Periodic Trajectories in Saltation Transport

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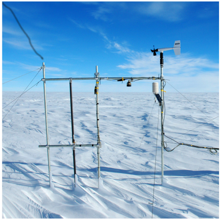
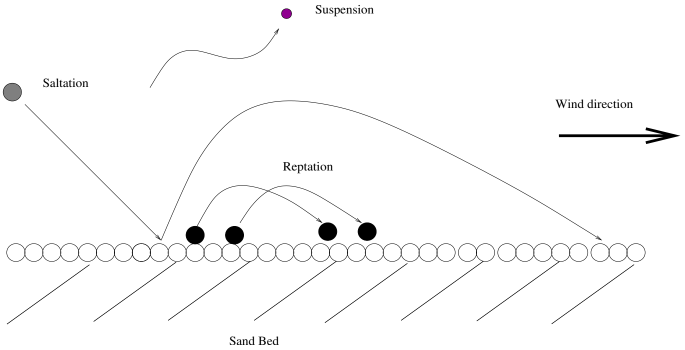
Saltation Transport

- Aeolian sand transport (air)
- Snow drift (air)
- Bed load transport (water)
- Saltation transport on extra-terrestrial planets

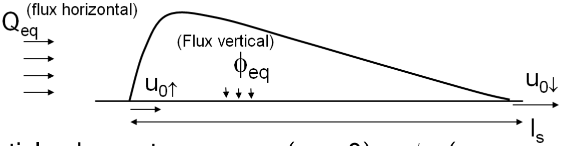

Aeolian saltation transport (Bagnold, 1941)



- Sand density : $\sigma = \rho_p/\rho_{air} = 2200$
- Mode of transport :
  Saltation : $d \approx 0.1 - 0.6\ mm$
  Suspension : $d < 0.1\ mm$

Mass flow rate : Bagnold law

- Hypothesis : Trajectories replaced by an 'averaged' trajectory of length l_s



  - Particle shear stress : $s_{grain}(y=0) = \phi_{eq}(u_{0\downarrow} - u_{0\uparrow})$

- Momentum conservation :
  $s_{grain}(y) + S_{air}(y) = S_\infty = \rho_{air} u^{*2}$
- Mass flow rate :
  $Q_{eq} = l_s\, \phi_{eq} = \frac{l_s}{(u_{0\downarrow} - u_{0\uparrow})}(S_\infty - S_{air}(0))$

Mass flow rate : Bagnold law

- Bagnold assumptions :
  - $S_{air}(y=0) \approx 0$
  - $(u_{0\downarrow} - u_{0\uparrow}) \approx u_{0\downarrow} \propto u^*$
  - $l_s \propto (u^{*2}/g)$
- Bagnold cubic scaling law :

$$Q_{eq} = \frac{l_s}{(u_{0\downarrow} - u_{0\uparrow})} S_\infty \propto \frac{\rho_{air}}{g} u^{*3}$$

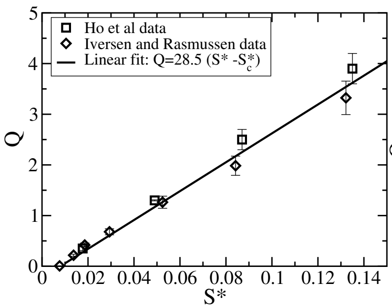
- Strong ressemblance with the Meyer-Peter and Müller law for bed-load transport in water :

$$Q \propto (S^* - S^*_c)^{3/2} \quad \text{where} \quad S^* = \rho_p u^{*2}/(\rho_p - \rho_f)gd \quad \text{(Shields)}$$

Experimental data : Aeolian Sand transport

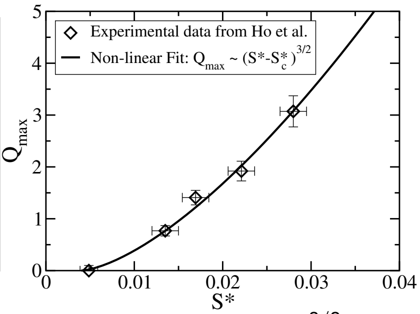
- Wind-tunnel experiments : Mass flow rate measurements
  (Iversen & Rasmussen 1990, Ho et al. 2011)

Erodible bed



$Q \propto (S^* - S_c^*)$

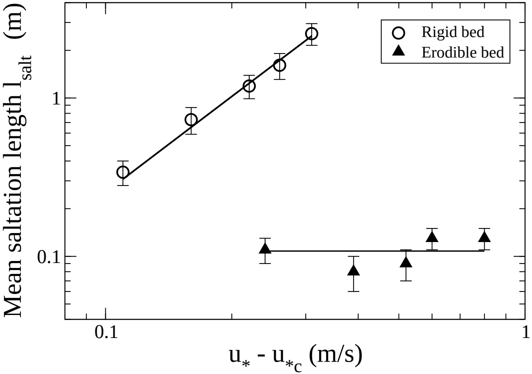
Rigid bed



$Q \propto (S^* - S_c^*)^{3/2}$

Experimental data : Aeolian Sand transport

- Mean saltation hop length : Erodible vs Rigid bed
  (Ho et al. 2011)



- Erodible bed : $l_s \approx cst$ & Rigid bed : $l_s \propto (S^* - S^*_c)$

Modeling Aeolian Sand Transport

- Different modeling approaches
  - Eulerian/Eulerian Approach :

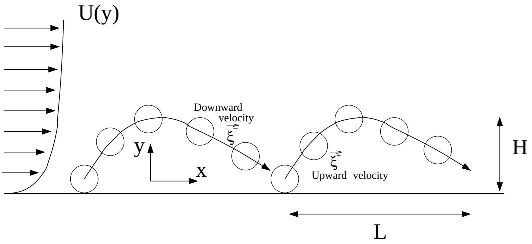
    Herrmann et al. 2001, Jenkins et al. 2011.
    Lammel et al. 2012, Pähtz et al. 2012.

  - Eulerian/Lagrangian Approach :

    Anderson and Haff 1988, Werner 1990
    Andreotti 2004, Cressels et al. 2009, Duran et al. 2012.

Motivation

- Develop the simplest Eulerian/Lagragian approach
- Idea : describe the Aeolian saltation transport in terms of a single periodic trajectory (instead of a distribution of trajectories)



- Consider the single trajectory to be the ensemble average of the trajectories that participate in a steady motion

Strategy and System parameters

- Phrase the motion equations as a two-point boundary value problem (solving simultaneously for ascending and descending motion)
- Characteristic scales :
  unit length : particle diameter d, unit velocity : $\sqrt{gd}$
  unit stress : $\rho_p gd$
- Dimensionless numbers :
  - Particle Reynolds number : $Re = d\sqrt{gd}/\nu$ ($\approx 1 - 10$)
  - Density ratio : $\sigma = \rho_p/\rho_{air}$ (≈ 2000)
  - Stokes number : $St = \sigma Re$ ($\approx 10^4$)
  - Shields number : $S^* = \rho_f {u^*}^2/(\rho_p - \rho_f)gd$ ($\approx 0.01 - 1$)

Particle Motion

Ascending Motion :

$$a_x^+ = \xi_y^+ \frac{d\xi_x^+}{dy} = D^+(U - \xi_x^+) \,, \quad (1)$$

$$a_y^+ = \xi_y^+ \frac{d\xi_y^+}{dy} = -D^+\xi_y^+ - 1 \,, \quad (2)$$

Descending Motion :

$$a_x^- = \xi_y^- \frac{d\xi_x^-}{dy} = D^-(U - \xi_x^-) \,, \quad (3)$$

$$a_y^- = \xi_y^- \frac{d\xi_y^-}{dy} = -D^-\xi_y^- - 1 \,, \quad (4)$$

Drag coefficient : $D = (0.3\sqrt{(U - \xi_x)^2 + \xi_y^2} + 18/Re)/\sigma$

Particle Motion

Downstream position x^+ and x^- are functions of y :

$$\xi_y^+ \frac{dx^+}{dy} = \xi_x^+ , \quad (5)$$

$$\xi_y^- \frac{dx^-}{dy} = \xi_x^- . \quad (6)$$

Fluid Motion

Steady and fully developped turbulent boundary layer

- Momentun conservation :

  $$S_{air}(y) + s_{particle}(y) = S^* = constant$$

- Mixing length turbulence model :

  $\sigma S_{air} = \kappa^2 l^2 |dU/dy|(dU/dy)$ with $l = (y + y_0)$

$$\Rightarrow \frac{dU}{dy} = \frac{\left[\sigma(S^* - s_{particle})\right]^{1/2}}{\kappa(y + y_0)}$$

Eulerian Velocity and Particle Shear Stress

We consider a steady state consisting in an ensemble of particles with periodic trajectories

We introduce the concentration of ascending and descending particles : c^+ and c^-

- Eulerian particle velocity :

$$u(y) = (c^+\xi_x^+ + c^-\xi_x^-)/(c^+ + c^-)$$

- Particle shear stress :

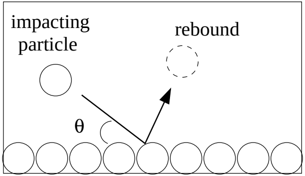
$$s(y) = -\left(c^+\xi_y^+\xi_x^+ + c^-\xi_y^-\xi_x^-\right)$$

Two-point boundary value problem

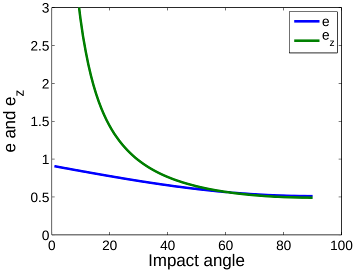
- Counting equations and unknown parameters
  - Seven first order differential equations for : ξ_x^+,ξ_x^-, ξ_y^+, ξ_y^-, x^+, x^- and U
  - Three additional unknown parameters :
    - the height H and the length L of the trajectory
    - the mass hold-up $M = \int_0^H c(y)dy$
- Continuity relations provide only 7 conditions
  - $\xi_x^+(H) = \xi_x^-(H)$, $\xi_y^+(H) = 0$ and $\xi_y^-(H) = 0$
  - $x^+(0) = 0$, $x^-(0) = L$ and $x^+(H) = x^-(H)$
  - $U(0) = 0$
- Need for additional conditions

Rebound on a rigid and bumpy surface

- Rebound law



Rigid and bumpy bed



$$e = \xi^+(0)/\xi^-(0) = (A - B\sin\theta)$$
$$e_y = \xi_y^+(0)/\xi_y^-(0) = -(A_y/\sin\theta - B_y)$$

$A = 0.9$, $B = 0.4$, $A_y = 0.5$ and $B_y = 0$

(values obtained from D.E.M simulation with $e_n = 0.8$)

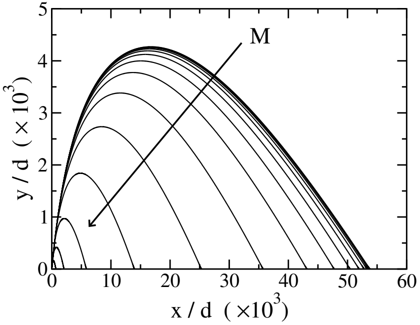
Additional boundary conditions

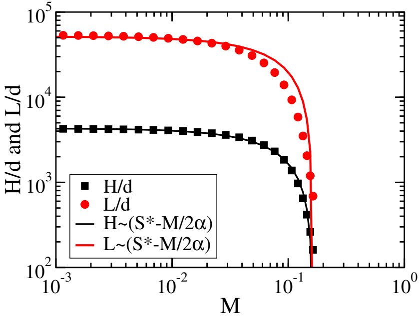
- Two additional conditions provided by the rebound law : $\xi^+(0) = e(\theta)\,\xi^-(0)$ and $\xi_y^+(0) = -e_y(\theta)\,\xi_y^-(0)$
- We have now $(7+2)$ boundary conditions for 7 first order differential equations + 3 unknown parameters
- For a given flow strength, there is therefore one free parameter (the mass hold-up M)

Model predictions : Particle trajectory

System parameters :
$Re = 0.73$ ($d = 0.23\ mm$), $\sigma = 2200$ and $S^* = 0.06$

- Varying the mass holdup at a fixed Shields number





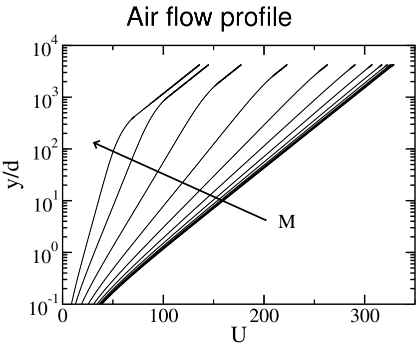
Trajectories decrease in size with increasing mass holdup

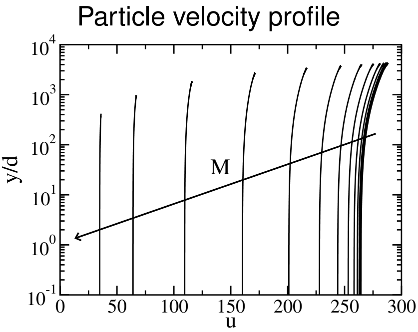
Model predictions : Air and particle velocity

System parameters :

$Re = 0.73$ ($d = 0.23\ mm$), $\sigma = 2200$ and $S^* = 0.06$

- Varying the mass holdup at a fixed Shields number





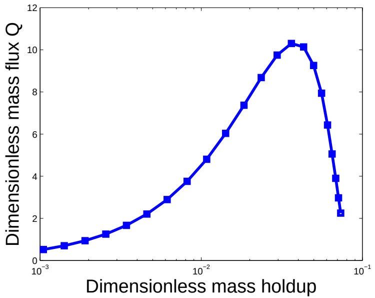
Air and particle velocity decrease with increasing mass holdup

Model Predictions : Mass flow rate

System parameters :
$Re = 0.73$ ($d = 0.23\,mm$), $\sigma = 2200$ and $S^* = 0.06$

- Varying the mass holdup at a fixed Shields number

Mass flow rate vs mass holdup



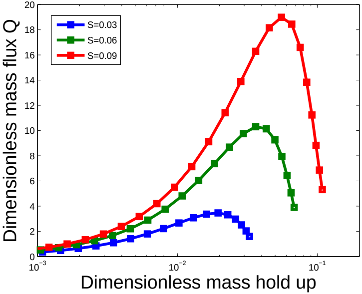
The mass flow rate exhibits a maximum

Model Predictions : Mass flow rate

System parameters :

$Re = 0.73$ ($d = 0.23\ mm$) and $\sigma = 2200$

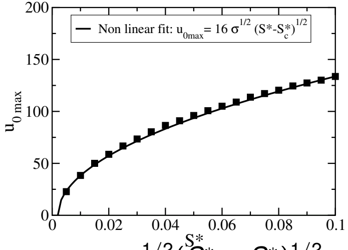
- Mass flow rate vs mass holdup for different Shields numbers

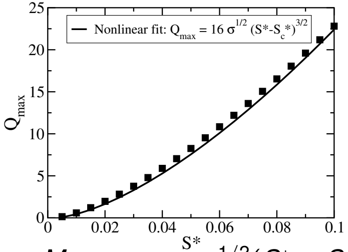


The mass flow rate and the mass holdup at maximum capacity both increase with increasing Shields number

Model Predictions : Flow at maximum capacity

- Trajectory height and length at maximum capacity : $L_{max} \propto H_{max} \propto (S^* - S_c^*)$
- Mass hold-up at maximum capacity : $M_{max} \propto (S^* - S_c^*)$
- Particle velocity and Mass flow rate at maximum capacity



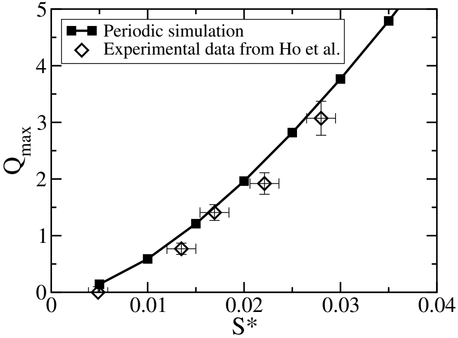


$u_{0max} \propto \sigma^{1/2}(S^* - S_c^*)^{1/2}$ $\quad Q_{max} \propto M_{max} u_{0max} \propto \sigma^{1/2}(S^* - S_c^*)^{3/2}$

Comparison with Experiments

- Wind-tunnel experiments on rigid and bumpy bed :
  (Ho, Phd Thesis 2012, Rennes)
  Parameters : $d = 0.230\ mm$ ($Re = 0.73$) and $\sigma = 2200$

Maximum capacity of transport vs Shields number



Extension of the Model for erodible beds

- Main differences between Rigid and Erodible bed :

  Finite supply & Inexhaustible source of particles

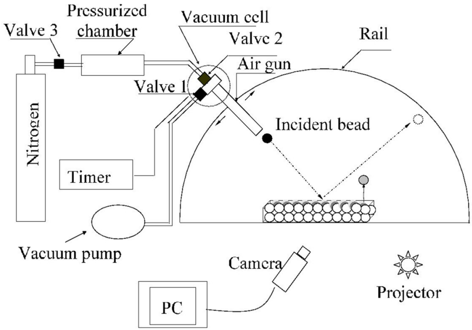
  No mass exchange between the bed and the flow & Mass exchange via the splash process

- Main issue : How to account for the splash process ?

Splash process

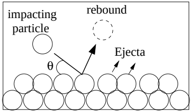
- Model Collision Experiment (Beladjine et al, PRE 2007)

  Particle parameters : $d = 6\ mm$ and $\rho_p = 2300\ kg/m^3$

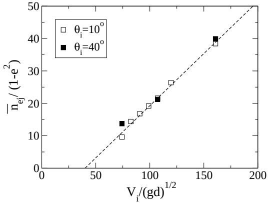


Splash process : Experimental outcomes

- Rebound : law similar to that on a rigid bed
- Ejections of particles



$N_{tot} = N_{rebound} + N_{ej}$



ξ_c Critical velocity for ejection

$$N_{tot}(\xi) = \begin{cases} 1 + N_0(1-e^2)\,(\xi/\xi_c - 1) & \text{if } \xi > \xi_c \\ 1 & \text{if } 1 \leq \xi \leq \xi_c \\ 0 & \text{if } \xi \leq 1 \end{cases}$$

- Steady State :

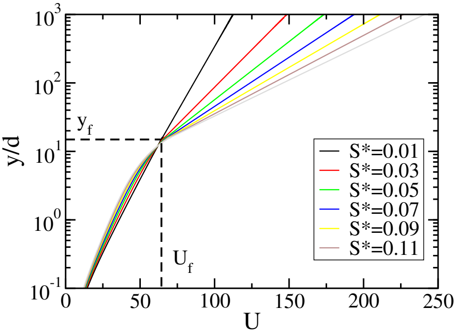
$$N_{tot} = 1 \Rightarrow 1 < \xi_0 < \xi_c \Rightarrow \xi_0 \approx (1+\xi_c)/2 \approx 20$$

Additional condition leaving the system without any free parameter

Model predictions : Air velocity

System parameters :
$Re = 0.73$ ($d = 0.23\,mm$) and $\sigma = 2200$

- Air velocity profiles : $0.01 < S^* < 0.1$

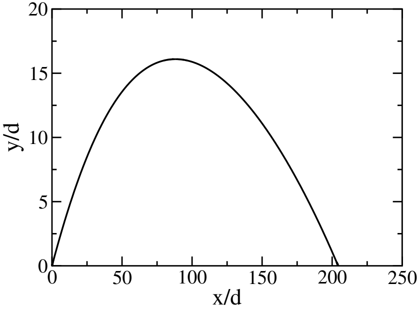


Log profiles with a focus point (Bagnold 1941)

Model predictions : Periodic trajectories

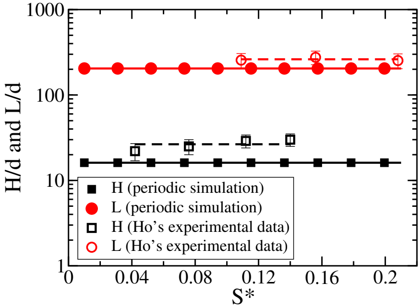
System parameters :
$Re = 0.73$ ($d = 0.23\ mm$) and $\sigma = 2200$

- Saltation trajectory



Trajectory invariant with Shields

- Saltation Height and Length

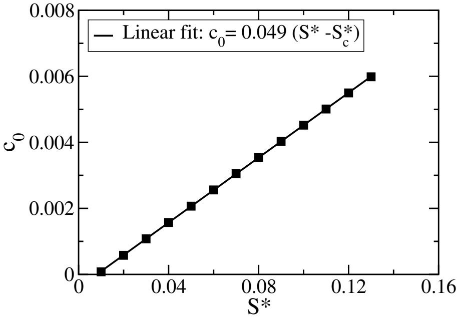


***H* and *L* invariant with Shields**

Model predictions : concentration and mass flux rate

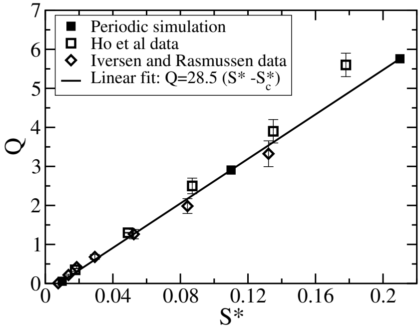
System parameters :
$Re = 0.73$ ($d = 0.23\,mm$) and $\sigma = 2200$

- Concentration at the bed



$$c_0 \propto (S^* - S^*_c)$$
and $M \approx c_0 H \approx (S^* - S^*_c)$

- Mass flow rate



$$Q \propto (S^* - S_c)$$

Conclusion and Perspectives

- Conclusion
  - Simple predictive model for saltation transport over erodible and rigid bed
  - Two different saltation regimes :
    -Unlimited saltation : $Q \propto (S^* - S_c^*)^{3/2}$
    -Splash-limited Saltation : $Q \propto (S^* - S_c^*)$
- Perspectives
  - Application to bed load-transport in water
  - Application to saltation transport on other planetary aeolian environments

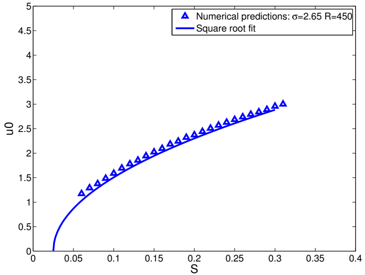
Sediment transport in water

- Differences with transport in Air :
  - $\sigma = \rho_p / \rho_{fluid} \approx 2.6$
  - $1 < St = \sigma Re < 1000$
- Model modifications :
  - Particle motion equations :
    Replace $g \to g' = (1 - 1/\sigma) g$ (Buoyancy)
  - Collision :
    $e \to e' = e - 6.9(1 + e)/St$ (Lubrication force)

Aquatic transport over Rigid bed

System parameters :
$Re = 450$ ($d = 0.23\ mm$), $\sigma = 2.65$ and $St = 1200$ (no lubrication)

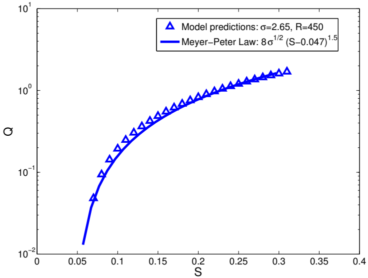
- Particle velocity at the maximum capacity



$$u_0 \propto (S^* - S_c^*)^{1/2}$$

$u_0 \ll \xi_c \approx 40 \;\Rightarrow$ Splash not triggered

- Mass flux at the maximum capacity Q



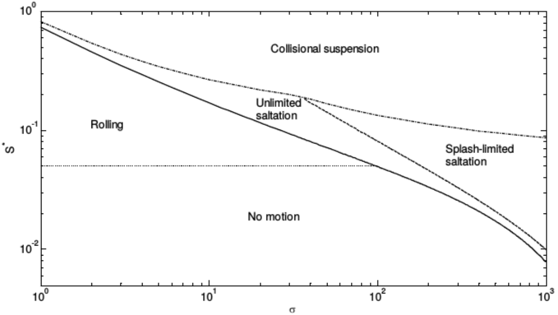
$$Q \propto (S^* - S_c)^{3/2}$$

- Aquatic bed-load transport Mass : Unlimited saltation transport

Saltation transport on extra-terrestial atmospheres

- General phase diagram for saltation transport (Berzi et al. JFM 2016)

$St = 1000$



- Venus :$\sigma = 80$ and Titan : $\sigma = 200$
  $\Rightarrow$ Expected transition from unlimilted to splash-limited saltation