Advanced Study Group 2018/2019

Forecasting with Lyapunov Vectors

(Convenor: Marcus Werner Beims, Federal University of Paraná, Brazil)

1 Background

The motivation of our ASG was to study the applicability of the alignment of covariant Lyapunov vectors for the prediction of extreme events in distinct complex systems, ranging from physical and chemical systems to biophysics and fluid dynamics. Covariant Lyapunov vectors can be used to quantify local hyperbolicity [1] in dynamical systems, to predict extreme events in chaotic systems [2] and are useful to describe spatial properties of complex systems. In addition to the alignment of Lyapunov vectors, we were also interested in times series data forecasting using machine learning techniques.

2 People and activities

To study the above subject from different perspectives, the three members of the ASG were chosen due to their expertise in related fields. JAC Gallas from the Federal University of Paraíba (Brazil), applied nonlinear dynamics: lasers physics, chemical and biochemical dynamics; I Jánosi from the Eötvös Loránd University (Hungary), environmental fluid dynamics and nonlinear time series analysis; and MW Beims from the Federal University of Paraná (Brazil), classical and quantum nonlinear dynamics.

The ASG started in November 2018 with the three team members and ended in February 2019. In the beginning of the ASG the three members discussed some strategies and possibilities to achieve the goal of the ASG. For this, we invited 8 short visitors from South and North America and from Europe. We also organized a 2-days miniworkshop which provided an opportunity to present a number of ideas and have a brainstorming session among participants and members of the MPIPKS. The program of the miniworkshop is presented in the table bellow. Many students and researchers from the MPIPKS, specially from the group of Prof Kantz, attended the miniworkshop. Two papers written during the ASG were already accepted for publication [3, 4], two submitted for publication [5, 6] and 11 are in preparation [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] .

	Tuesday Jan 22nd 2019
9:30-10:00	Daniel Weingärtner
Title:	Using Machine Learning to predict changes, duration and "the future" in the Chaotic Lorenz System
10:00-11:00	Brian Hunt
Title:	Reservoir Computing: a Machine-Learning Approach to Forecasting and Attractor Reconstruction from Time Series Data
11:00-11:30	Jonathan Brisch
Title:	Scale dependent error growth
	Wednesday Jan 23th 2019
9:30-10:30	Imre Jánosi
Title:	Lagrangian analysis of ocean surface flows + A few questions about vortex census
10:30-11:30	Támas Tél
Title:	Death and revival of chaos

3 Scientific results

The scientific results for the period of four months can be presented in two main parts related to forecasting.

In the first part we discussed extensively the applicability of the alignment property of covariant Lyapunov vectors to predict extreme events in the Lorenz systems, in tumor growth models, in chemical models, in Lagrangian manifolds and its relation to the stability of power grids. We have found that the alignment is able to predict events in time series obtained from the integration of differential equations from the mentioned models when treated in the chaotic regime. On the other hand, the angle between the covariant Lyapunov vectors does not give additional information to the dynamics of Lagrangian manifolds, as nicely demonstrated in the talk of I Janosi (see miniworkshop program). A definitive answer about the relation between the alignment of Lyapunov vectors and the stability of power grids is still missing, but under analysis.

In the second part we were interested in applying machine learning techniques to predict the evolution of time series in the dynamical systems mentioned above. Machine learning techniques used directly for the prediction of times series is an actual problem and of special interest. About this subject we had the overviews from B Hunt and D Weingärtner (see miniworkshop program). In distinction to Hunt's methodology, we transformed a reservoir computing network in a classification problem. Transitions between the seasons and their duration can be predicted with great accuracy by means of counting and classification strategies, for which we train Multi-Layer Perceptrons ensembles. Even the longest seasons can have predicted their occurrences and duration. We also show the use of Recurrent Neural Networks architectures as Long Short-Term Memory and Echo State Network to generate data with accuracy up to few hundreds time steps.

4 Summary and outlook

Many new projects were started as a consequence of our ASG. These projects combine the analysis of covariant Lyapunov exponents and machine learning techniques to predict events (extreme or not) in chaotic dynamical systems. These includes physical, chemical and biophysical systems and Lagrangian manifolds in fluid dynamics. The expectation for the future is to improve the ability to reproduce (and predict) chaotic times series for longer times. Nowadays, the longest reproducible time in this context is around eight Lyapunov times. Comparisons between the usual machine learning architectures and the classification technique will be analysed in more details.

References

[1] M.W.Beims and J.A.C.Gallas, *Alignment of Lyapunov Vectors: A Quantitative Criterion to Predict Catastrophes?* Sci.Rep., **6**, 3710 (2016).

[2] M.W.Beims and J.A.C.Gallas, *Predictability of the onset of spiking and bursting in complex chemical reactions*, PCCP **20**, 18539 (2018).

[3] Gonzalo Marcelo Ramírez-Ávila, Imre M. Jánosi, and Jason A. C. Gallas *Two-parameter areal scaling in the Hénon map*, Europhysics Letters **125**, ?? accepted (2019).

[4] A.Calderon-Cardenas, F.W.Hartl, J.A.C. Gallas, and H.B.V. de Albuquerque *Modeling the triple-path electro-oxidation of formic acid on platinum: cyclic voltammetry and oscillations* Catalysis Today **330**, ?? accepted (2019).

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[7] G.M. Ramirez-Avila, I.M. Jánosi, and J.A.C. Gallas, *Two-parameter areal scaling in the Hénon map.*

[8] G.M. Ramírez-Ávila, S. Depickére, I. M. Jánosi, M. R. Gallas, and J. A. C. Gallas, *Arithmetic progressions of spiking and bursting in Rulkov's neuron model.*

[9] M.W. Beims, I. M. Jánosi, H. Varela, and J.A.C. Gallas, *Alignment of Lyapunov vectors on the triple-path electro-oxidation of formic acid on platinum.*

[10] Imre M. Jánosi, D. Silhavy, J. Tamás, and P. Csontos, *Perennials precisely measure the length of winter and adjust flowering dates.*

[11] E.L.Brugnano, J.A.C.Gallas and M.W.Beims, *Prediction of seasonal changes in the Lorenz system.*

[12] M.Ramírez-Ávila, M.W.Beims and J.A.C.Gallas, *Alignment of Lyapunov vectors and tumor growth.*

[13] J.M.Garreau and M.W.Beims, *Distance correlation in the quantum standard map.*

[14] P.Lind, J.A.C.Gallas, and M.W.Beims, *Applying Lyapunov vectors to assess the stability of power grids.*

[15] S. Sokołowski and J.A.C. Gallas, *Molecular dynamics simulation of particle deposition in large channels.*

[16] A.Calderón-Cárdenas, H. Varela, and J.A.C. Gallas, *Stability diagrams for the triple-path electro-oxidation of formic acid on platinum.*

[17] E.L.Brugnago and T.A.Hild, *Generating chaos: using machine learning to predict changes and duration of seasons and generate chaotic series in Lorenz systems.*