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Random Walk Model with Waiting Times Depending on the Preceding Jump Length

Vasily Yu. Zaburdaev[1]

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In the present paper, the generalized continuous time random walk model with a coupled transition kernel is considered. The coupling occurs through the dependence of the waiting time probability distribution on the preceding jump length. For the description of this model, a method is suggested that includes the details of the microscopic distribution over the waiting times and arrival distances at a given point. A close analogy to the problem of a random walk with finite velocity is demonstrated for the particular case of coupling, when a waiting time is a simple function of a preceding jump length. With its help an analytical solution for the generalized random walk model is found, including both effects (finite velocity and jump dependent waiting times) simultaneously.

KEY WORDS: continuous time random walk model, coupled transition kernel, green's function, fractional derivatives, levy flights.

1. INTRODUCTION

A wide variety of Random Walk models have been studied in great detail and in different contexts, providing a deep understanding of their properties.[(1,2)] The continuous time random walk model (CTRW)[(3)] is probably the most advanced and flexible one. Anomalous diffusion transport phenomena, including Levy flights and possible trapping of particles, significantly broadened its field of applications into biology, economics and social sciences. They also served as the basis for the distribution of the language of the fractional derivatives. This is becoming a powerful and useful tool in modern physics.[(4,5)] Various generalizations and complications of the CTRW were proposed recently, e.g. truncated Levy

[1] Max-Planck-Institute for Dynamics and Self-Organization, Bunsenstr. 10, 37073 Göttingen, Germany; e-mail: Vasily.Zaburdaev@ds.mpg.de

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flights,[6] or the finite velocity random walks.[7−11] From the point of view of the general theory of random walks, the latter case is especially interesting. It has a transition probability kernel, which can not be factorized in terms depending only on space or time coordinates alone.[8,11] The most thorough and complete results for this particular problem appeared in ref. 12 and almost simultaneously in refs. 13, 14. Later the results were partially repeated in ref. 15 (see also ref. 16). A recent attempt to describe such phenomena in a general form was made in ref. 17. However, the chosen initial equations were too rough to capture all the peculiarities of the process. Refs. 13 and 14, where CTRW with a coupled transition probability was discussed, share the same disadvantage. The present paper is devoted to the detailed analysis of the generalized CTRW model with a coupled transition probability. Here, the waiting time distribution function is chosen to be an arbitrary function of the length of the preceding jump. One can bear in mind a natural "physiological" analogy. After making a jump one needs time to rest and recover. The longer the jump distance, the longer are the recovery and the waiting time. This is just one of many possible models that can be treated by the general approach discussed here.

2. STANDARD CTRW MODEL

First we briefly describe the standard CTRW.[3] Consider one dimensional motion of independent noninteracting particles. Each particle can make a jump of length x, with a probability density $g(x)$. It is usually chosen to be symmetric $g(x) = g(-x)$. After a particle arrives at some point, it waits for a time τ, distributed with another probability density $f(\tau)$, and makes a subsequent jump. Functions g and f are characteristics of the current model and responsible for the macroscopic transport properties. This model gives a microscopic description of the diffusion process. However, classical diffusion is just one particular case of a general set of possible regimes. If, for example, g has a slow decaying power law tail and its second moment, $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 g(x)dx$, is infinite (so called Levy flights), it leads to the superdiffusive behavior. If in turn f is heavy tailed and the mean waiting time, $\langle\tau\rangle = \int_0^{\infty} tf(t)dt$, is infinite ("traps" of particles), we are in the situation of a slower subdiffusive transport. When two effects are present simultaneously, a self-similarity of the problem is defined by their interplay. It is now well known that such anomalous transport can be described in terms of the fractional differential equations:[4]

$$\frac{\partial^\gamma n}{\partial t^\gamma} = -K(-\triangle)^\beta n, \tag{1}$$

with a spreading of a cloud of particles according to the scaling law $\overline{x} \propto t^{\gamma/(2\beta)}$. The constant K and exponents $\gamma < 1$, and $\beta < 1$ are defined by the power law tails of f and g (see below). It should be noted, that a passage from the microscopic details to the asymptotic transport equation is not trivial at all. For the discussion and peculiarities of this derivation we refer to a series of works[12,18,19] and already cited reviews. Some of the important aspects will be revisited in this paper as well.

3. GENERALIZED CTRW WITH COUPLED TRANSITION PROBABILITY

Now we introduce a more general model. Suppose that a waiting time probability distribution is a function of a preceding jump length. It means that particles, which jump from x to $x + y$, wait before making the next jump for a time τ, determined by the probability density function $f(\tau, |y|)$. The most natural and one of the simplest is exponential, although it is not important for the following formulae. We use it here only for the illustration:

$$f(\tau, |y|) = \frac{1}{\tau_r(|y|)} \mathrm{e}^{-\tau/\tau_r(|y|)}.$$

The mean waiting time dependence $\tau_r(|y|)$ is defined by the concrete problem, for which one wishes to apply the model. Referring again to the hypothetical physiological example, we can say that each step requires some energy, thus, a corresponding time of recovery is necessary before making the next step. A natural assumption is that the resting time is proportional to the spent energy or the length of the preceding jump:

$$\tau_r(|y|) = \tau_0 + \alpha|y|^\gamma, \quad \gamma, \alpha > 0, \quad \tau_0 \geq 0. \tag{2}$$

The probability to stay in a given point until time τ, provided that a particle has arrived there from a distance $|y|$, is easily expressed as $F(\tau, |y|) = 1 - \int_0^\tau f(\tau', |y|) d\tau'$ for any $f(\tau, |y|)$. For the jump lengths probability density we use the standard power law form, $g(x) = \beta(1 + |x|)^{-2\beta - 1}$, $\beta > 0$. It allows us to access different regimes of diffusion by varying the value of β: $\beta > 1$ classical diffusion, $\beta < 1$ Levy flights and superdiffusion (provided there are no traps of particles).

Particles resting at a given point have different life times and distances from which they arrived. In order to distinguish between these particles we introduce the microscopic density function N, depending on four parameters: coordinate x, time t, current resting time τ, and travelled distance y (there are only the first three of them in the standard model[18]).

It should be stressed, that this additional step to the microscopic level is absolutely necessary for correct and complete description of the CTRW model. For example, it is well known that the asymptotic transport equations for the CTRW of the form (1) (and even initial equations in the integral form, widely used in the literature) do not conserve the continuity of evolution. This feature of transport equations is also closely related to the problem of aging in CTRW models.[20,21] Suppose we take some initial condition at $t = 0$ and let it evolve according to (1) for some time $t_1 > 0$. Then we take its profile at $t = t_1$ as a new initial condition for the same equation and wait for another time t_2. It is easy to check that the resulting density profile would be different from that started originally at $t = 0$ and stopped at $t = t_1 + t_2$ without interruption (i.e. solutions of (1) do not possess the semigroup property). This indicates inherently asymptotic nature of Eq. (1), and the reason for that is the ignorance of the microscopic details. In ref. 19 it was shown that it was possible to derive the transport equation, which preserves the continuity of evolution, but only with the microscopic details explicitly taken into account. This concept is followed in the present paper as well.

The macroscopic density, $n(x, t)$, is given by the integral of $N(x, t, \tau, y)$ over all possible waiting times and travelled distances:

$$n(x,t) = \int_{-\infty}^{+\infty} \int_{0}^{t} N(x, t, \tau, y)\, d\tau\, dy. \qquad (3)$$

By writing t as an upper limit of the time integral, we assume that all particles were introduced to the system simultaneously at time $t = 0$ with zero resting times. This is done for simplicity and also to exclude the possible memory effect (see ref. 19 for details). By the same reason we assume that in the initial distribution all particles have zero arrival distances. The balance equation for a number of particles in a given point has a standard form (cf. ref. 18):

$$n(x,t) = \int_{-\infty}^{+\infty} g(y) \int_{0}^{t} F(\tau, |y|) Q(x - y, t - \tau)\, dy\, d\tau + F(t, 0) n_0(x). \qquad (4)$$

The density is a sum of outgoing particles from all other points at different times given by the flow Q, weighted by jump length probability, and provided they survived after their arrival till the time t. The last term on the right hand side is just the influence of the initial distribution. The survival time distribution, and therefore the transition probability kernel, depend now on both the waiting time and jump length. This is the crucial difference as compared to the standard CTRW model. A similar situation arises in the problem of a finite velocity of random walks.[(12)] The finite velocity of a moving particle provides an effective additional delay time, which also depends on the travelled distance.

The expression for the outgoing flow Q is given by the conditional probability formula[(18)]:

$$Q(x,t) = \int_{-\infty}^{+\infty} \int_{0}^{t} \frac{N(x, t, \tau, y) f(\tau, |y|)}{F(\tau, |y|)}\, dy\, d\tau. \qquad (5)$$

It has a simple physical interpretation. At a given moment of time t, $N(x, t, \tau, y)$ is a survived part of particles initially arrived to x from $x - y$ time τ ago. It means that there were $N(x, t, \tau, y)/F(\tau, |y|)$ of such particles at the time $t - \tau$. Now multiplying it by $f(\tau, |y|)$ we find its part, which is ready to fly away at time t (after the waiting time τ), and therefore gives the contribution to the outgoing flow (5).

In a majority of cases, one usually searches an effective transport equation for the macroscopic density of particles $n(x, t)$ observable in experiments. This does not appear to be an easy task, because of the complicated interrelation of micro- and macroscopic densities, N and n, through the set of Eqs. (3–5). Nevertheless, by using the self-similar dependence of N on its variables, the balance equation (4) can be rewritten in terms of n only.[12,18] In the present paper we use ideologically the same technique as in ref. 12, but in a slightly different realization. By equating the right hand sides of Eqs. (3) and (4), it is easy to determine that N has a self-similar dependence on its parameters:

$$N(x, t, \tau, y) = F(\tau, |y|)\,[g(y)Q(x - y, t - \tau) + n_0(x - y)\delta(y)\delta(t - \tau)]\,. \quad (6)$$

Strictly speaking, the above equation represents the microscopic model of the generalized random walk process and serves the basis for writing down the balance equation (4) in the integral form. In some sense, an inverse derivation of this result is due to the adopted standard "physical" point of view. It is more natural to introduce first the macroscopic density and flow and then, if necessary, investigate its microscopic details. Here, we want to outline that for random walk problems the microscopic details (3) should be considered necessarily prior to writing down the balance equation for the macroscopic density.

When N is substituted into the formula for the flow (5) in the self-similar form (6), it eliminates the denominator in (5):

$$Q(x, t) = \int_{-\infty}^{+\infty}\int_{0}^{t} [g(y)Q(x - y, t - \tau) + n_0(x - y)\delta(y)\delta(t - \tau)]\, f(\tau, |y|)\, dy\, d\tau.$$

The above equation can be solved with respect to $Q(x, t)$ by using the Fourier and Laplace transforms. They convert the space and time convolution type integrals into a simple product of the Laplace–Fourier transforms of the integrands:

$$Q_{k,p} = \frac{n_{0,k} f_p(0)}{1 - \{g(y) f_p(|y|)\}_k}. \quad (7)$$

Here indexes k and p correspond to the Fourier and Laplace components respectively. Applying the same transformation to the balance equation (4) we find:

$$\begin{aligned} n_{k,p} &= \{g(y)F_p(|y|)\}_k Q_{k,p} + F_p(0) n_{0,k} \\ &= \frac{n_{0,k}\{g(y)F_p(|y|)\}_k f_p(0)}{1 - \{g(y) f_p(|y|)\}_k} + F_p(0) n_{0,k}. \end{aligned} \quad (8)$$

Actually, this is the answer to the problem of generalized random walks with a coupled transition probability. It should be noted here that to obtain this result, we used just a few assumptions about the initial conditions and never referred to the explicit form of the density $f(\tau, |y|)$.

4. EXAMPLES AND FINITE VELOCITY OF RANDOM WALK

To demonstrate the strength of the method and to compute some formulas in usual time and space coordinates, we take the simplest waiting time distribution function:

$$f(\tau, |y|) = \delta(\tau - \tau_r(|y|)), \quad \tau_r(|y|) = \alpha|y|, \quad \alpha > 0.$$

The linear dependence of waiting times on the travelled distance makes this model very similar to that with the finite velocity of walking particles.[12] Indeed, now we have an effective constant velocity of jumps $1/\alpha$.

The combination $\{g(y) f_p(|y|)\}_k$ in (8) becomes $\{g(y)\mathrm{e}^{-\alpha p|y|}\}_k$, and the result can be written as:

$$n_{k,p} = \frac{n_{0,k}\{g(y)F_p(|y|)\}_k}{1 - \{g(y)\mathrm{e}^{-\alpha p|y|}\}_k} + n_{0,k}. \tag{9}$$

In the above expression the initial delta-like condition, $n_0(x)\delta(t)$, was separated. After a first time step of the random walk, all particles would be distributed according to the jump length distribution g, and then the first term on the right hand side of (9) would come into play. To give an illustrative interpretation of this result (9), it is necessary to summarize in brief the peculiarities of random walks with finite velocity (for details see ref. 12).

It is well known that the classical diffusion equation, derived from the standard random walk model, possess one unphysical property. At any small $t > 0$ the density of particles is nonzero for any distant x even for the delta-like initial distribution of particles, and, therefore, implies an infinite velocity of their motion. That is why the introduction of the finite velocity of walks is an important generalization, which is much closer to the realistic picture of physical processes. The CTRW model with finite velocity has been studied in detail in the context of the radiative transfer in plasmas.[12]

Finite velocity brings significant changes to the corresponding Green's function when the superdiffusive spreading of a cloud of particles, $x \propto t^{1/(2\beta)}$, is faster then the light front $|x| = vt$. This corresponds to a slow decaying tail of g with $\beta < 1/2$. We suppose that in the model of the finite velocity there is no anomalously long waiting times, and the asymptotic regime is determined by the jump length's distribution alone. For $\beta > 1/2$ the spreading of the cloud is distorted only at far tails leaving the superdiffusive self-similarity unchanged. For $\beta > 1$, when $\langle x^2 \rangle$ is finite, the classical diffusive behavior takes place. It is useful to give here an expression for the Fourier–Laplace transform of the Green's function of the finite velocity problem, G_v:

$$G_{v,k,p} = \frac{F_p}{1 - f_p\{g(y)e^{-p|y|/v}\}_k}.$$

Notations here are similar, $g(y)$ and $f(\tau)$ (not to confuse with $f(\tau, |y|)$) denote the jump length and the waiting time distributions. Since we are interested in the asymptotic (macroscopic) behavior of the density profile, rather then using full Laplace and Fourier transforms, it is possible to take their expansion in Taylor series with respect to small k and p. In the absence of traps, but in the presence of extremely long jumps ($\beta < 1/2$), it is sufficient to retain first terms in expansion of f_p and F_p. They are 1 and $\langle\tau\rangle$ respectively. The main contribution in the expansion stems from the term $\{g(y)e^{-p|y|/v}\}_k$:

$$G_{v,k,p} = \frac{\langle\tau\rangle}{1 - \{g(y)e^{-p|y|/v}\}_k} = \frac{2\langle\tau\rangle\pi^{-1}\sin(2\pi\beta)\Gamma(2\beta)}{(p/v + ik)^{2\beta} + (p/v - ik)^{2\beta}}. \tag{10}$$

The combination in the denominator, $(p/v + ik)^{2\beta} + (p/v - ik)^{2\beta}$, was also obtained in[13–15]. When transformed to the usual coordinates, it can be referred to as the fractional material derivative $(v^{-1}\partial/\partial t \pm \partial/\partial x)^{2\beta}$. It also guaranties the absence of particles beyond the light front. By setting k to be equal zero in (10) we can see that the total number of resting particles is decreasing according to $\int_{-\infty}^{+\infty} G_v dx \propto t^{2\beta-1}$. There is an irreversible transformation of resting particles into flying ones (expression is given below). In the problem of the radiative transfer, these two sorts have quite different nature. Resting particles are excitations of ions in plasma while flying ones are the emitted γ quanta. The total number of all particles, which is the sum of resting and flying once, is, of course, always conserved.

It is easy to see, that the Green's function of the final velocity random walks (10) is an essential part of the answer for our initial problem (9) with $v = 1/\alpha$. Its solution in common space and time variables is given by the convolution integral of $G_v(x, t)$ with F and g. It is important that for some particular β, $G_v(x, t)$ can be calculated explicitly (inverse Laplace and Fourier transform of (10) can be found). For $\beta = 1/4$ it has an extremely simple asymptotic form[12]:

$$G_v(x, t) \propto \frac{\theta(vt - |x|)}{v^{1/2}t^{3/2}}, \quad v = 1/\alpha.$$

The density of particles, given by (9), can now be easily plotted (see Fig. 1).
The total number of particles is conserved (there are only resting particles so far) and they are resting before and beyond the light front. Peaks on the graph reflect the ballistic self-similarity of the problem $|x| \propto t/\alpha$.

Another interesting question is the collective effect of the finite velocity and the waiting times proportional to the jump lengths. The answer can be found in a

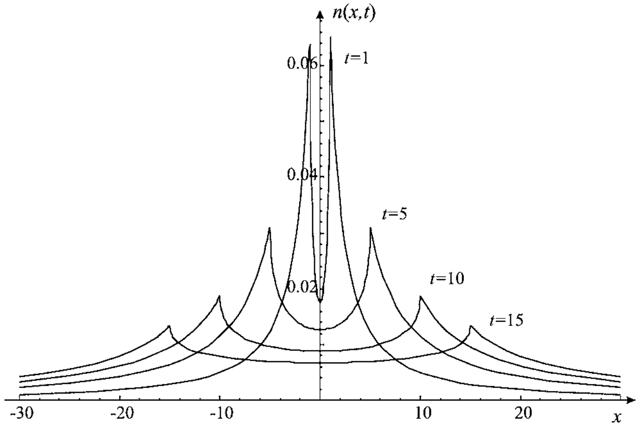

Fig. 1. The density of particles (9) at different times for the random walk model with waiting time being proportional to the preceding jump length: $\tau_r(|y|) = \alpha|y|$. We set $\alpha = 1$ and take delta-like initial condition $n_0(x) = \delta(x)$.

straightforward way and looks very similar. Only slight changes in (9) occur:

$$n_{k,p} = \frac{n_{0,k}\{g(y)\mathrm{e}^{-p|y|/v}F_p\}_k}{1 - \{g(y)\mathrm{e}^{-\alpha p|y| - p|y|/v}\}_k} + n_{0,k}. \tag{11}$$

Now this is an expression for the resting particles only. As already mentioned, a finite velocity of moving particles leads to their separation into two groups – resting and flying. At a given point there are always sitting particles, described by (11), and flying to somewhere else. The density of flying particles in a given point is determined by the flow Q (7) from all other points, with a corresponding delay time[12]:

$$n_{\mathrm{fly}}(x,t) = \frac{1}{v}\int_{-\infty}^{+\infty} Q\left(x - y, t - \frac{|y|}{v}\right)\int_{|y|}^{+\infty} g(z)\,dz\,dy. \tag{12}$$

It is remarkable that a new effective velocity $v_{\mathrm{eff}} = v/(1+\alpha v)$ appears in the denominator of (11). For example, if $\alpha = v = 1$, it would be equal to $1/2$. Nevertheless, the light front border moves with $v = 1$, and it is the real physical limitation on the particle's positions (see the additional exponential in the numerator in (11)). That is why we expect bordered by $|x| = vt$ density profile with the local maxima corresponding to the self-similar scaling $|x| \propto v_{\mathrm{eff}}t$. In the (Fig. 2) the total normalized density, which is the sum of resting (11) and flying (12) particles $n_{\mathrm{total}} = n + n_{\mathrm{fly}}$, is plotted. Although it has discontinuities at $|x| = vt$, indicating the accumulation of particles at this border, they are integrable and the total number of particles is conserved $\int_{-\infty}^{+\infty} n_{\mathrm{total}}dx = \mathrm{const}$.

Only the simplest case of the linear proportionality of the waiting times to the travelled distance was considered above. Of course, other dependencies are also allowed. The main obstacle on the way to the solution is finding the Fourier transform of the $g(y) f_p(|y|)$ combination. Its expansion in Taylor series with respect to small k and p in denominator of (9) gives the asymptotic scaling relation between k and p. For the considered examples, $k \propto p$ corresponds to the ballistic regime $x \propto t$. If the resting time is quadratic in jump length $\tau_r = \alpha |y|^2$, then such a long recovery time is not only bounding superdiffusion to the ballistic scaling, but suppresses it to the classical diffusion $k^2 \propto p$ $(x \propto \sqrt{t})$. For the general dependence (2) the question remains open and probably, can be answered fully only with the help of the numerical analysis.

5. CONCLUSIONS

The generalized CTRW model was considered, taking into account the waiting time's dependence on the jump length. The method, based on the self-similar properties of the microscopic density distribution, allowed us to find the Green's function of the corresponding transport equation analytically. For the model example, with the waiting time proportional to the preceding step length, we demonstrated its close analogy with the finite velocity problem. We believe that considered examples indicate a possibility to apply the developed approach to the biological systems, where the random walk together with the recovery processes and the finite velocity are present. Such problems could be the foraging movements of animals[22,23] or the motion of zooplankton.[24,25]

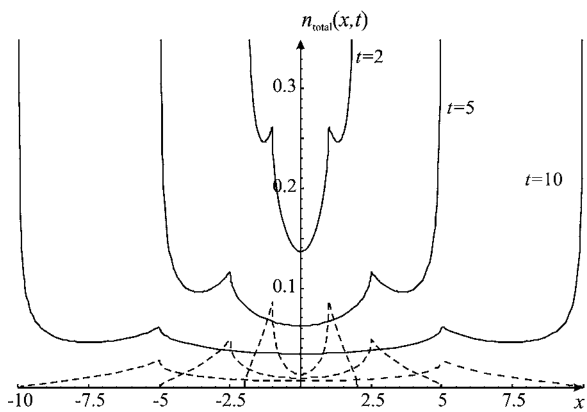

Fig. 2. The total density of particles $n_{total}(x, t)$(solid line) and density of resting particles $n(x, t)$(dashed line) at different times for the generalized random walk model. Both effects, jump depending resting times and finite velocity, are taken into account simultaneously. $\alpha = v = 1$, $v_{eff} = 1/2$, $n_0(x) = \delta(x)$.

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