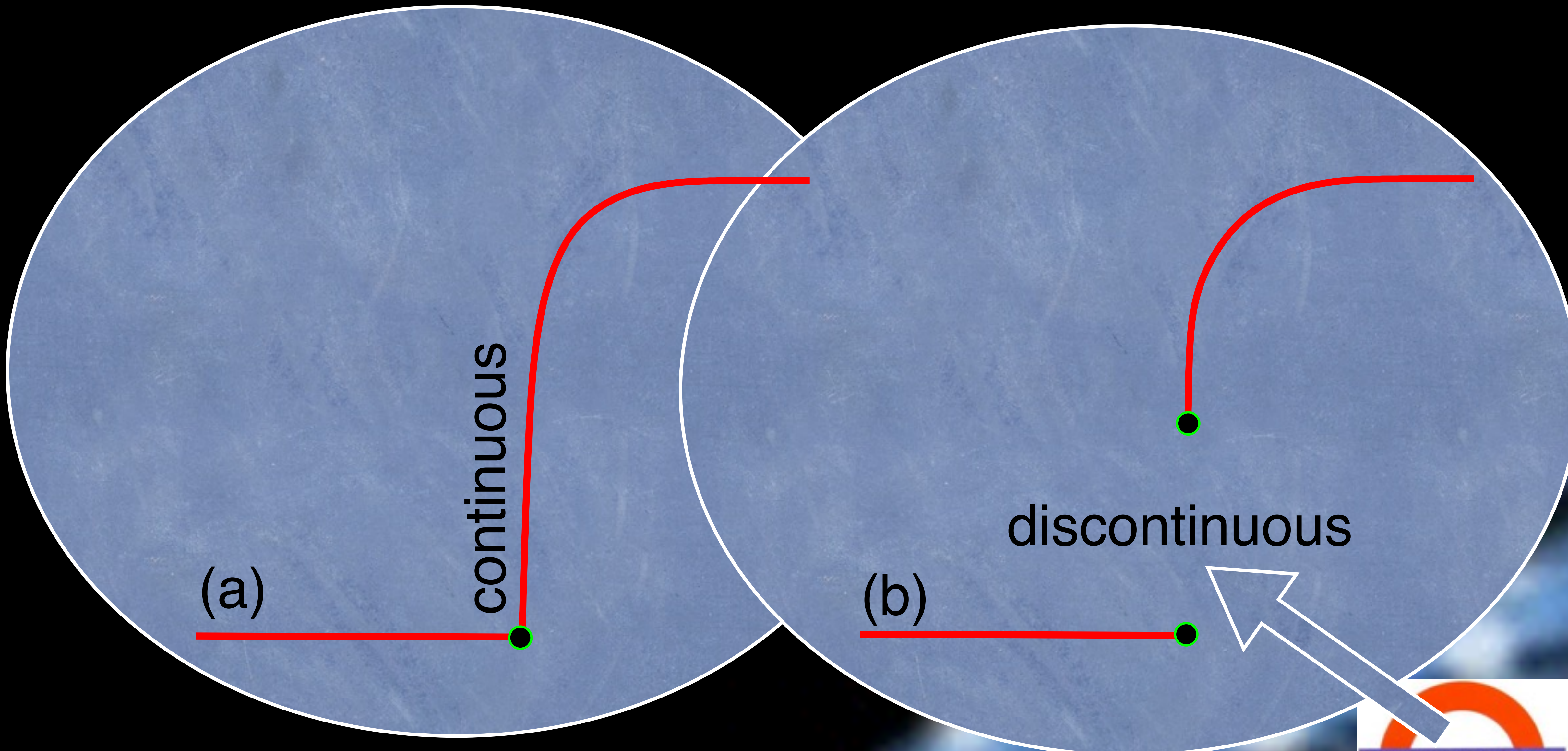
no competition



link competition

Achlioptas, D'Souza, Spencer, Science 2009

Is there a real gap for competitive rules ?



When there a real gap for competitive rules ?

> 200 papers since 2009

„Explosive percolation" is continuous

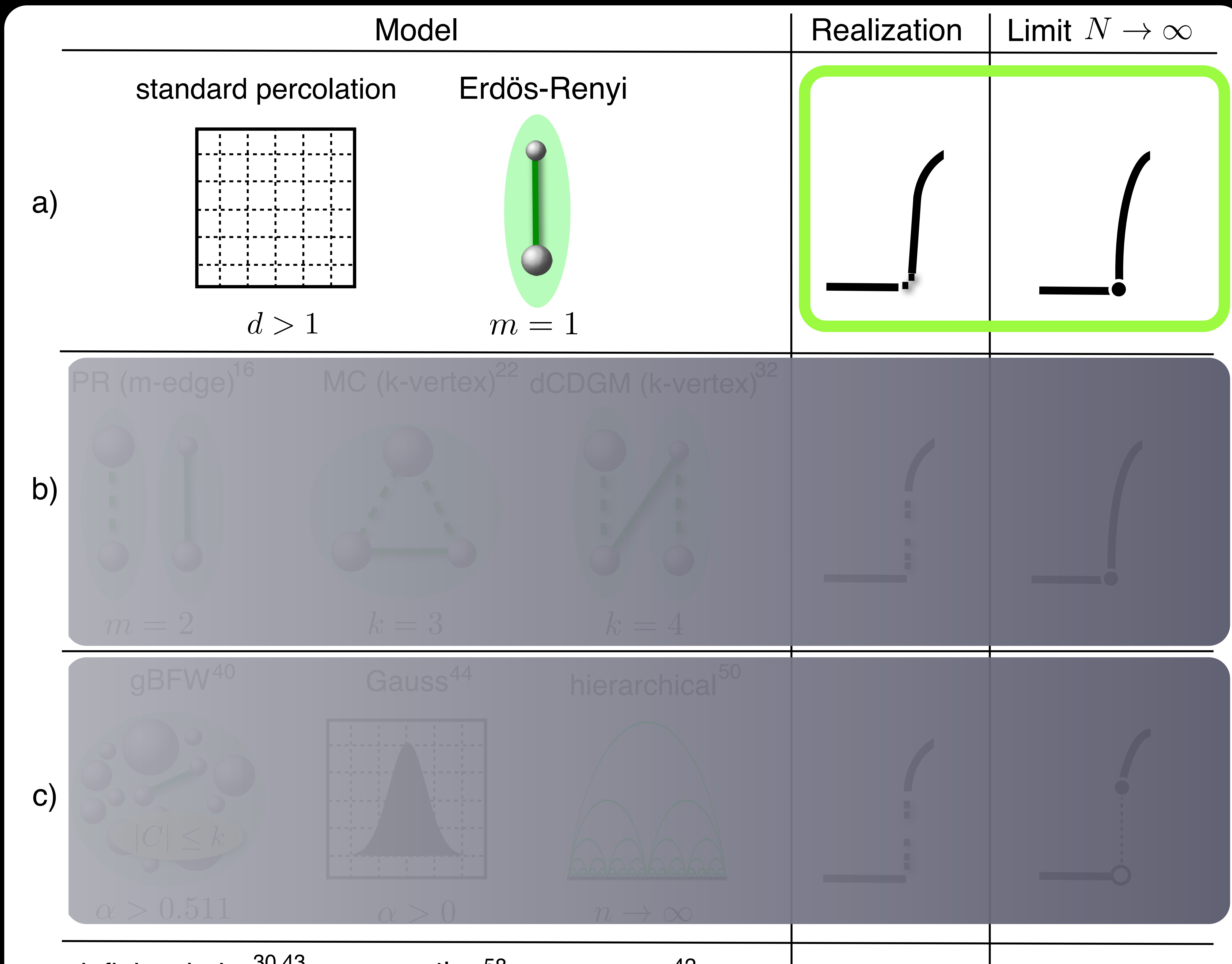
breakthrough

main conclusion of rigorous proof:

„any percolation process based on picking a fixed number of random vertices gives a continuous transition"

Riordan & Warnke, Science 333, 322 (2011)

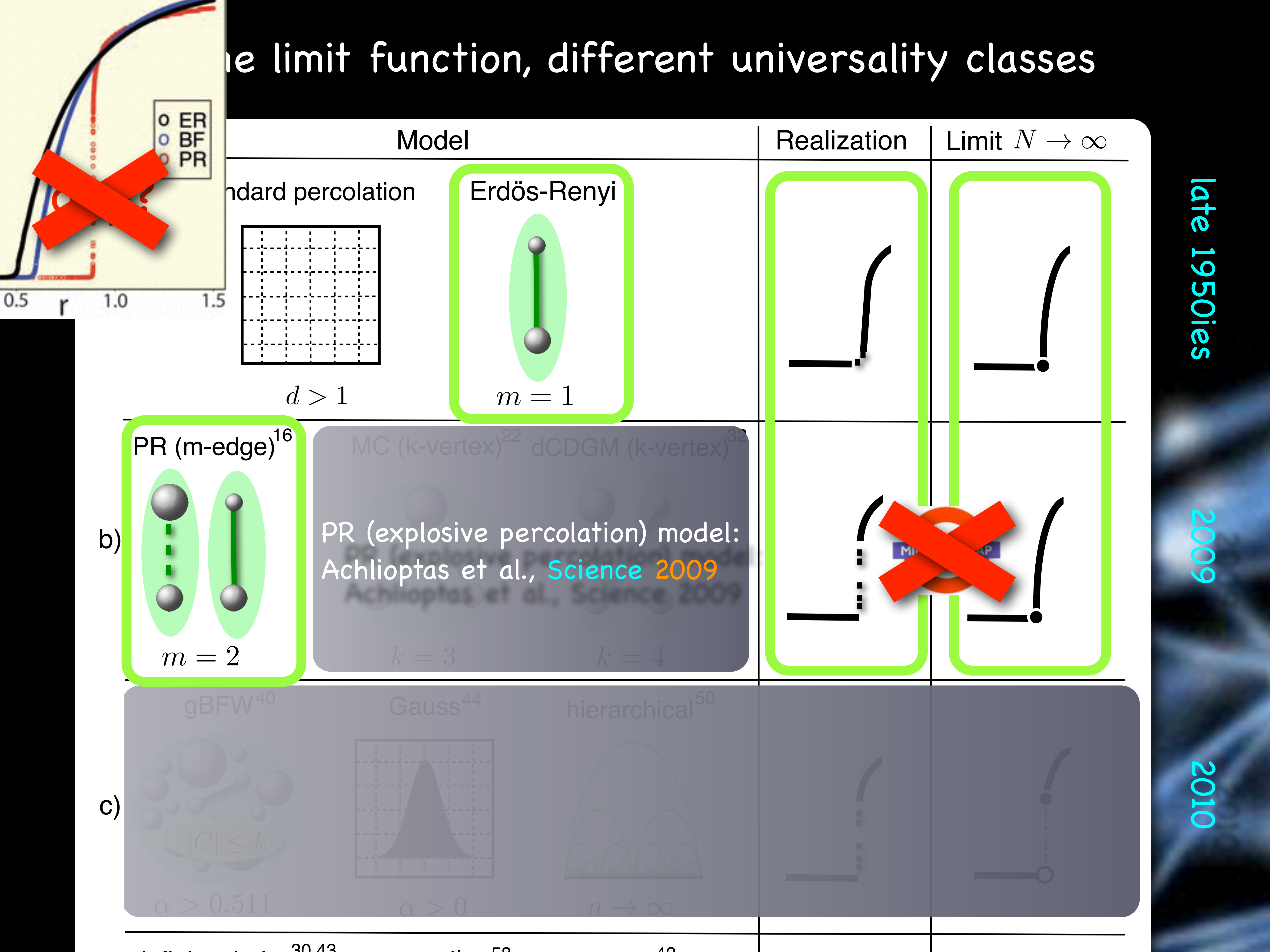
Percolation from late 1950ties until 2016 (=today)



late 1950ies

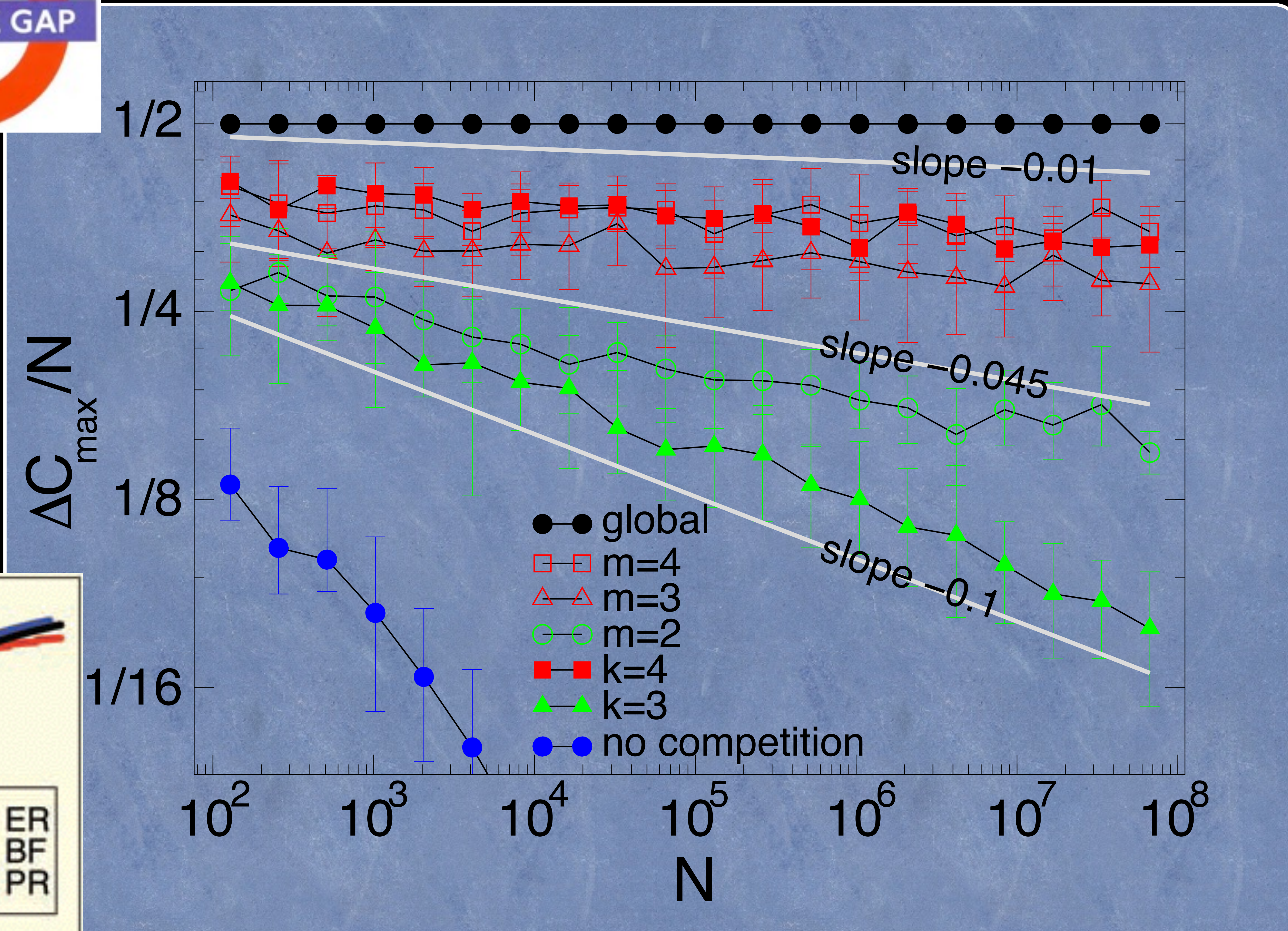
2009

2010



...e limit function, different universality classes
Model
Realization
Limit $N \to \infty$
...ndard percolation
$d > 1$
Erdös-Renyi
$m = 1$
b)
PR (m-edge)[16]
$m = 2$
MC (k-vertex)[22]
dCDGM (k-vertex)[32]
PR (explosive percolation) model:
Achlioptas et al., Science 2009
$k = 3$
$k = 4$
c)
gBFW[40]
Gauss[44]
hierarchical[50]
$|C| \leq k$
$\alpha > 0.511$
$\alpha > 0$
$n \to \infty$
ER
BF
PR
0.5
r
1.0
1.5
late 1950ies
2009
2010

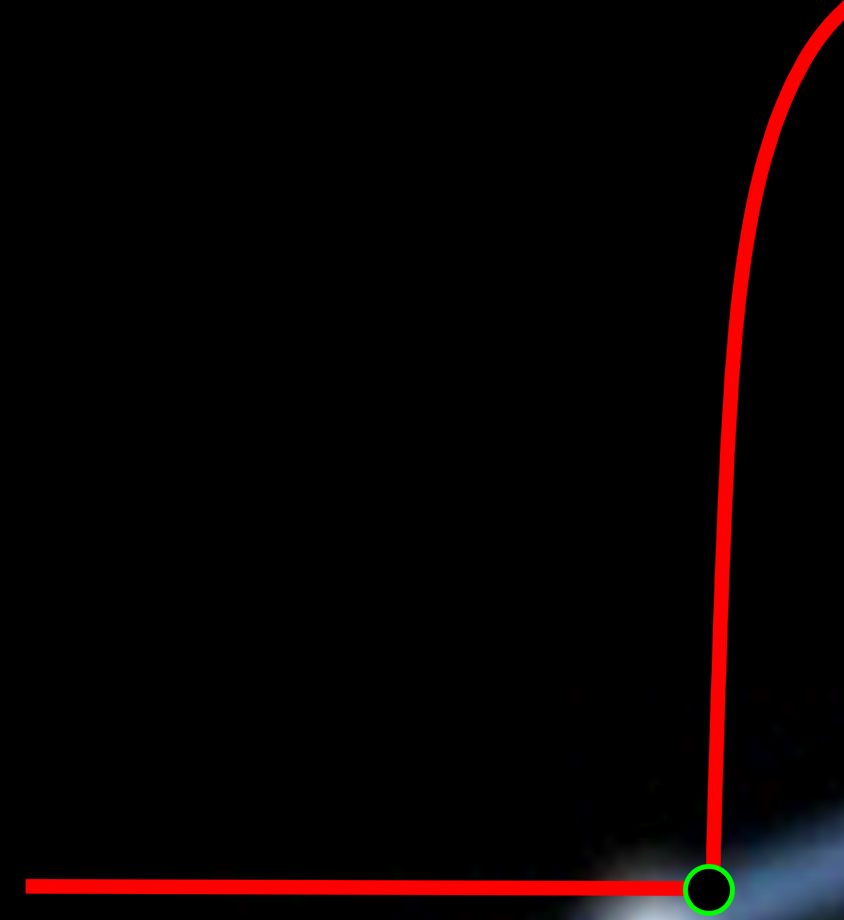
Scaling of largest gap

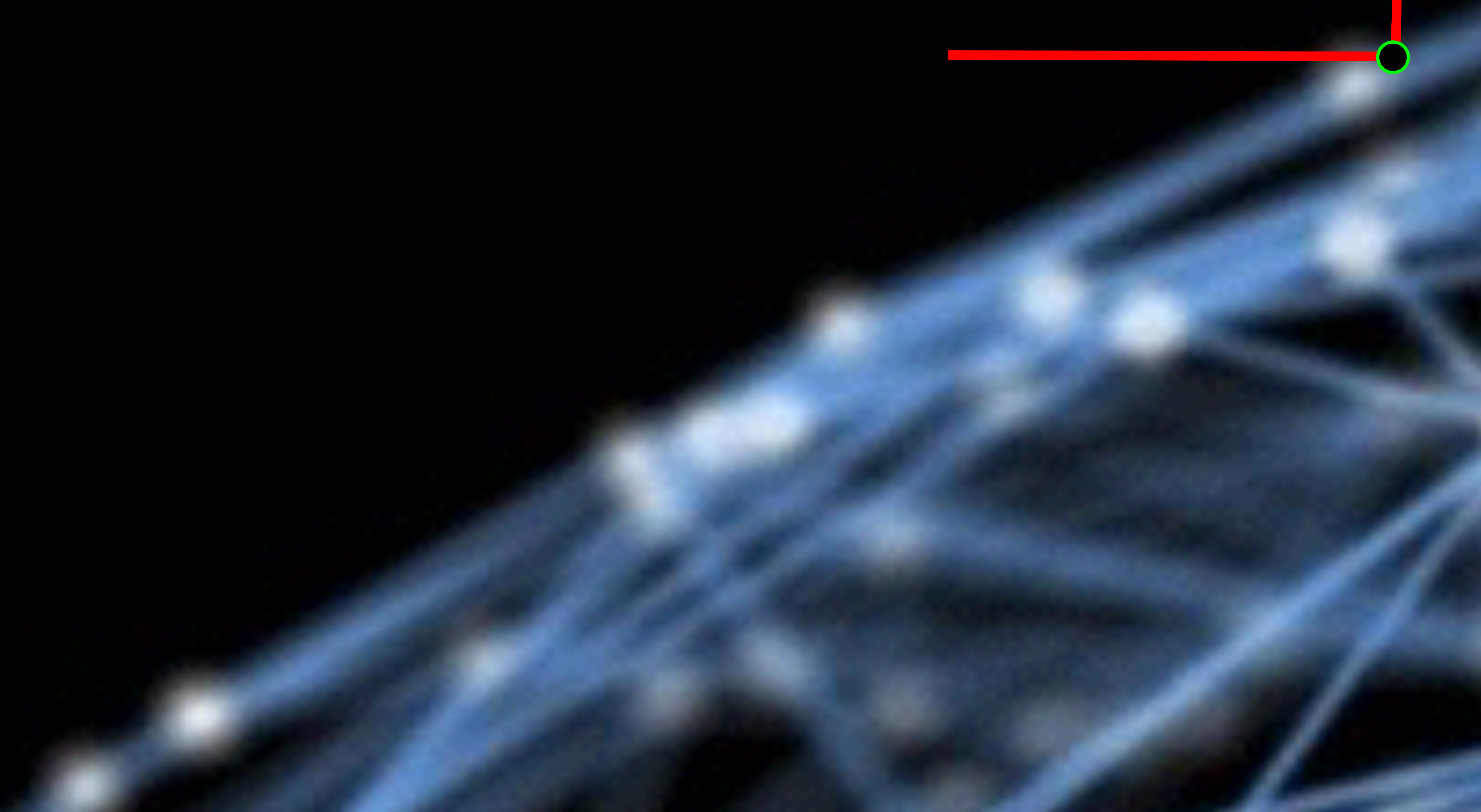


$$\Delta C_{\max}/N \sim N^{-\beta}$$

Nagler, Levina & Timme, Nat. Phys. 2011

Does it always look like this?
(in thermodynamic limit)





What is fractional percolation?

Preferentially merge components of similar size

macroscopic component

microscopic components

(cartoon taken from www)

Schröder, Rahbari, Nagler, Nat. Commun. 2013

Fragmentation

What is fractional percolation?

Random network version, ‘counter example’:

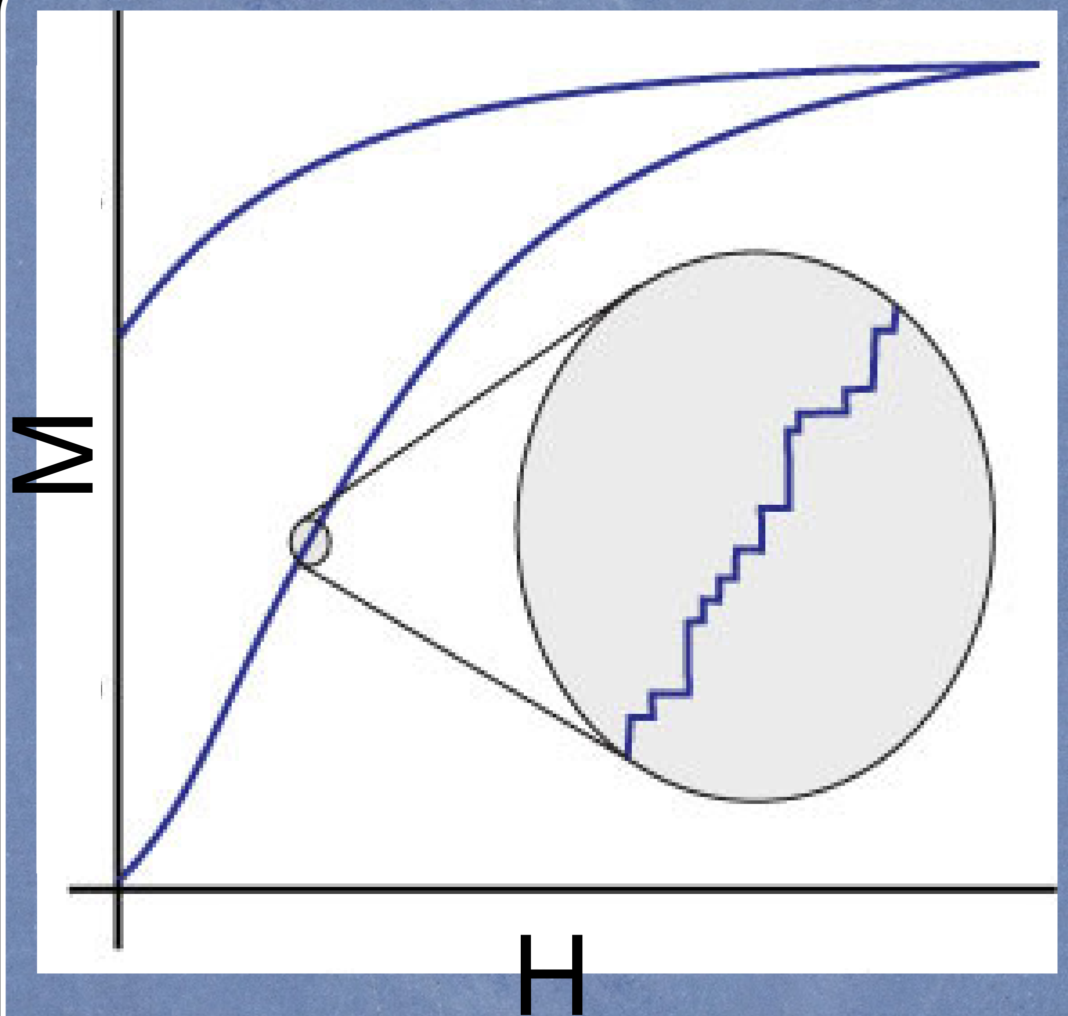
„Pick 3 vertices at random, join the two vertices inside the most commensurate clusters“

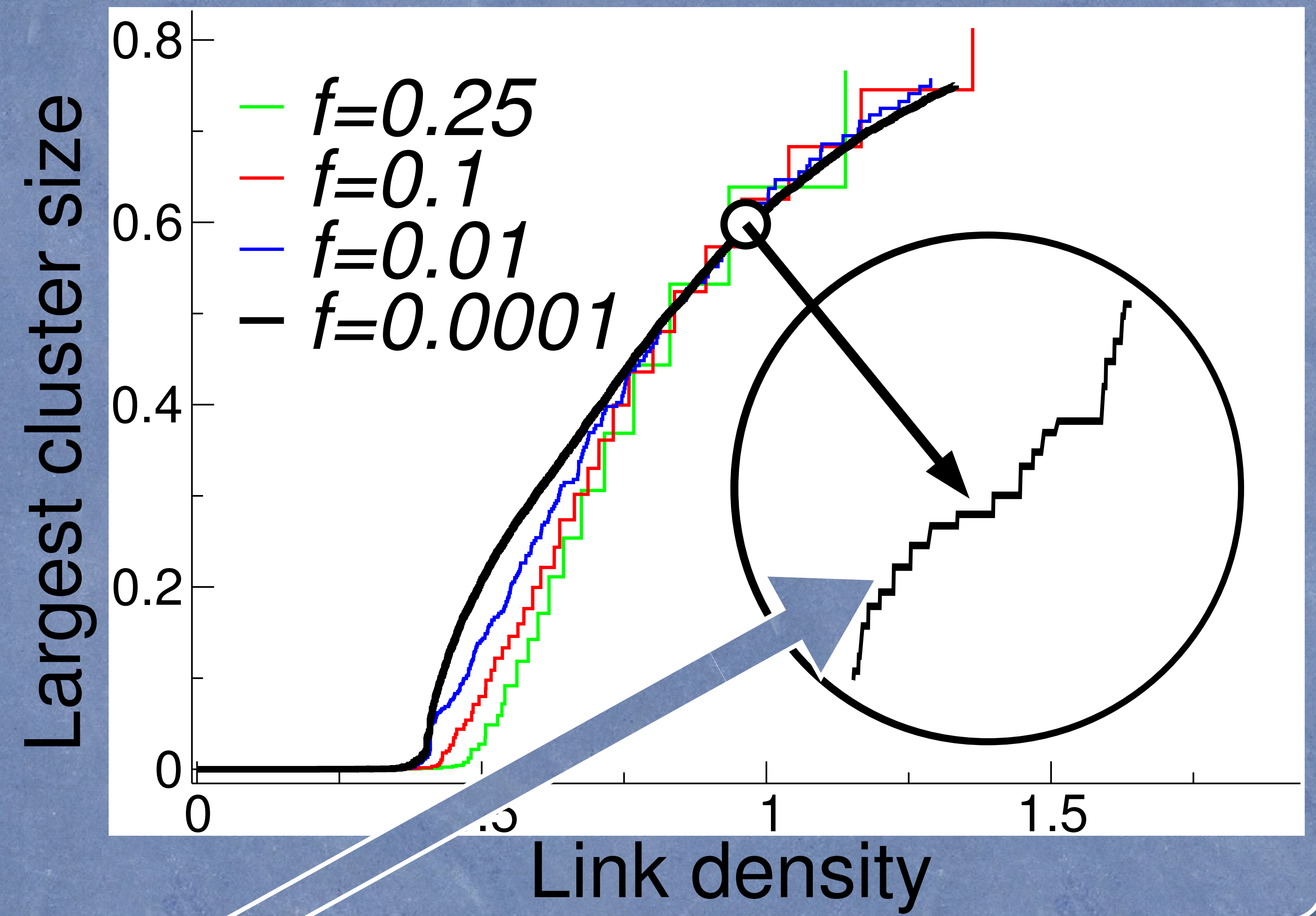
(DS model)

Nagler, Tiessen & Gutch, Phys. Rev. X 2012
Schröder, Rahbari, Nagler, Nat. Commun. 2013

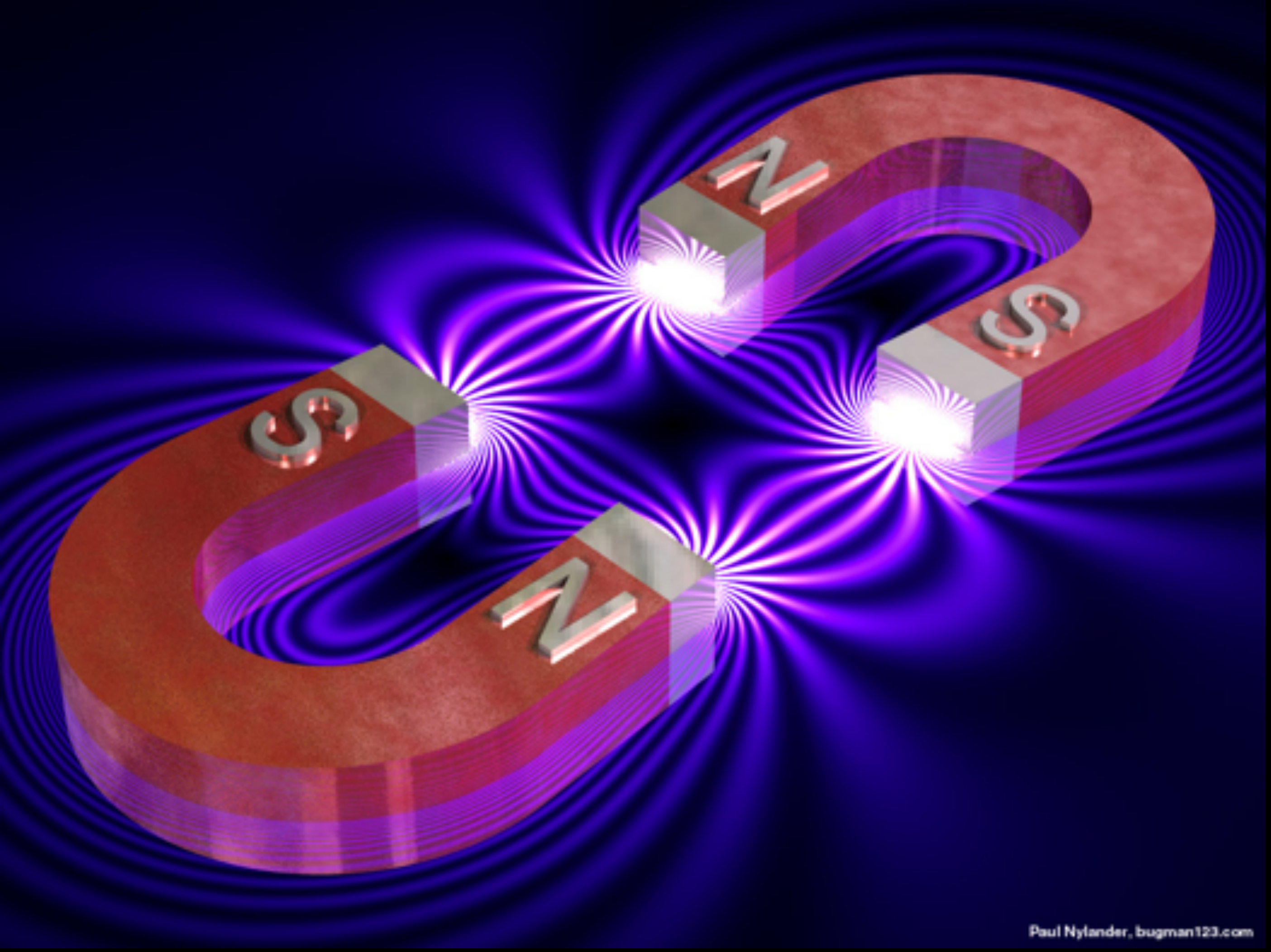
Fractional percolation & Barkhausen noise

Non-finite stochastic discontinuous phase transitions



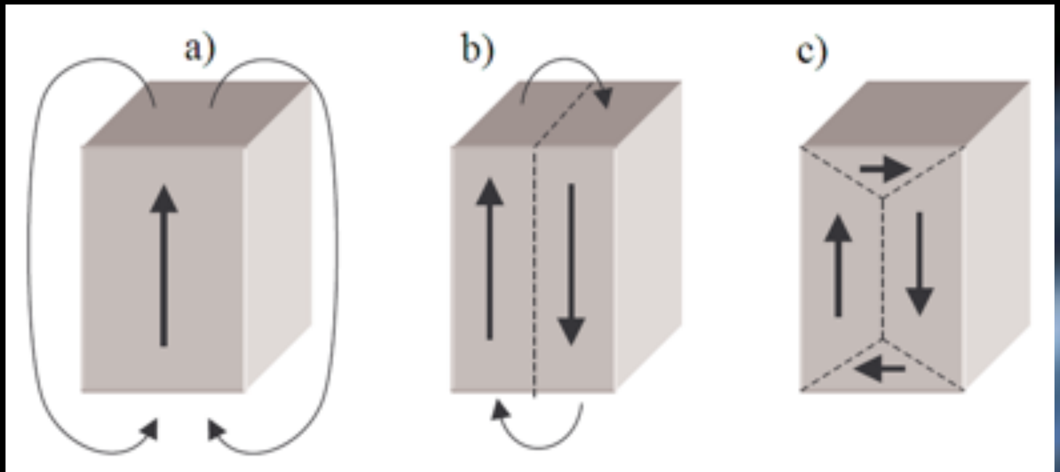


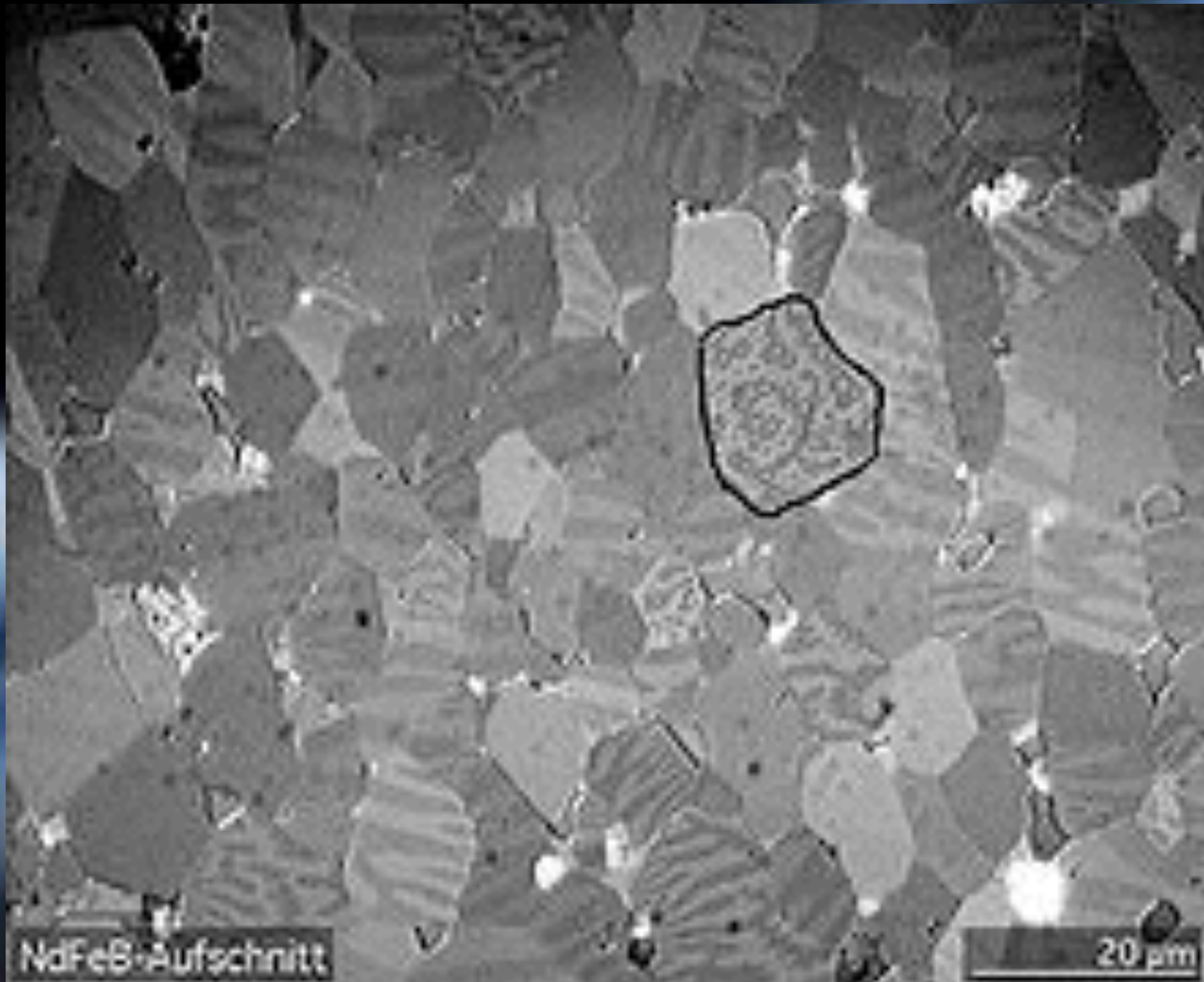
Schröder, Rahbari, Nagler, Nat. Commun. 2013



Ferromagnetism

fragmentation & domain wall motion



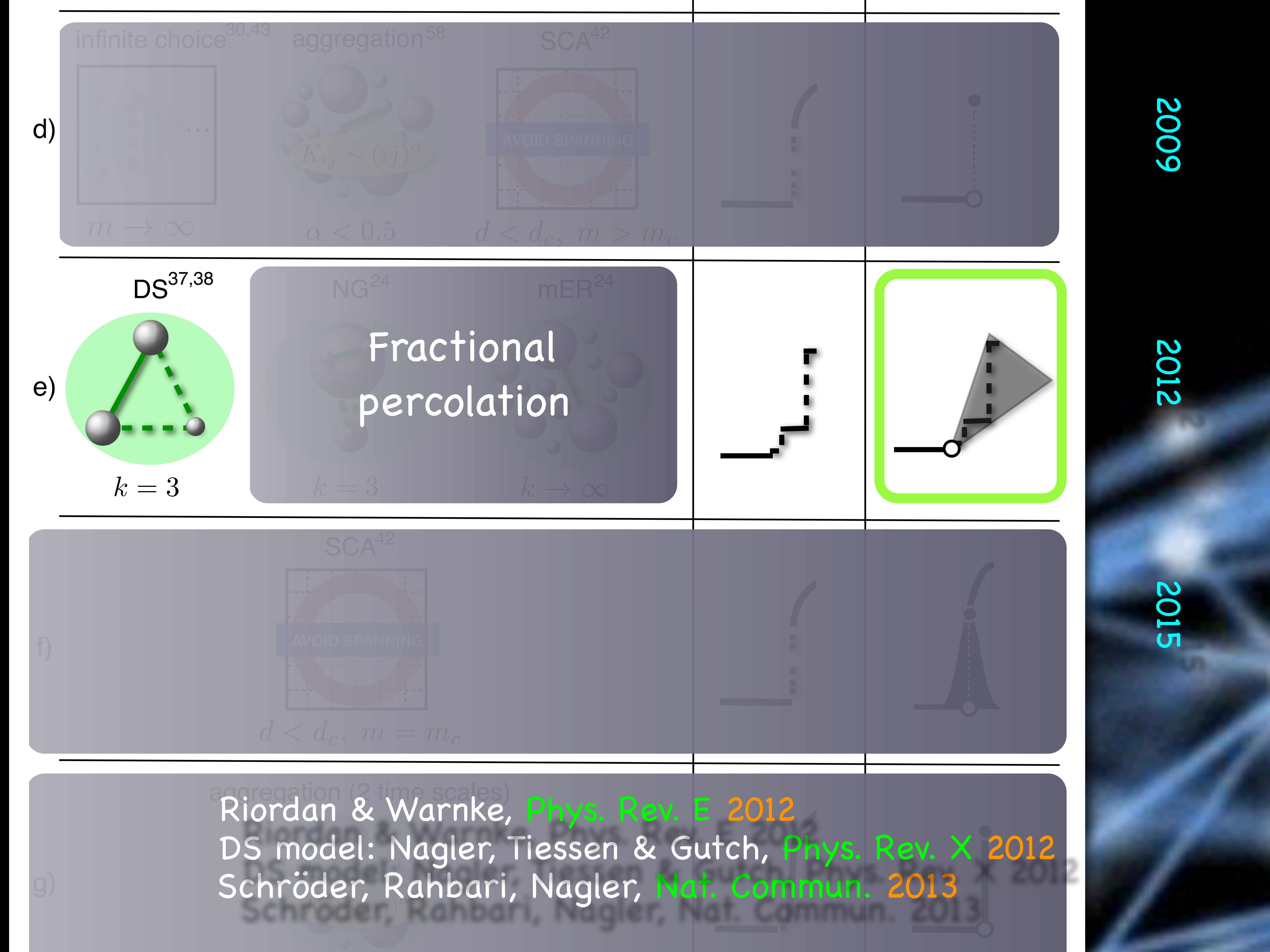


rigorous result

power law fluctuations

$$D(s) \sim 1/s$$

s=jump size of largest component


d)
infinite choice[30,43]
aggregation[58]
SCA[42]
AVOID SPANNING
$m \to \infty$
$\alpha < 0.5$
$d < d_c,\ m > m_c$
e)
DS[37,38]
$k = 3$
NG[24]
mER[24]
Fractional percolation
f)
SCA[42]
$d < d_c,\ m = m_c$
g)
aggregation (2 time scales)
Riordan & Warnke, Phys. Rev. E 2012
DS model: Nagler, Tiessen & Gutch, Phys. Rev. X 2012
Schröder, Rahbari, Nagler, Nat. Commun. 2013
2009
2012
2015

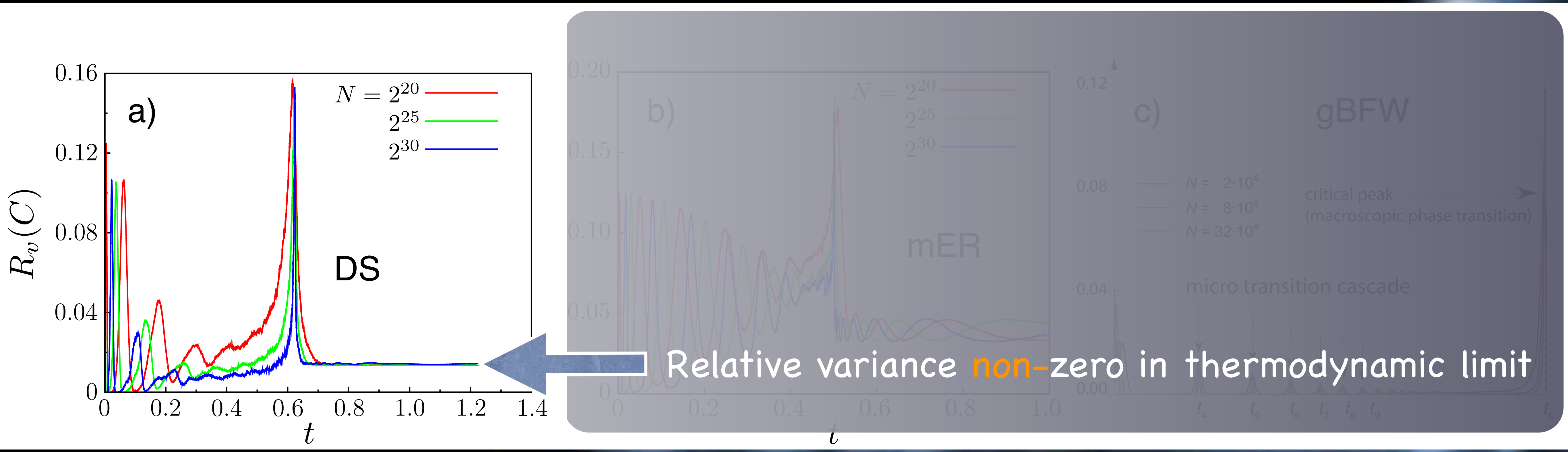
Anomalous critical and supercritcial behavior: non-self-averaging

Fluctuations survive in the thermodyn. limit!

M=order parameter

$$\mathcal{R}_v := \frac{\langle M^2 \rangle - \langle M \rangle^2}{\langle M \rangle^2}$$

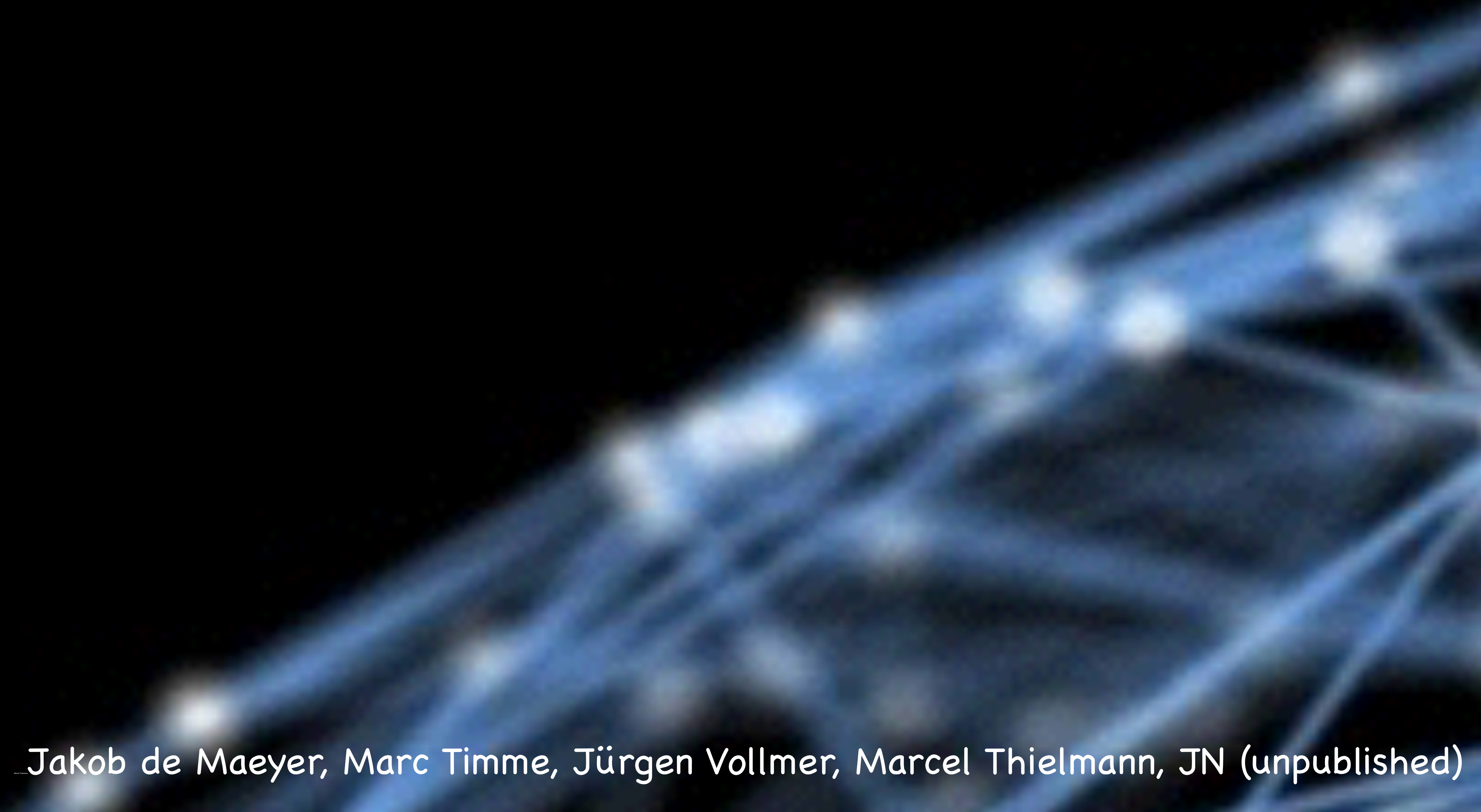
M=Size of largest cluster



Relative variance non-zero in thermodynamic limit

fluctuation function =
relative variance, Rv, of largest cluster size as a function of time

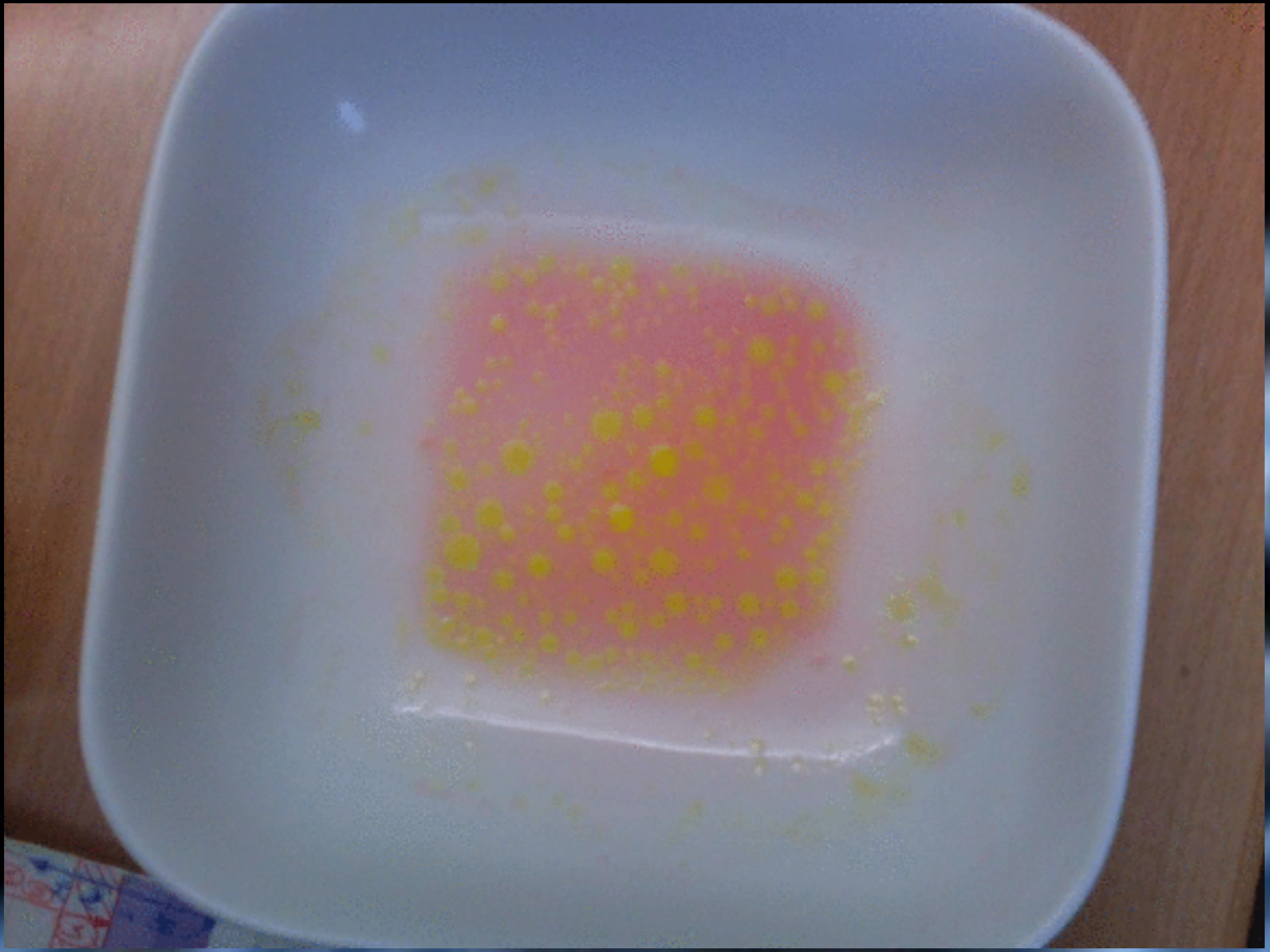
PART III: droplets



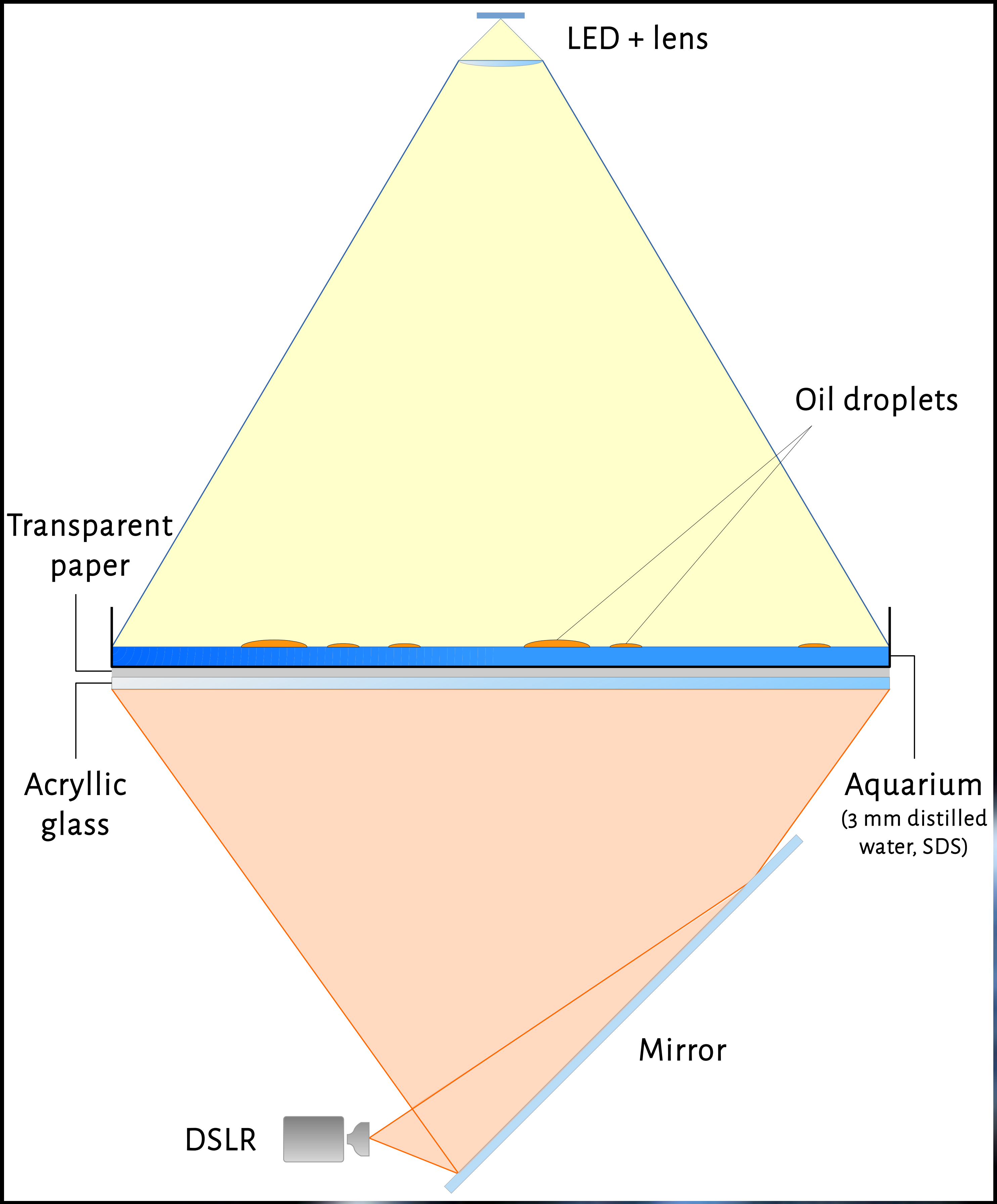
Jakob de Maeyer, Marc Timme, Jürgen Vollmer, Marcel Thielmann, JN (unpublished)

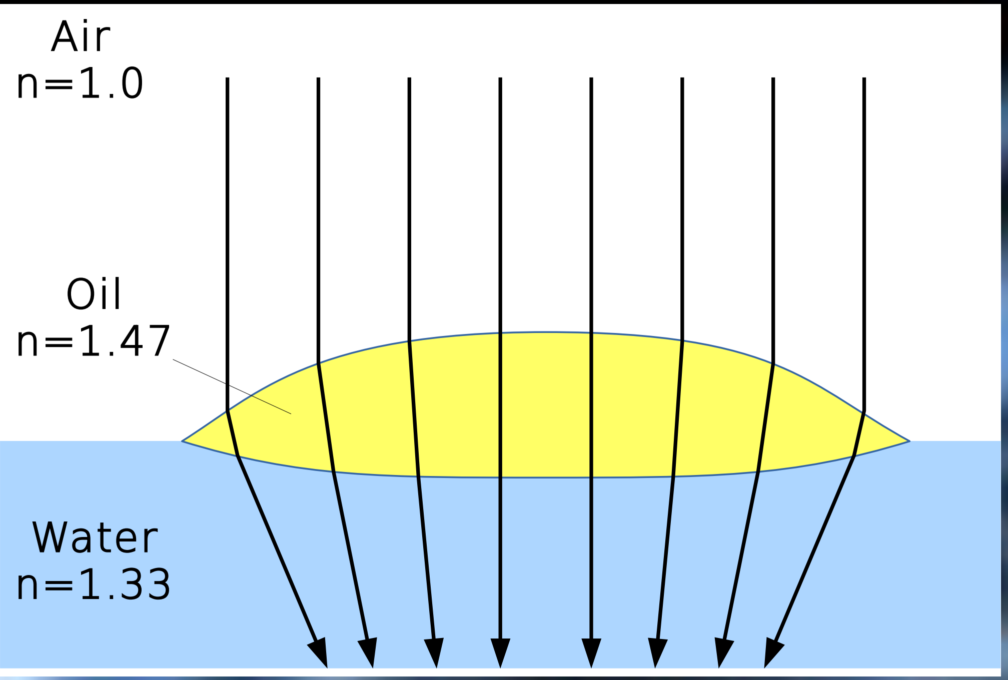
Experiments

Beetroot-Carrot salad



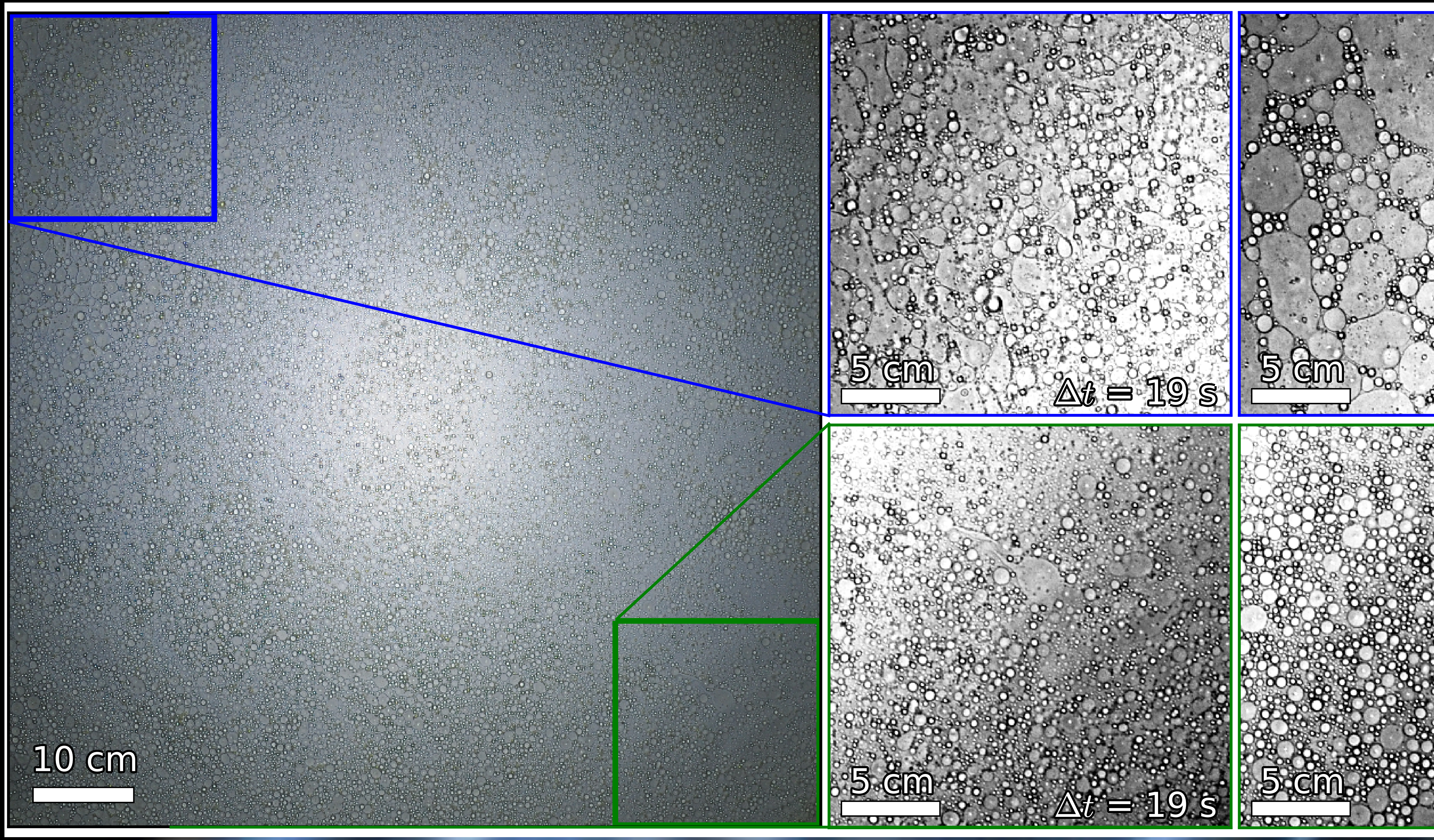
Experimental setup





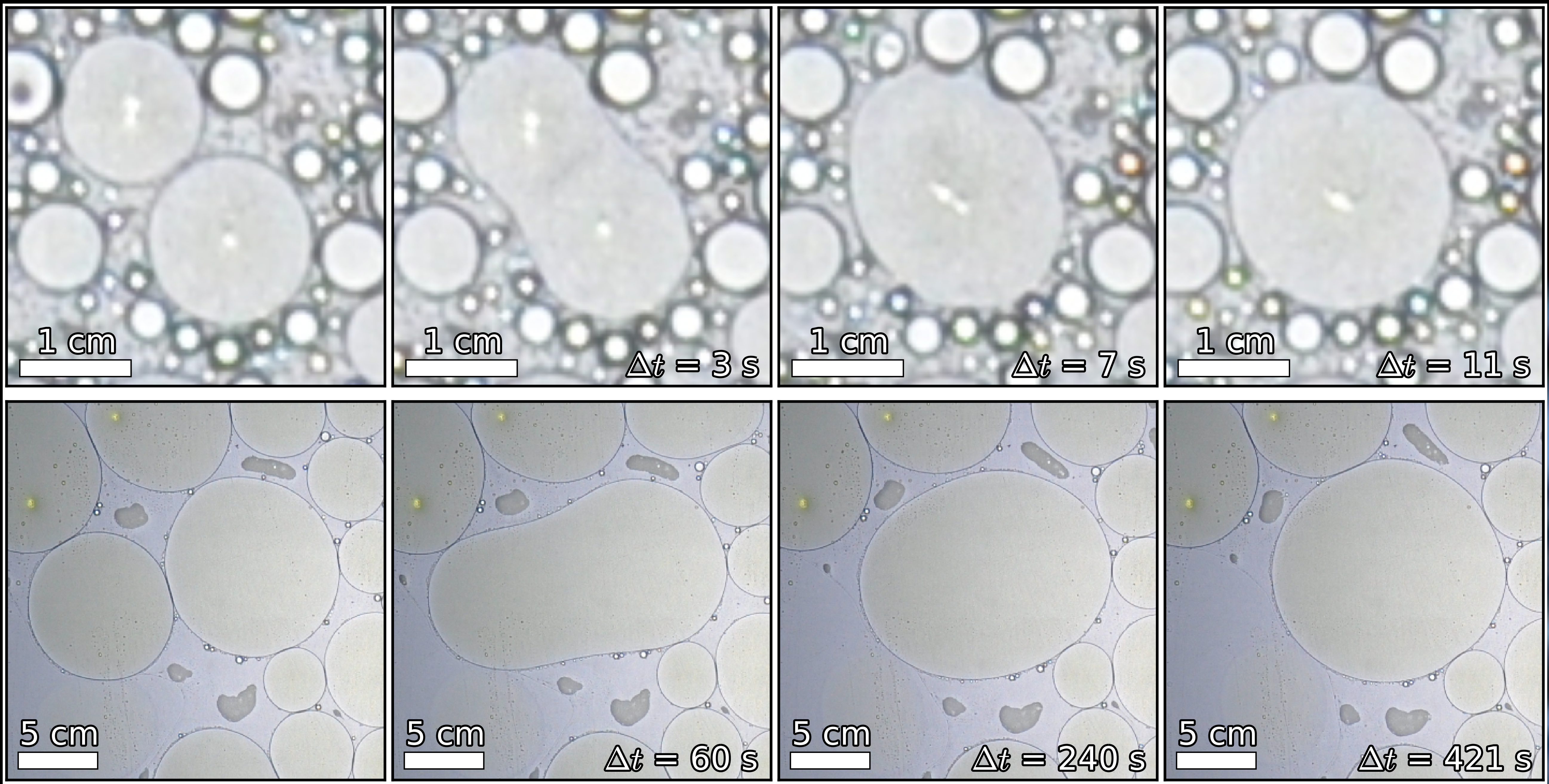
Experiment Jakob de Maeyer (MPI DS)

1m x 1m, 30.000 droplets

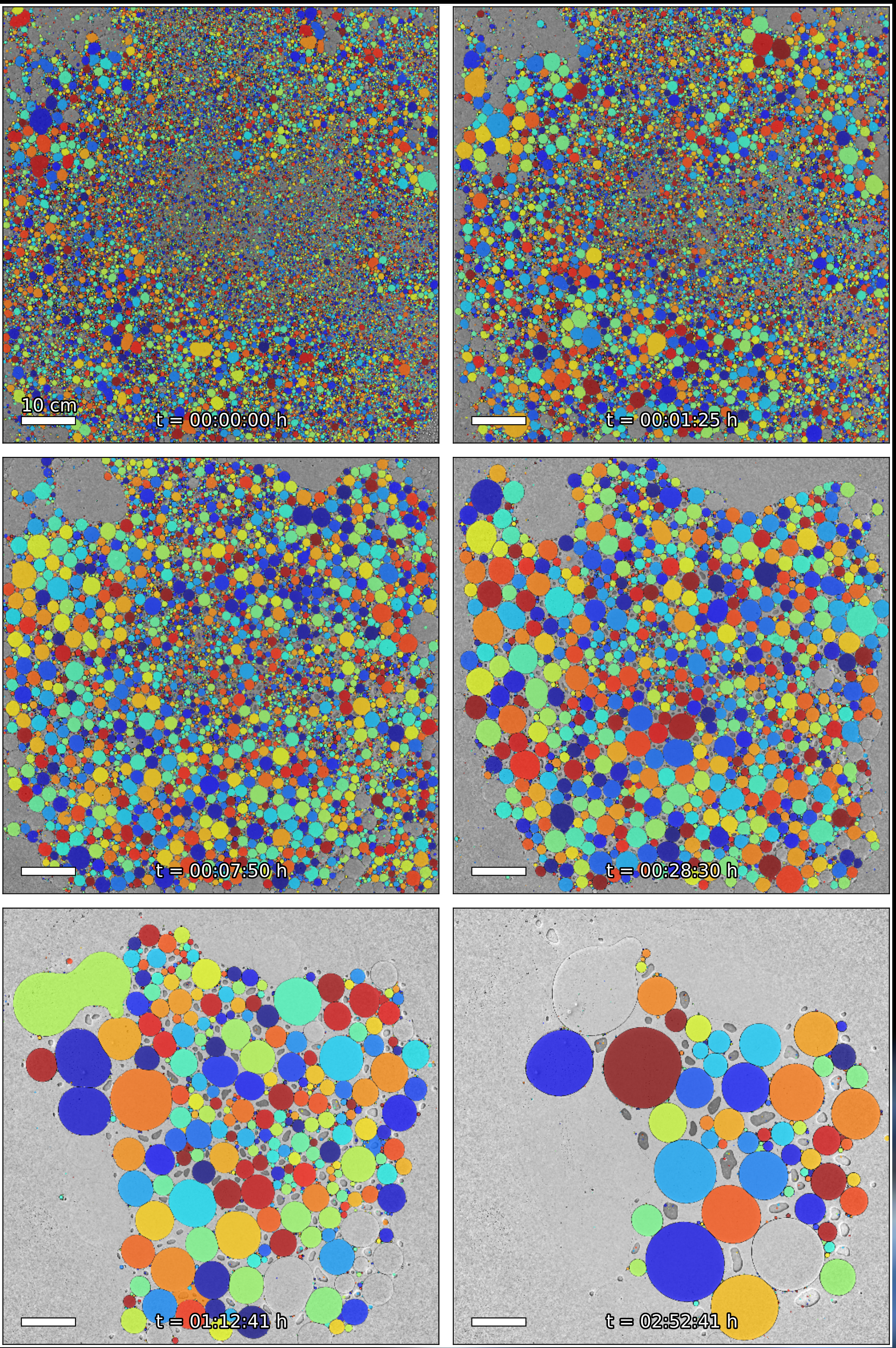


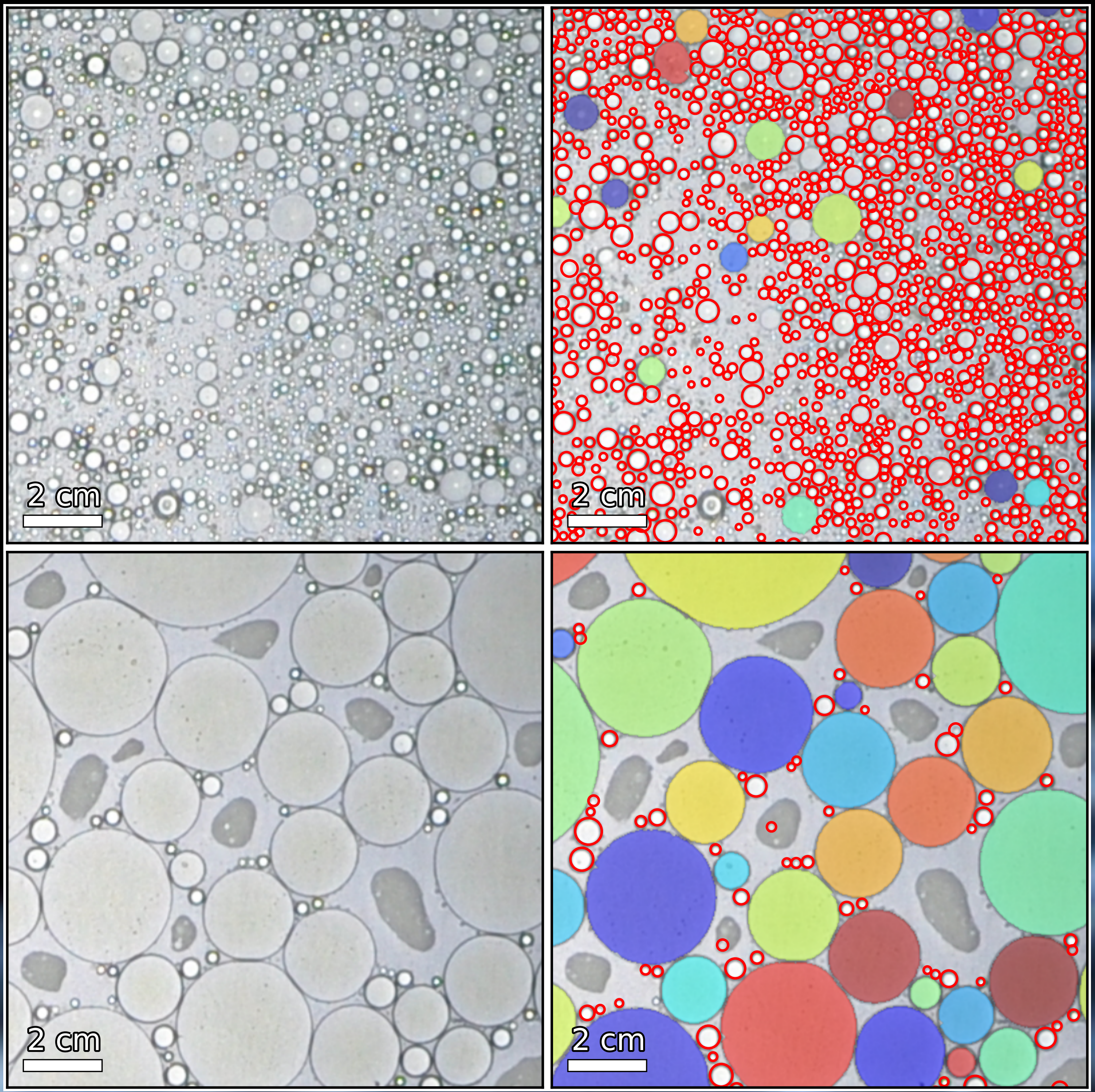
Experiment Jakob de Maeyer (MPI DS)

Merger of two large clusters

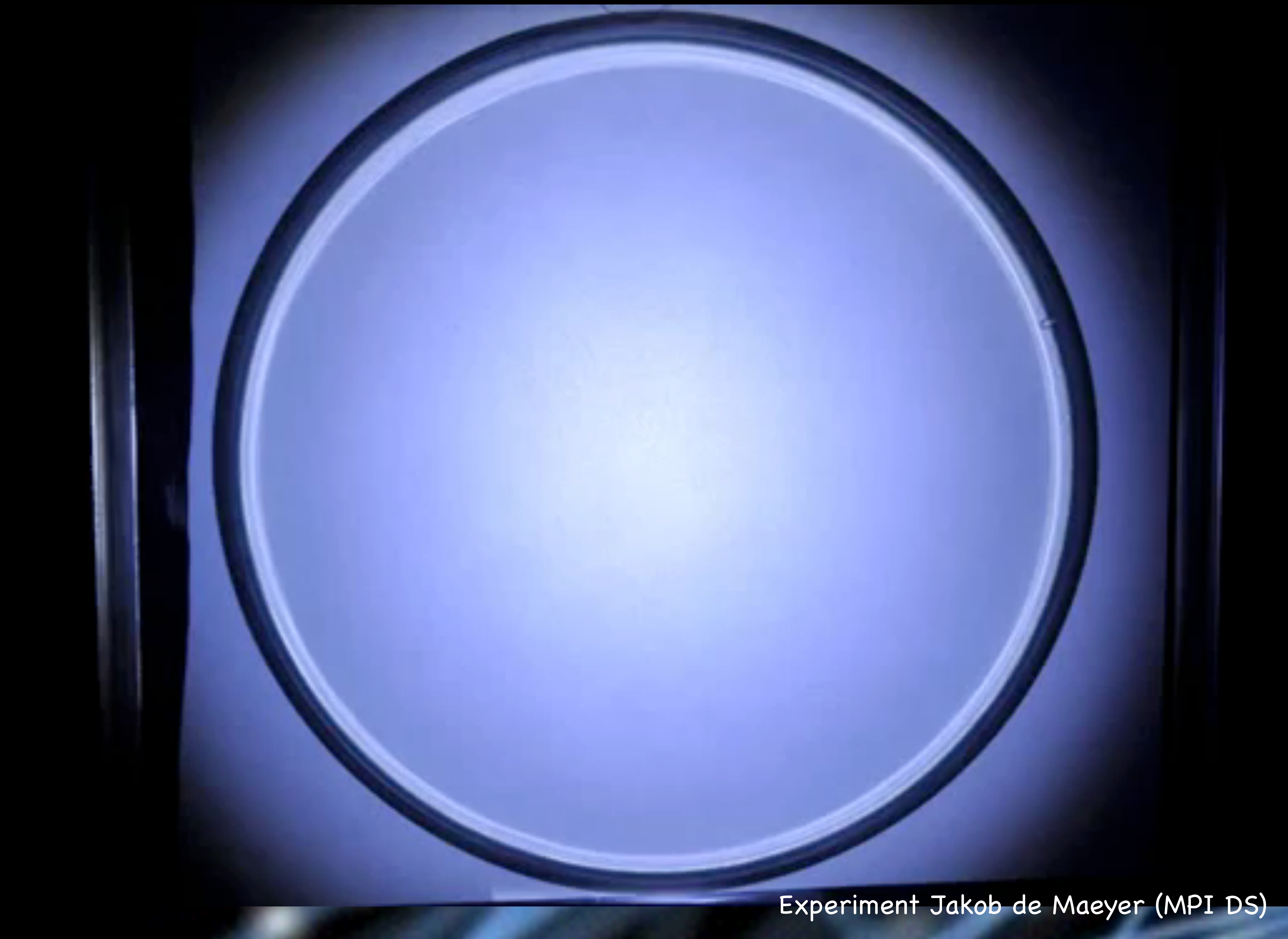


Experiment Jakob de Maeyer (MPI DS)

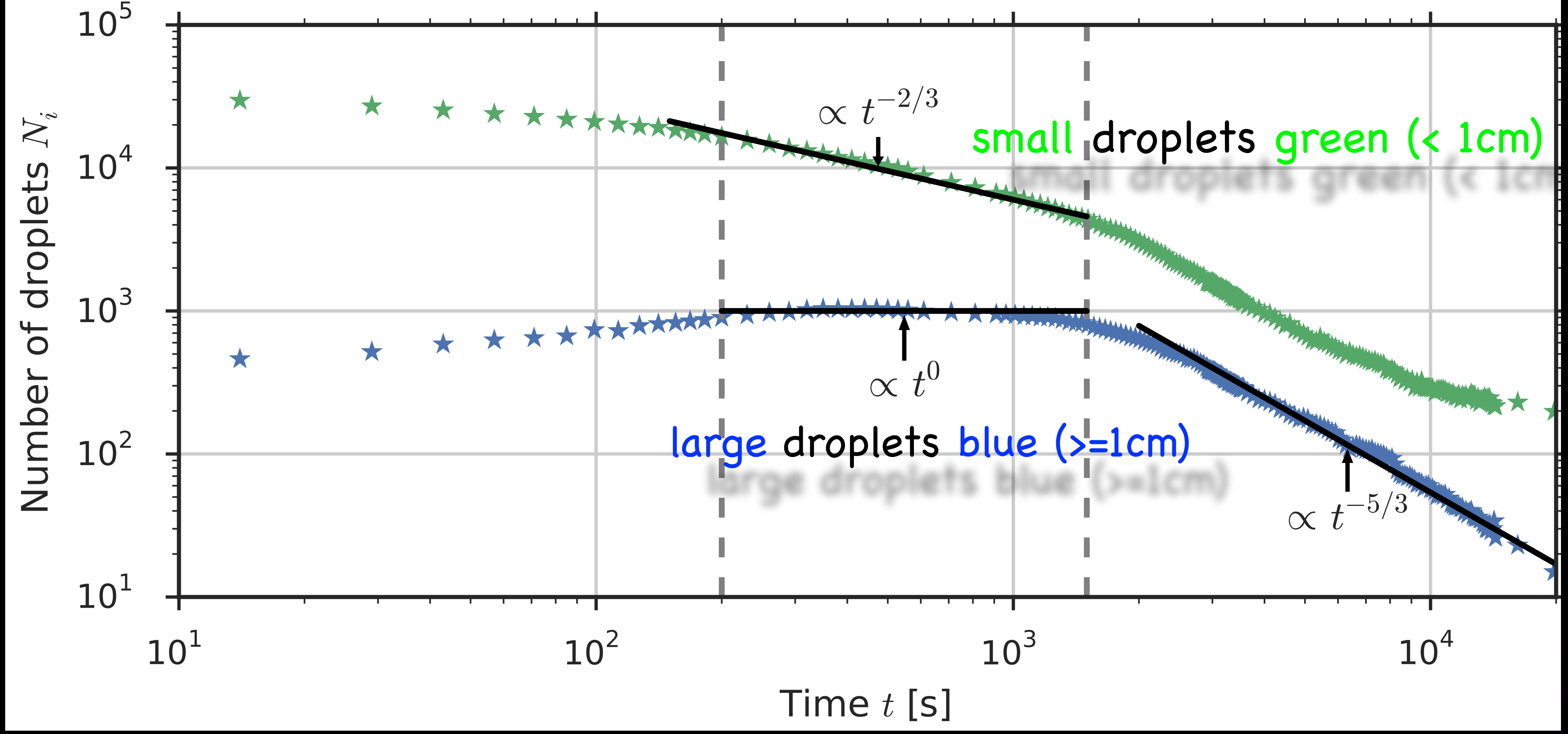




Experiment Jakob de Maeyer (MPI DS)



Experiment Jakob de Maeyer (MPI DS)



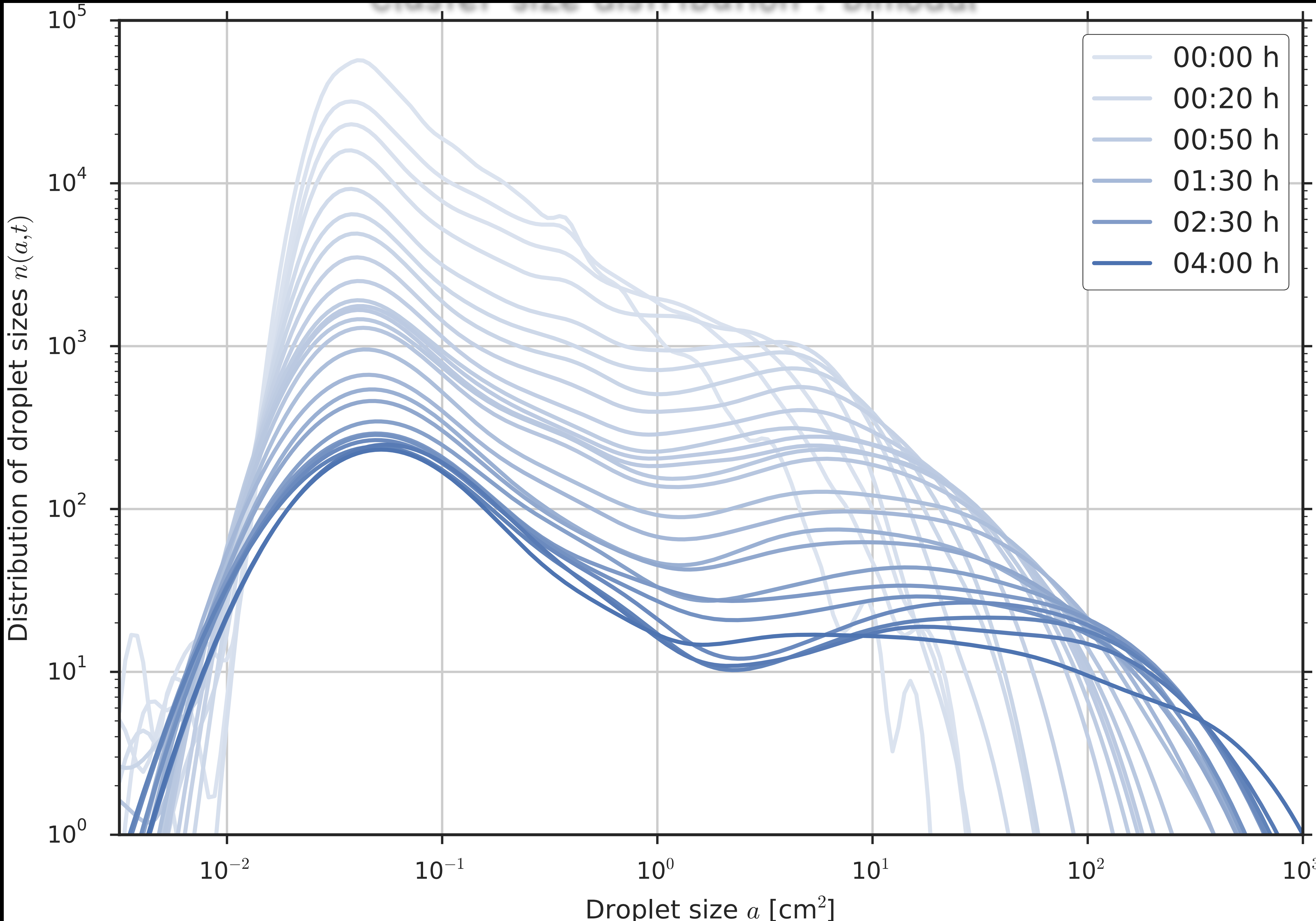
preliminary result: two time scales for different cluster sizes(*)

(modelled by composite kernel model)

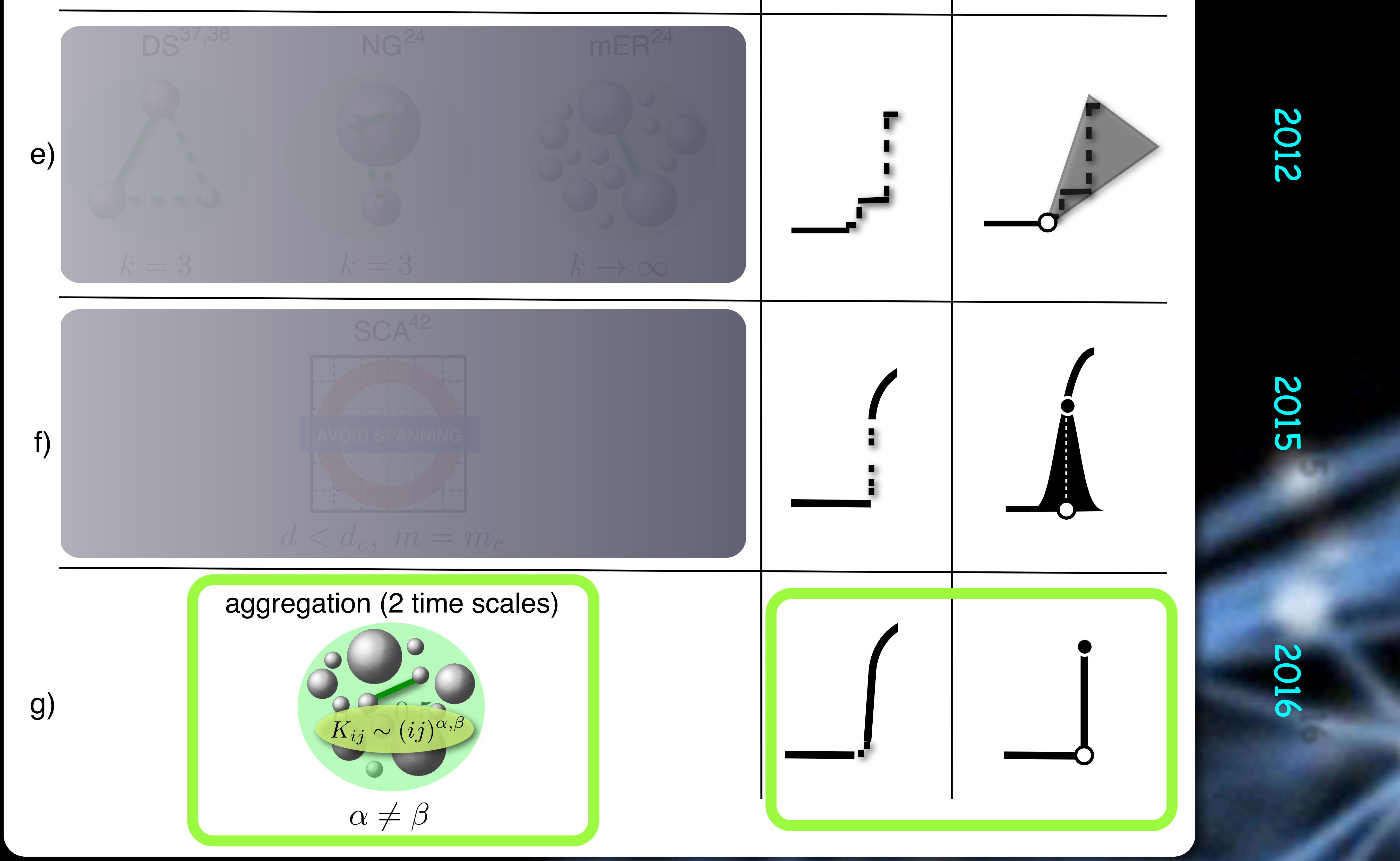
(*) expected for other systems:
aggregation under gravity,
aggregation in planetary systems,
polymerization in a thermal gradient

C. B. Mast et al., PNAS 110: 8030 (2013)
N. Brilliantov et al., PNAS 112: 9536 (2015)
Cho, Mazza, Kahng, Nagler (under review)

Cluster size distribution : bimodal



Experiment Jakob de Maeyer (MPI DS)

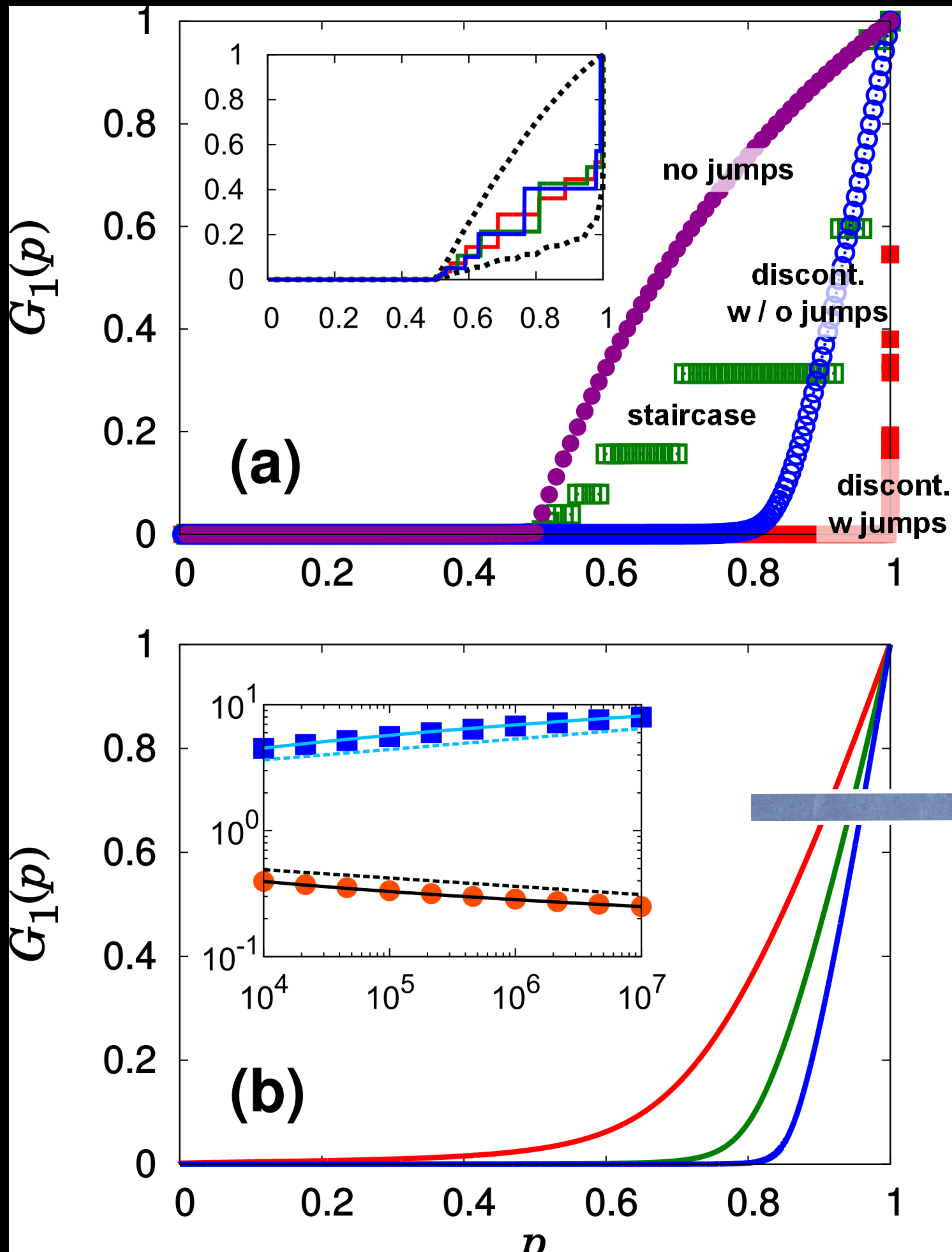


Genuine non-self-averaging and ultra-slow convergence in gelation,
Cho, Mazza, Kahng, Nagler (under review)

Aggregation with composite kernel

Two time scales -> rich phenomenology

G1=order parameter=size of largest cluster

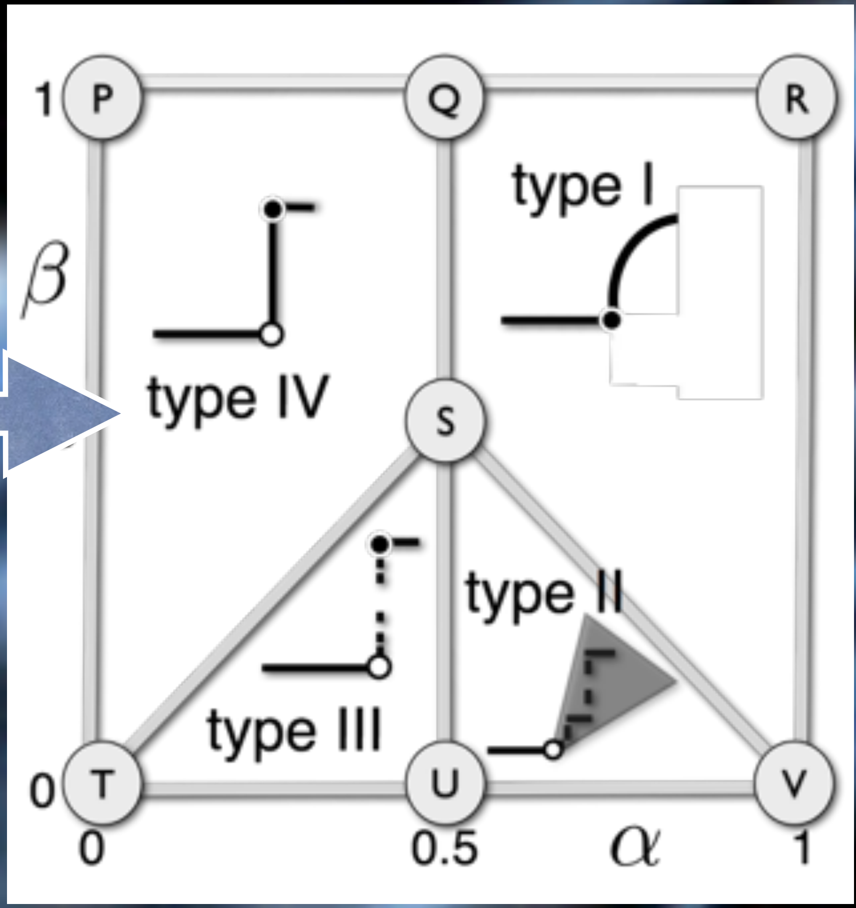


Smoluchowsky equation

$$\frac{\mathrm{d}n_k}{\mathrm{d}t} = \sum_{i+j=k} K_{ij} n_i n_j - 2n_k \sum_j K_{kj} n_j$$

$$K_{ij} = (ij)^{\omega}$$

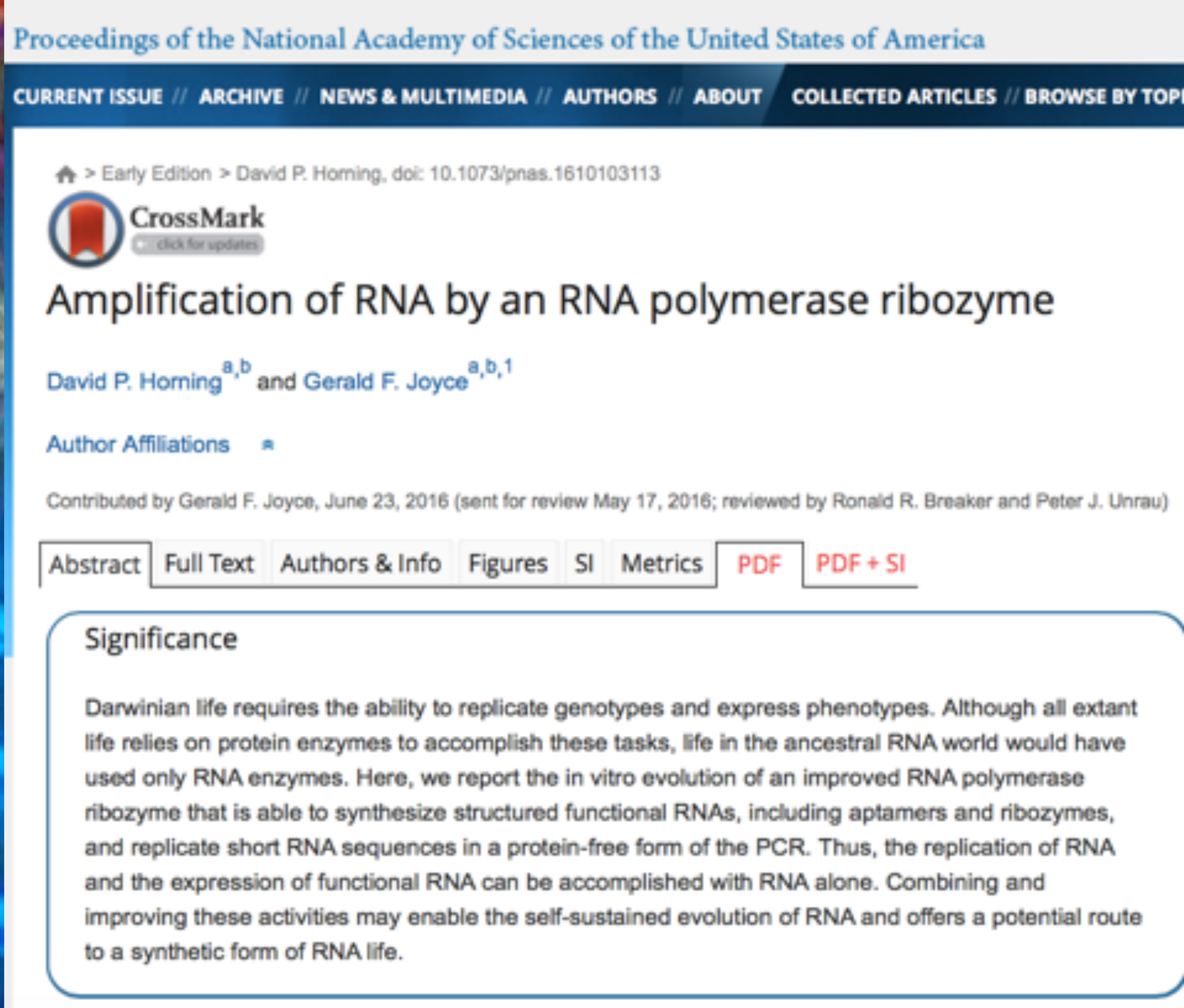
$$\omega = \begin{cases} \alpha & \text{if } i \neq G_1, \\ \beta & \text{otherwise} \end{cases}$$



Genuine non-self-averaging and ultra-slow convergence in gelation, Cho, Mazza, Kahng, Nagler (accepted, Phys. Rev. E)

PART IV

Early molecular evolution: DNA replication?



Proceedings of the National Academy of Sciences of the United States of America

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> Early Edition > David P. Horning, doi: 10.1073/pnas.1610103113

CrossMark

Amplification of RNA by an RNA polymerase ribozyme

David P. Horning[a,b] and Gerald F. Joyce[a,b,1]

Author Affiliations

Contributed by Gerald F. Joyce, June 23, 2016 (sent for review May 17, 2016; reviewed by Ronald R. Breaker and Peter J. Unrau)

Abstract | Full Text | Authors & Info | Figures | SI | Metrics | PDF | PDF + SI

Significance

Darwinian life requires the ability to replicate genotypes and express phenotypes. Although all extant life relies on protein enzymes to accomplish these tasks, life in the ancestral RNA world would have used only RNA enzymes. Here, we report the in vitro evolution of an improved RNA polymerase ribozyme that is able to synthesize structured functional RNAs, including aptamers and ribozymes, and replicate short RNA sequences in a protein-free form of the PCR. Thus, the replication of RNA and the expression of functional RNA can be accomplished with RNA alone. Combining and improving these activities may enable the self-sustained evolution of RNA and offers a potential route to a synthetic form of RNA life.

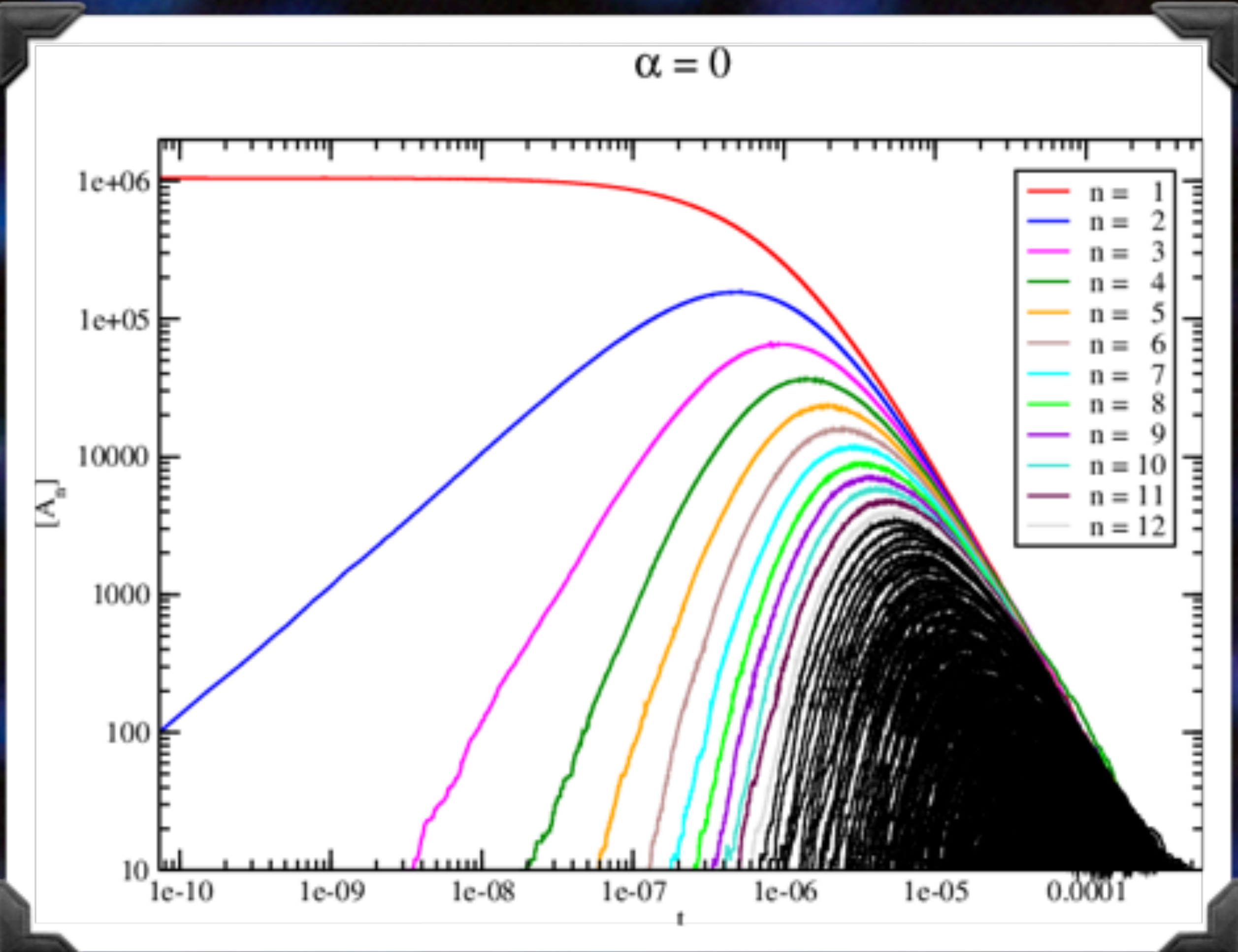
Eigen, Selforganization of matter and the evolution of biological macromolecules. *Naturwissenschaften* 58:465-523 (1971)
Mast et al., PNAS 110: 8030 (2013)
Worst, Zimmer, Wollrab, Kruse, Ott (under review)

DNA replication and ligation

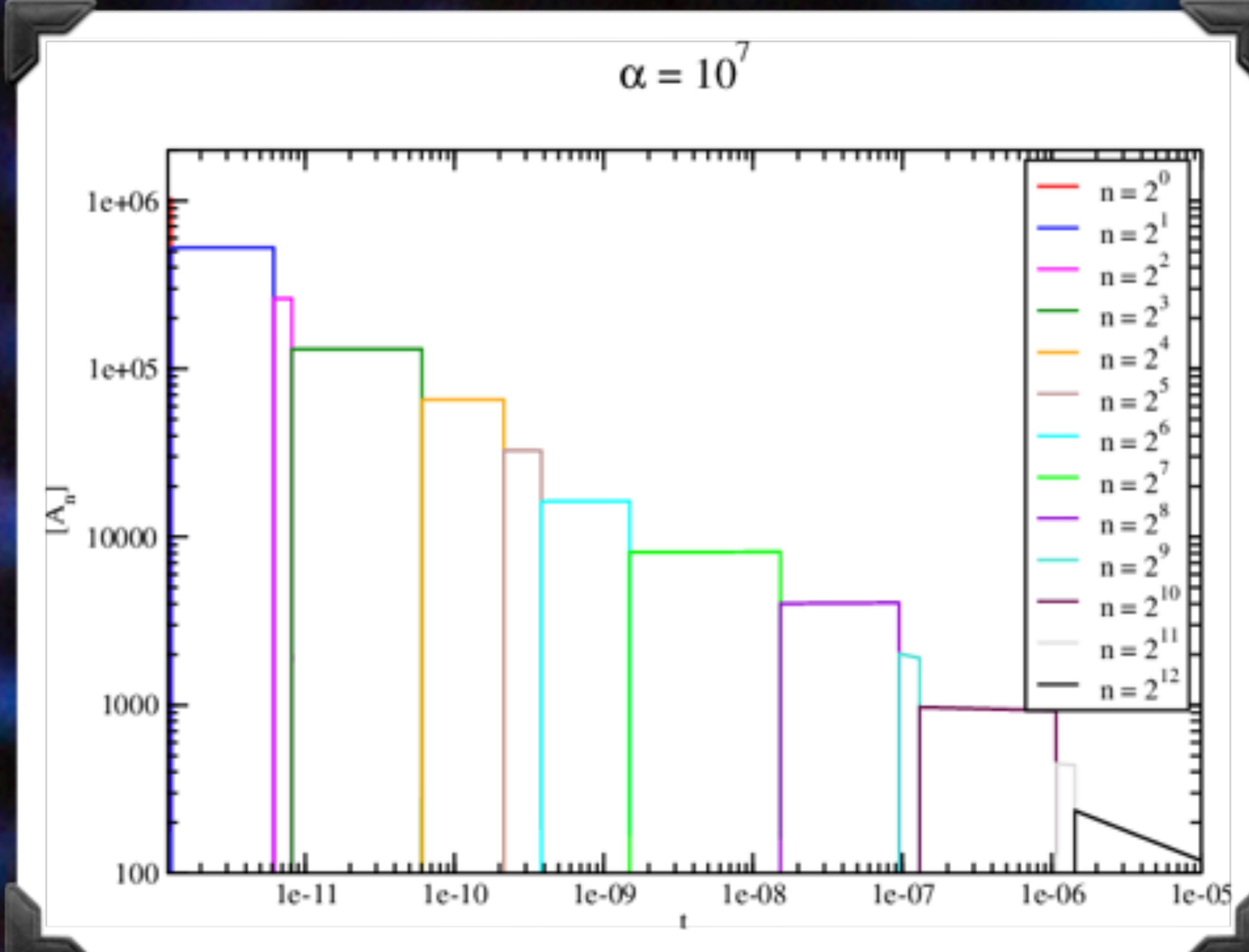
First results from a model with autocatalysis (replication) and spontaneous concatenation (ligation)

$$A_n + A_m \xrightarrow[\alpha[A_{n+m}]+\beta]{} A_{n+m}$$

Concentration of polymers of length n as function of time



ligation only

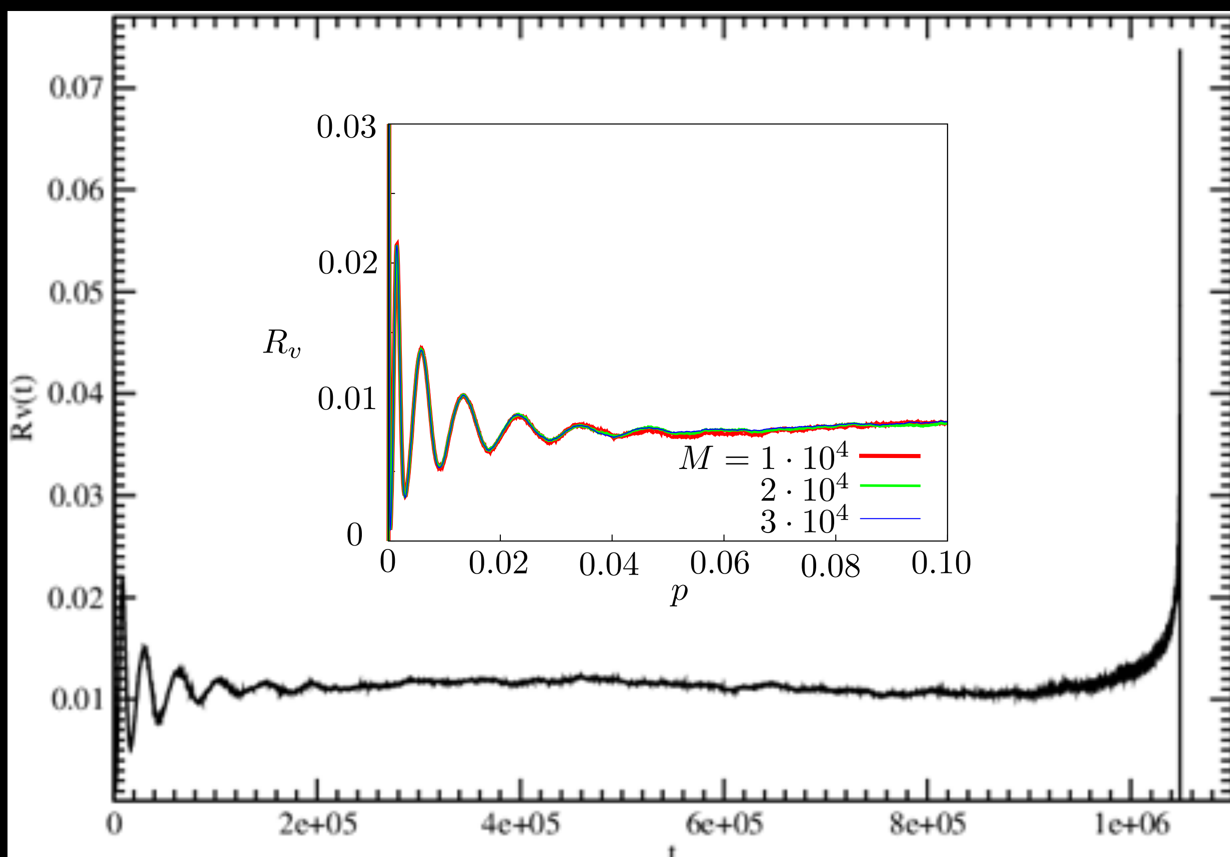


ligation and replication

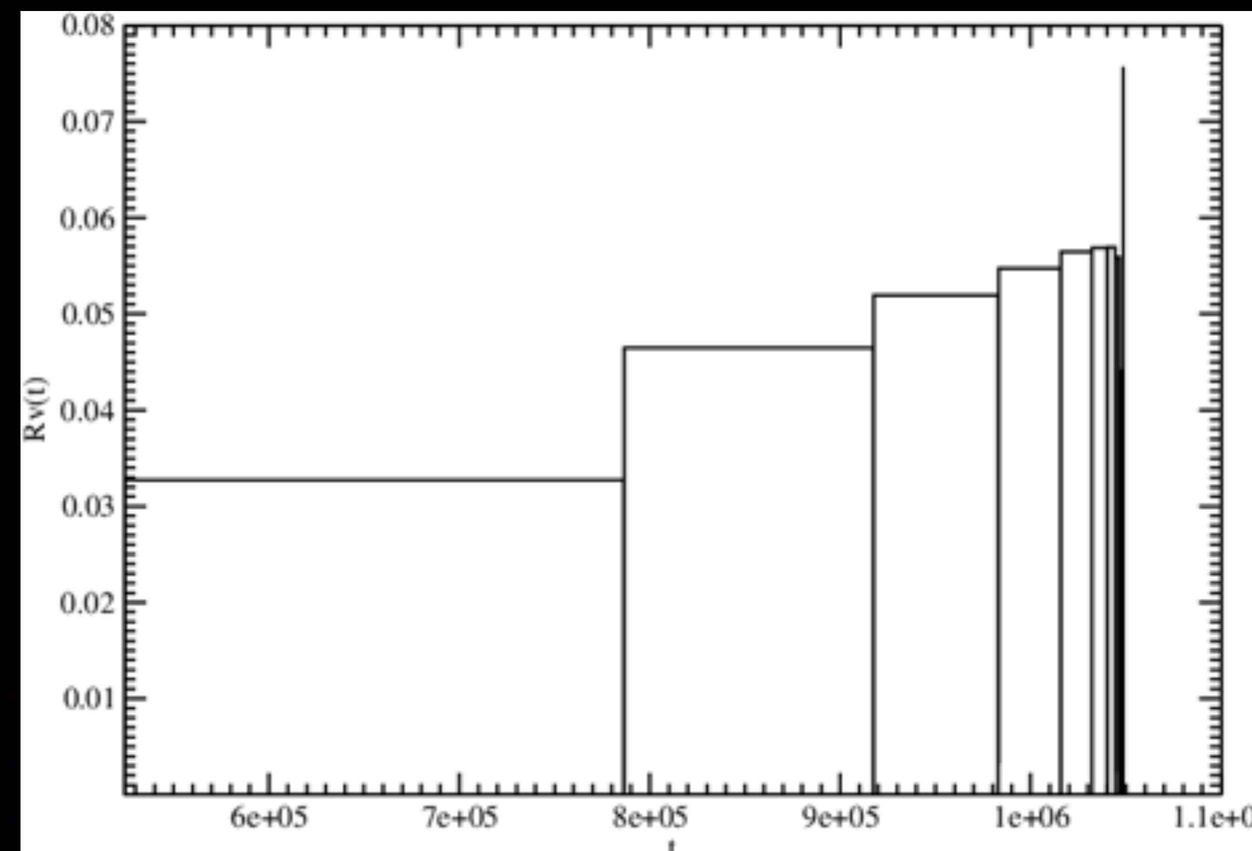
Zimmer, Kruse & Nagler (in prep.)

Non-self-averaging behaviors
(Rv>0)

ligation only

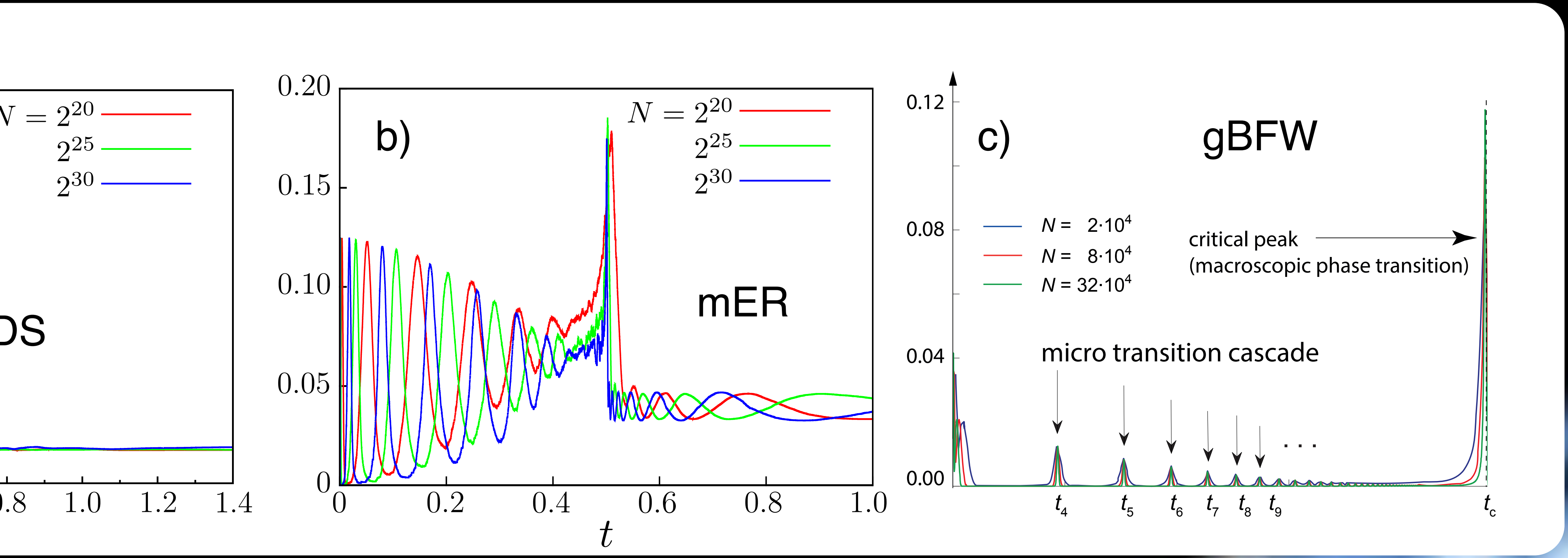


ligation and replication



fluctuation function =
relative variance, Rv, of order parameter as a function of time

Anomalous critical and supercritcial behavior in other models



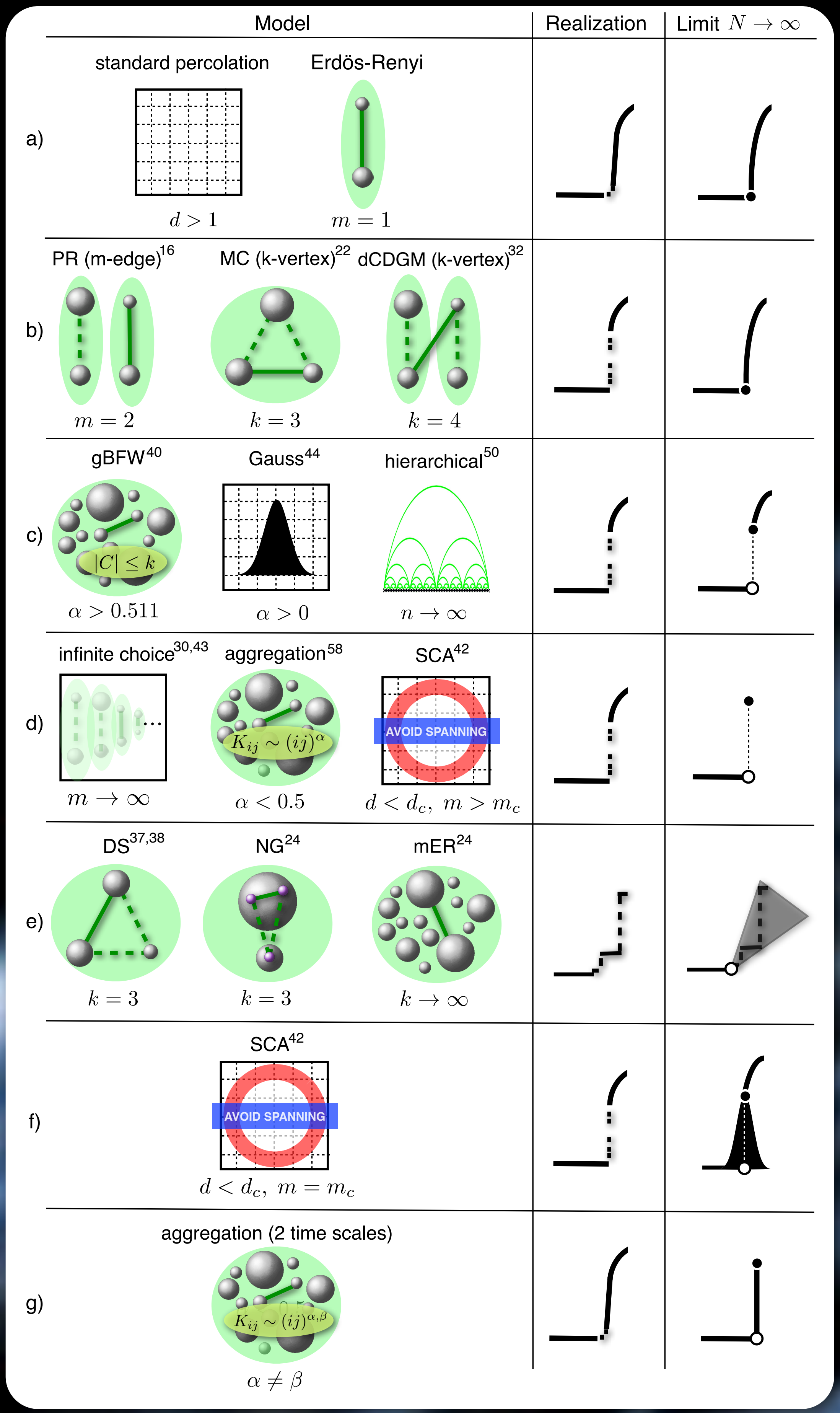
fluctuation function =
relative variance, Rv, of order parameter as a function of time

Fluctuations survive in the thermodyn. limit!

D'Souza & Nagler, Novel critical and supercritical phenomena in Explosive Percolation, Nature Physics 11:3378 (2015)

Thanks! (jnagler@ethz.ch)

Phase transition types



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