Quantum oscillations in insulators with neutral Fermi surfaces

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Contents

- Theory of quantum oscillations of insulators with neutral fermi surfaces.
- The “composite exciton fermi liquid” in SmB_6.

D. Chowdhury

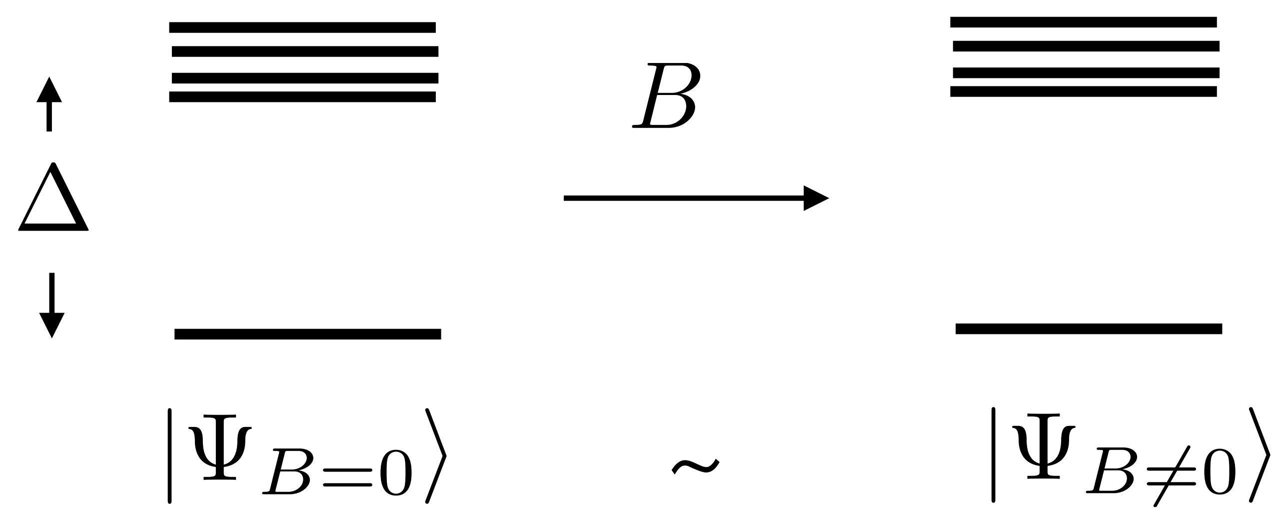
T. Senthil

Debanjan Chowdhury, Inti Sodemann, T. Senthil, arXiv:1706.00418 (2017)

Inti Sodemann, Debanjan Chowdhury, T. Senthil, arXiv:1708.06354 (2017)

Insulators in magnetic fields

- Consider a band insulator at zero T:



Adiabatically connected

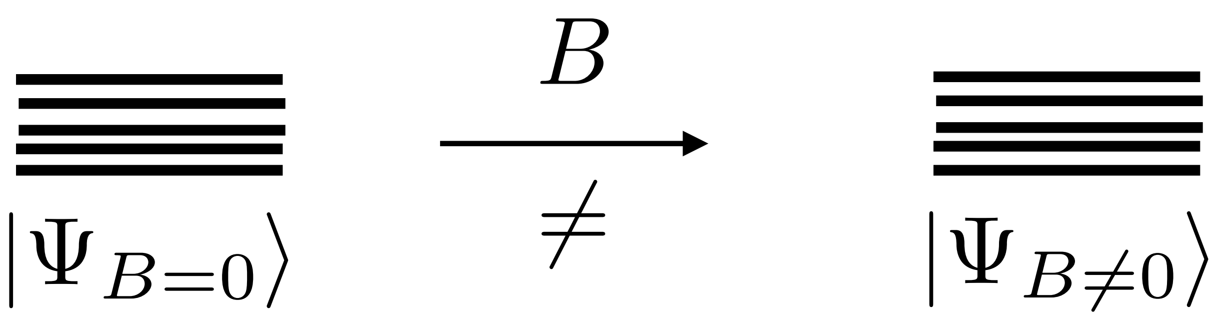
- Adiabaticity implies linear response:

$$E_\Psi(B) = E_0 + \chi \frac{B^2}{2} + \cdots \quad \Rightarrow \quad 4\pi M = -\chi B + \cdots$$

$$4\pi M = -\frac{dE}{dB}$$

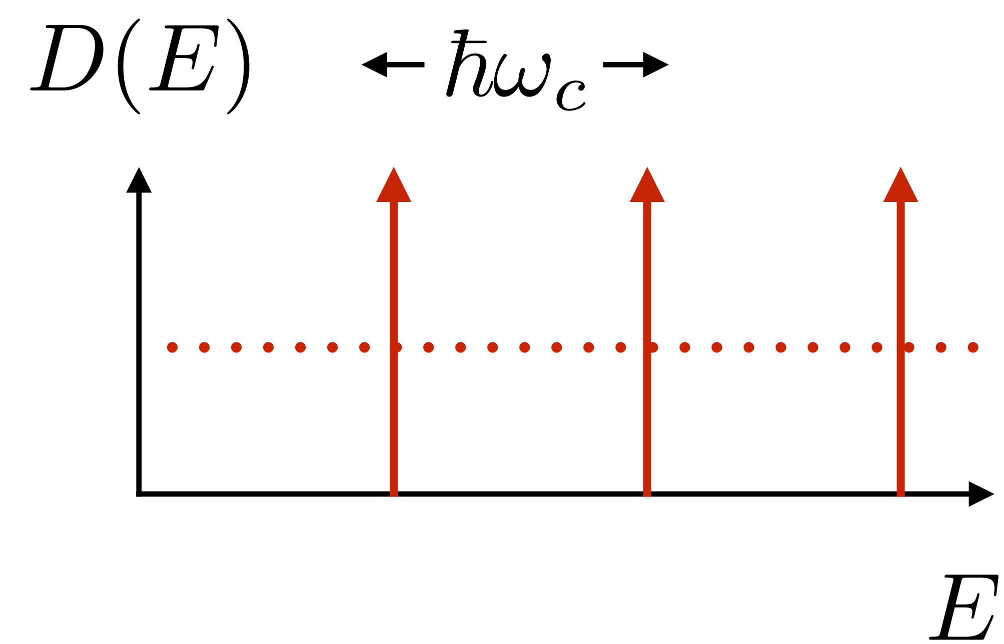
Metals in magnetic fields

- Consider a metal at zero T:

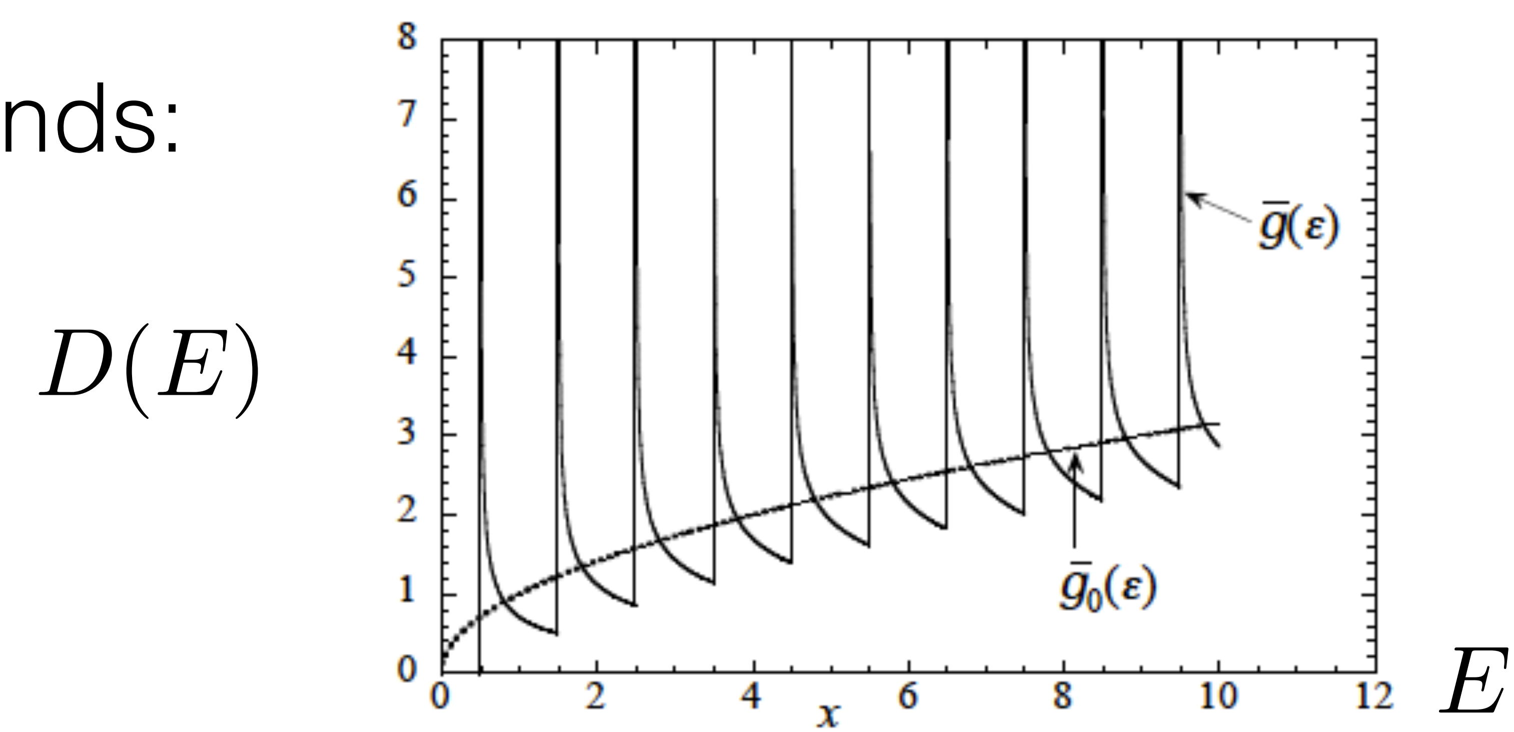


NOT adiabatically connected!

2D Landau levels:

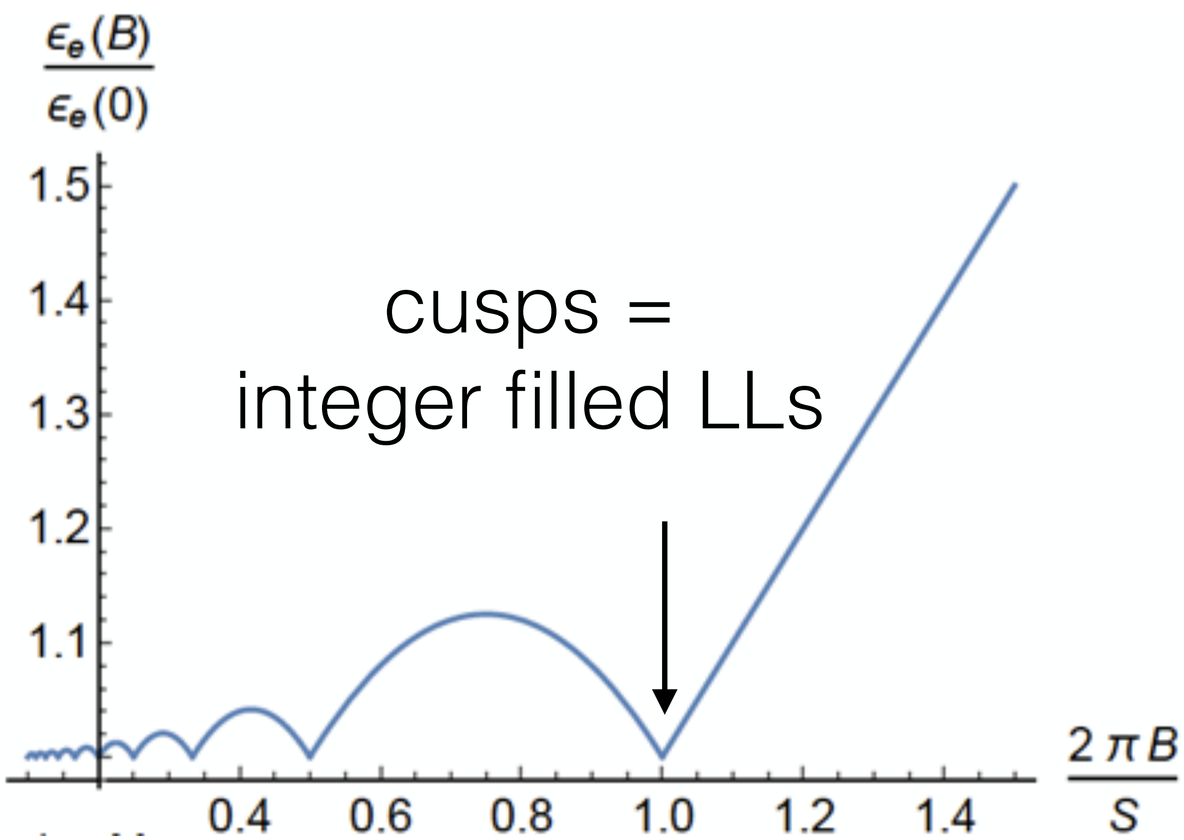


- 3D Landau bands:



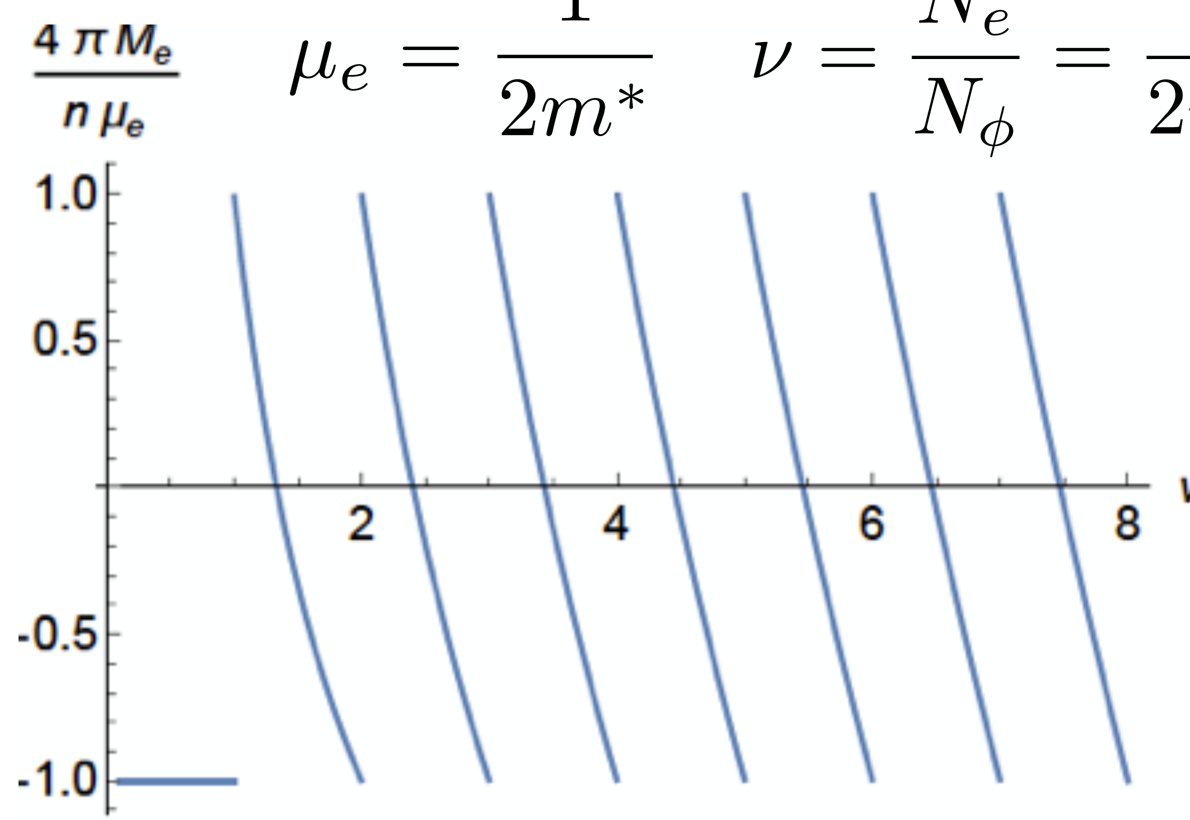
Metals in magnetic fields

- Energy of 2D metal as function of field:



- Magnetization of 2D metal as function of field:

$$\mu_e = \frac{1}{2m^*} \qquad \nu = \frac{N_e}{N_\phi} = \frac{S}{2\pi B}$$



Amplitude $(T = 0, B \to 0)$

2D metal $\quad 4\pi\delta M_{osc} \sim n_e\mu_e \sim \mathrm{const}$

3D metal $\quad 4\pi\delta M_{osc} \sim \chi_L S\sqrt{B/S} \sim B^{1/2}$

Beyond band insulators and metals

- **Q**: is there a phase of matter that is an insulator but has quantum oscillations?

$$\lim_{T\to 0} \sigma(T) = 0 \qquad j = \sigma E$$

Beyond band insulators and metals

- **Q**: is there a phase of matter that is an insulator but has quantum oscillations?

$$\lim_{T\to 0} \sigma(T) = 0 \qquad j = \sigma E$$

- **A**: yes! a fractionalized phase of matter: the spinon fermi surface is an electrical insulator displaying quantum oscillations.

O. I. Motrunich, PRB (2006)

Inti Sodemann, Debanjan Chowdhury, T. Senthil, arXiv:1708.06354 (2017)

The spinon fermi surface

P.W. Anderson

- Fractionalized state

Spinon

Chargon (holon)

Electron → $$c_\sigma^\dagger = f_\sigma^\dagger \, b^\dagger$$

Spinfull neutral fermion

Spinless charged boson

- Spinon forms a fermi sea
- Chargon forms a bosonic Mott insulator

Florens & Geoges, PRB (2004)

The spinon fermi surface

Electron Spinon Chargon (holon)

$$c^{\dagger}_{\sigma i} = f^{\dagger}_{\sigma i} \, b^{\dagger}_{i}$$

Physical states satisfy: $n_c = n_f = n_b$

Explicit trial wave-functions can be written as:

$$\Psi_c(r_1\sigma_1, ..., r_N\sigma_N) = \Psi_f(r_1\sigma_1, ..., r_N\sigma_N)\Psi_b(r_1, ..., r_N)$$

fermi sea boson Mott insulator

The spinon fermi surface

Explicit trial wave-functions can be written as:

$$\Psi_c(r_1\sigma_1, ..., r_N\sigma_N) = \Psi_f(r_1\sigma_1, ..., r_N\sigma_N)\Psi_b(r_1, ..., r_N)$$

In half-filled band

$$N_f = N_{\text{sites}}$$

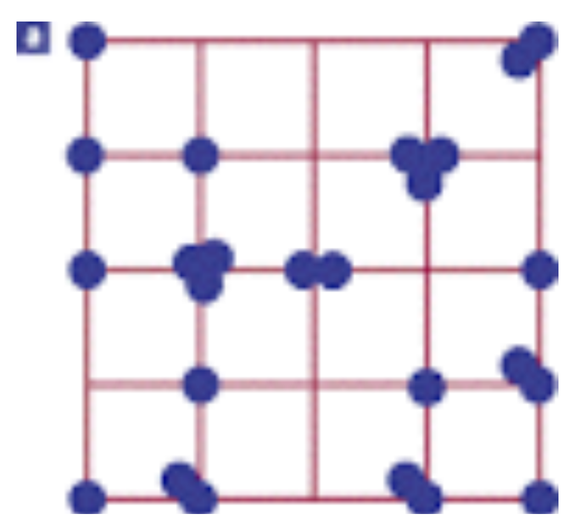
$$N_b = N_{\text{sites}}$$

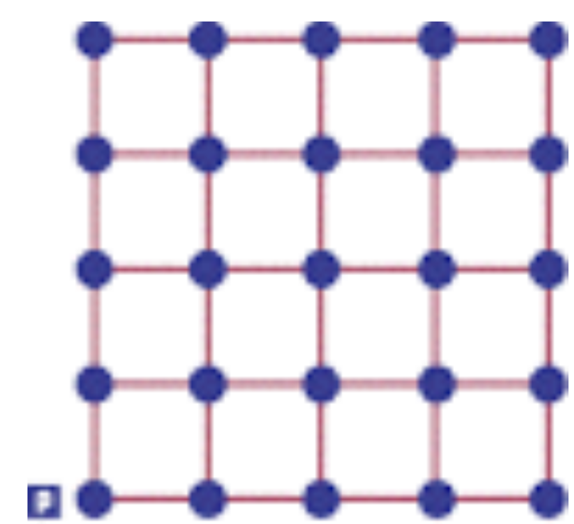
Bosons can form simple Mott state

Superfluid

$\langle b \rangle \neq 0$ ⟶ $\langle b \rangle = 0$

Mott





The spinon fermi surface

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In half-filled band

$$N_f = N_{\text{sites}}$$

$$N_b = N_{\text{sites}}$$

Bosons can form simple Mott state

Superfluid → Mott

$$\langle b \rangle \neq 0 \quad \longrightarrow \quad \langle b \rangle = 0$$

$$\langle c^\dagger_{i\sigma} c_{j\sigma} \rangle \approx \langle f^\dagger_{i\sigma} f_{j\sigma} \rangle \langle b^\dagger_i b_j \rangle$$

Metal → gapped

Florens & Geoges, PRB (2004)

Senthil, PRB (2008)

Electric response of spinon fermi sea

Electron → $c^\dagger_{\sigma i}$

Spinon → $f^\dagger_{\sigma i}$

Chargon (holon) → $b^\dagger_i$

$$c^\dagger_{\sigma i} = f^\dagger_{\sigma i}\, b^\dagger_i$$

U(1) gauge field

$$q_i = n_{fi} - n_{bi}$$

Gauge invariant terms

$$\mathcal{L} = \mathcal{L}_f(p-a) + \mathcal{L}_b(p-A+a) + \cdots$$

$\langle b \rangle \neq 0$ ⟶ ★ ⟶ $\langle b \rangle = 0$

Metal

Gauge field higgsed

Spinon and chargon linearly confined

spinon FS

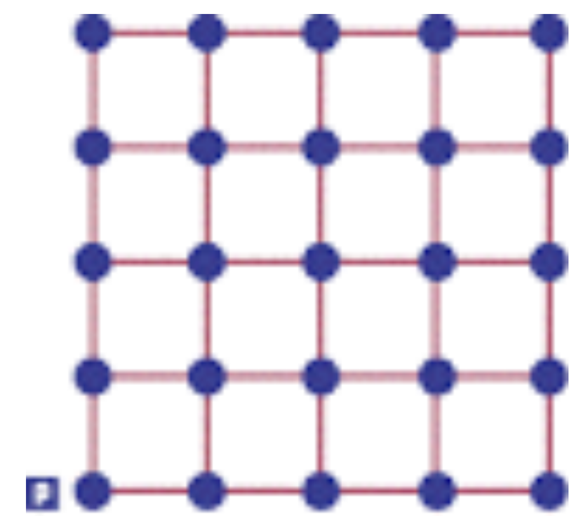
Gauge field Landau damped

Spinon and chargon de-confined

Electric response of spinon fermi sea

Mott

$$\langle b \rangle = 0$$



Electron → Spinon → Chargon (holon)

$$c^\dagger_{\sigma i} = f^\dagger_{\sigma i}\, b^\dagger_i$$

$$\mathcal{L} = \mathcal{L}_f(p-a) + \mathcal{L}_b(p-A+a) + \cdots$$

DC insulator $\lim_{\omega\to 0}\sigma(\omega) = 0$ $\lim_{T\to 0}\sigma(T) = 0$

Boson is a “dielectric” $\sigma_b(\omega) \approx i\omega \frac{1-\epsilon}{4\pi}$

Constraint $j_f = j_b$ Charge is not fully “frozen”

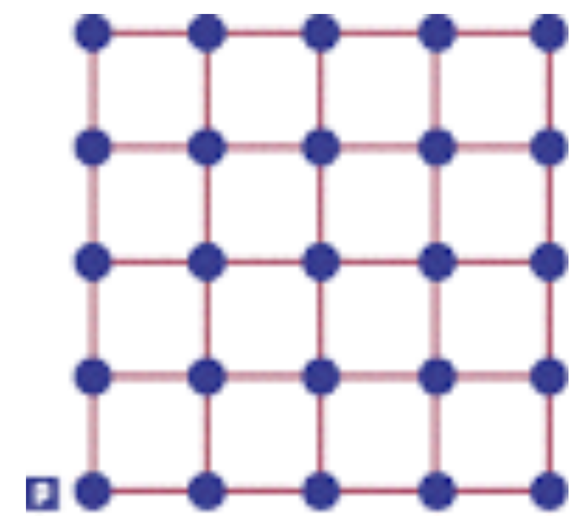
Electric response of spinon fermi sea

Electron → Spinon ↓ ← Chargon (holon)

$$c^\dagger_{\sigma i} = f^\dagger_{\sigma i}\ b^\dagger_i$$

Mott

$$\langle b \rangle = 0$$



$$\mathcal{L} = \mathcal{L}_f(p-a) + \mathcal{L}_b(p-A+a) + \cdots$$

DC insulator $\lim_{\omega\to 0} \sigma(\omega) = 0$

Ioffe-Larkin rule: $\rho = \rho_s + \rho_b$ Ioffe & Larkin, PRB (1989).

$$Re[\sigma(\omega)] = \omega^2 \left(\frac{\epsilon_b - 1}{4\pi}\right)^2 \frac{1}{Re[\sigma_s(\omega)]}$$

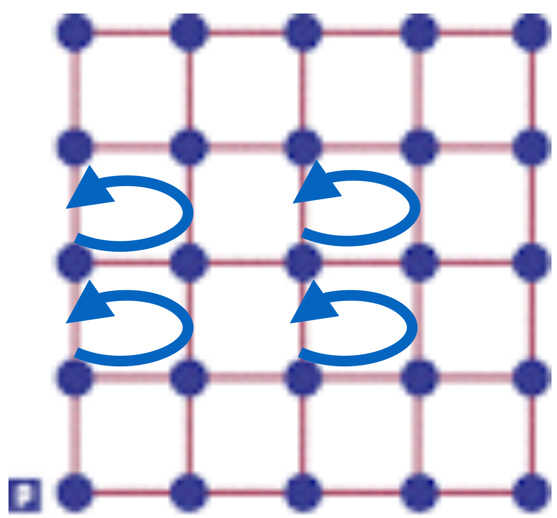
T-K Ng & PA Lee, PRL (2007).

Magnetism of spinon fermi surface

$$\mathcal{L} = \mathcal{L}_f(p - a) + \mathcal{L}_b(p - A + a) + \cdots$$

$\mathbf{B} = \nabla \times \mathbf{A}$

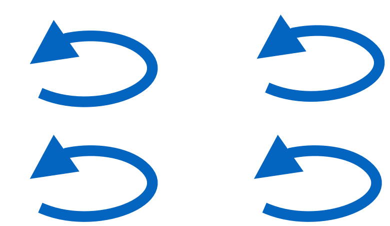
$j_b \neq 0$



$\mathbf{b} = \nabla \times \mathbf{a}$

$j_f = j_b$

$f^\dagger$



$M_f = M_b$

$$\epsilon = \epsilon_f(b) + \epsilon_b(B - b) + \cdots \approx \frac{\chi_f b^2}{2} + \frac{\chi_b (B - b)^2}{2} + \cdots$$

$$\frac{\partial \epsilon}{\partial b} = \frac{\partial \epsilon_f}{\partial b} + \frac{\partial \epsilon_b}{\partial b} = 0 \quad \Rightarrow \quad M_f = M_b$$

Equilibrium
equal
magnetizations

Magnetism of spinon fermi surface

Electron → Spinon → Chargon (holon)

$$c^\dagger_{\sigma i} = f^\dagger_{\sigma i}\, b^\dagger_i$$

$$\epsilon = \epsilon_f(b) + \epsilon_b(B-b) + \cdots \approx \frac{\chi_f b^2}{2} + \frac{\chi_b (B-b)^2}{2} + \cdots$$

$$\frac{\partial \epsilon}{\partial b} = \frac{\partial \epsilon_f}{\partial b} + \frac{\partial \epsilon_b}{\partial b} = 0 \quad \Rightarrow \quad M_f = M_b$$

Equilibrium equal magnetizations

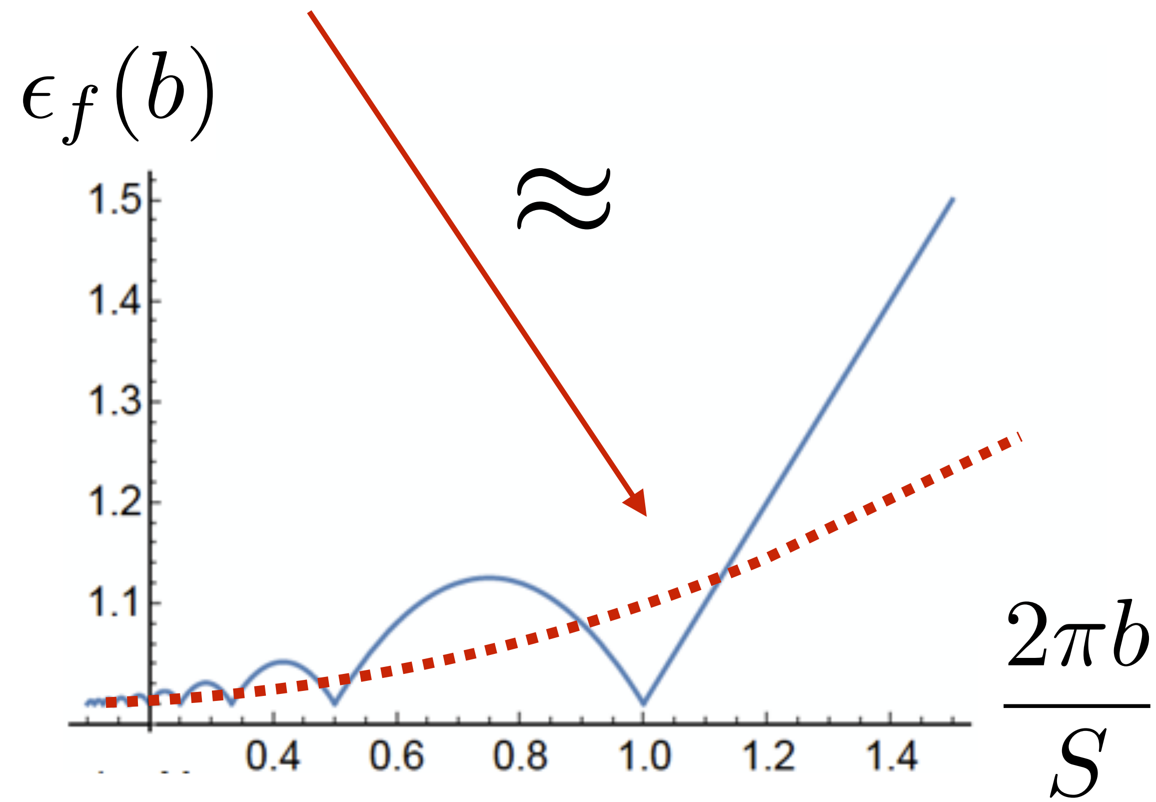
$$b_{\text{eq}} = \frac{\chi_b}{\chi_f + \chi_b} B$$

Metal		**spinon FS**
$\langle b \rangle \neq 0$	→ ★ →	$\langle b \rangle = 0$
$\chi_b = \infty$		$\chi_b < \infty$
$b_{\text{eq}} = B$		$b_{\text{eq}} = \alpha B$

Quantum oscillations of spinons

$$\epsilon = \epsilon_f(b) + \epsilon_b(B-b) + \cdots \approx \frac{\chi_f b^2}{2} + \frac{\chi_b (B-b)^2}{2} + \cdots$$

Spinon spectrum is non-perturbative

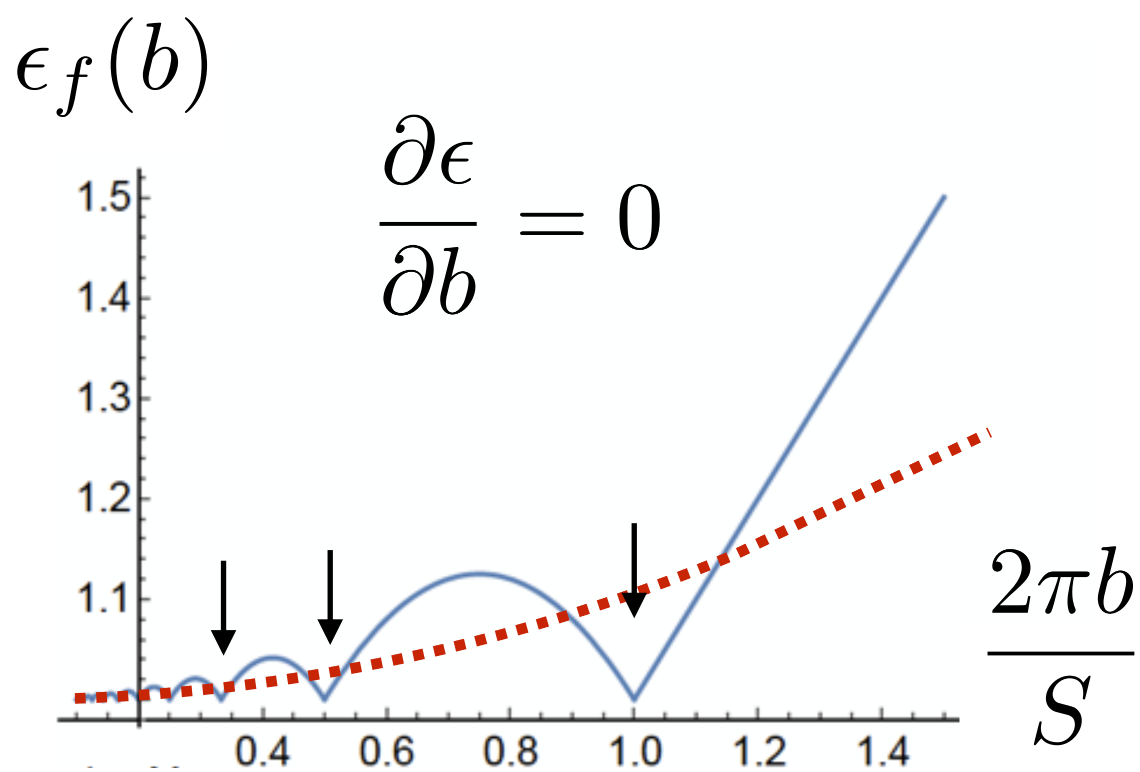


$$\frac{\partial \epsilon}{\partial b} = \frac{\partial \epsilon_f}{\partial b} + \frac{\partial \epsilon_b}{\partial b} = 0$$

Two dimensions:

$$\epsilon^f_{\text{osc}}(b) \approx \frac{\chi_{\text{osc}} b^2}{2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \frac{\frac{kTS}{2\pi b}}{\sinh(\frac{kTS}{2\pi b})} \cos\left(\frac{kS}{b}\right).$$

Quantum oscillations of spinons



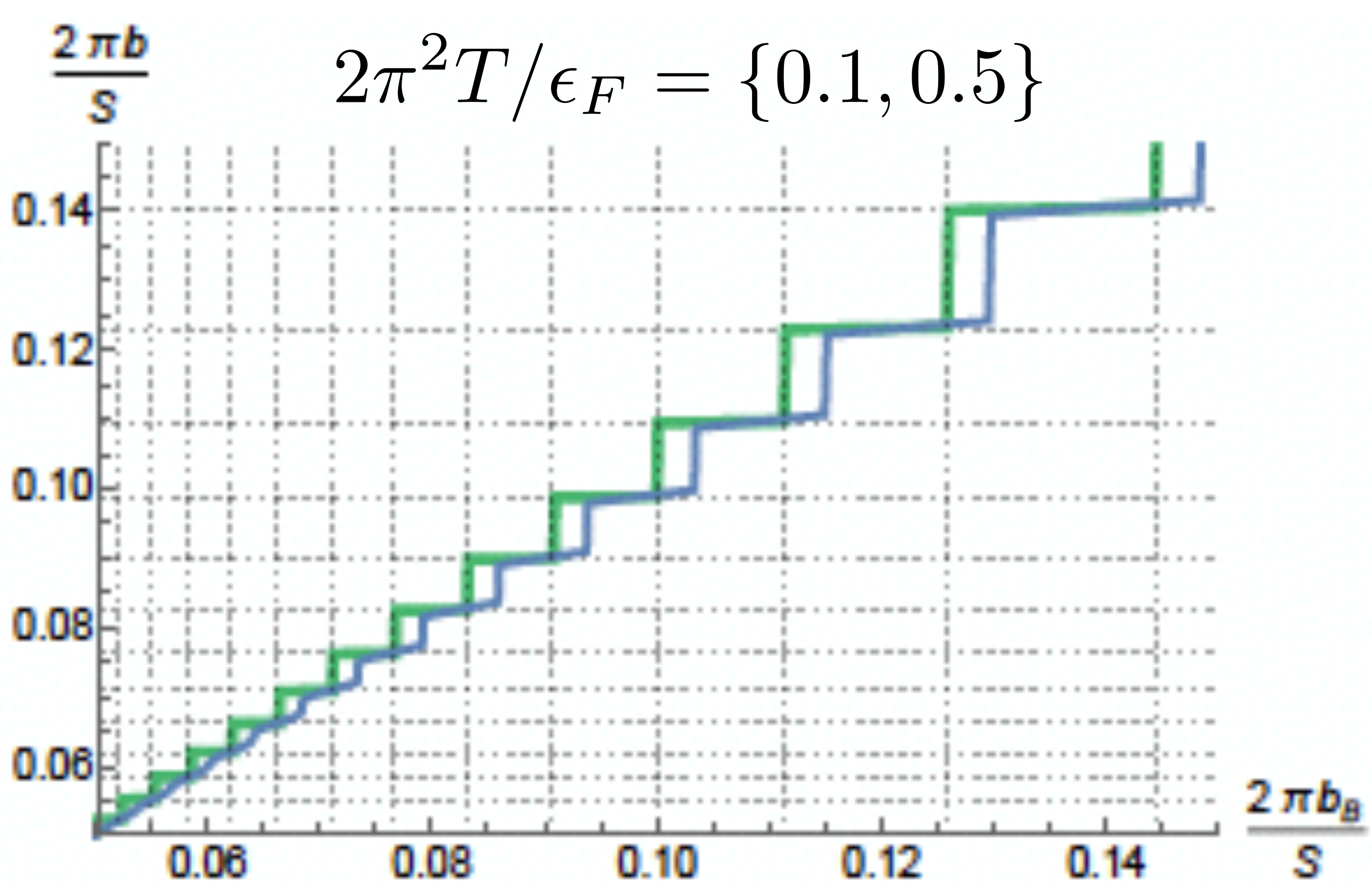
$$\epsilon = \epsilon_f(b) + \epsilon_b(B - b)$$

Two regimes:

$$T \gg \omega_\phi = \frac{b}{m_f} \qquad b_{\text{eq}} \approx \alpha B$$

$$T \ll \omega_\phi = \frac{b}{m_f}$$

several metastable states



Oscillation period in 3D:

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi\alpha}{S_\perp}$$

$$\alpha = \frac{\chi_b}{\chi_f + \chi_b}$$

Quantum oscillations of spinons

Magnetization oscillations of spinons vs metals

Spinons	**2D**	**3D**
period (higher T)	S_F/α	$S_{F\perp}/\alpha$
period (low T)	S_F/α'	$S_{F\perp}/\alpha$
Amplitude	B^2	B^2
Metals		
period	S_F	$S_{F\perp}$
Amplitude	const	$\sqrt{B}$

$$\alpha = \frac{\chi_b}{\chi_f + \chi_b}$$

Inti Sodemann, Debanjan Chowdhury, T. Senthil, arXiv:1708.06354 (2017)

Our proposal

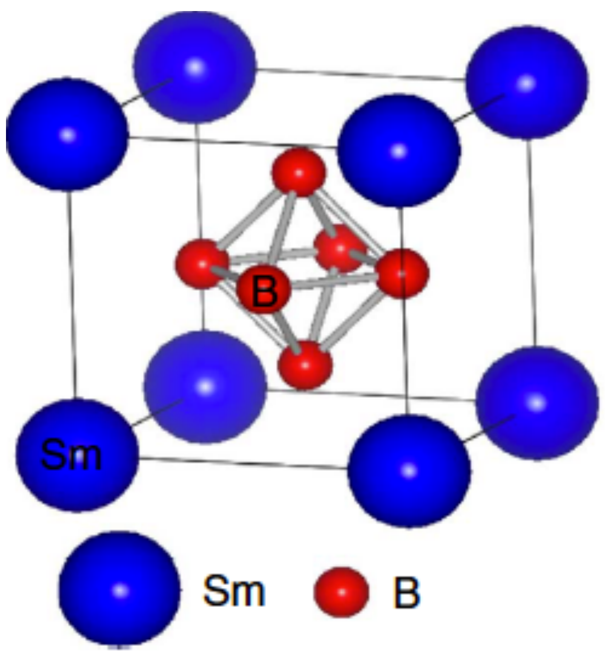
“Composite exciton Fermi liquid” (proposed phase for SmB_6 and mixed valence insulators):

- **Fractionalized** phase with gapped charge degrees of freedom (insulator) and a **neutral fermi surface** (fermionic composite excitons) that **displays quantum oscillations**.

A new mechanism within periodic Anderson model is needed because we don’t have a half filled band, as in the case of spinon fermi surface.

Introduction to SmB_6

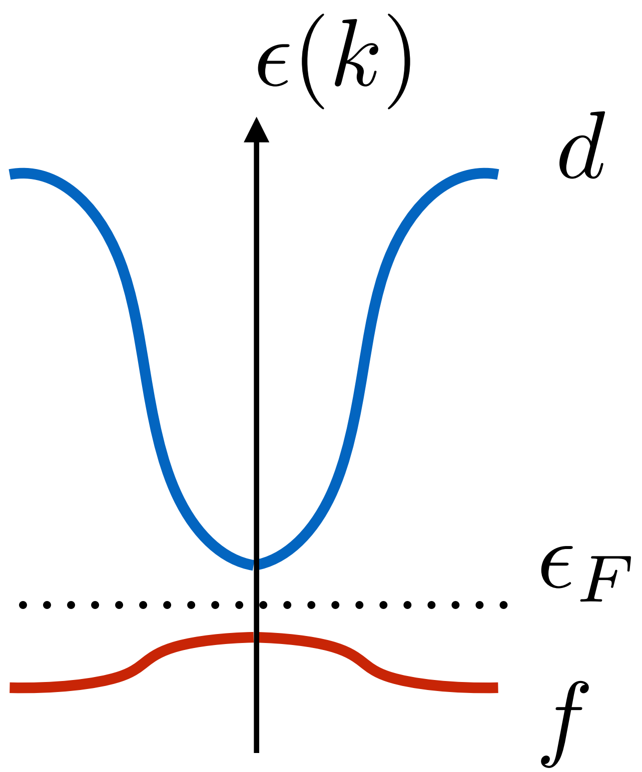
- Simple cubic structure.
- All action happens in Samarium.
- Traditional picture of mixed valence insulator:



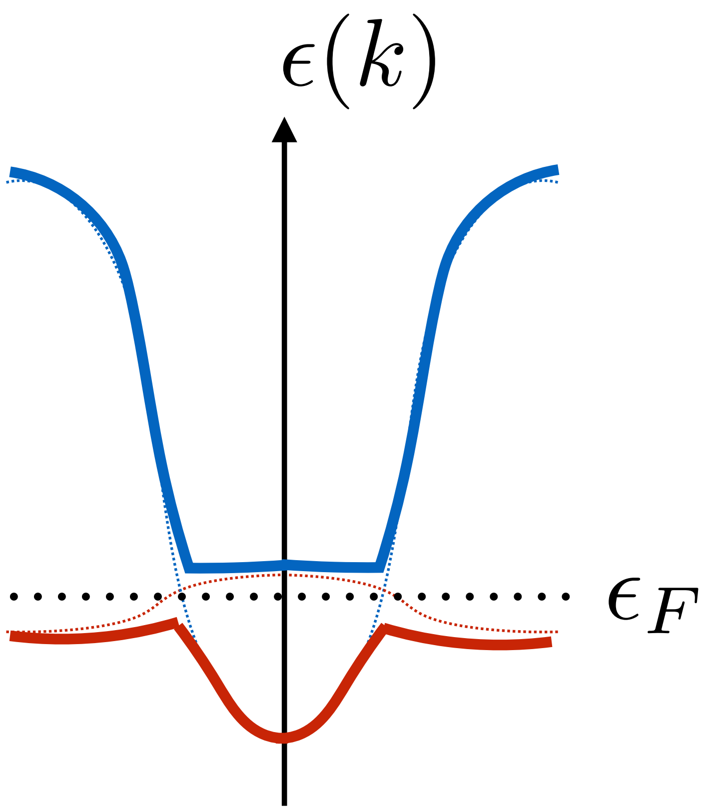
$$[\text{Xe}]4f^6 5d^0 6s^2$$

$$5d^1 + 4f^5 \leftrightharpoons 4f^6$$

Atomic limit



Mixed valence

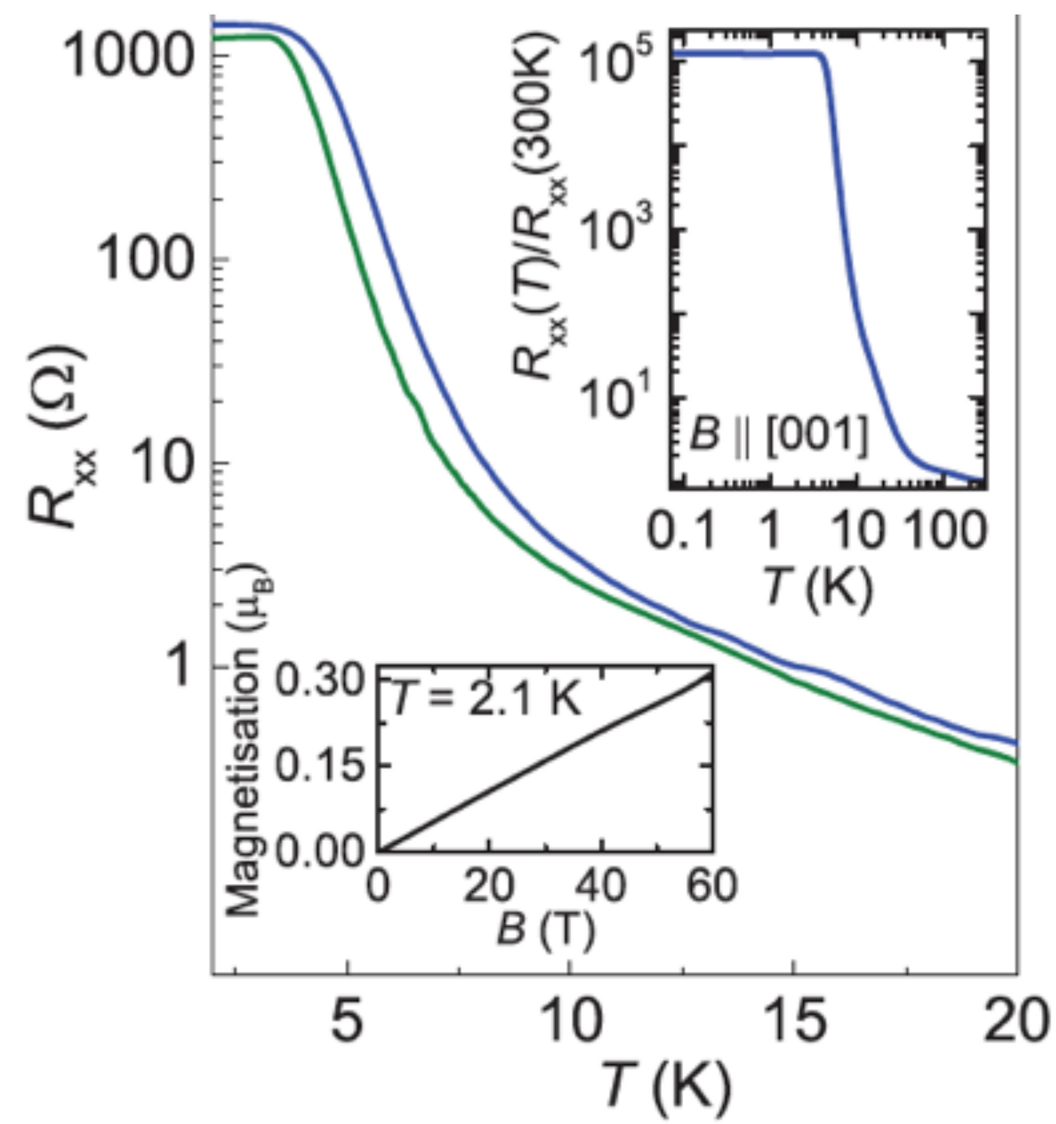


SmB_6 puzzling behavior

- Insulating behavior from charge transport:

$$\rho \approx \rho_0 e^{\frac{\Delta}{T}}$$

$$\Delta \approx 10 meV$$



- Surface is metallic (proposed to be topological).

M. Dzero et al. Ann. Rev. CMP (2016)

B. S. Tan et al., Science (2015).

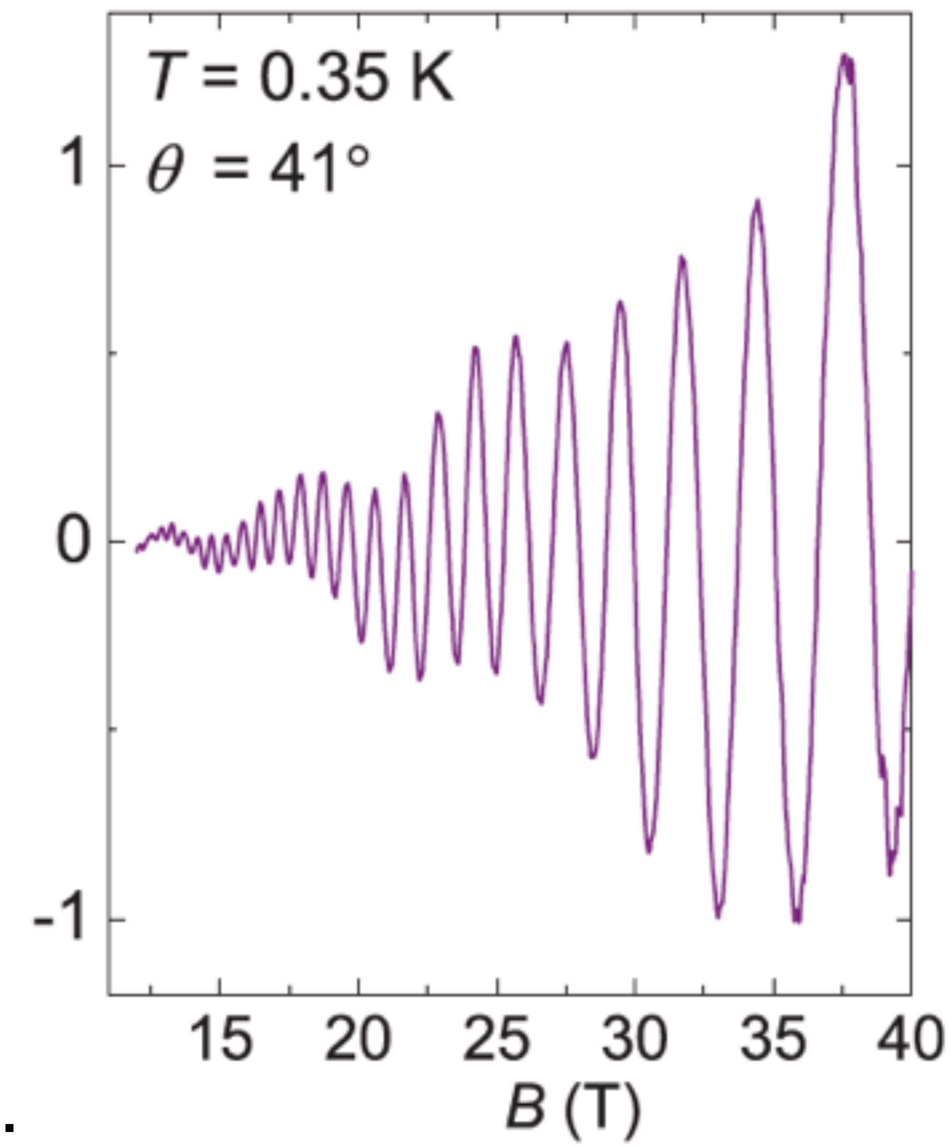
SmB_6 magnetic oscillations

- De Haas-van Alphen effect visible at $B \sim 5T$

- Surface vs bulk picture of dHvA effect:

- G. Li et al. ***Science*** 346, 1208 (2014).
- B. S. Tan et al. ***Science*** 349, 287 (2015).
- J. D. Denlinger et al., arXiv:1601.07408 (2016).

$M(\mathrm{a.u.})$

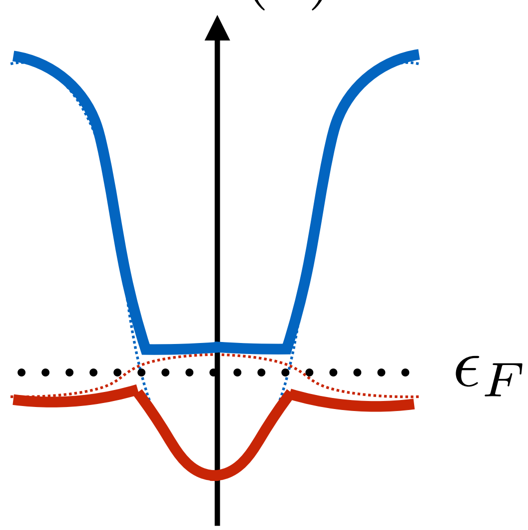


B. S. Tan et al., Science (2015).

SmB_6 puzzles

- Could be magnetic breakdown?

Zhang, Song, Wang, PRL (2016).
Knolle and Cooper, PRL (2015).



Gap:

$$\Delta \sim 10meV$$

Cyclotron:

$$\omega_c \approx 0.2meV\ \ B[T]$$

Theory oscillations visible at $B \sim 50T$

Experiment oscillations visible at $B \sim 5T$

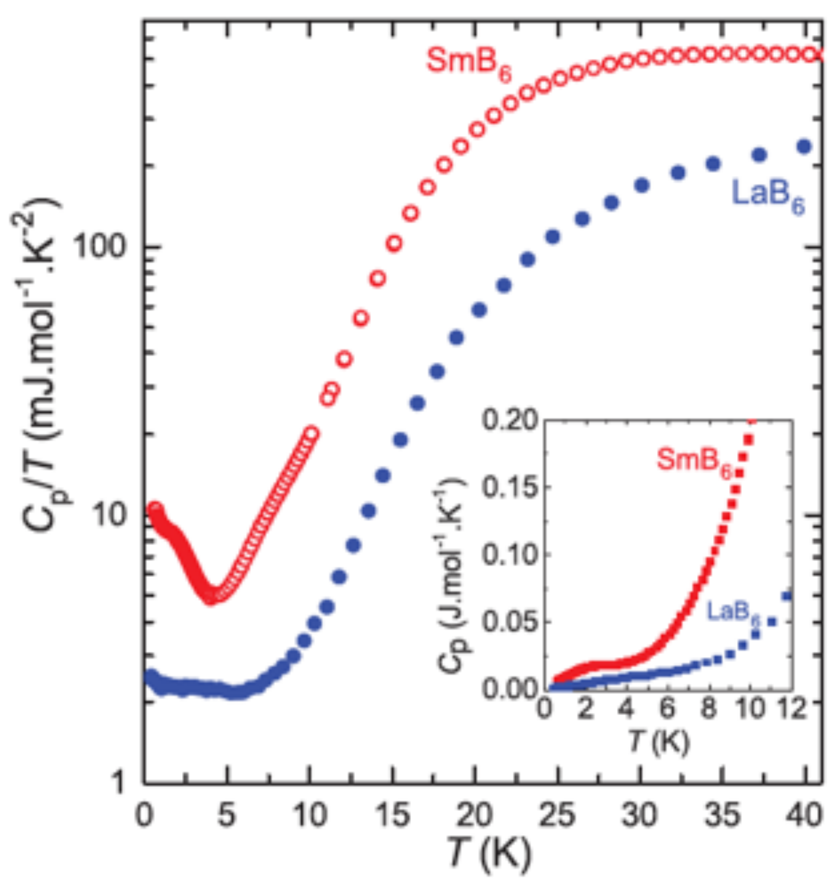
- Other anomalies:

  Specific heat to temperature ratio has finite intercept:

$$\gamma = \frac{C}{T}$$

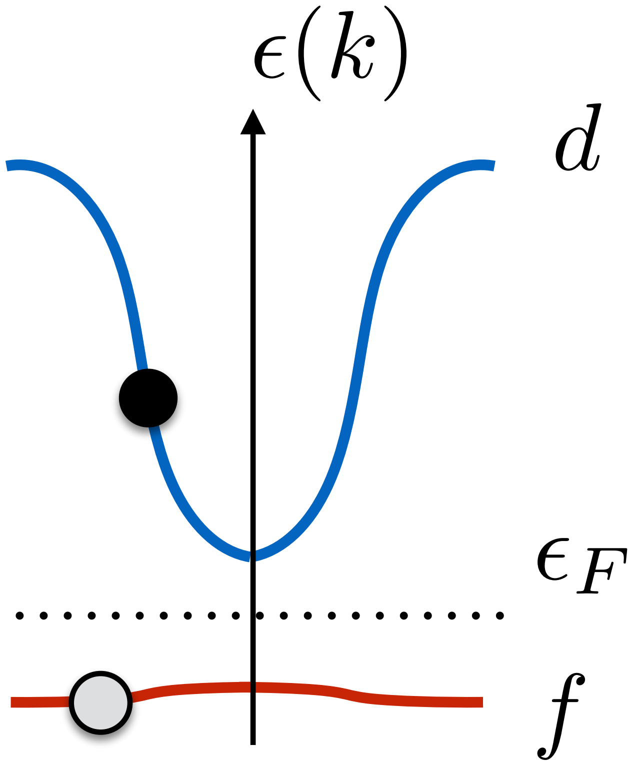
Like in a fermi sea

$$C_{\text{fermions}} \propto \gamma T \qquad C_{\text{phonon}} \propto T^3$$

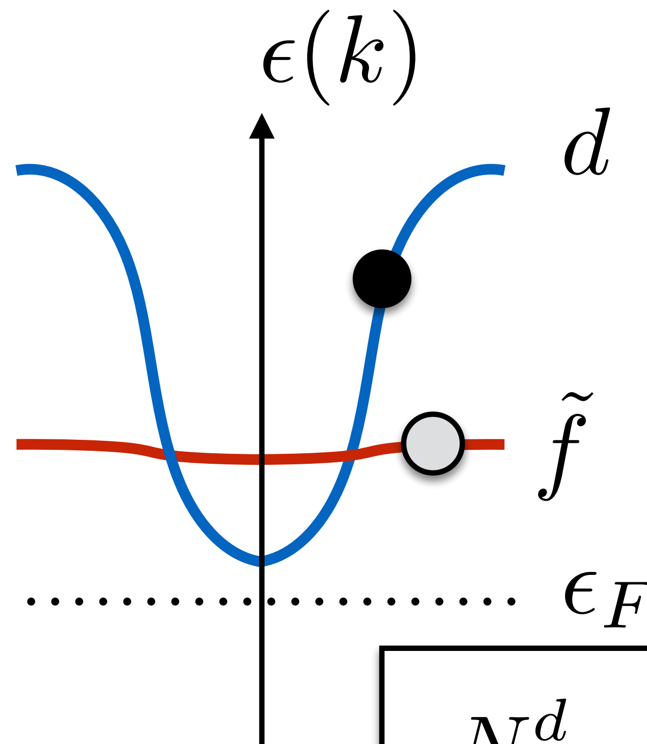


"Composite exciton Fermi liquid"

Atomic limit



f part-hole conjugation



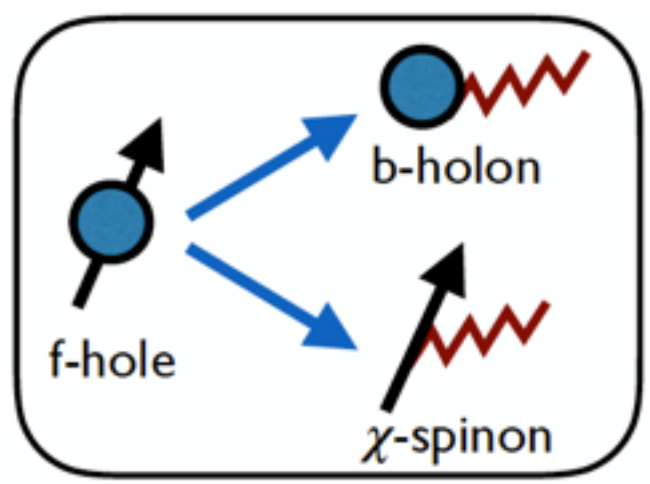
$$N^d_{electrons} = N^f_{holes}$$

f-holes have
strong on-site repulsion

$$U_{ff} \sum_i n_i^f (n_i^f - 1)$$

$U_{ff} \to \infty$ Hard-core constraint

$$n_i^f \leq 1$$



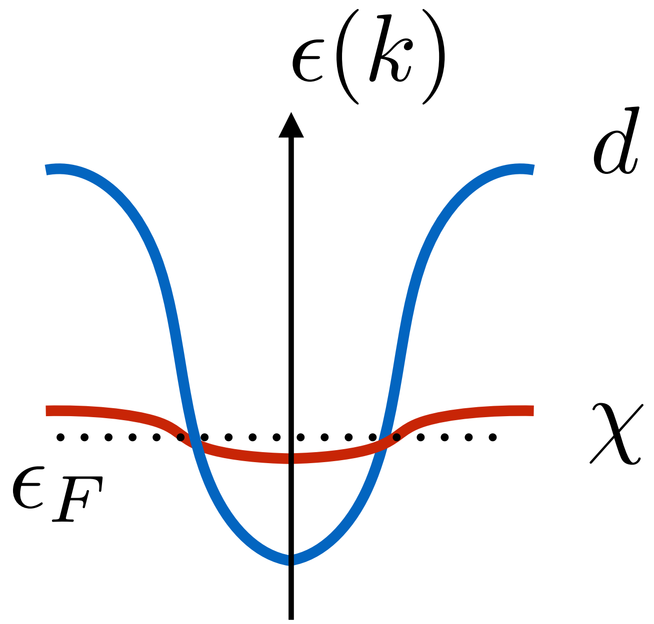
Slave bosons:

$$f_\sigma^\dagger = \chi_\sigma^\dagger b^\dagger$$

$b^\dagger$: spinless boson

$\chi_\sigma^\dagger$: neutral spinfull fermion

"Composite exciton Fermi liquid"



One option:
bosons condense

$$\langle b \rangle \neq 0$$

=> Metal ("boring")

$$\boxed{N^d_{electrons} = N^b = N^\chi}$$

Fermi-bose mixture:

$b^\dagger$: spinless boson

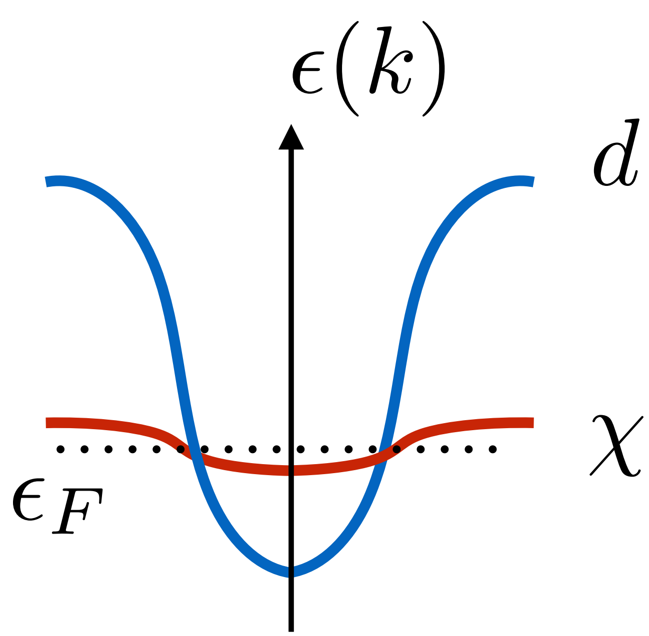
$\chi^\dagger_\sigma$: neutral spinfull fermion

$d^\dagger_\sigma$: d-electron

Slave bosons:

$$f^\dagger_\sigma = \chi^\dagger_\sigma b^\dagger$$

"Composite exciton Fermi liquid"



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Fermi-bose mixture:

$b^\dagger$: spinless boson

$\chi^\dagger_\sigma$: neutral spinfull fermion

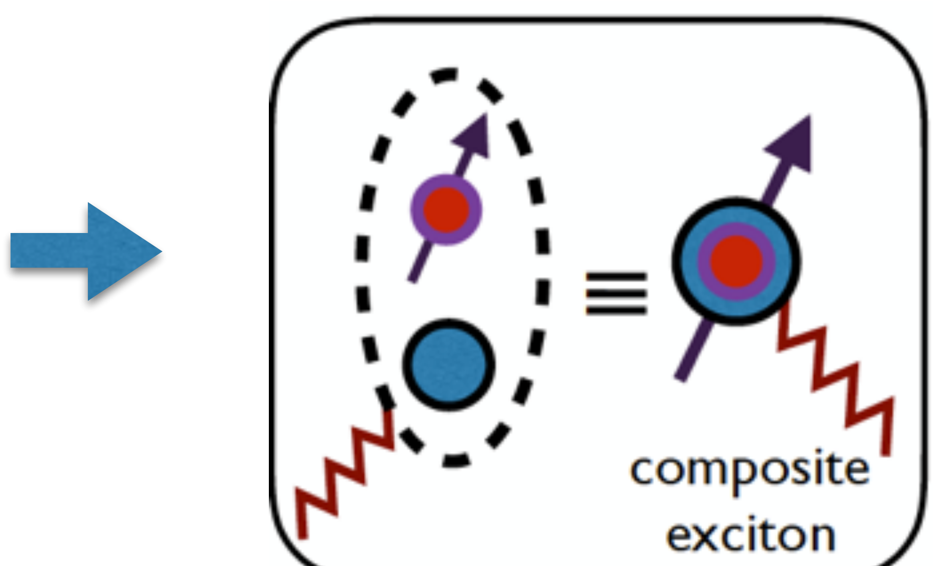
$d^\dagger_\sigma$: d-electron

More "interesting" option:

Bosons bind with d electrons

b and d attract:

$$-U_{df} \sum_i n_i^f n_i^d$$

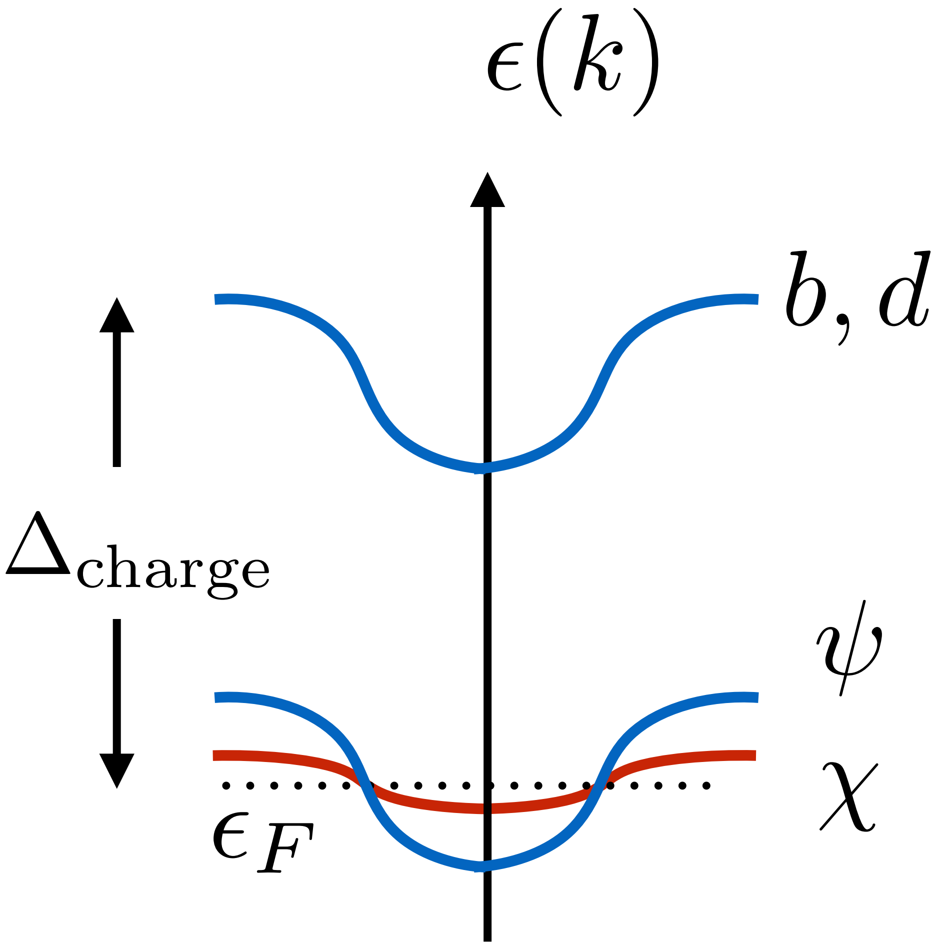


Composite fermionic exciton:

$$\psi_{k\alpha} \equiv b\, d_{k\alpha}, \quad \psi^\dagger_{k\alpha} \equiv b^*\, d^\dagger_{k\alpha}$$

Bound state of "f-holon" and d electron.

“Composite exciton Fermi liquid”



Fermi-bose mixture:

$b^\dagger$: spinless boson

$\chi^\dagger_\sigma$: neutral spinfull fermion

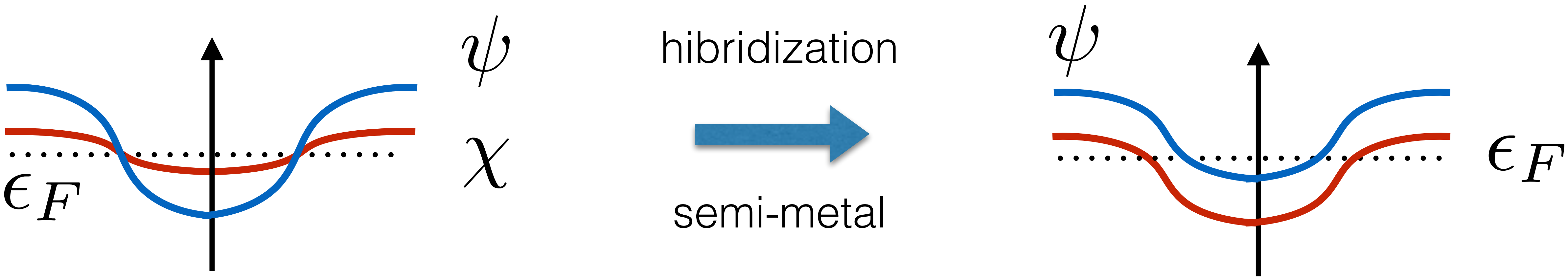
$d^\dagger_\sigma$: d-electron

$\psi^\dagger_\sigma = b^\dagger d^\dagger_\sigma$: fermionic exciton (charge neutral)

Δ_{charge} : “ionization” energy to un-bind fermionic exciton.

charge carrying degrees of freedom gapped: **electrical insulator**

gapless surface of **spin-carrying neutral fermions**



“Composite exciton Fermi liquid”

- Fractionalized phase with low energy description:
  - Neutral (spinful) fermion: ψ_σ forms fermi surface.
  - Charge 1 (spinless) boson: b (gapped).
  - Fermion/boson couple minimally to a gauge field a_μ

$$\mathcal{L} = \sum_i \psi_i^\dagger \left(i\partial_t + \mu_i - a_0 - \frac{(p-a)^2}{2m_i^\psi} \right) \psi_i +$$

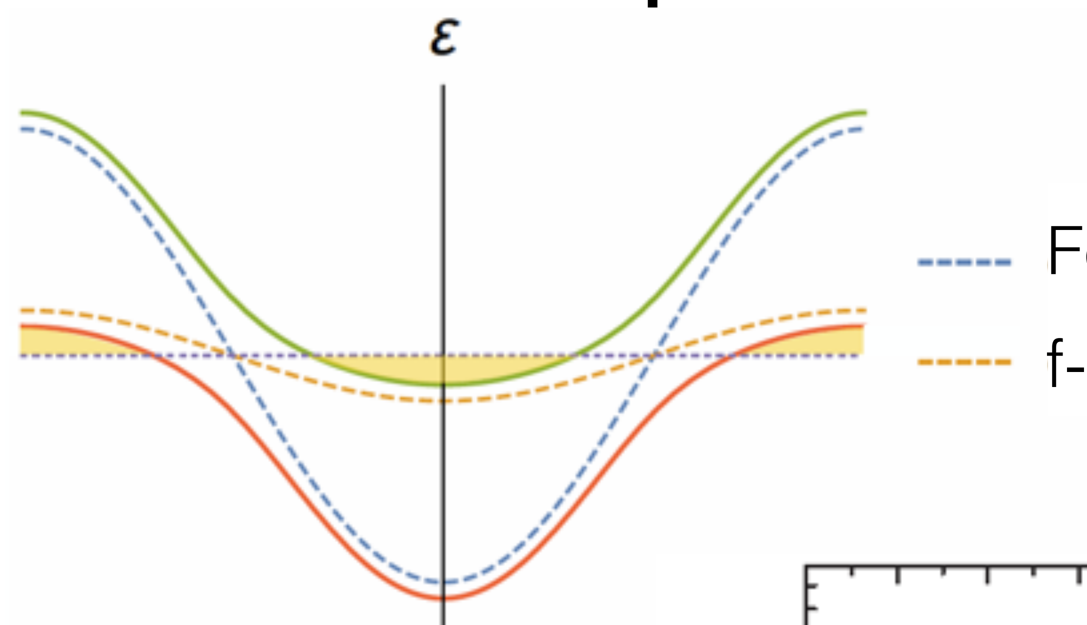
$$+ |(i\partial_\mu + a_\mu - A_\mu)b|^2 - u|b|^2 - \frac{g}{2}|b|^4 + \cdots$$

  - Above description implies it is an insulator with a form of de Haas-van Alphen effect.

- Low energy description is similar to spinon fermi surface although very different in microscopic origin.

Properties of "Composite exciton Fermi liquid"

Fractionalized
fermi sea with two pockets
("semi-metal")



Some properties:

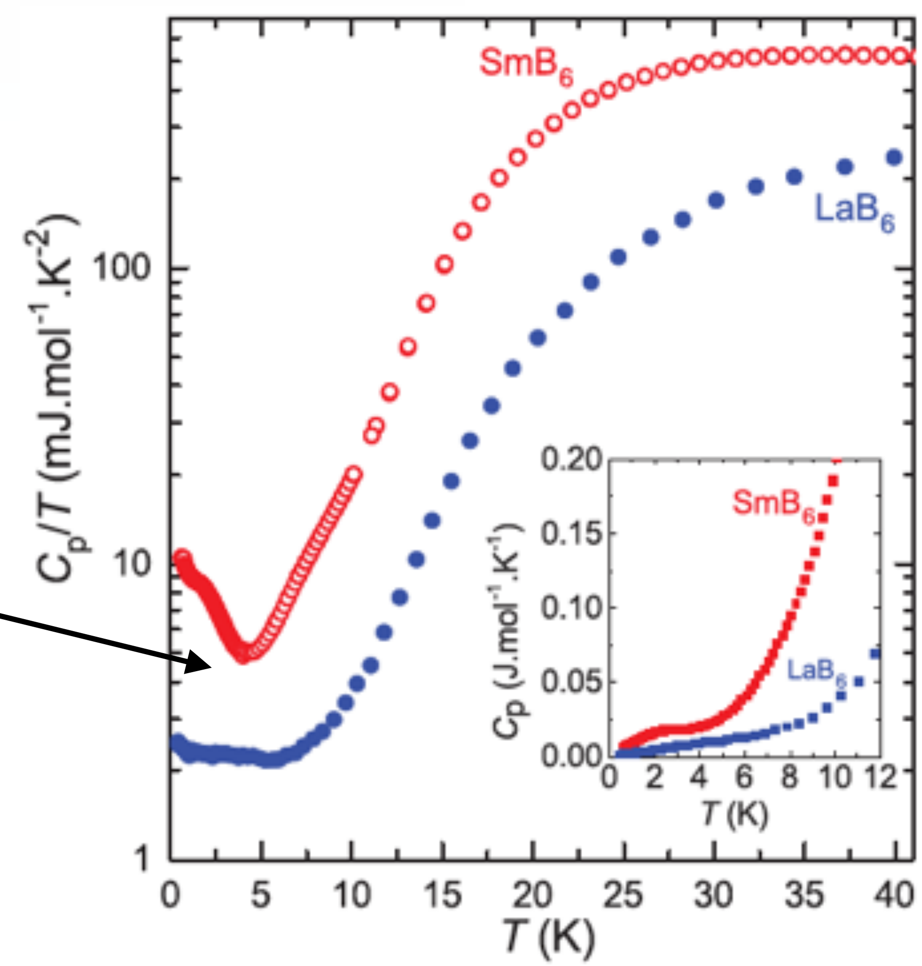
- **Essentially linear specific heat:**

$$C = \gamma T \qquad \gamma \sim \ln(1/T)$$

- **Sub-gap optical conductivity:**

$$\mathrm{Re}[\sigma(\omega)] = \omega^2 \left(\frac{\epsilon_b - 1}{4\pi}\right)^2 \frac{1}{\mathrm{Re}[\sigma_{ce}(\omega)]}$$

Upturn might indicate other physics at lower temperature

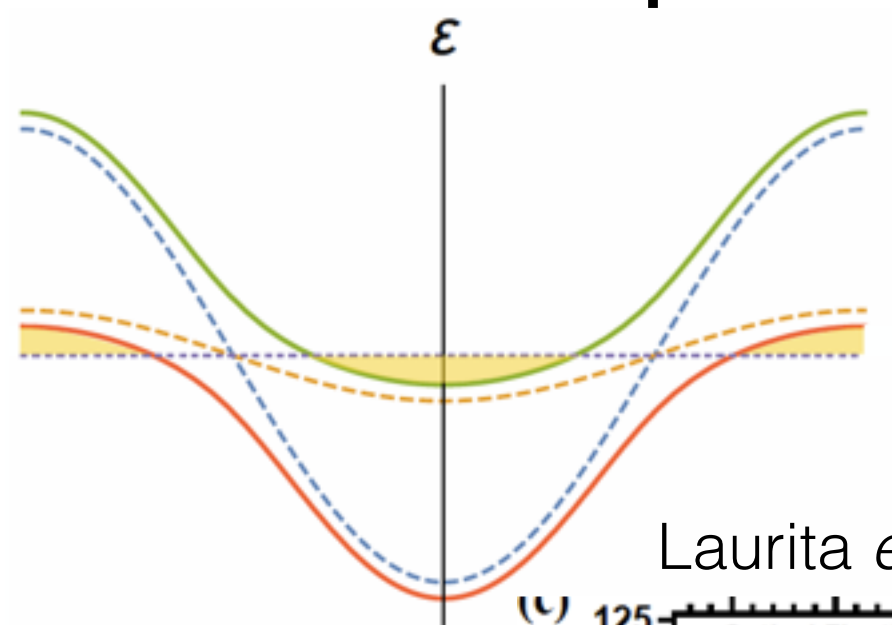


B. S. Tan et al., Science (2015).

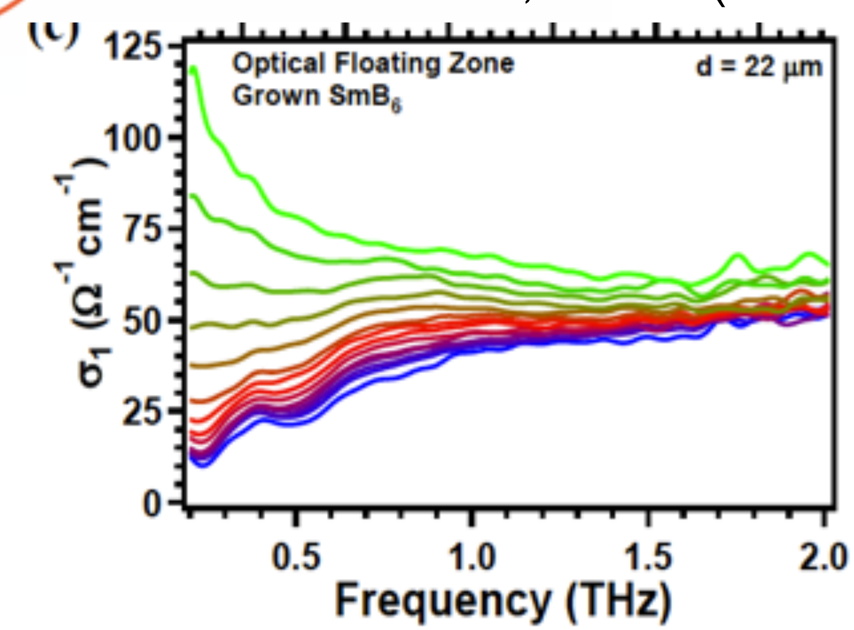
D. Chowdhury, I. Sodemann, T. Senthil, arXiv:1706.00418 (2016).

Properties of “Composite exciton Fermi liquid”

Fractionalized
fermi sea with two pockets
(“semi-metal”)



Laurita *et al.*, **PRB** (2016)



Some properties:

- Essentially linear specific heat:

$$C = \gamma T \qquad \gamma \sim \ln(1/T)$$

- Sub-gap optical conductivity:

$$\mathrm{Re}[\sigma(\omega)] = \omega^2 \left(\frac{\epsilon_b - 1}{4\pi}\right)^2 \frac{1}{\mathrm{Re}[\sigma_{\mathrm{ce}}(\omega)]}$$

Disordered: $\mathrm{Re}[\sigma(\omega)] \sim \omega^2$

Clean: $\mathrm{Re}[\sigma(\omega)] \sim \omega^{2.33}$

$\Delta \sim 1\mathrm{THz}$

D. Chowdhury, I. Sodemann, T. Senthil, arXiv:1706.00418 (2016).

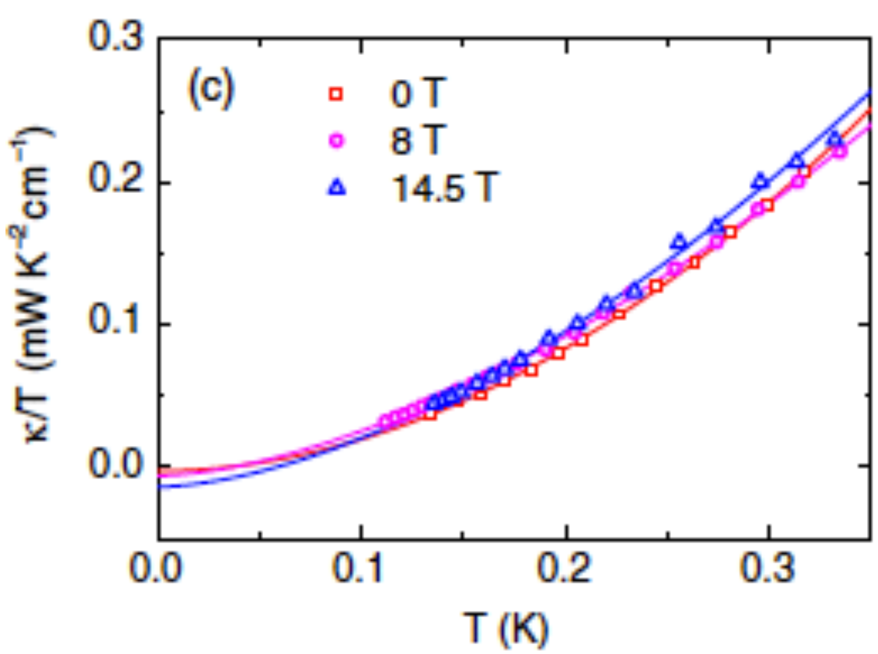
Properties of "Composite exciton Fermi liquid"

Linear T heat conductivity: $$\kappa_{xx} = \sum_{i=1,2} \frac{k_B^2 \tau_i}{9 m_i} \left(\frac{2 m_i \varepsilon_F}{\hbar^2} \right)^{3/2} T$$

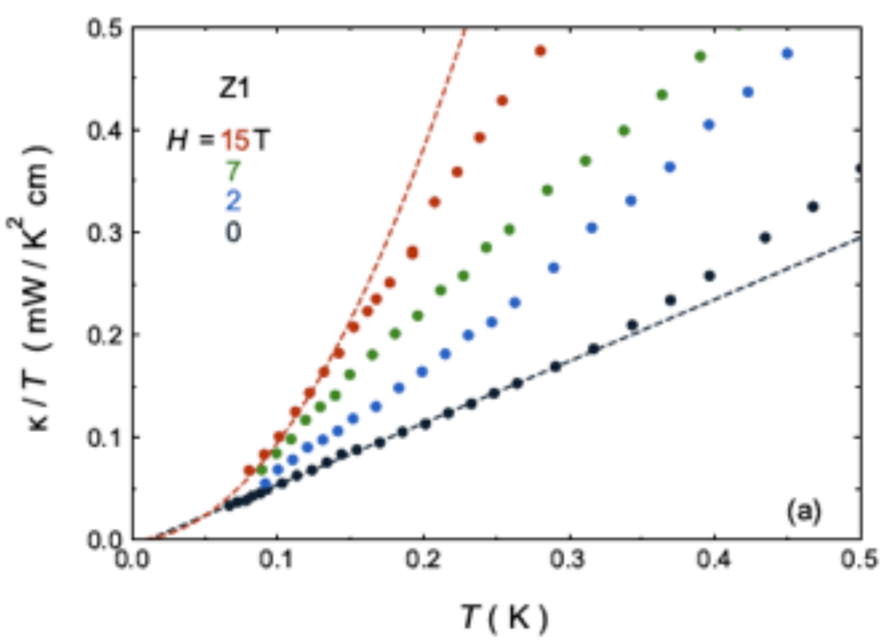
Linear T transverse heat conductivity: $$\kappa_{xy}^i = (\omega_{c,i} \tau_i) \kappa_{xx}^i$$

$$\omega_{c,i} = e|\vec{b}|/m_i = \alpha e|\vec{B}|/m_i$$

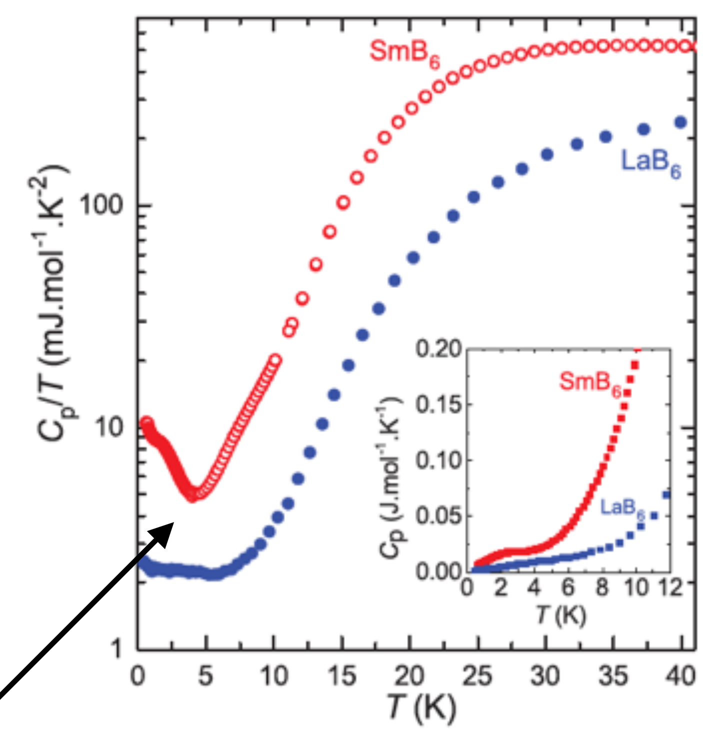
Measured heat conductivity:



Y. Xu *et al.*, **PRL** (2016)



Boulanger *et al*, *arXiv:1709.10456*



B. S. Tan et al., Science (2015).

Useful to extend measurements to higher T !

D. Chowdhury, I. Sodemann, T. Senthil, arXiv:1706.00418 (2016).

Summary

- Strong correlations in mixed valence insulators, such as SmB_6, can give rise to state with gapped charge degrees of freedom but with a surface of gapless neutral fermions.
- Neutral fermion is a superposition of fermionic exciton (bound state of d electron and f holon) with f spinon.
- Insulators with neutral fermi surfaces coupled minimally to internal gauge field give rise to oscillations reminiscent of de Haas-van Alphen effect.

D. Chowdhury, I. Sodemann, T. Senthil, arXiv:1706.00418 (2017).
Inti Sodemann, Debanjan Chowdhury, T. Senthil, arXiv:1708.06354 (2017).