Plasma Physics Reports, Vol. 26, No. 5, 2000, pp. 462–464. Translated from Fizika Plazmy, Vol. 26, No. 5, 2000, pp. 494–496.

BRIEF COMMUNICATIONS

Theory of Nondiffusive Penetration of a Magnetic Field into a Conducting Medium

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Received July 8, 1999

Abstract—The penetration of a current and, accordingly, a magnetic field into the plasma of pulsed systems characterized by short temporal and spatial scales can be investigated in electron magnetohydrodynamics. A study is made of the rapid penetration of the magnetic field of an injected high-current ion beam into a plasma.

This work is a continuation of studies devoted to the rapid penetration of a magnetic field into a plasma or plasmalike media. An investigation of this phenomenon in the electron magnetohydrodynamic (EMHD) model revealed many interesting processes that were not captured with the help of the classical theory of the skin effect. Among the works on this problem, we should mention an important paper by Kingsep *et al.* [1], in which it was predicted that the magnetic field could penetrate into the plasma in the form of a nonlinear constant-amplitude wave moving at a constant velocity.

Our purpose here is to study the characteristic behavior of the magnetic field of a high-current charged-particle beam injected into a plasma. The short temporal and spatial scales of the problem, τ and a, allow us to apply the EMHD approach [1], which is valid under the conditions

$$\omega_{pe}, \omega_{He} \gg \tau^{-1} \gg \omega_{pi}, \omega_{Hi}, \tag{1}$$

$$c/\omega_{pi}, \rho_{Hi} \gg a \gg c/\omega_{pe}, \rho_{He}, \tag{2}$$

$$v_{Te}, v_{Ae} \gg j/ne \gg c_s, v_A, \tag{3}$$

where

$$\omega_{H\alpha} = Z_\alpha eB/m_\alpha c, \quad \omega_{p\alpha}^2 = 4\pi n Z_\alpha^2 e^2/m_\alpha,$$

$$c_s = (ZT_e/MA)^{1/2}, \quad v_A = B/\sqrt{4\pi n_i MA}.$$

The beam can be modeled merely by the external current j_b, because the mechanical component of the beam-particle generalized momentum dominates over its field component, $|\mathbf{p}| \gg \left|\frac{Ze}{c}\mathbf{A}\right|$ (or, in other words, the Larmor radius of the beam electrons substantially exceeds the spatial scale a). Analogously, we can neglect the friction between the beam and plasma particles in comparison with the Ohmic resistance, because the effective Coulomb collision frequency is proportional to $E_b^{-3/2}$, where E_b is the energy of the beam particles. Under conditions (1)–(3), the plasma ion velocity is much lower than the plasma electron velocity, so that the plasma ions can be assumed to be immobile.

The geometry of the problem is illustrated in Fig. 1. The z-axis is directed along the external current $z \parallel \mathbf{j}_b$; the plasma occupies the half-space $0 < z < a$; and the system is uniform along the y-axis, $\partial/\partial y \equiv 0$. At the initial time $t = 0$, the reverse current in the plasma completely neutralizes the external (beam-driven) current and $\mathbf{B} \equiv 0$. Outside the plasma, at any instant, we have $\mathbf{B} = \mathbf{B}_0$, where $\mathbf{B}_0$ is the self-magnetic field of the beam. Under the assumptions adopted, the ion and electron beams can be treated in the same manner; we should only keep in mind that, in the case of an electron beam, the beam current flows in the direction opposite to that of the beam.

We start with the set of equations

$$\mathbf{curl E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

$$\mathbf{curl B} = \frac{4\pi}{c}(\mathbf{j} + \mathbf{j}_b),$$

$$\mathbf{j} = -en_e\mathbf{v}_e$$

and the equation of electron motion

$$m\frac{d\mathbf{v}_e}{dt} = -e\mathbf{E} - \frac{1}{n_e}\nabla p_e - \frac{e}{c}[\mathbf{v}_e\mathbf{B}] + \frac{e}{\sigma}\mathbf{j}.$$

Assuming, for simplicity, that n = const and σ = const and performing the necessary manipulations, we obtain

$$\frac{\partial}{\partial t}\left(\mathbf{curl v}_e - \frac{e}{mc}\mathbf{B}\right) = \mathbf{curl}\left[\mathbf{v}_e, \mathbf{curl v}_e - \frac{e}{mc}\mathbf{B}\right] - \frac{ce}{4\pi m\sigma}\Delta\mathbf{B} - \frac{e}{m\sigma}\mathbf{curl\, j}_b, \tag{4}$$

$$\mathbf{v}_e = \frac{\mathbf{j}_b}{ne} - \frac{c}{4\pi}\mathbf{curl}\mathbf{B}. \quad (5)$$

We should point out the following important property of equation (4). In the limit of infinitely high conductivity, equation (4) passes over to the familiar frozen-in equation in which the frozen-in quantity is the curl of the generalized plasma-electron momentum $\mathbf{P} = \mathbf{p} - \frac{e}{c}\mathbf{A}$.

The problem of the transport of a magnetic field by an external current was solved by Kingsep *et al.* [2] without allowance for electron inertia. The physical model they developed can be outlined as follows. The magnetic field penetration is described by the dynamic equation

$$\frac{\partial B}{\partial t} + v\frac{\partial B}{\partial z} = D\frac{\partial^2 B}{\partial z^2}, \quad v = j_b/ne,$$
$$D = \frac{c^2}{4\pi\sigma}, \quad B = B_y. \quad (6)$$

In the initial stage, when the profile of B is steep, the magnetic field penetrates into a plasma due to diffusion. In later stages, when the profile of B becomes sufficiently smooth, the magnetic field becomes frozen in the current-carrying electrons and is transported by them. The magnetic field enters the plasma through the boundary $z = 0$ with the velocity $v = j/ne$. The exact solution to equation (6) is

$$B = \frac{B_0}{2}\left(\exp\frac{vz}{D}\operatorname{erfc}\frac{z+vt}{2\sqrt{Dt}} + \operatorname{erfc}\frac{z-vt}{2\sqrt{Dt}}\right). \quad (7)$$

As a result of the competition between diffusion and the linear transport of the magnetic field out of the plasma, the steady-state magnetic-field profile

$$B = B_0\exp\frac{v}{D}(z-a)$$

is established at the boundary $z = a$ (Fig. 2).

In our problem, we take into account electron inertia, insert (5) into (4), and perform simple but rather laborious manipulations to obtain the following one-dimensional equation, describing this physical model:

$$\frac{\partial}{\partial t}\left(B - \frac{c^2}{\omega_{pe}^2}\frac{\partial^2 B}{\partial z^2}\right) + v\frac{\partial}{\partial z}\left(B - \frac{c^2}{\omega_{pe}^2}\frac{\partial^2 B}{\partial z^2}\right) = D\frac{\partial^2 B}{\partial z^2}, \quad (8)$$

$$B(z, 0) = B_0\theta(-z), \quad (9)$$

$$B = B_y,\ v = j_b/ne,\ a^2 = c^2/\omega_{pe}^2,\ D = \frac{c^2}{4\pi\sigma}.$$

Following [2], we neglect the effects at the beam boundary; i.e., we omit the term $\mathbf{curl}\frac{\mathbf{j}_b}{\sigma}$, which accounts for the magnetic field generation.

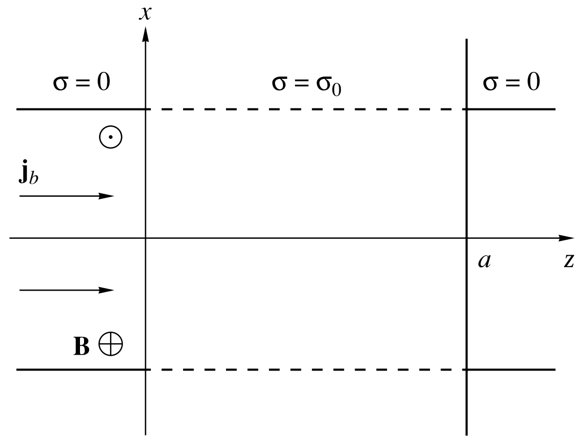

Fig. 1. Geometry of the problem.

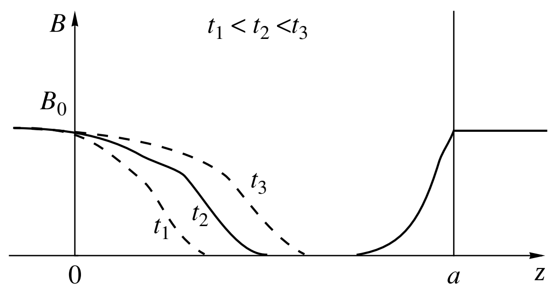

Fig. 2. Penetration of a magnetic field into the plasma without allowance for electron inertia.

Taking the Fourier transformation of (8) in the z-coordinate, we arrive at a linear differential equation. This equation can be easily integrated to yield the time dependence of the Fourier transformed magnetic field. With allowance for the initial conditions, we can represent the exact solution to equation (8) in terms of the Fourier integral,

$$B(z, t) = B_0\left[\frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\sin(k(z-vt))}{k}\exp\left(-\frac{Dtk^2}{1+a^2k^2}\right)dk\right]. \quad (10)$$

The specific form of both the initial condition (9) and equation (8) allows us to follow the penetration of the initial jump (9) in the magnetic field into a plasma using the Lax method, i.e., expanding the solution into a series in functions with different smoothness [3]. To do this, we represent the exact solution (10) as the sum of discontinuous and smooth functions, $B = B_{\text{sing}} + B_{\text{con}}$. As the discontinuous function B_{sing}, we adopt $B_{\text{sing}} = \varphi(z, t)\theta(S(z, t))$, where $\theta(x)$ is the Heaviside step function. We substitute B_{sing} into equation (8) and collect the factors in the generalized functions $\theta(S(z, t))$, $\delta(S(z, t))$, $\delta'(S(z, t))$, and $\delta''(S(z, t))$. If we succeed in finding the

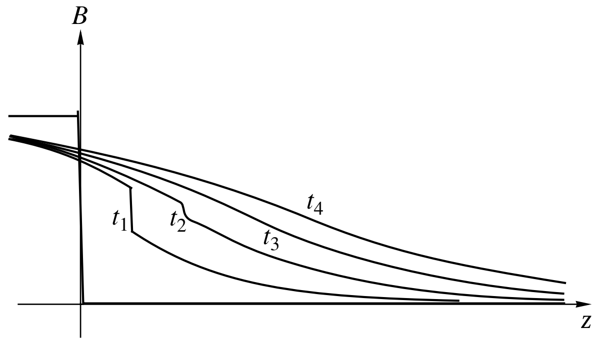

Fig. 3. Penetration of a magnetic field into the plasma with allowance for electron inertia.

functions $\varphi(z, t)$ and $S(z, t)$ with which to force the factors in the delta function and its derivatives to zero, then we could state that the remaining function $B_{\rm con}$ would be at least continuous. The desired functions $\varphi(z, t)$ and $S(z, t)$ satisfy the equations

$$S'_t + vS'_x = 0, \tag{11}$$

$$\varphi_t + \varphi_x = -\frac{D}{a^2}\varphi. \tag{12}$$

Equation (11) implies that $S = f(x - vt)$. Equation (12) can be integrated by the method of characteristics. With allowance for the fact that, at $t = 0$, the function $B_{{\rm sin}g}$ should satisfy the initial condition (9), we obtain the final expression for $B_{{\rm sin}g}$:

$$B_{{\rm sin}g} = B_0\theta(vt - z)e^{-\frac{D}{a^2}t}. \tag{13}$$

Analyzing (13), we can see that the initial discontinuity (9) propagates with the current velocity v and is exponentially damped as time elapses. The diffusion acts to reduce the jump rather than smooth the profile. Applying the same procedure, we can show that the remaining function $B_{\rm con} = B - B_{{\rm sin}g}$ is infinitely differentiable. Of course, the solutions obtained and the boundary conditions are discontinuous because we work in the EMHD theory. In reality, the magnetic field changes sharply on a spatial scale of about c/ω_{pe}. On infinitely long time scales, the discontinuity disappears and the magnetic field profile becomes smooth, in which case neglecting the highest derivative in equation (8) yields equation (6). The time evolution of the solution is illustrated in Fig. 3.

Thus, in our problem, unlike in the nonlinear problems treated by Gordeev *et al.* [4, 5] with allowance for electron inertia, no small-scale solitons are generated: we deal with a discontinuity (rather than a soliton) that appears on a spatial scale of about c/ω_{pe} and is exponentially damped with time. The effective distance over which the jump propagates is equal to $l_{\rm eff} \approx \frac{4\pi\sigma}{\omega_{pe}^2}\frac{j_b}{ne} = \frac{j_b}{ne\nu_{ei}}$.

ACKNOWLEDGMENTS

I am grateful to A.S. Kingsep for his guidance and support throughout the work and to K.V. Chukbar and V.V. Below for valuable discussions. This work was supported in part by the Russian Foundation for Basic Research (project no. 99-2-16659), the program of the Ministry of Science and Technology of the Russian Federation "Problems of Nonlinear Dynamics," and INTAS (grant no. 21-1998).

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Translated by I. A. Kalabalyk