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Dynamics, Synchronization and Inverse problem in Neural Networks with synaptic plasticity

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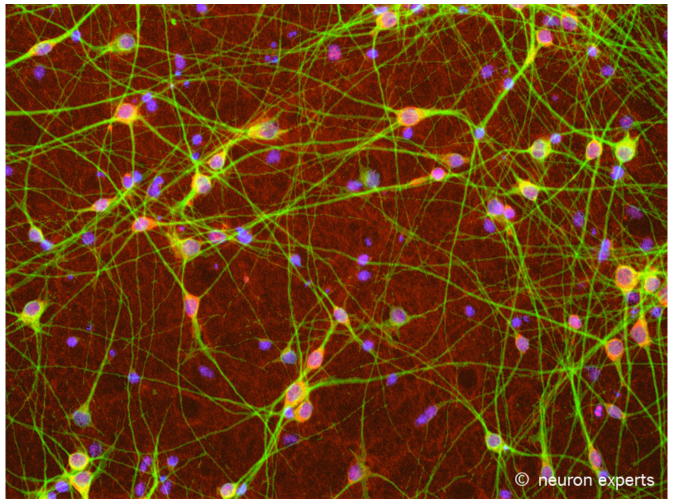
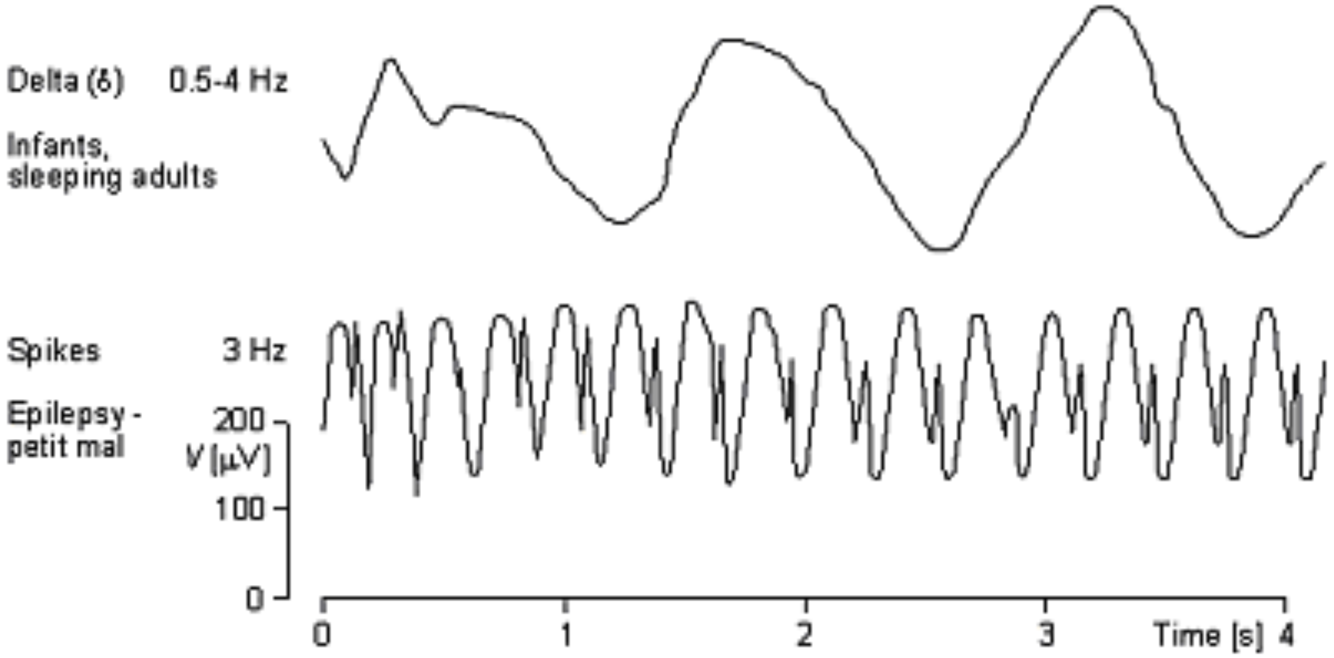
From Microscopic to collective dynamics in Neural Circuits
Dresden, 2016

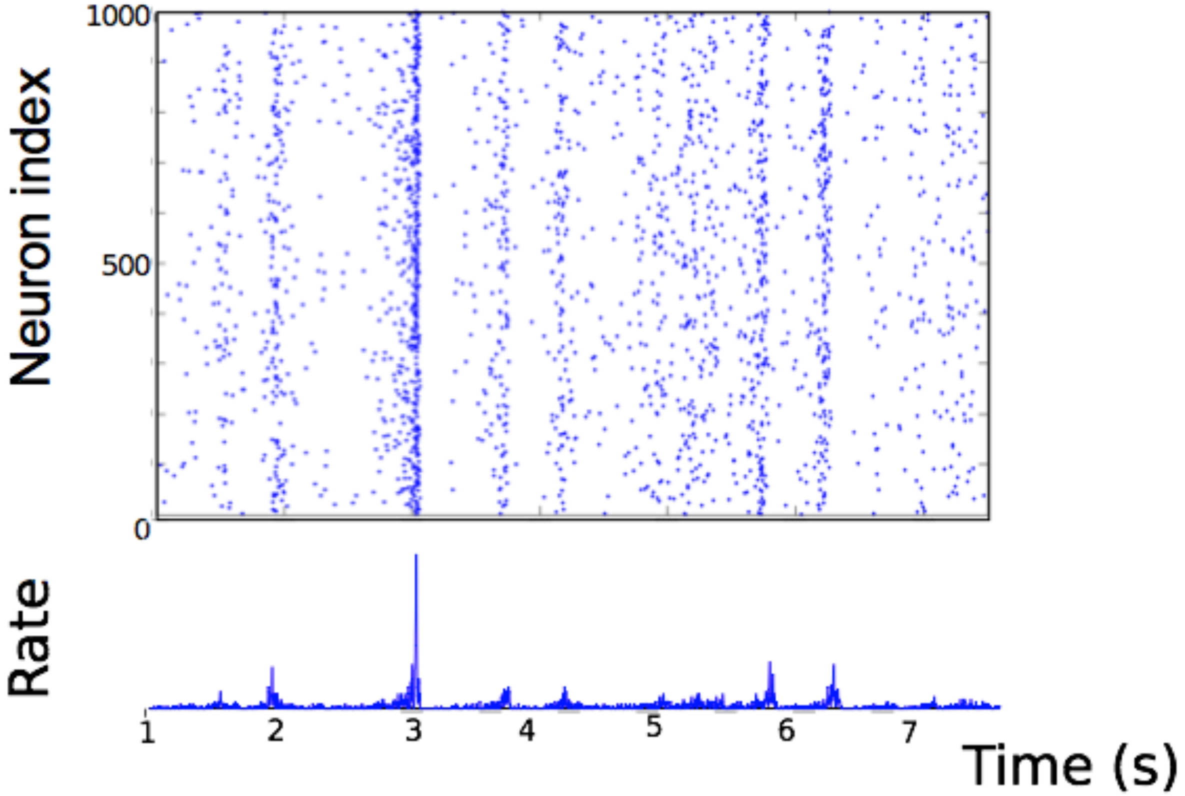
Neural Network



$Y(t)$ Electrical activity



Avalanches activity



Outline

- LIF neurons with synaptic plasticity
- Heterogeneous Mean Field
- Partially synchronous and asynchronous regimes
- From collective activity to network structure

LIF Neuronal Model

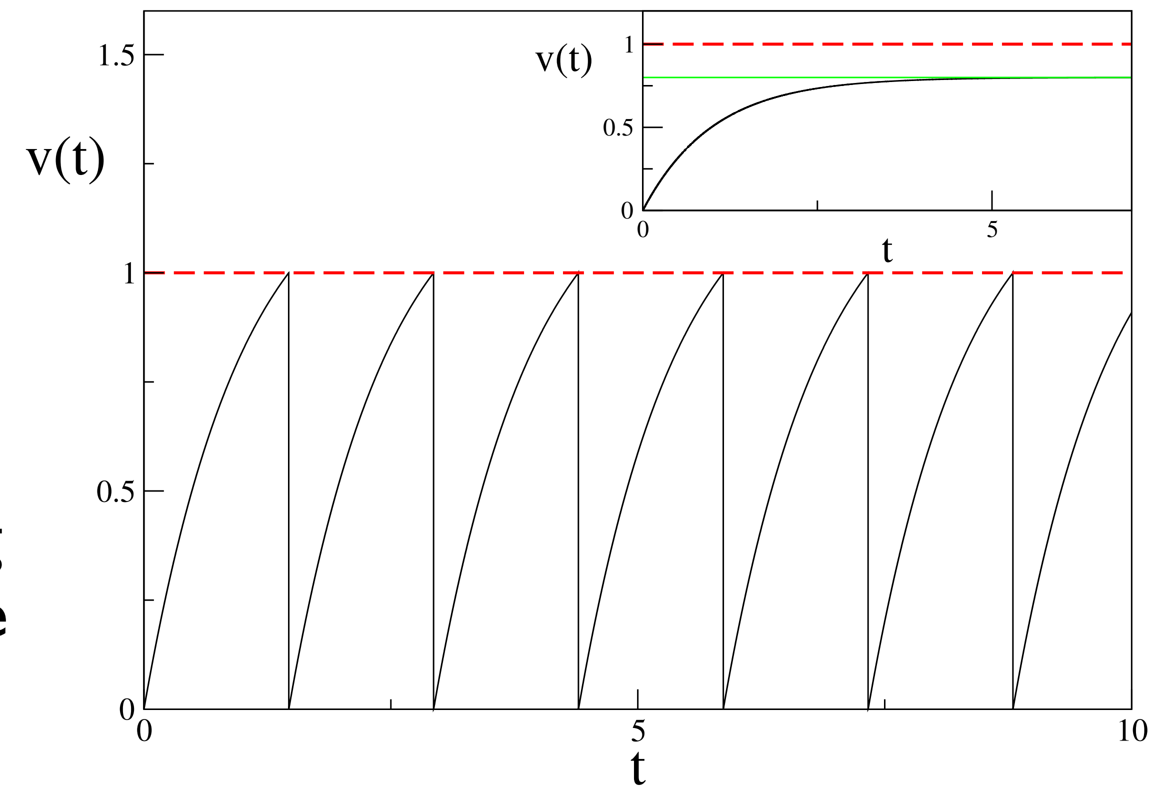
$$\dot{v} = a - v + I_{\text{syn}}$$

$$v > v_{\text{th}} = 1 \begin{cases} \text{spike} \\ v = 0 \end{cases}$$

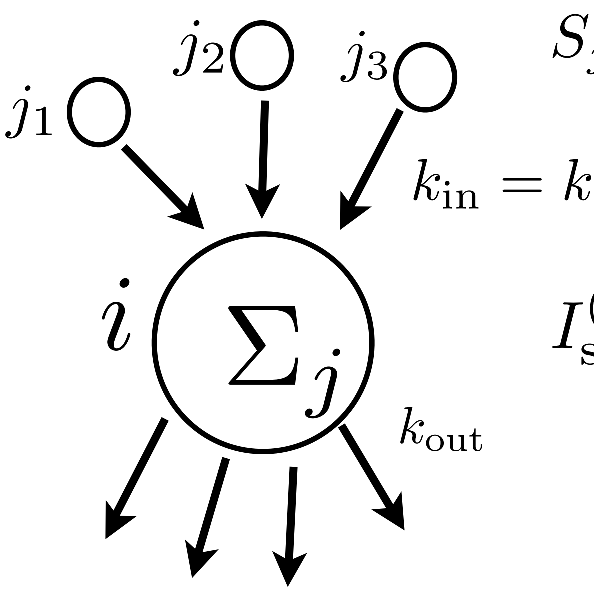
$a > 1$

↓

Spiking Regime



Short term plasticity: TUM model for excitatory neurons



$$S_j(t) = \sum_{n|t_n<t} \delta(t - t_n(j))$$

$$I^{(i)}_{\text{syn}}(t) = \tilde{g} \sum_{j \in \text{presyn}.i} y_j(t)$$

$$\dot{y}_j = -\frac{y_j}{\tau_{\text{in}}} + u x_j S_j$$

$$\dot{x}_j = \frac{z_j}{\tau_{\text{r}}} - u x_j S_j$$

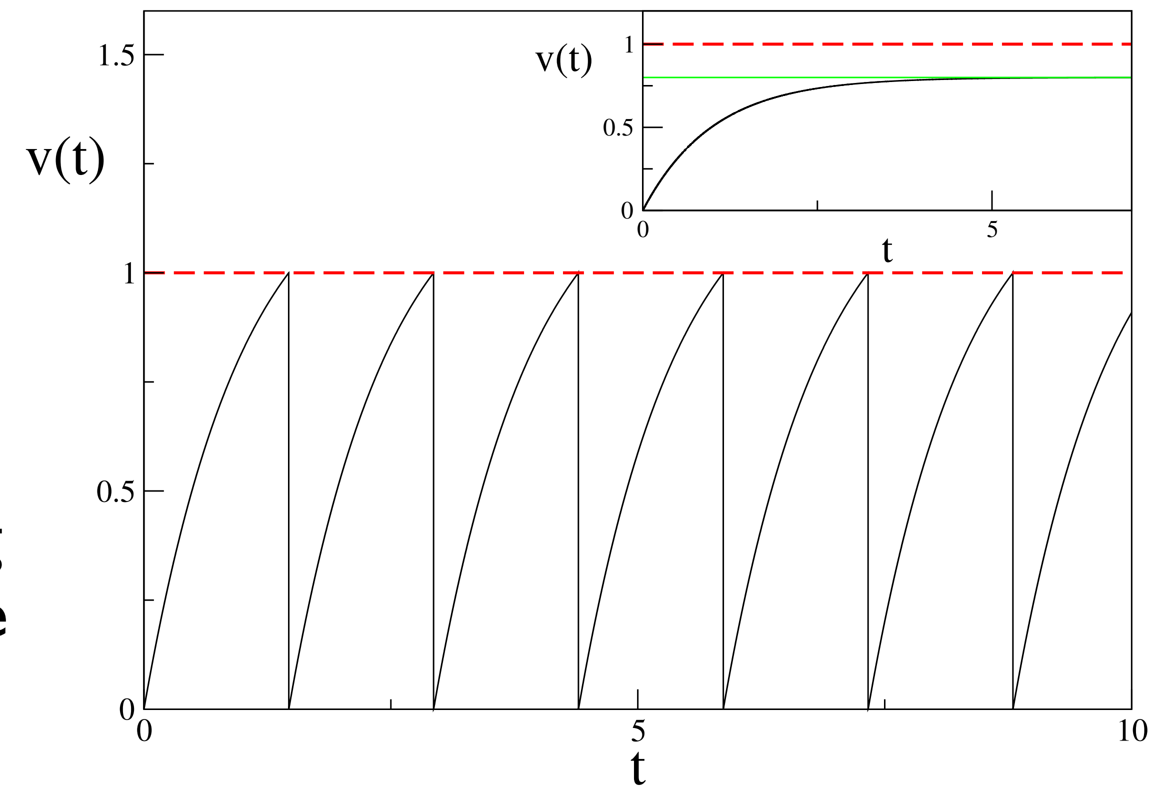
$$x_j + y_j + z_j = 1$$

LIF Neuronal Model

$$\dot{v} = a - v + I_{\text{syn}}$$

$$v > v_{\text{th}} = 1 \begin{cases} \text{spike} \\ v = 0 \end{cases}$$

$a > 1$ → Spiking Regime



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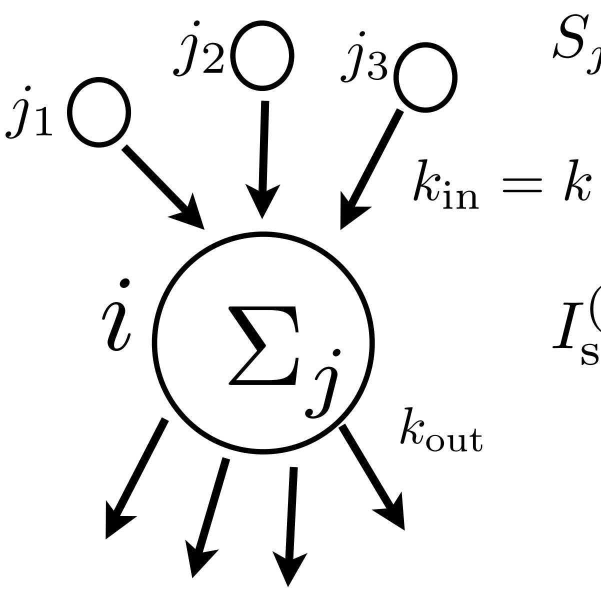

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LIF Neuronal Model

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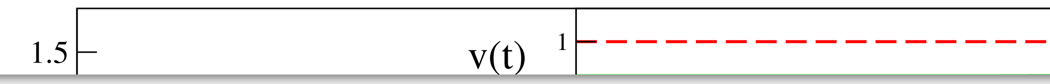


Finite size network of N neurons

$$\dot{v}_i = a - v_i + \frac{g}{N} \sum_{j \neq i} g_{ij} y_j$$

$$\dot{y}_i = -\frac{y_i}{\tau_{\mathrm{in}}} + u x_i S_i$$

$$\dot{x}_i = \frac{1 - x_i - y_i}{\tau_{\mathrm{r}}} - u x_i S_i$$

$\tau_{\mathrm{in}} = 0.2$; $\tau_{\mathrm{r}} = 26.6$

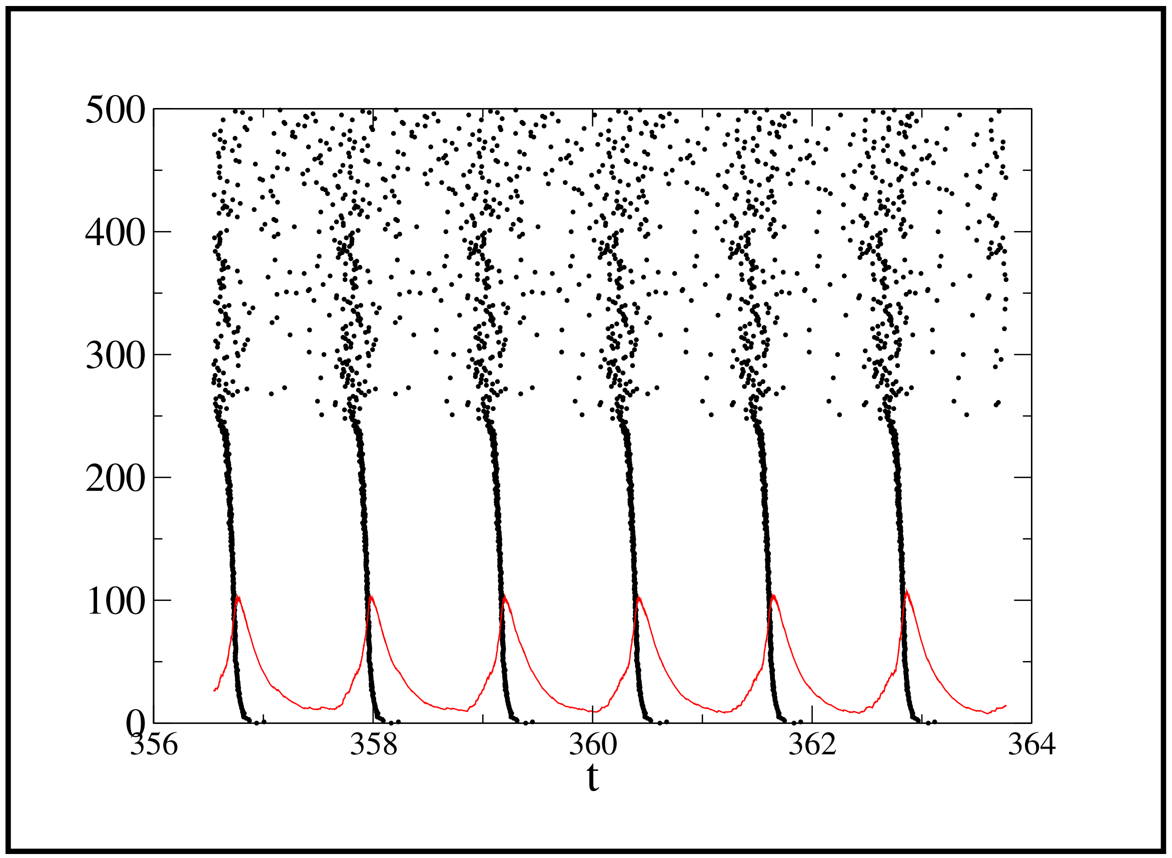
$u = 0.5$; $a = 1.3$

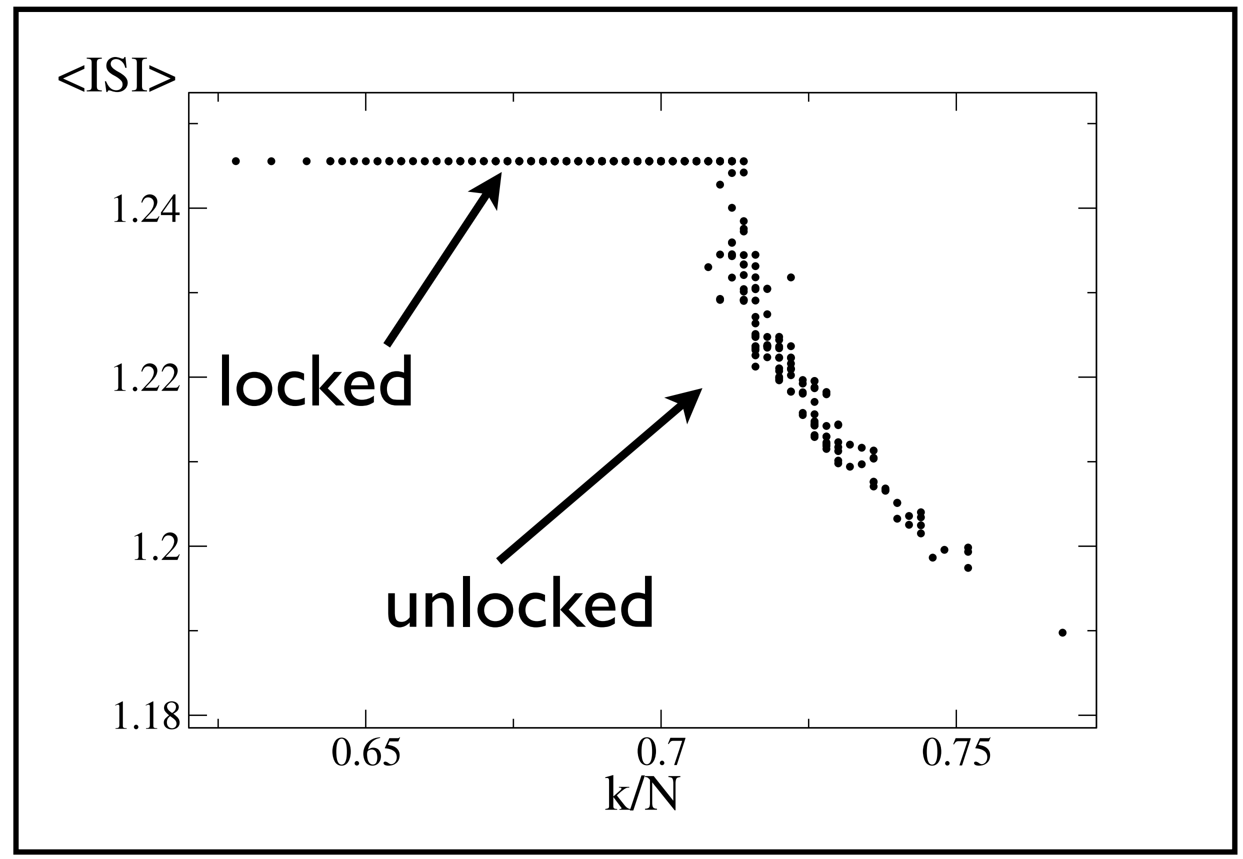
$g = 30$

Erdös–Renyi random Network

each link connected with probability p

large N : $P_N(k) = G(Np, Np(1-p))$





$$Y(t) = \frac{1}{N} \sum_i y_i(t)$$

$$\tilde{k} = \frac{k}{N}$$

Thermodynamic limit

Erdös–Renyi: $P(\tilde{k}) = G(p, p(1-p)/N)$

fluctuations $\sigma_{\tilde{k}} \sim 1/\sqrt{N}$

$$\text{Dynamics } (N \to \infty) \neq \text{Dynamics (finite } N)$$

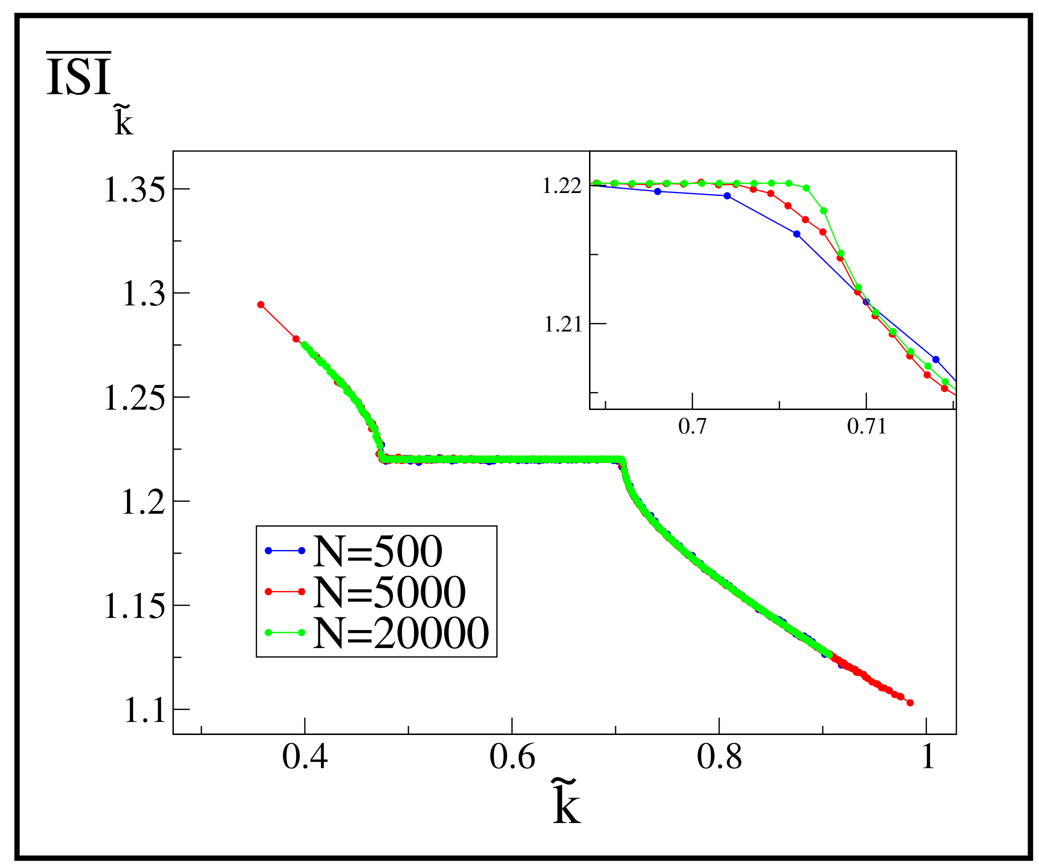
New Network construction

$P(\tilde{k})$ fixed

extract $\tilde{k}_i$ from $P(\tilde{k})$

&

assign randomly $N\tilde{k}_i$ inputs



Gaussian $P(\tilde{k})$: $\langle \tilde{k} \rangle = 0.7$ $\sigma_{\tilde{k}} = 0.06$

Heterogeneous Mean Field

$$\dot{v}_i = a - v_i + \frac{g}{N}\sum_j \epsilon_{ij} y_j$$

$$\frac{1}{k_i}\sum_j g_{ij} y_j(t) \simeq \frac{1}{N}\sum_j y_j(t) = Y(t) \implies F_i(t) = \frac{g}{N}\sum_j g_{ij} y_j(t) \to g\tilde{k}_i Y(t)$$

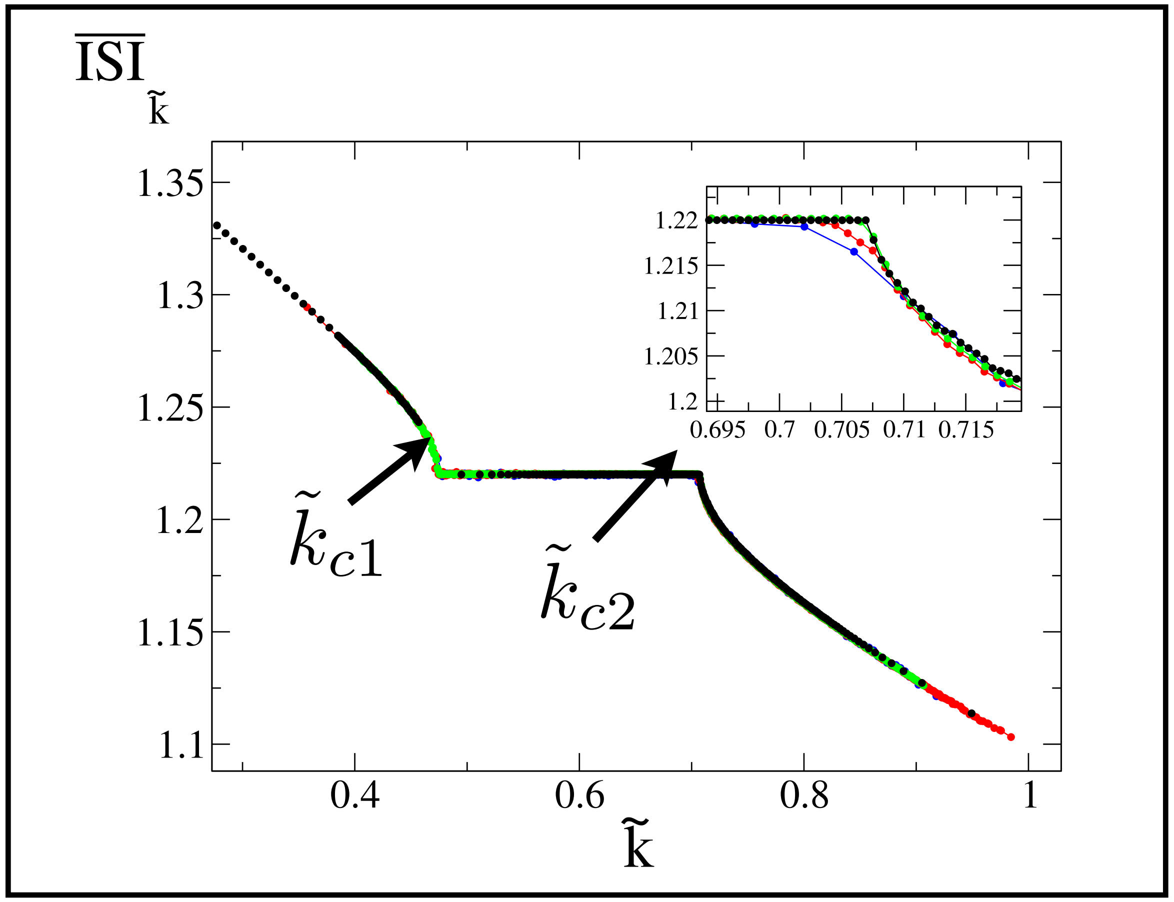
MF

$$\dot{v}_{\tilde{k}}(t) = a - v_{\tilde{k}}(t) + g\tilde{k}Y(t)$$

$$\dot{y}_{\tilde{k}}(t) = -\frac{y_{\tilde{k}}(t)}{\tau_{\rm in}} + u x_{\tilde{k}}(t) S_{\tilde{k}}(t)$$

$$\dot{x}_{\tilde{k}}(t) = \frac{(1 - y_{\tilde{k}}(t) - x_{\tilde{k}}(t))}{\tau_{\rm r}} - u x_{\tilde{k}}(t) S_{\tilde{k}}(t)$$

$$Y(t) = \int_0^1 P(\tilde{k}) y_{\tilde{k}}(t) d\tilde{k}$$



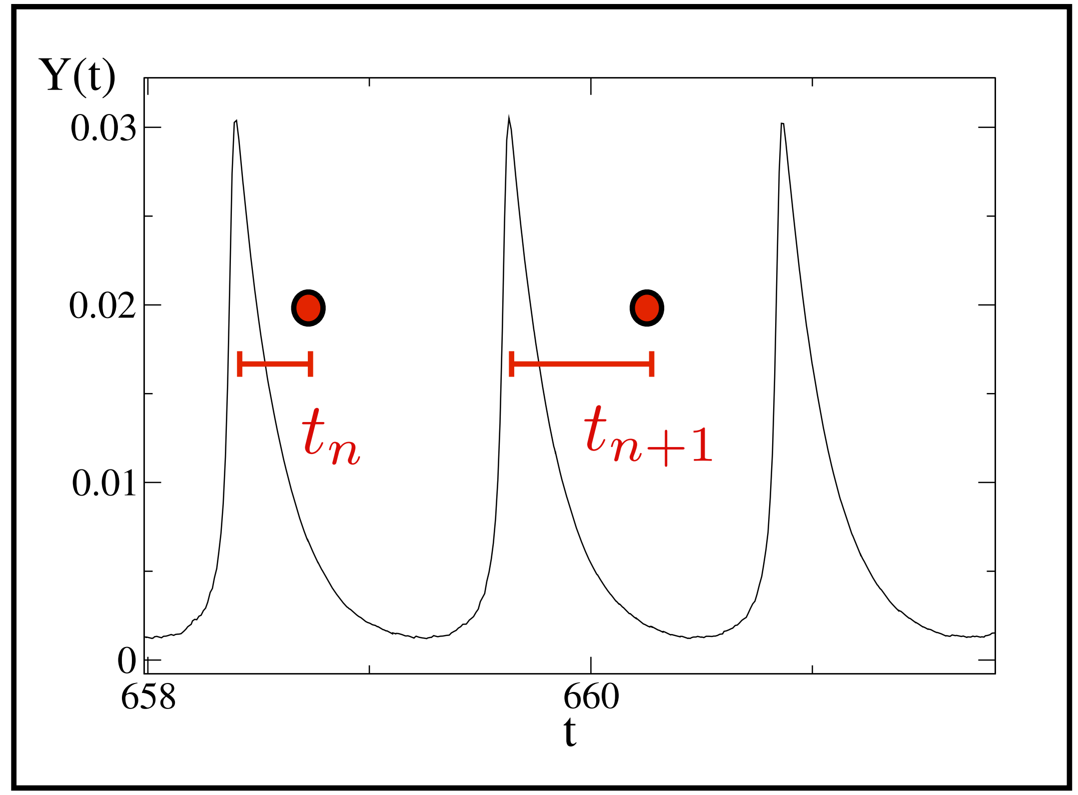
Gaussian $P(\tilde{k})$: $\langle \tilde{k} \rangle = 0.7$ $\sigma_{\tilde{k}} = 0.06$

Stability Analysis

Take $Y(t)$ from HMF simulation

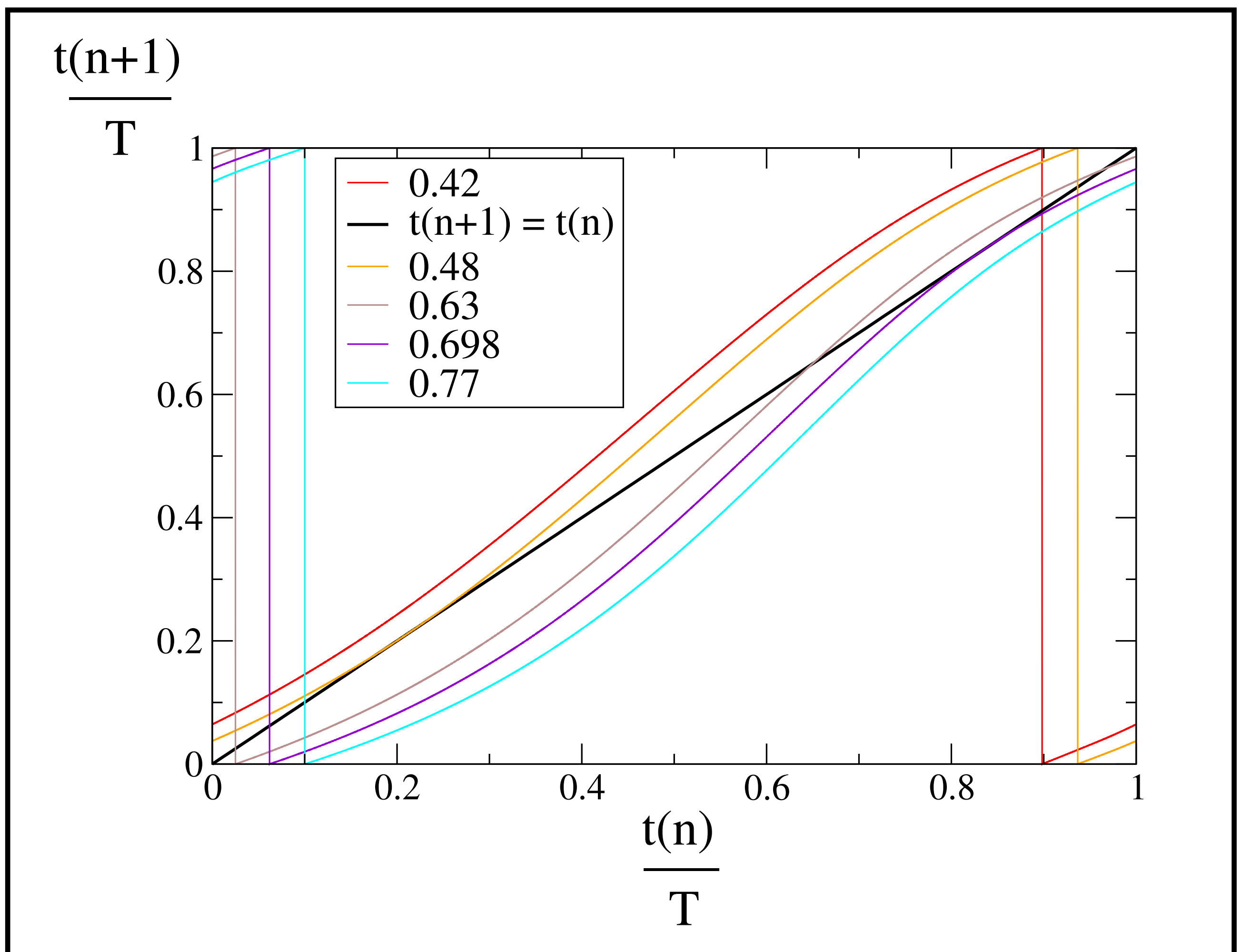
$Y(t)$ periodic of period T

$$\dot{v}_{\tilde{k}}(t) = a - v_{\tilde{k}}(t) + g\tilde{k}Y(t)$$



Obtain a map:

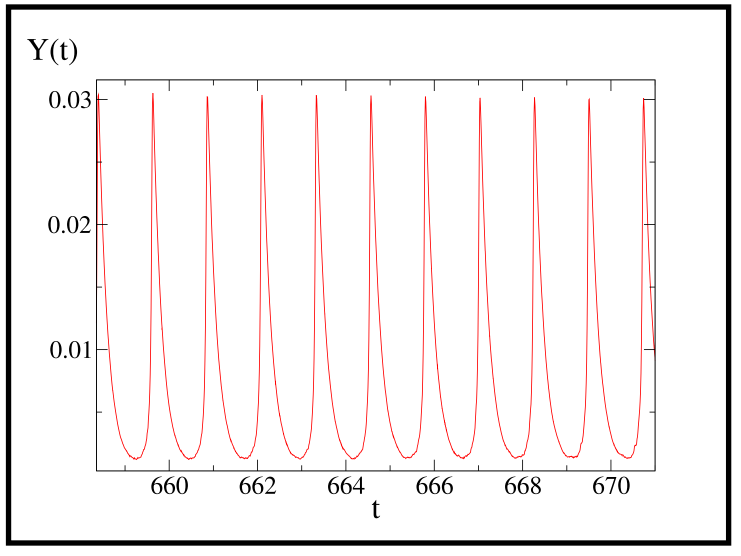
$$t_{n+1}(\tilde{k}) = M_{\tilde{k}} t_n(\tilde{k})$$



From partial synchrony to asynchronous phase: the role of degree disorder

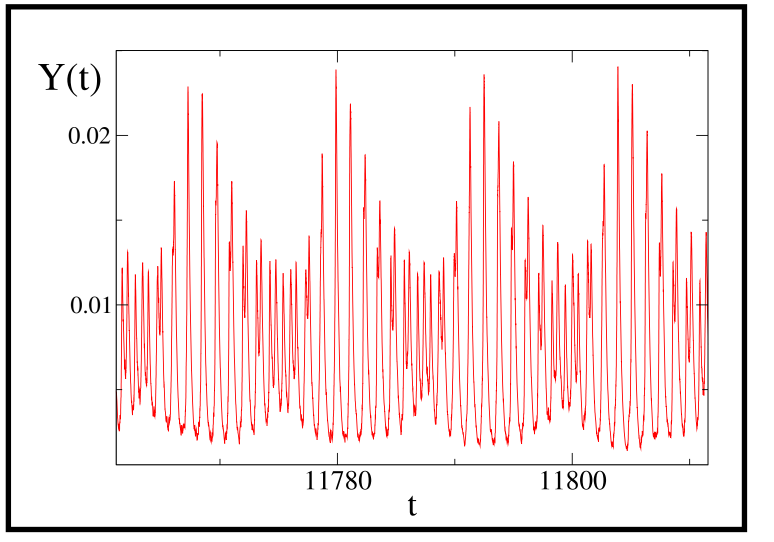
single peak $P(\tilde{k})$

Ω



double peak $P(\tilde{k})$

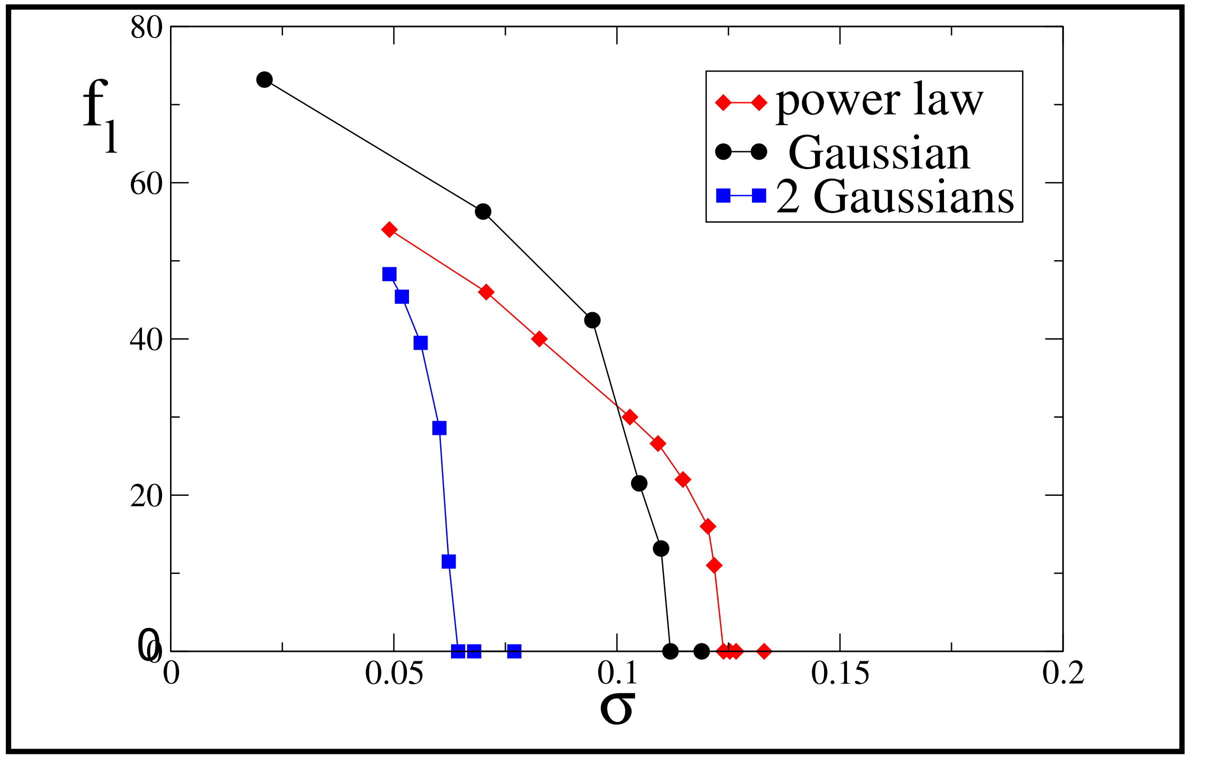
(Ω_1, Ω_2)



$$R = \left\langle \frac{1}{N} \left| \sum_{j=1}^{N} e^{\imath \phi_j(t)} \right| \right\rangle$$

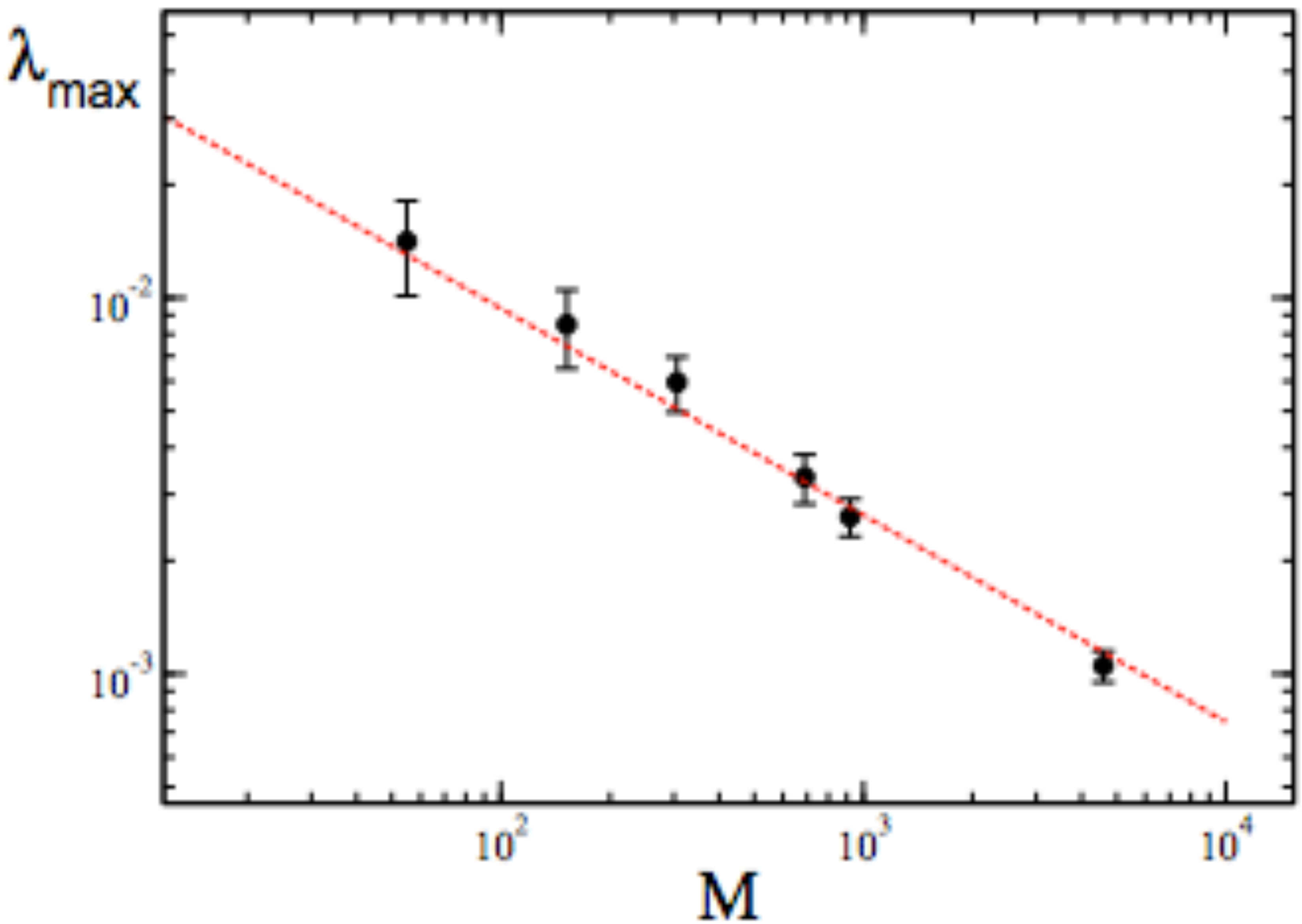
$$\phi_i(t, m) = 2\pi \frac{t - t_i(m)}{t_i(m+1) - t_i(m)}$$

Transition to asynchrony

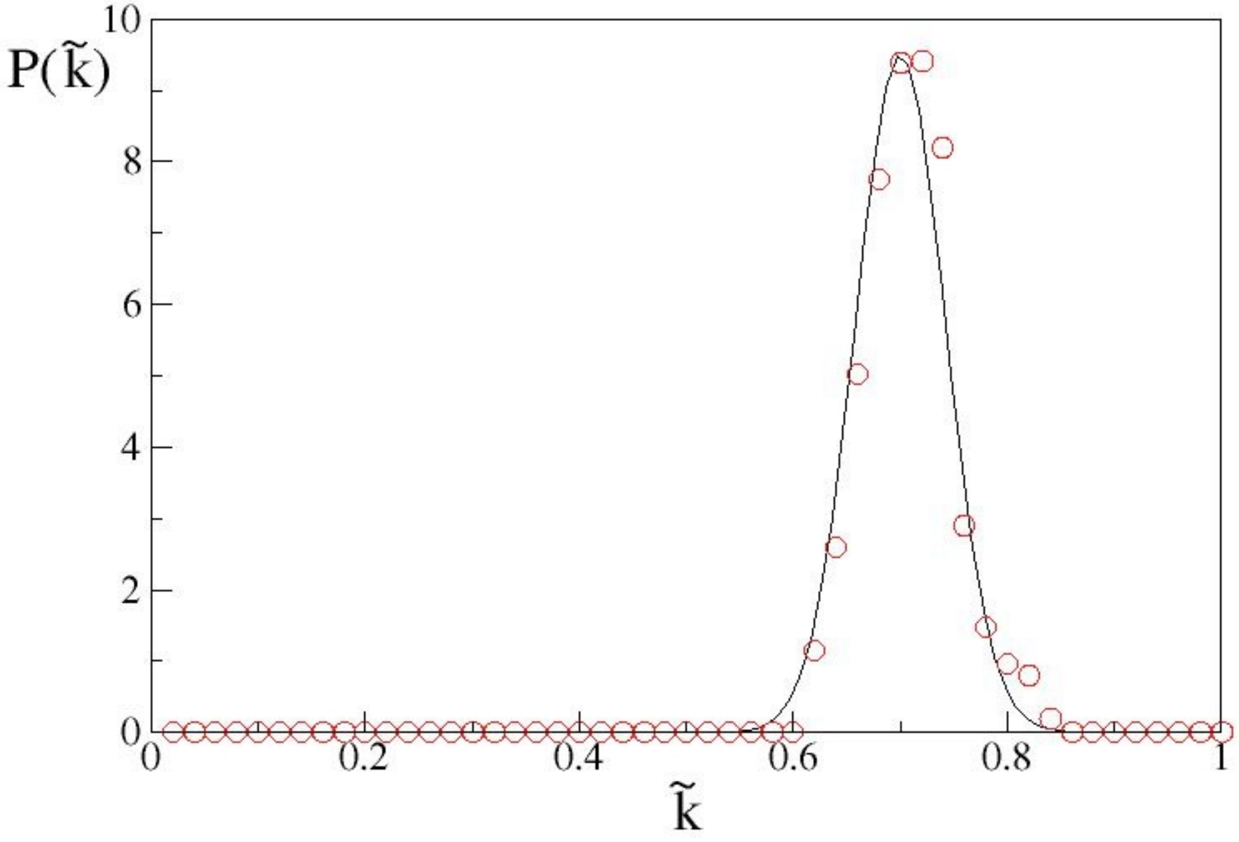


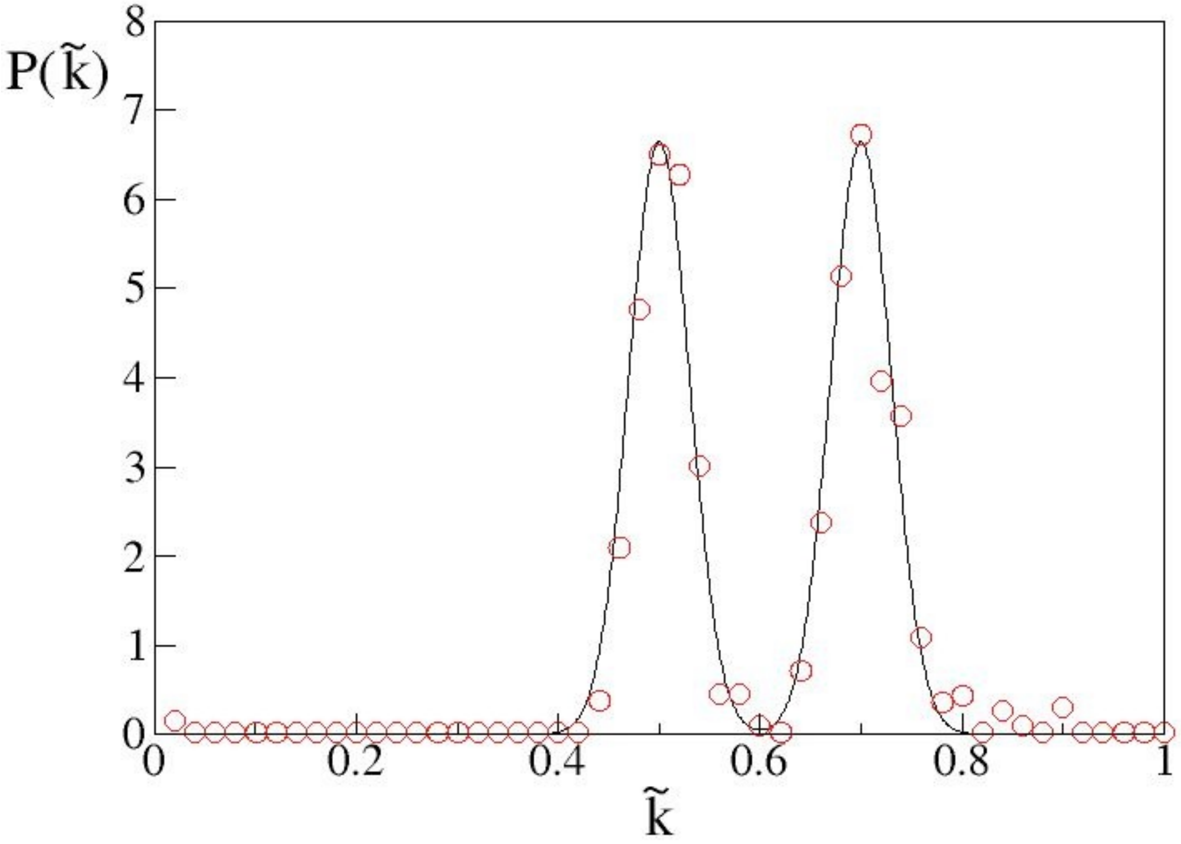
HMF model is non-chaotic

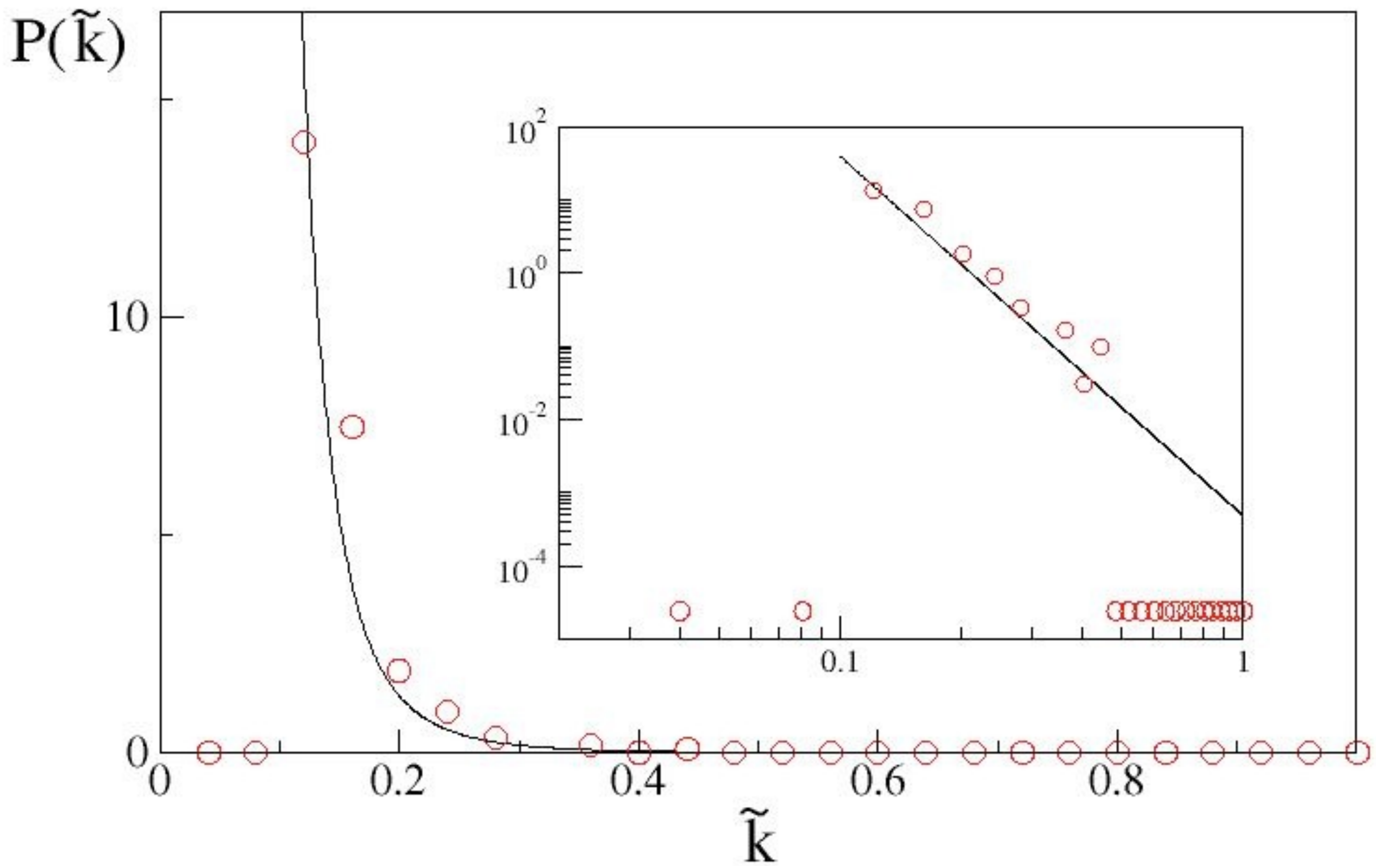
$$\lambda_{max} \sim 1/\sqrt{M}$$



Global Inverse Problem: $Y(t) \to P(\tilde{k})$





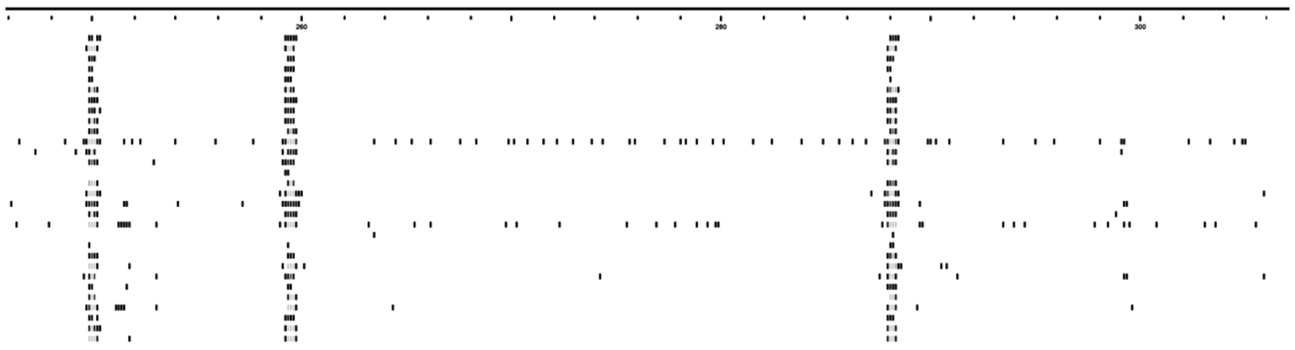


Different setups: the disorder in neurons excitability

in vitro experiments

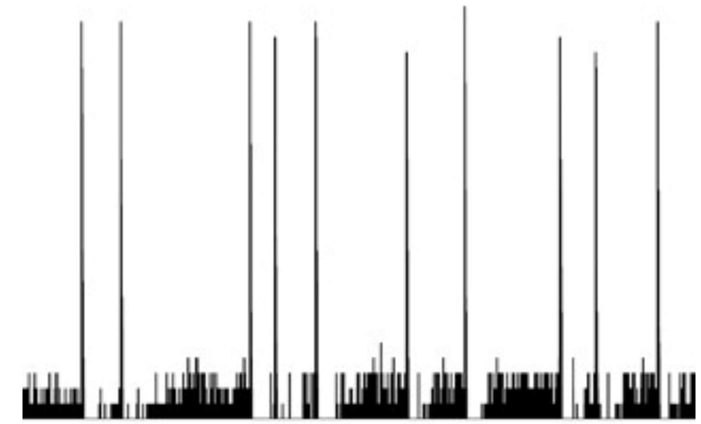
Robinette et al., Front. Neuroeng. (2011)

neuron #



time

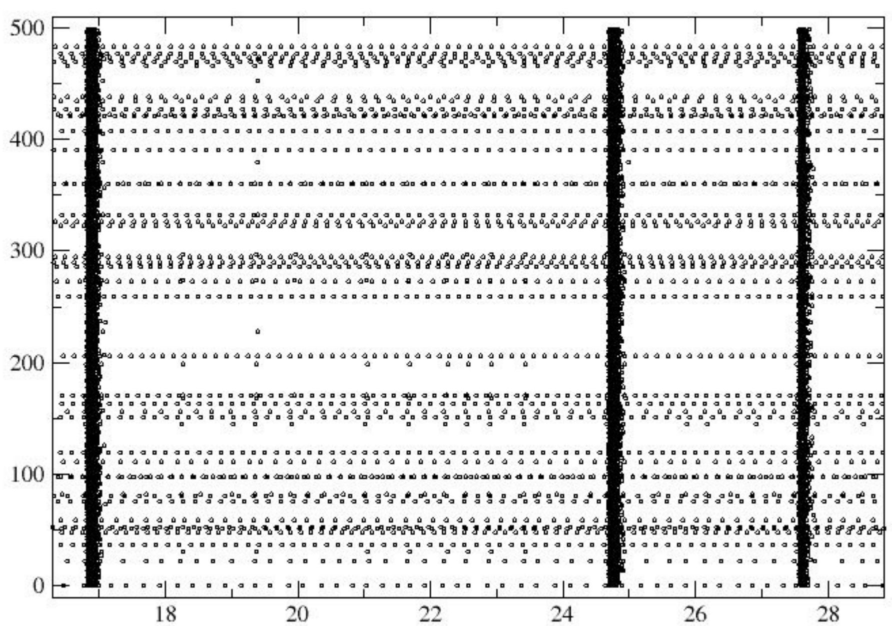
Y



time

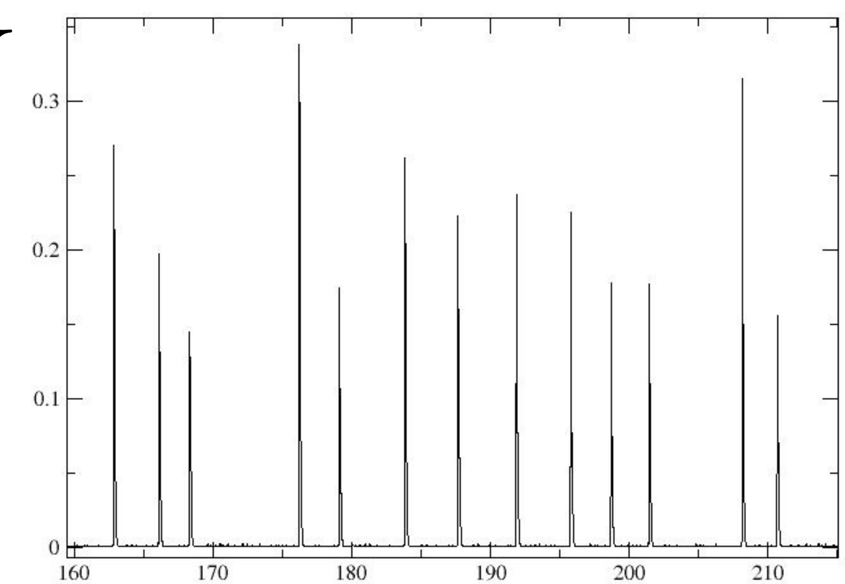
Model: disorder on a_i around $a_c = 1$

neuron #



time [s]

Y



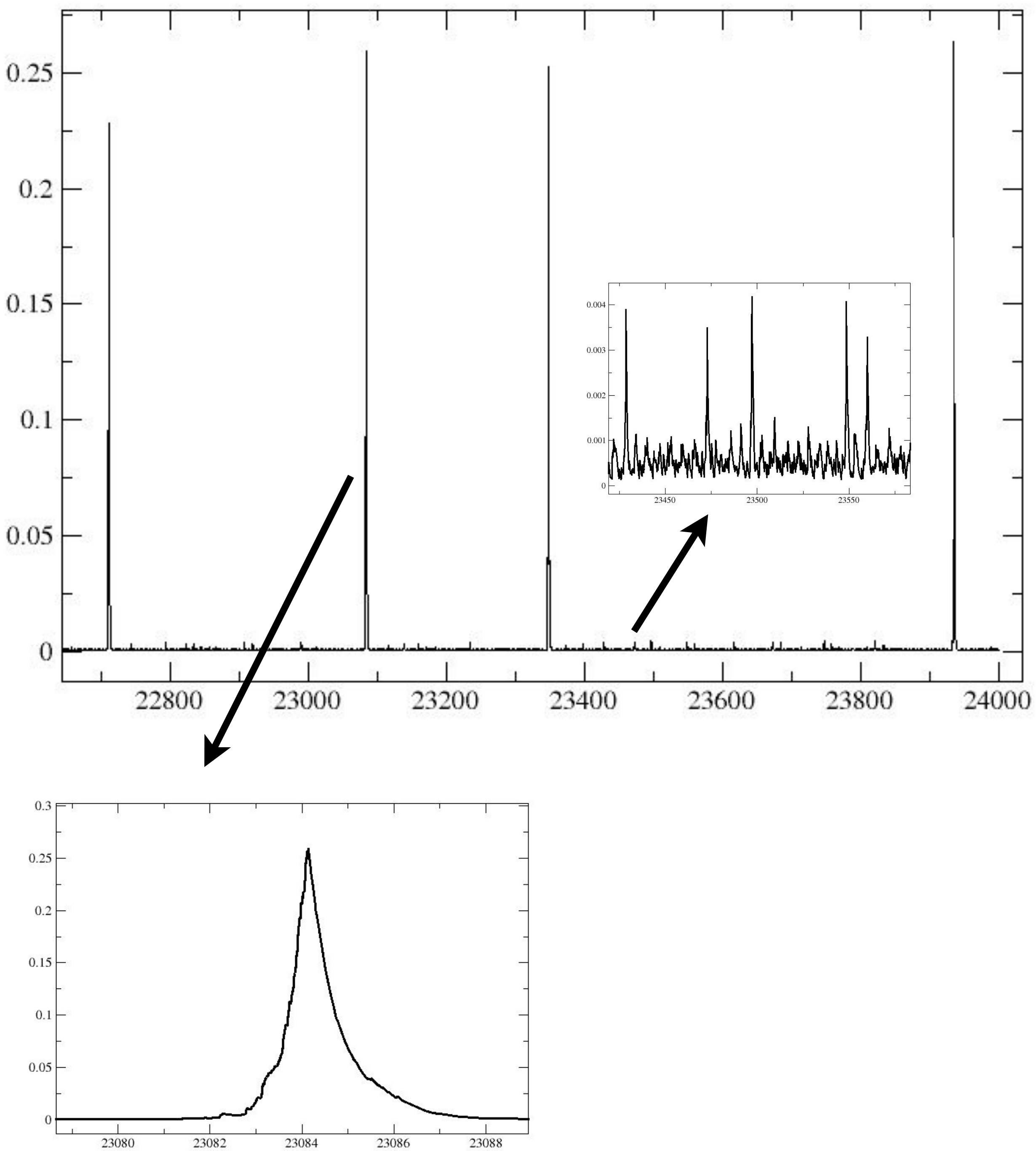
time [s]

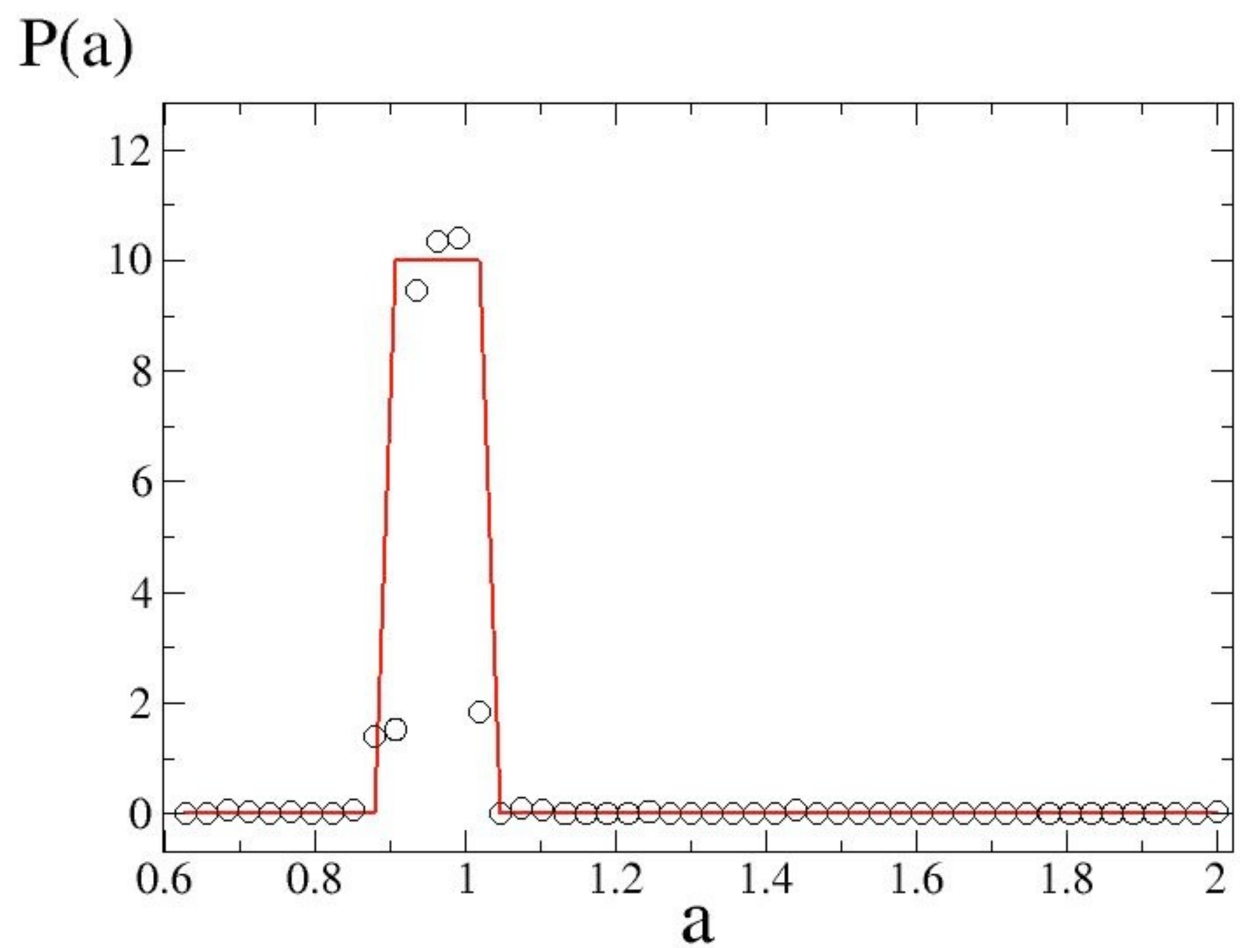
Different setups: the inversion procedure

Uniform distribution P(a) around threshold, All-to-All network

$$Y(t) = \int P(a) y_a(t) da$$

$Y(t)$





Conclusions

- Heterogeneus Mean Field reproduces finite size dynamics
- Rich dynamical phase
- Connectivity distribution from global signals

Collaborators:

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University of Florence: R. Livi