

# **How distributed is „distributed information“ in complex systems?**

**Marc Timme**

in collaboration with Christoph Kirst and Demian Battaglia



**Network Dynamics**

Max Planck Institute for Dynamics & Self-Organization



Technical University of Darmstadt



Bernstein Center for Computational Neuroscience Göttingen



Georg August University Göttingen

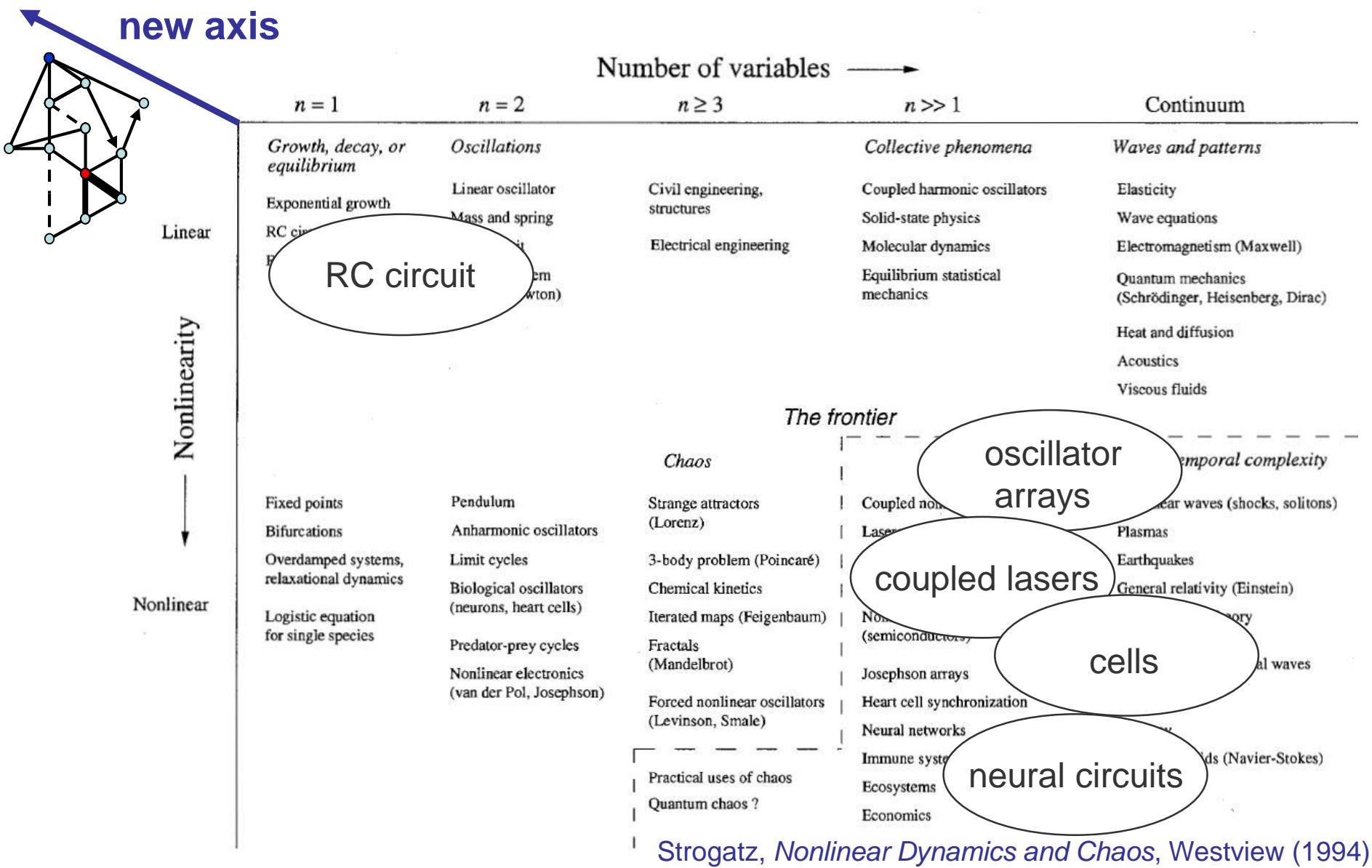
# **Networks are everywhere - most are dynamic**

Social Networks  
Neural Circuits  
Gene Regulatory Networks  
Traffic Networks  
Communication Networks  
,Smart' Power Grids

...

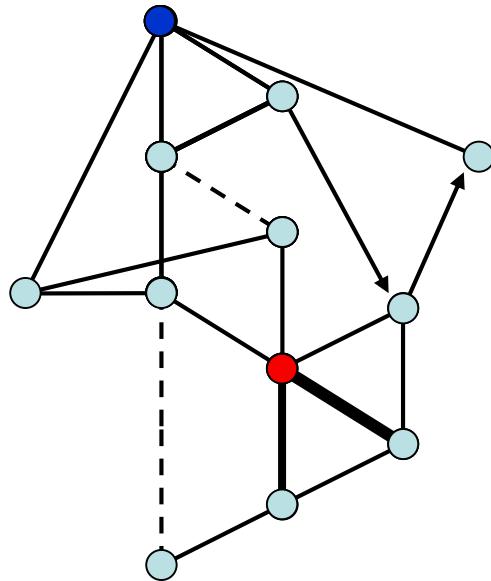
**All these systems are essential in everyday life!**

# From simple & low-dimensional to complex systems



# ... to complex *networked* systems

Graph/**connectivity as new axis**: complex networks



linear:  
Kirchhoff's law,  
Random walks,...

In general  
**no continuum limit**

nonlinear



Limited knowledge,  
Few general methods

Network Topology co-acts with nonlinearities:  
Topological Speed Limits, Remote Action, ...

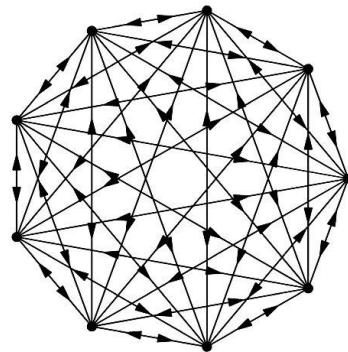
# Mathematical challenges for theory

Simultaneous occurrence of:

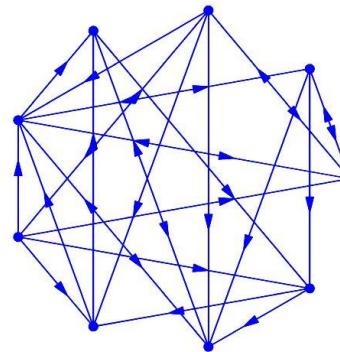
- **Nonlinearity**
- **High dimensionality**
- Complicated Network **Connectivity**
- Interaction **Delays**
- Strong **Heterogeneities**
- **Stochasticity**

common approach:

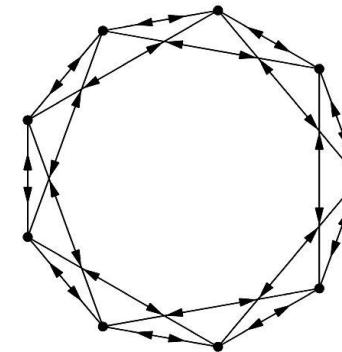
**Mean Field Theories, Statistical Description**, e.g. averaging over network



all-to-all  
(regular)



general  
(irregular)



local  
(regular)

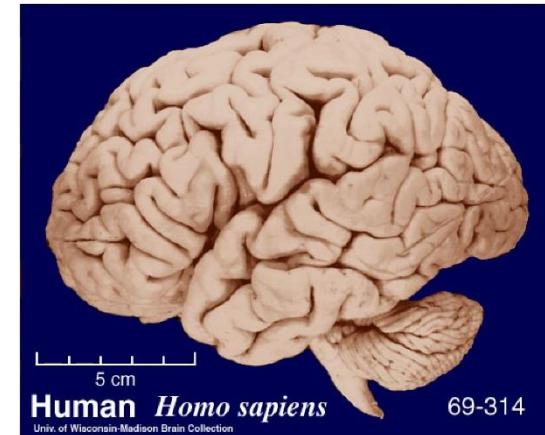
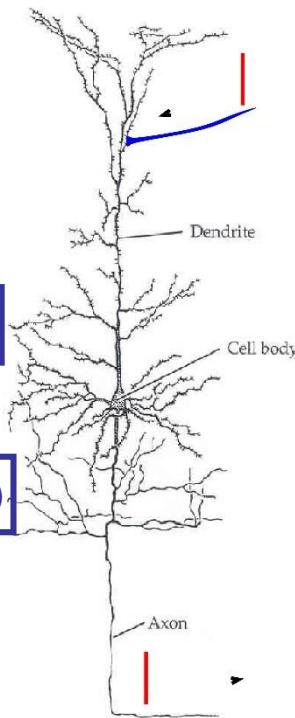
**Mind the specifics (links, events, realizations, ...) !**

# Distributed collective dynamics of networks

## Dynamics of biological networks

( $10^{-3} - 10^{10}$  s;  $10^{-5} - 10^{-1}$  m)

- Selective links support sensing
- Connectomics from dynamics
- model-free network inference
- information transfer [Nature Comm. \(2016\)](#)
- protein scaling laws from geometry



## Dynamics of physical networks

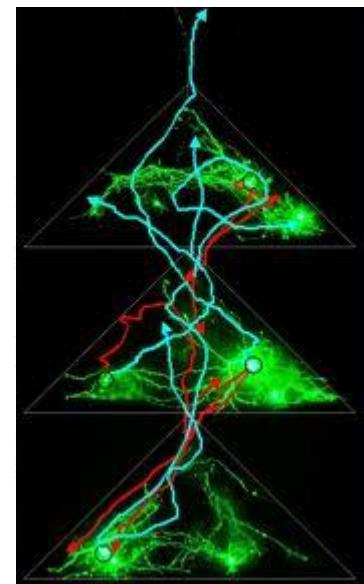
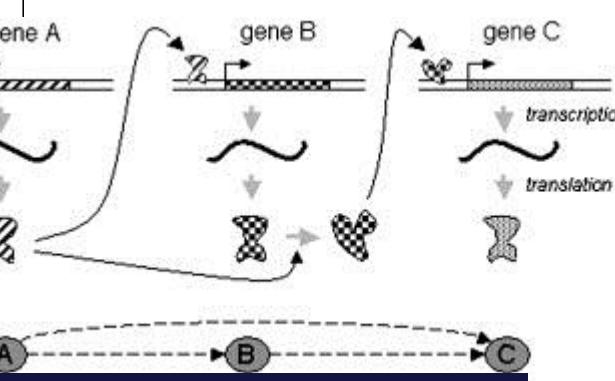
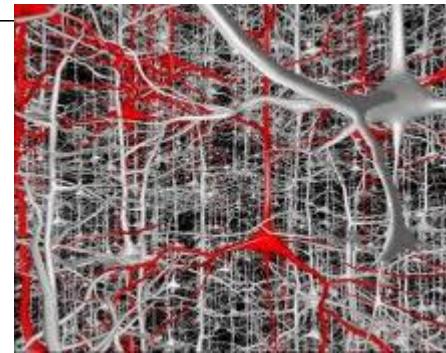
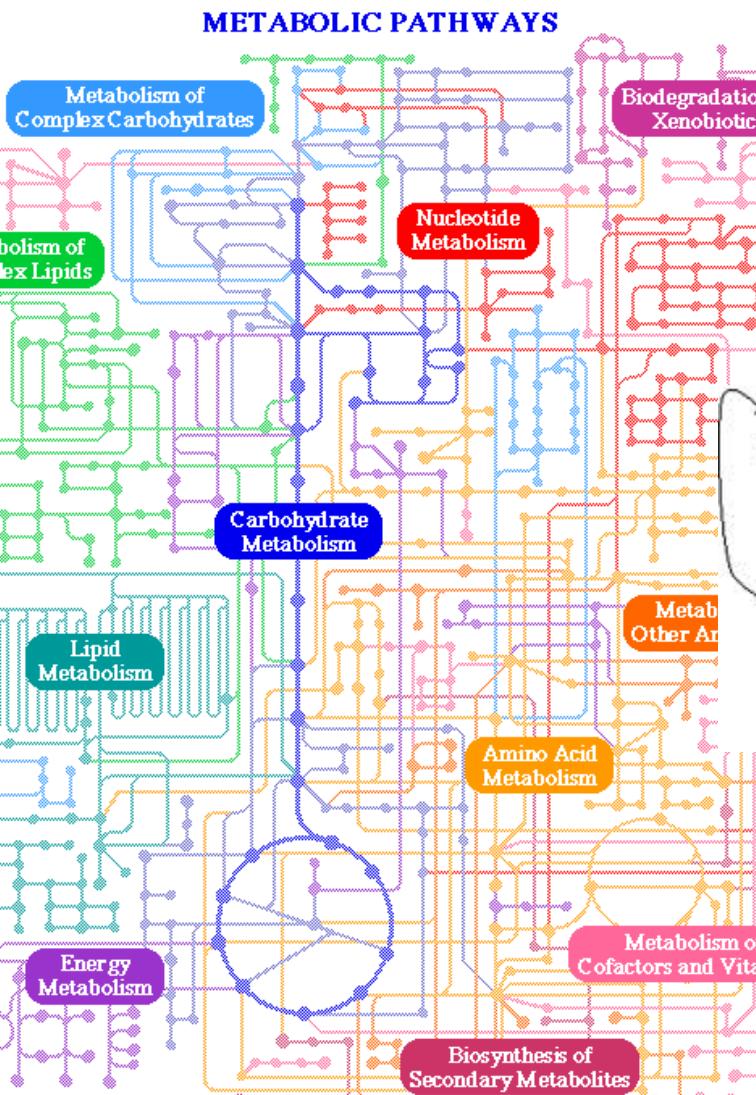
( $10^{-2} - 10^{10}$  s;  $10^{-9} - 10^6$  m)

- Network growth & disordered media
- Dynamical system's computation & network control
- Nonlocal rerouting and dynamically smart grids
- Flexible networked mobility



# Information routing essential for function

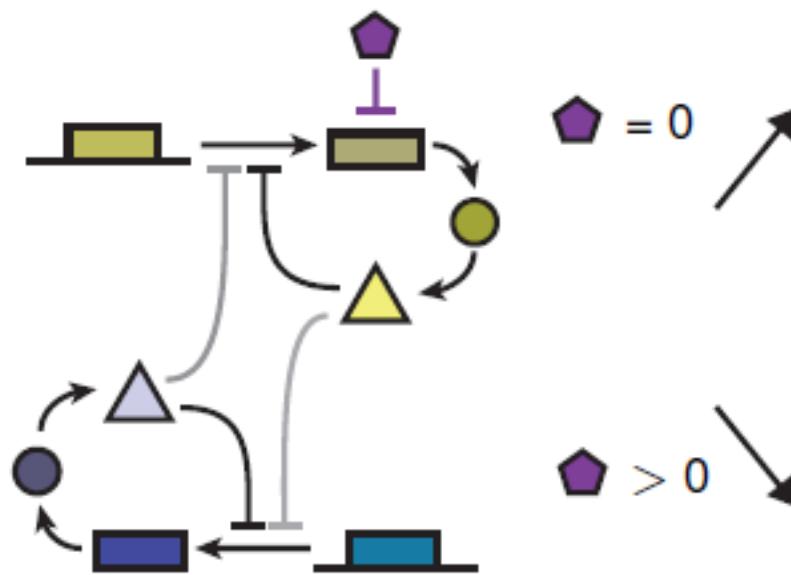
## How does it work?



Biosystems, 96:86 (2009);  
Brockmann et al, Nature (2006)  
Google Inc. (2010)  
Feinerman et al,  
Nature Phys. (2008)

# Collective Dynamics → Information Routing Patterns I

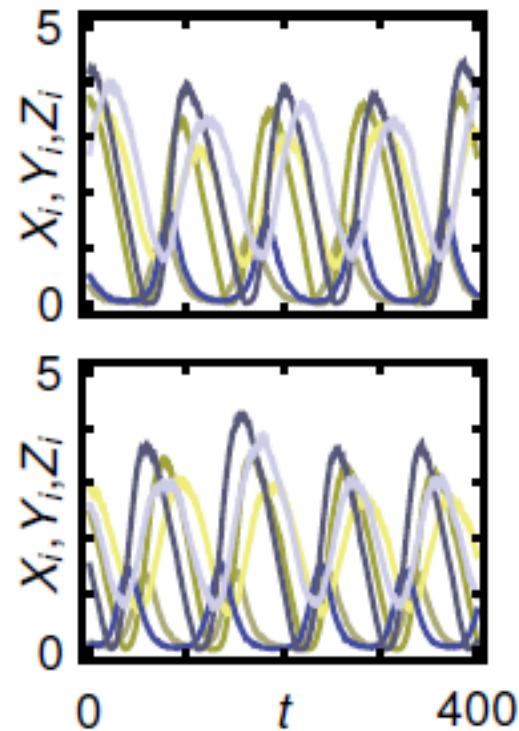
**a** Gene regulatory network



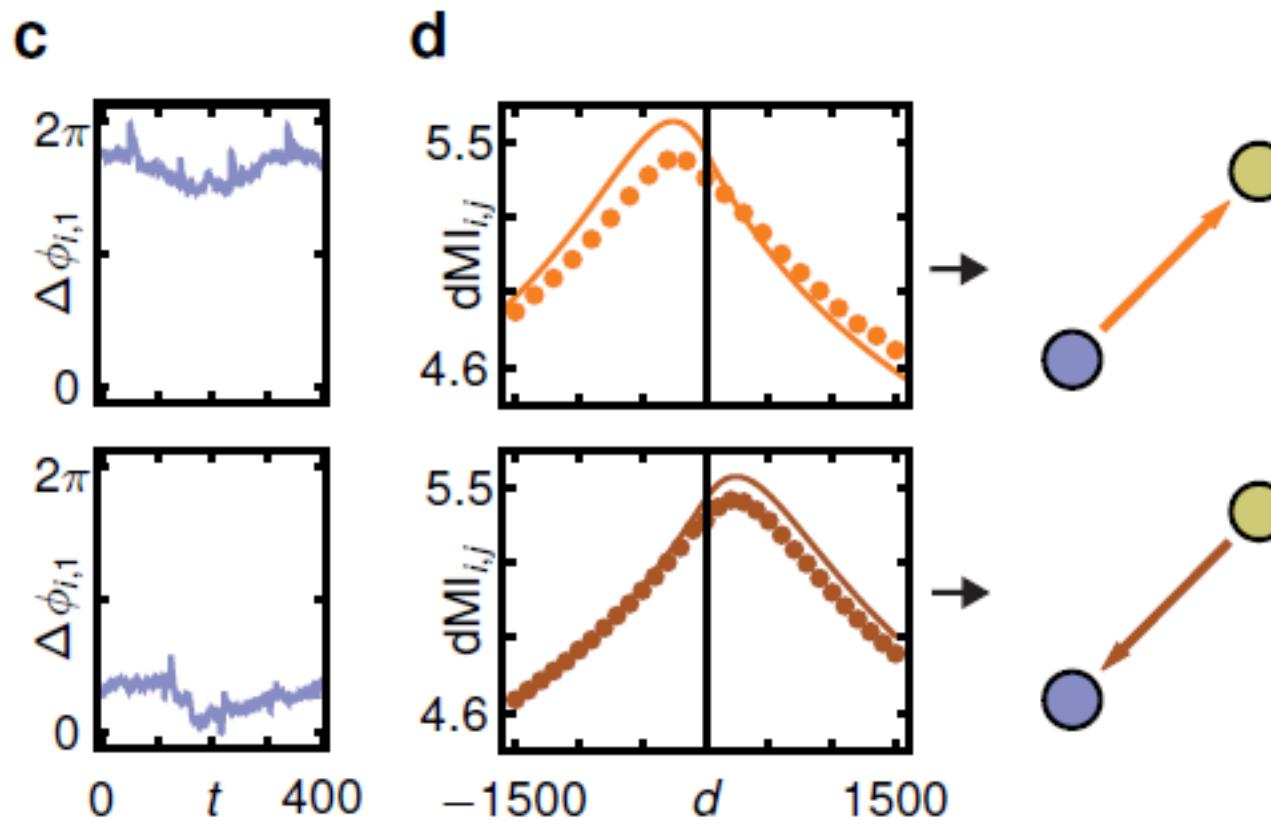
$$\text{purple pentagon} = 0$$

$$\text{purple pentagon} > 0$$

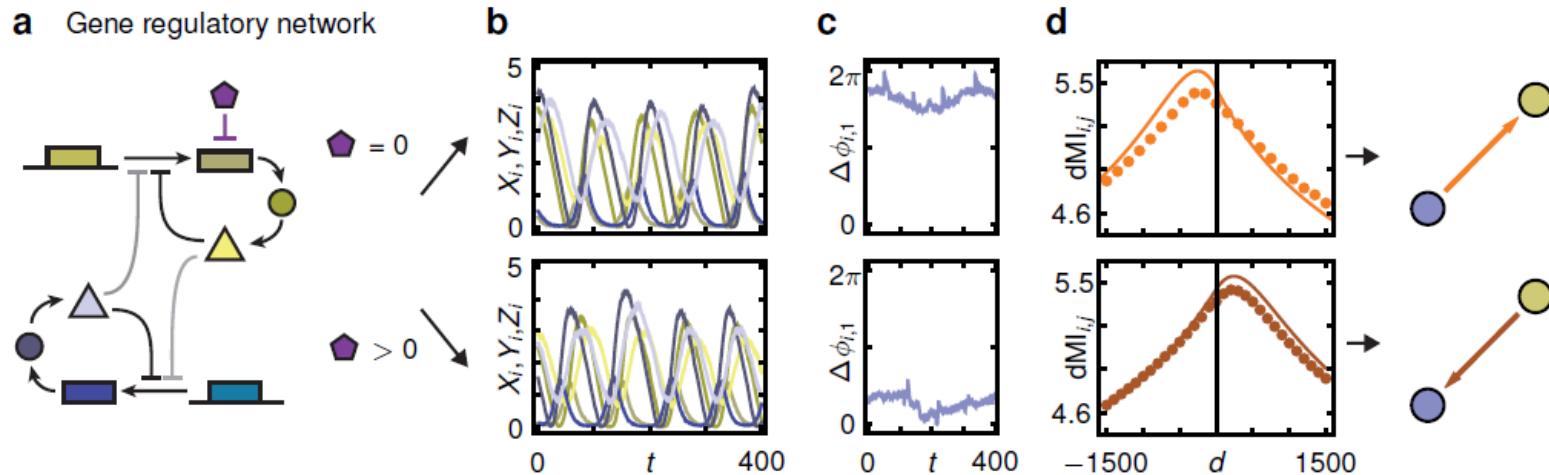
**b**



# Collective Dynamics → Information Routing Patterns I



# Collective Dynamics → Information Routing Patterns I



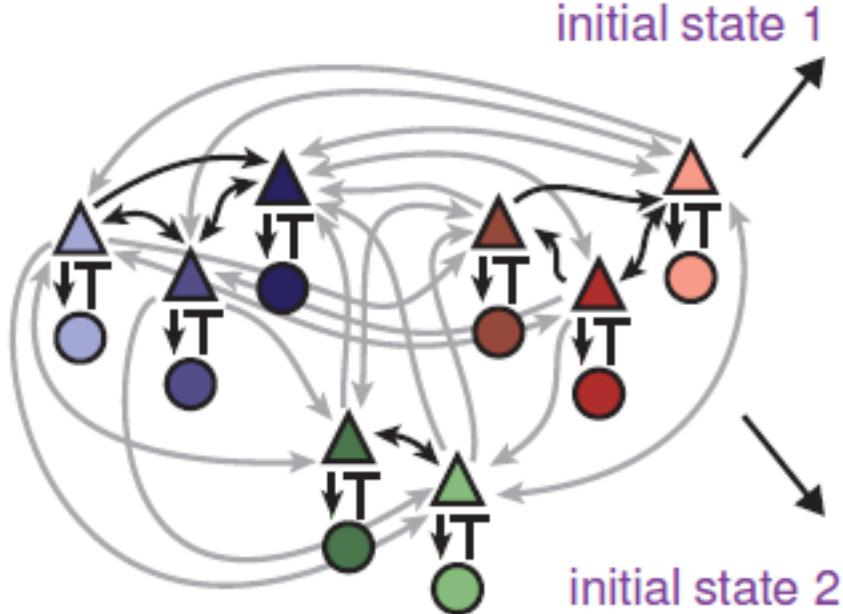
**External signals change information routing pattern**

Information sharing, quantified by delayed mutual information (dMI)

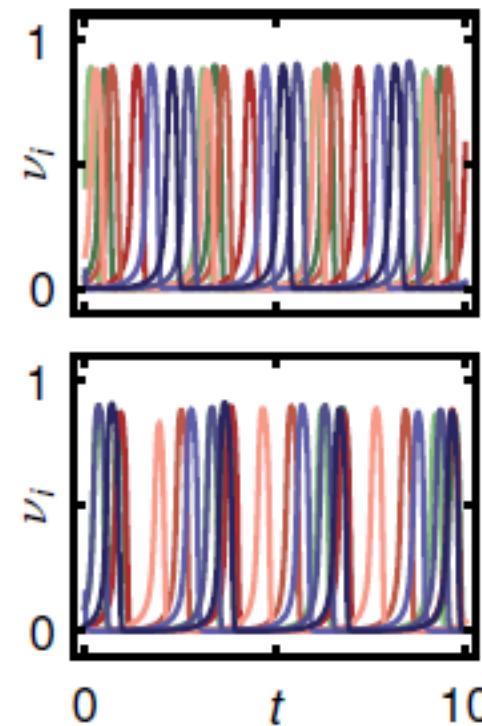
$$dMI_{i,j}(d) = \int p(\phi_{i,t}, \phi_{j,t+d}) \log \left( \frac{p(\phi_{i,t}, \phi_{j,t+d})}{p(\phi_{i,t}) p(\phi_{j,t+d})} \right) d\phi_{i,t} d\phi_{j,t+d}$$

# Collective Dynamics → Information Routing Patterns II

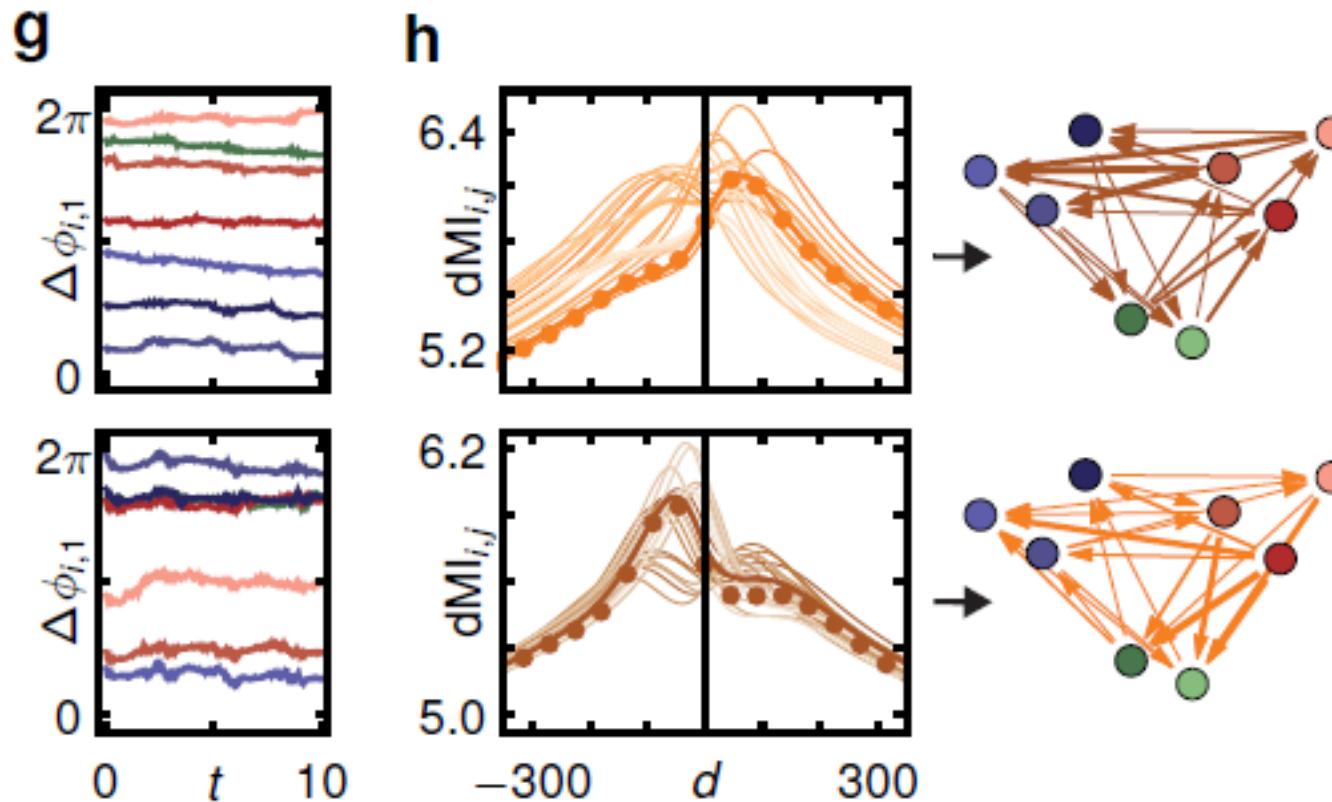
e Neuronal network



f

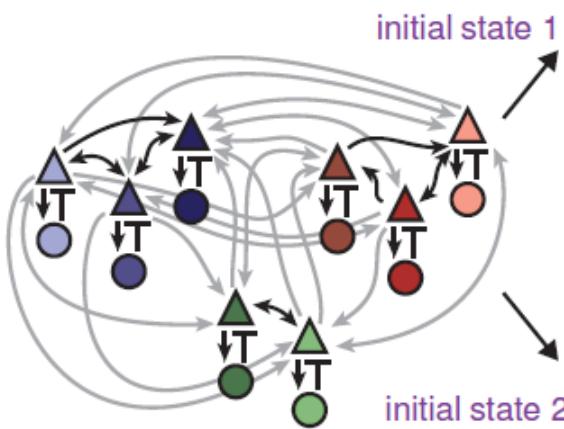


# Collective Dynamics → Information Routing Patterns II

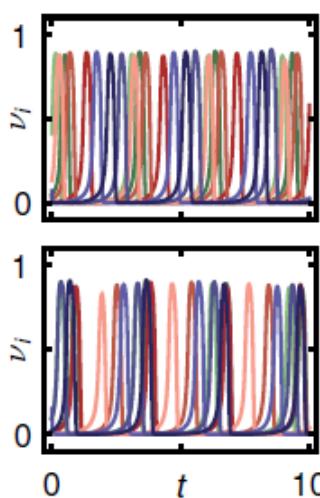


# Collective Dynamics → Information Routing Patterns II

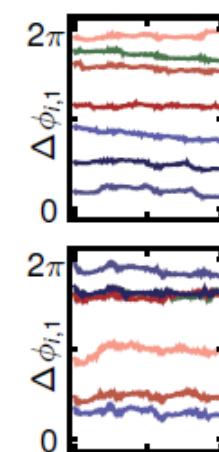
e Neuronal network



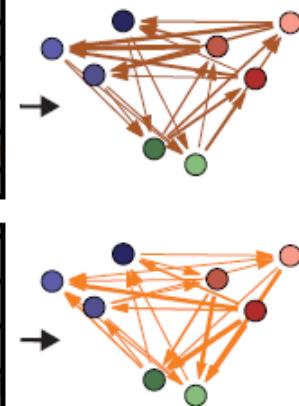
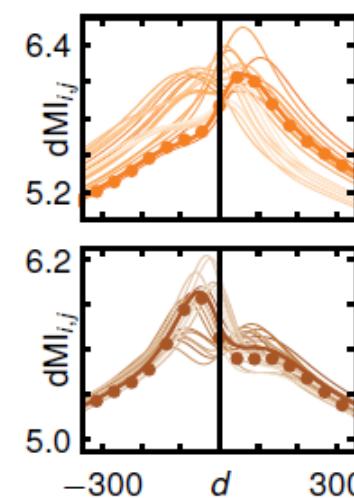
f



g

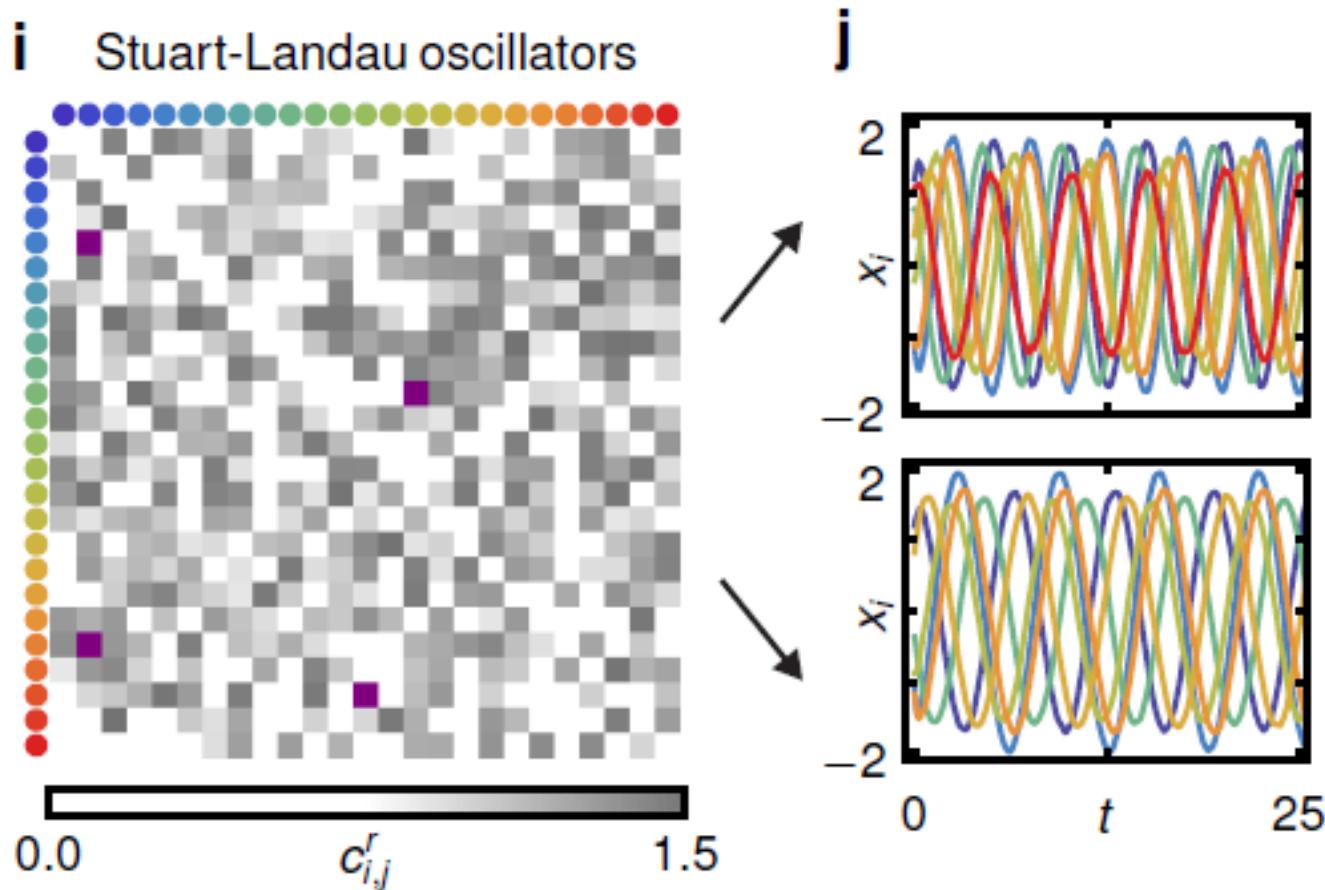


h

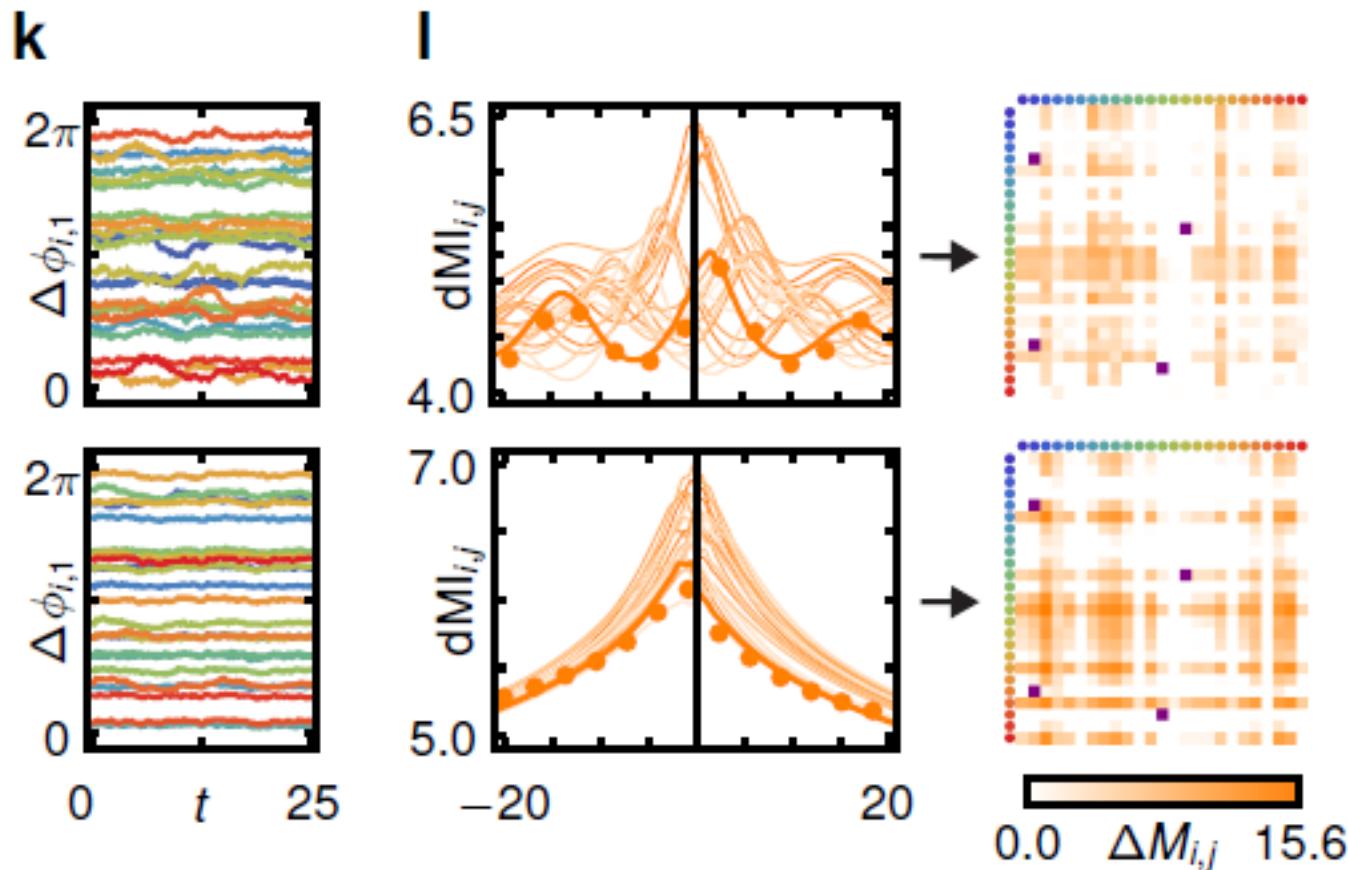


**Initial states determine information routing pattern**

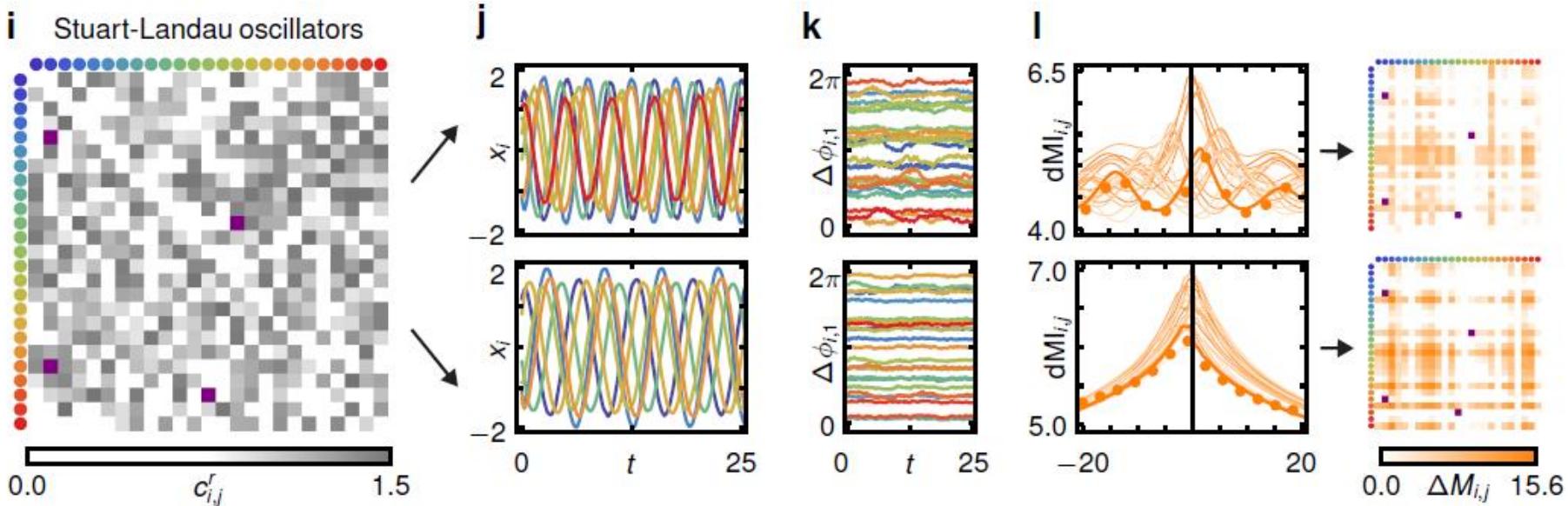
# Collective Dynamics → Information Routing III



# Collective Dynamics → Information Routing III



# Collective Dynamics → Information Routing III



Local changes modify information routing patterns

Dynamic & flexibly controllable patterns of IRPs  
in large networks

# Analytical Prediction of Information Routing Patterns

1. Phase reduction of oscillators
2. Averaging coupling (over cycle)

$$\frac{d}{dt}\phi_i = \omega_i + \sum_{j=1}^N \gamma_{ij}(\phi_i - \phi_j) + \sum_{k=1}^N \varsigma_{ik}\xi_k$$

3. Joint probabilities
4. Integration of dMI

$$dMI_{ij}(d) = \frac{k_{ij(d)} I_1(k_{ij(d)})}{I_0(k_{ij(d)})} - \log(I_0(k_{ij(d)}))$$

$I_n(k)$  : n-th modified Bessel function

$k_{ij(d)}$  : inverse of variance of von Mises distribution  
„Gaussian on circle“

# Information Routing Patterns (IRPs) between subnetworks

$$\frac{d}{dt} \phi_i = \omega_i + \sum_{j=1}^N \gamma_{ij} (\phi_i - \phi_j) + \sum_{k=1}^N \varsigma_{ik} \xi_k$$

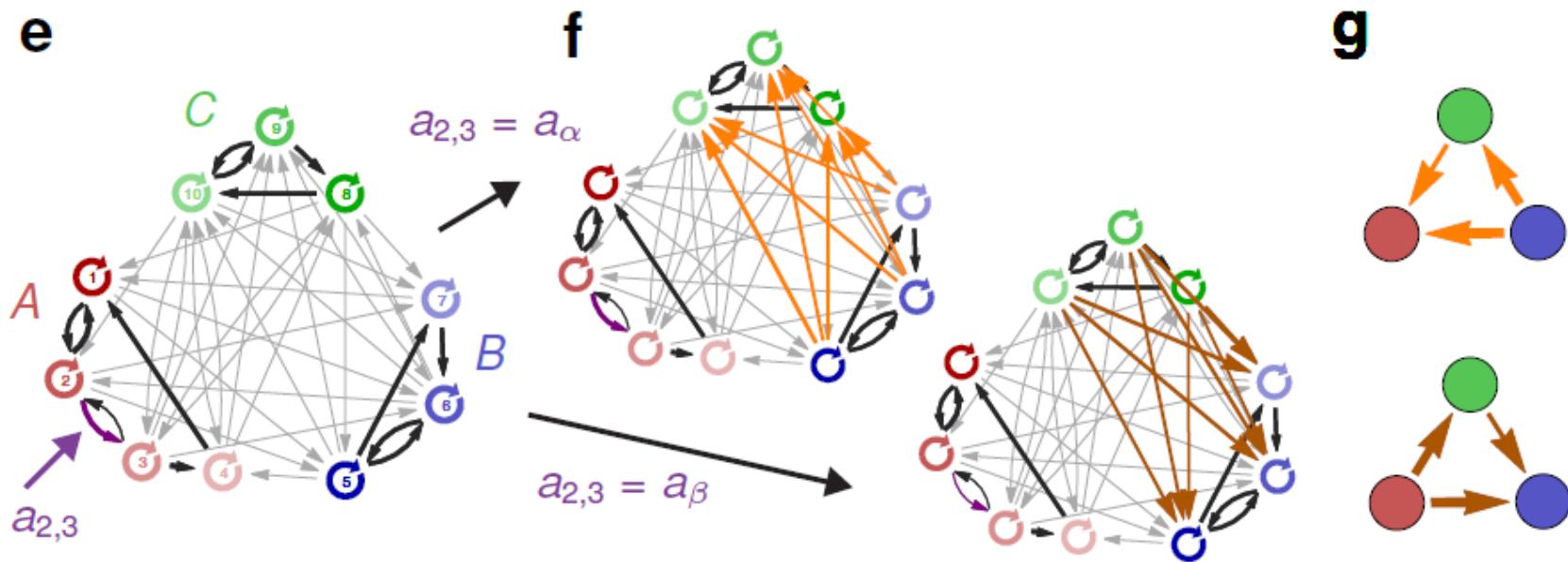
$\Phi_X$  : effective phase of subnetwork X

... other effective quantities ...

$$\frac{d}{dt} \Phi_X = \Omega_X + \sum_{Y=1}^M \Gamma_{X,Y} (\Phi_X - \Phi_Y) + \sum_{Y=1}^M \Sigma_{X,Y} \Xi_Y$$

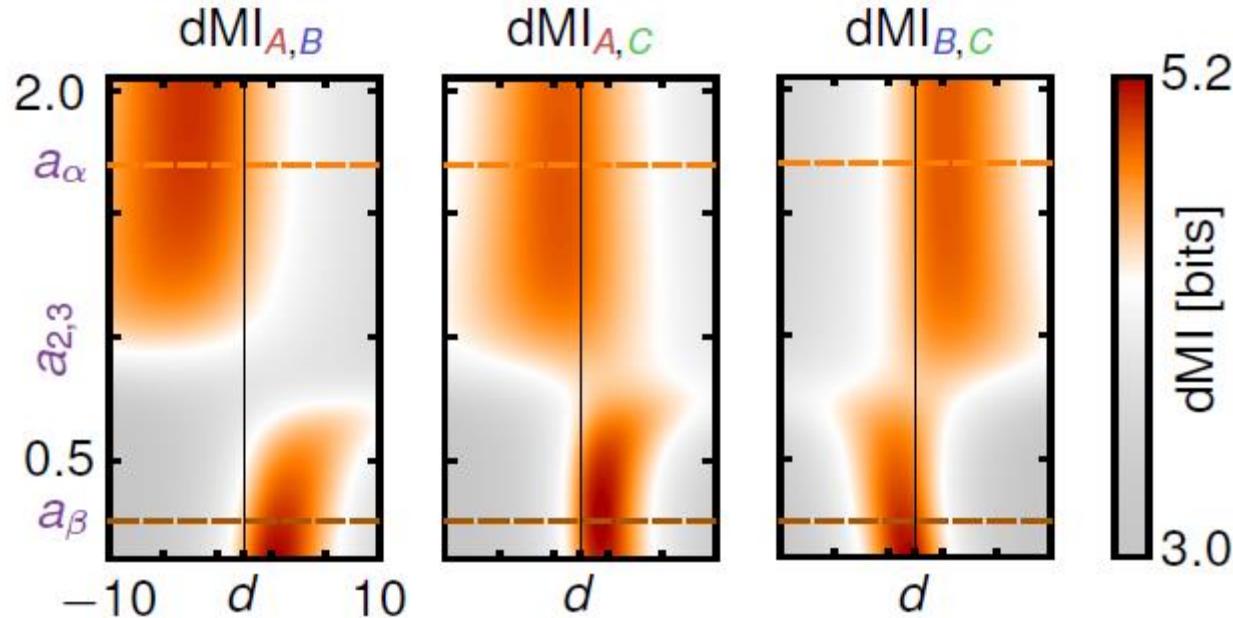
Theory naturally generalizes to predict IRPs  
between entire subnetworks

# Remote Control of Information Routing



**Local Structural Changes → Nonlocal Changes in IRP**

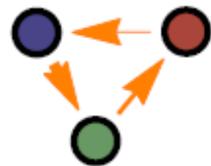
# Remote Control of Information Routing



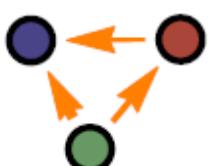
**Local Structural Changes → Nonlocal Changes in IRP**

# Combinatorial #IRPs possible

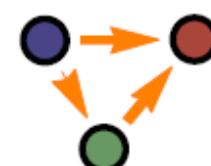
$[\alpha_A \alpha_B \alpha_C]$



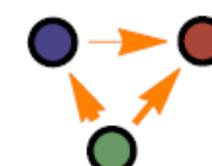
$[\alpha_A \alpha_B \beta_C]$



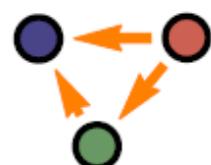
$[\alpha_A \alpha_B \beta_C]'$



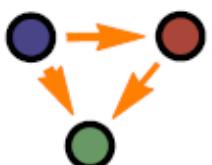
$[\alpha_A \beta_B \alpha_C]$



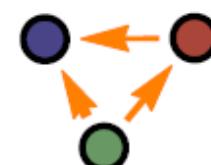
$[\alpha_A \beta_B \alpha_C]'$



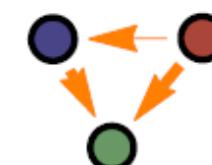
$[\alpha_A \beta_B \beta_C]$



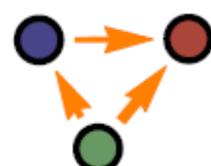
$[\beta_A \alpha_B \alpha_C]$



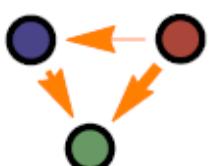
$[\beta_A \beta_B \alpha_C]$



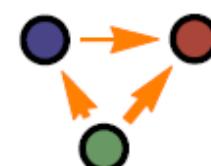
$[\beta_A \beta_B \alpha_C]'$



$[\beta_A \beta_B \alpha_C]''$

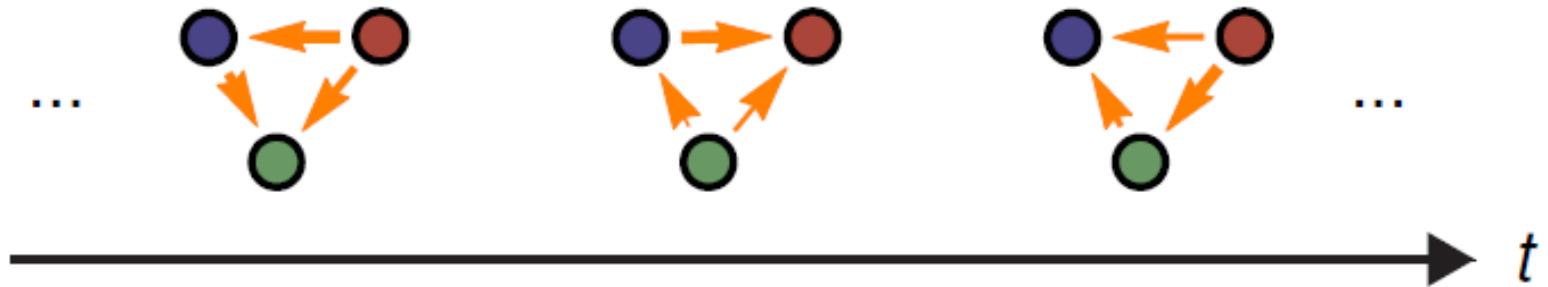


$[\beta_A \beta_B \beta_C]$



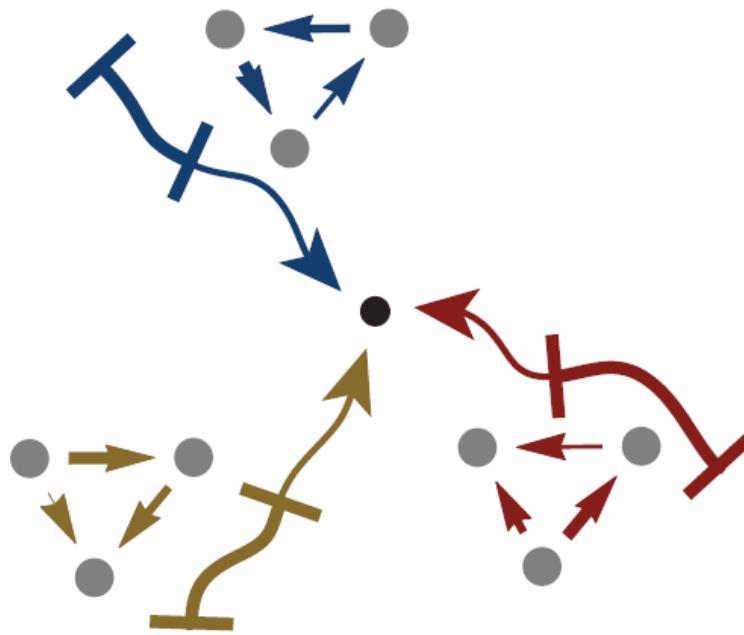
# Temporally Changing Routing

$[\beta_A \alpha_B \beta_C]$



Collective dynamics close to complex periodic orbit  
→ temporally changing phase order  
→ temporally changing IRPs

# Flexible Routing via Transients



**Each transient** may exhibit **individual IRP**  
despite the fixed point („ground state“) exhibits one IRP

# Summary

## Collective states determine information routing patterns (IRP)

IRP selected by:

- **local unit/coupling features**
- **network connectivity**
- **initial condition**
- **external driving signals**

Analytic description for phase oscillator networks

Coarse-graining: theory generalizes to IRP between entire subsystems

Possible features of network dynamics:

Remote control

Combinatorial #IRPs

Temporally changing IRPs

**Theory of Dynamic Information Routing in Networks**

# Network Dynamics

## Dynamics of Biological (& Bioinspired) Networks:

distributed processing & self-organized control

*Nature Phys.* (2010);

*Nature Comm.* (2016);

*J. Neurosci.* (2015);

*Phys. Rev. Lett.* (2012c);

*PLoS Comput. Biol.* (2011, 2012, 2013, 2015);

*New J. Phys.* (2011);

*Frontiers Neurosci.* (2011, 2012, 2013)

*Phys. Rev. X* (2013, 2014);

**spike-based processing & non-additive neural circuits:**

2 BMBF + 1 DFG grants (2004-2016)

## Dynamics of Networks in Engineering:

communication, power grids & flexible mobility

*Phys. Rev. Lett.* (2016, 2012b); *New J. Phys.* (2012, 2015); *Chaos* (2014); *EPJST* (2013);

*EPJB* (2014); *IEEE Trans. Power Syst.* & *IEEE Trans. Mobile Comput.* (tbp, 2016)

~~power grids: 2 BMBF grant (2013), Mio Euros (2013-2014)~~ 1.6-1955 (2016)

~~flexible mobility: EFRE grant 2.96 Mio Euros (pending, 2016-2018)~~

## Theoretical & Mathematical Foundations:

*Phys. Rev. Lett.* (2015, 2012a);

*Nature Phys.* (2011);

*SIAM J. Appl. Math.* (2010);

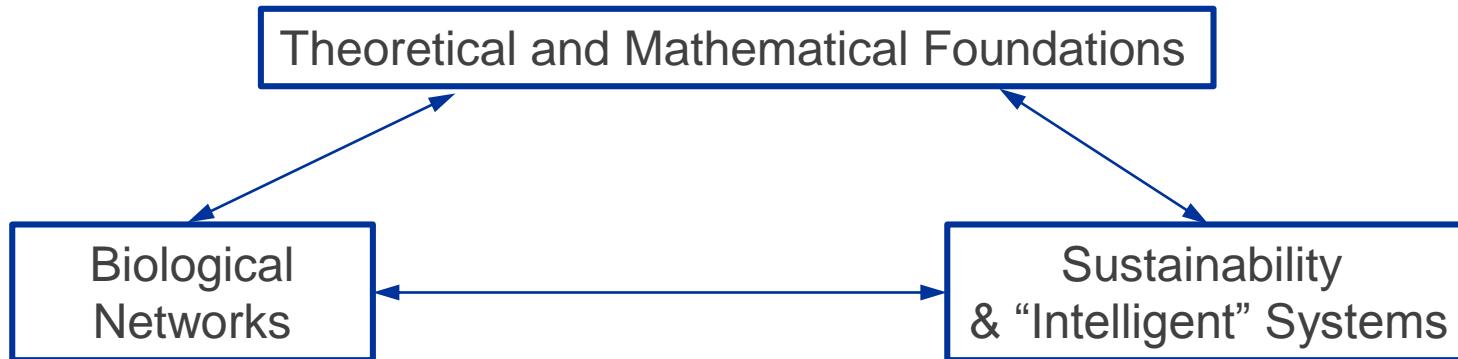
*Chaos* (2014)

*Discr. Cont. Dyn. Syst.* (2010)

*SIAM J. Appl Dyn. Syst.* (2012);

*New J Phys.* (2012)

# Current Topics in Network Dynamics



## Selected recent publications

*Nature Phys.* (2010 + 2011)  
*Phys. Rev. Lett.*  
(2010, 2011, 2012a,b,c, 2015, 2016)  
*Phys. Rev. X* (2013 + 2014)  
*New J. Phys.* (2012a,b, 2015);

*Nature Comm.* (2016)  
*Frontiers in Comput. Neurosci.*  
(2009a,b, 2011, 2012, 2013)  
*PLoS Comput. Biol.*  
(2011, 2013, 2014, 2015)  
*J. Neurosci.* (2015)

*Discr. Cont. Dyn. Syst.* (2010)  
*SIAM J. Appl Dyn. Syst.* (2012)  
*SIAM J. Appl. Math.*, (2010)

*patent* WO 2013178237 (2013)  
*patent registration* 16154441.6-1955 (2016)  
*IEEE Trans. Mobile Comput.* (tbp 2016);  
*IEEE Trans. Power Syst.* (tbp 2016); *Chaos* (2014);  
*Eur. Phys. J. Special Topics* (2014);  
*Eur. Phys. J. B* (2013);

physics  
bio- & neuro-  
science

appl. mathem.

engineering

# Collaboration & Support

## Network Dynamics

Jose Casadiego

Dimitra Maoutsas

Florencia Noriega

Malte Schroeder

Wen-Chuang Chou

Debsankha Manik

Diemut Regel

Nahal Sharafi

Hauke Haehne

Nora Molkenthin

Benjamin Schaefer

Xiaozhu Zhang

S. Herminghaus (**MPIDS**) A. Fiala, F. Wörgötter (**Göttingen**) J. Nagler (**ETH Zurich**)

Christoph Kirst (**Rockefeller**) Demian Battaglia (**Marseille**) Martin Greiner (**Aarhus**)

R.-M. Memmesheimer (**Columbia**) Mor Nitzan (**Hebrew U**) S. G. Shandilya (**Yale**)

Dirk Witthaut (**FZ Julich**) Martin Rohden (**JU Bremen**) S. Hallerberg (**IBM Dublin**)

Rudolf Sollacher (**Siemens**) Th. Walter (**EasySmartGrid**) Joh. Klinglmayr (**LCM**)



Deutsche  
Forschungsgemeinschaft



Bundesministerium  
für Bildung  
und Forschung



Forschungsinitiative der Bundesregierung

Marc Timme, Network Dynamics, TU Darmstadt & U Göttingen & MPI f. Dynamics & Self-Organization

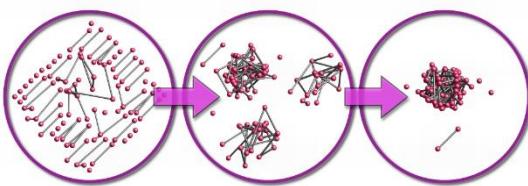
Thank you to  
my colleagues, collaborators,  
my research group **Network Dynamics**,

[www.networkdynamics.info](http://www.networkdynamics.info)



and you all for your attention & interest  
Questions and comments welcome!

# Recent Progress Theoretical and Mathematical Foundations

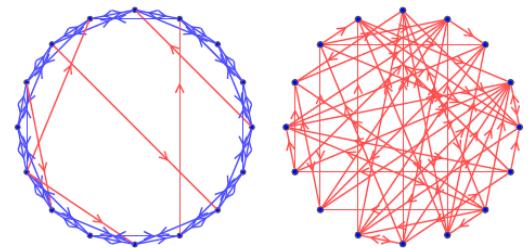


## Explosive Transition in Network Growth

competition → transition between 1st & 2nd order  
**single link matters!**

*Nature Phys.* (2011);

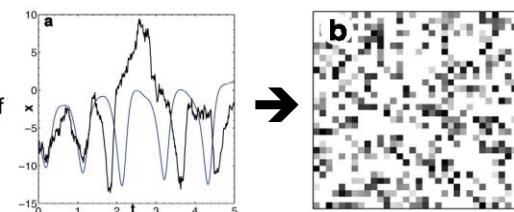
see also Nagler & Co, *Nature Comm.* (2013), *Phys. Rev. X* (2013)



## Small World Networks

**structure:** Watts & Strogatz, *Nature* (1998), >21000 citations  
now relaxation **dynamics**

*Phys. Rev. Lett.* (2012a)



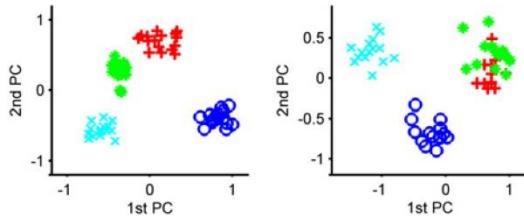
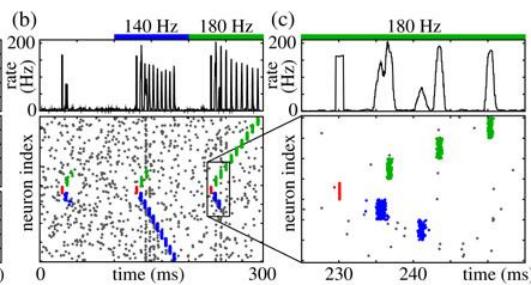
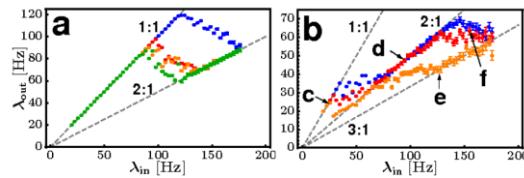
## Network Inverse Problem: Structure from Dynamics

Methods for **inference** and **design**

*Front. Comput. Neurosci.* (2011); *New. J. Phys.* (2011);

**Invited Topical Review:** *J. Phys. A* (2014)

# Recent Progress Neuro- and Biophysics



## Spike Sequence Processing in Neurons

Resource limitations

→ generic non-monotonic response

*BMC Neurosci.* (2013); *PLoS Comput. Biol.* (under review)

## Non-additive Coupling & Plasticity

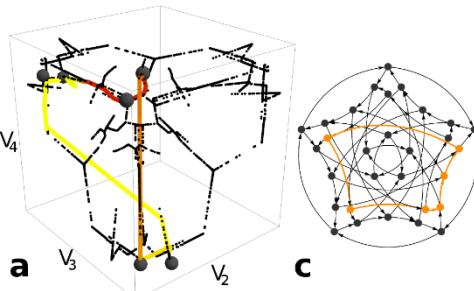
→ spike patterns & memory

*PLoS Comput. Biol.* (2012, 2013); *Phys. Rev. X* (2012)

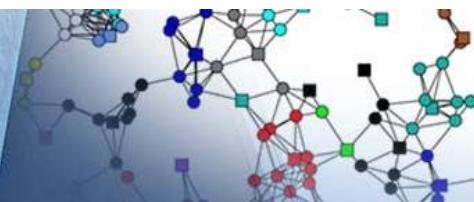
## Combinatorial Neural Processing exploiting network heterogeneities

joint PhD students with A. Fiala (Uni-Bio); *PLoS ONE* (2011);

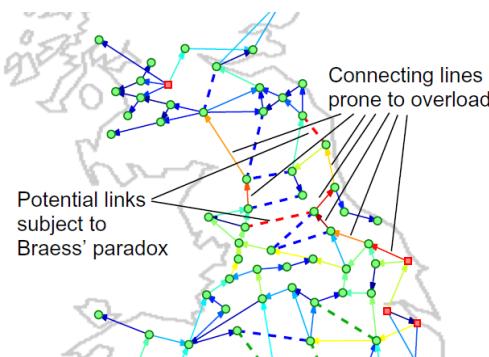
# Recent Progress – Intelligent Dynamical Systems f. Computation & Engineering



Intelligent Dynamical Systems  
universal **heteroclinic computing**  
with exponential capacity  
*Phys. Rev. Lett. (2012b)*



Communication Networks & Robotics  
distributed, stochastic & adaptive control  
→**self-organize versatile functions**  
*Nature Phys. (2010); New J. Phys. (2012a); patent (2013)*



Dynamically Smart Power Grids  
**decentralization helps, Braess' paradox hinders**  
*Phys. Rev. Lett. (2012c), New J. Phys. (2012b)*