Fractionalization in quantum matter: past, present and future

1) Unraveling the hidden link between composite fermions and exciton condensate

2) Quantum oscillations in insulators with neutral Fermi surfaces

IMPRS RETREAT

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Debye and the birth of quasiparticles

- Debye model (1912)

- Quantized a la Planck-Einstein black body photons.

- Sound ~ Light. Phonon ~ Photon.

Bohr model (1913)    Bose paper (1924)
Landau and the quasiparticle paradigm

• Charged quasiparticles: Fermions.

\[ \Delta Q = +1 \]

\[ t = -\infty \]
\[ V_{int} = 0 \]

• Neutral quasiparticles (quanta of collective oscillations): Bosons.

Magnon
Spin waves

Phonon
Sound
Quantum Hall revolution

- Electrons under strong magnetic fields display the quantum Hall effect:

\[ \sigma_{xy} = n \frac{e^2}{h} \]

\[ n = 1, 2, 3... \]

Landau level filling:

\[ \nu = \frac{N_e}{N_\phi} = n \]

\[ N_\phi = \frac{BA}{\Phi_0} \]
Fractional Quantum Hall effect

- Plateau at 1/3?

\[ \sigma_{xy} = \frac{e^2}{3h} \]

Tsui, Stormer, Gossard, PRL (1982).


Stormer  Tsui  Gossard
Fractional Quantum Hall effect

\[ \Delta Q = +1 \]
\[ t = -\infty \]
\[ V_{int} = 0 \]
\[ \Delta Q = +1 \]
\[ V_{int} \neq 0 \]

- Laughlin liquid at filling 1/3

\[ \Psi = \prod_{i<j} (z_i - z_j)^3 e^{-\frac{|z_i|^2}{4l^2}} \]
\[ z = x + iy \]

\[ \nu = \frac{N_e}{N_\phi} = \frac{1}{3} \]

\[ + \]

\[ + \]

\[ + \]

\[ \cdots \]
Fractional Quantum Hall effect

- Laughlin liquid at filling 1/3 \( \nu = \frac{N_e}{N_\phi} = \frac{1}{3} \)

\[
\begin{align*}
\sigma_{xy} &= \frac{e^2}{3h} \\
\Phi_0 &= \frac{h}{e} \\
\Delta Q &= \sigma_{xy} \Delta \Phi \\
\Delta Q_0 &= \pm \frac{e}{3}
\end{align*}
\]
Fractional Statistics

- Only fermions and bosons in 3D:

\[ (e^{i\varphi})^2 = 1 \]

\[ e^{i\varphi} = \pm 1 \]

Bosons or Fermions

- In 2D “any-ons” are allowed:

\[ \varphi = \pi/3 \]

Laughlin anyons
Fractionalization and topology

- Non-trivial degeneracy on closed manifolds:
  \[ D = (\text{nontrivial quasiparticles} + 1)^G \]
  \[ G = 3 \]
  \[ G = 0 \]
  \[ G = 1 \]

- Non-abelian anyons: “irrational” size of Hilbert space.

  \[ D_{\gamma_1 \gamma_2} = 2 \quad \rightarrow \quad D_\gamma = \sqrt{2} \]
  Majorana fermions

Experimentally realized in:
- GaAs at \( \nu = 5/2 \)
- 1D chains
  superconducting p-wave.
The hidden link between composite fermions and the exciton condensate

- BEC - BCS crossover a powerful unification in physics of quantum matter:

![Diagram showing BEC and BCS](image)

- Bose Einstein condensate molecules
- Superconductor Fermions

- Unification between two celebrated quantum Hall phases of matter: the exciton condensate and the composite fermion metal.
Exciton condensate

- No tunneling but strong interactions

\[ \nu = \nu_{\text{top}} + \nu_{\text{bottom}} = 1/2 + 1/2 \]

- Exciton condensate:

\[ |\text{top}\rangle + e^{i\phi} |\text{bottom}\rangle \]

\[ \langle c_{\text{bottom}}^\dagger c_{\text{top}} \rangle \propto e^{i\phi} \]

Long range XY order
Properties of exciton condensate

• Superfluidity for charge imbalance:
  \[ Q_- = Q_{\text{top}} - Q_{\text{bottom}} \quad [Q_-, \phi] = i \]

• Linearly dispersing Goldstone mode of \( \phi \) (pseudo-spin wave).

• Half-charged vortices (merons):
  \[ v = 1 \]
  \[ Q_+ = e/2 \]
  \[ 2\pi \text{ winding} \]
  \[ Q_+ = (vn_z) \frac{e}{2} \]
  \[ v \in \mathbb{Z} \]
  \[ n_z = \pm 1 \]

Composite fermion metal

- Fractionalized metal for half filled landau level:
  \[ N_e = \frac{1}{2} N_\phi \]

- Composite fermion: electron bound to two vortices

\[ \vec{B} \neq 0 \quad \Rightarrow \quad \vec{B}_{eff} = 0 \]

- Emergent 2-dimensional “gauge field” (analogous to the electro-magnetic field in 2D).

Duality in 1+1D QFT's

1 + 1 Sine – Gordon
\[ \frac{1}{2} (\partial \phi)^2 + (m/\beta)^2 \cos(\beta \phi) \]

\[ \frac{4\pi}{\beta^2} = 1 + \frac{g}{\pi} \]

1 + 1 Massive – Thirring
\[ \bar{\psi}(i\partial - m)\psi - \frac{g}{2} (\bar{\psi} \gamma_\mu \psi)^2 \]

\[ \psi^\dagger \psi = \frac{\beta}{2\pi} \partial_x \phi \]

S. Coleman, PRD 1975
Fermion vortex duality

**Physical Dirac fermion**

\[ \mathcal{L}_e = \bar{\psi}_e (i \partial - A) \psi_e + \mathcal{L}_{\text{int}} \]

\[ \delta n_{\text{elec}}(r) = \frac{\nabla \times \vec{a}}{4\pi} \]

\[ \psi_e^\dagger \leftrightarrow M_{4\pi} \]

**Dirac composite fermion vortex**

\[ \mathcal{L}_{cf} = \bar{\psi}_{cf} (i \partial - \phi) \psi_{cf} + \frac{a d A}{4\pi} + \mathcal{L}_{\text{int}} \]

\[ \hat{\mathbf{z}} \times \mathbf{j}_{\text{elec}}(r) = \frac{\nabla a_0 + \partial_t \vec{a}}{4\pi} \]

Electron creation is flux insertion operator

Bilayer exciton condensate and Composite fermion metal

- Are zero and infinite distance connected?

\[ d = 0 \quad \text{Exciton condensate} \quad ? \quad \text{Two C-Fermion metals} \quad d = \infty \]

- Precedents

**Theory**

Paired Quantum Hall State

Exciton condensate

Bonesteel et al. PRL (1996)

**Numerics**

\[ \nu = \nu_{top} + \nu_{bottom} = \frac{1}{2} + \frac{1}{2} \]

**Experiment**

Eisenstein, ARCMP (2014)
Bilayer exciton condensate and Composite fermion metal

- A special particle-hole invariant “cooper pairing” of composite fermions is equivalent to exciton condensate:

\[
\Delta = i\psi^\dagger \sigma_y \tau_x \psi^\dagger \sim i\psi_{\text{top}}^\dagger \sigma_y \psi_{\text{bottom}}^\dagger
\]

Exciton condensate from CF pairing

- Symmetric gauge field is gapped via Higgs.

- Anti-symmetric gauge field remains gapless. 2+1 Maxwell theory has a spontaneously broken symmetry:

\[
\begin{align*}
a_+ &= \frac{a_1 + a_2}{2} \\
\quad \text{a}_- &= \frac{a_1 - a_2}{2}
\end{align*}
\]

\[
\begin{aligned}
\langle \mathcal{M}_-(r) \mathcal{M}^\dagger_- (0) \rangle &\xrightarrow{|r| \to \infty} \text{const} \\
n^e_{\text{top}} - n^e_{\text{bottom}} &= \frac{\nabla \times \vec{a}_-}{2\pi}
\end{aligned}
\]

\[
\langle c^\dagger_{\text{bottom}} c_{\text{top}} \rangle \propto e^{i\phi}
\]

The state is an exciton condensate!
Relative $u(1)$ photon = Goldstone mode

- Photon is exciton condensate “spin-wave”.
- Electric charges under field $a_-$ are vortices of condensate order parameter:

$$4\pi q_- \leftrightarrow \text{vorticity}$$

$$\hat{z} \times (\vec{j}_{\text{top}}(r) - \vec{j}_{\text{bottom}}(r)) = \frac{\nabla a_0 + \partial_t \tilde{a}}{4\pi}$$
Abrikosov vortices = merons

- Abrikosov vortices carry half charge:

\[ n_{\text{top}}^e + n_{\text{bottom}}^e = \frac{\nabla \times \vec{a}_+}{2\pi} \rightarrow Q_\pi = \pm \frac{1}{2} \]

- Abrikosov vortices have a complex fermion zero mode:

\[
\begin{align*}
|0\rangle & \rightarrow |1\rangle \\
|1\rangle & \equiv \psi_0^\dagger |0\rangle \\
\end{align*}
\]

Layer X-change

\[
\begin{array}{ccc}
|0\rangle & \rightarrow & |1\rangle \\
|1\rangle & \rightarrow & |0\rangle \\
\end{array}
\]

\[
\begin{array}{ccc}
q_- & \quad (\text{vorticity}) \\
1/2 & \quad 2\pi \\
-1/2 & \quad -2\pi \\
\end{array}
\]
Abrikosov vortices = merons

- Two $\pi$ Abrikosov vortices of opposite vorticity are mutual semions
  \[ \pi - \text{vortex} \]
  \[ Q = 1/2 \]

- Their fusion is a fermion:

The electron (with layer charge imbalance neutralized by condensate).
Bogoliubov fermion

- Consider fusing two Abrikosov vortices of opposite flux but same charge (order parameter vorticity):

\[ \frac{4\pi}{\alpha} \]

Neutral Vortex

- Fusion is Bogoliubov fermion

\[ Q = \frac{1}{2}, \quad Q = -\frac{1}{2} \]
**Dictionary**

**Exciton condensate**

**Spin-wave**

\[ Q = 1/2 \]

2\(\pi\) winding

**XY vortex**

\[ Q = 1/2 \]

**Composite fermion superconductor**

**Photon**

\[ E \]

\[ a_1 - a_2 \]

**Abrikosov vortex**

\[ \pi \text{ flux} \]

\[ Q = 1/2 \]

**4\(\pi\) neutral vortex**

\[ Q = 1/2 \]

\[ Q = -1/2 \]

**Composite fermion**

Charge neutral Dipole carrying
Fractionalization w/out magnetic fields

- Spin liquids in frustrated magnets.

- Bosonic Laughlin state can be viewed as chiral spin liquid after mapping bosons to spins.

- “Smoking gun” experimental signatures?

- Fractionalization beyond the realm of frustrated magnets or quantum Hall?
Puzzles of SmB$_6$

- Simple cubic structure.
- All action happens in Samarium.
- Traditional picture of mixed valence insulator:

\[ [\text{Xe}] 4f^6 5d^0 6s^2 \]

\[ 5d^1 + 4f^5 \rightleftharpoons 4f^6 \]
SmB$_6$ puzzling behavior

- Insulating behavior from charge transport:

\[ \rho \approx \rho_0 e^{\frac{\Delta}{T}} \quad \Delta \approx 10\text{meV} \]

- De Haas-van Alphen effect visible at \( B \sim 5T \)

SmB$_6$ puzzles

• Could be magnetic breakdown?  

\[ \epsilon(k) \]

\[ \epsilon_F \]

△ ~ 10meV

\[ \omega_c \approx 0.2\text{meV} \ B[T] \]

Theory oscillations visible at \( B \sim 50T \)

Experiment oscillations visible at \( B \sim 5T \)

• Other anomalies:

Specific heat to temperature ratio has finite intercept:

\[ \gamma = \frac{C}{T} \]

Like in a fermi sea

\[ C_{\text{fermions}} \propto \gamma T \]

\[ C_{\text{phonon}} \propto T^3 \]
“Composite exciton Fermi liquid”

\[ \epsilon(k) \]

\[ d \text{ electron} \]

One option: bosons condense

\[ \langle b \rangle \neq 0 \]

\[ \Rightarrow \text{Metal ("boring")} \]

Fermi-bose mixture:

\[ b^\dagger : \text{spinless boson} \]

\[ \chi^\sigma_\sigma : \text{neutral spinfull fermion} \]

\[ d^\dagger_\sigma : \text{d-electron} \]

More “interesting” option:

Bosons bind with d electrons

- \[ U_{df} \sum_i n_i^f n_i^d \]

Composite fermionic exciton:

\[ \psi_{k\alpha} \equiv b \ d_{k\alpha}, \ \psi^\dagger_{k\alpha} \equiv b^* \ d^\dagger_{k\alpha} \]

Bound state of “f-holon” and d electron.
Properties of “Composite exciton Fermi liquid”

Fractionalized fermi sea with two pockets ("semi-metal")

Some properties:

- Essentially linear specific heat:
  \[ C = \gamma T \quad \gamma \sim \ln(1/T) \]

- Sub-gap optical conductivity:
  \[ \text{Re}[\sigma(\omega)] = \omega^2 \left( \frac{\epsilon_b - 1}{4\pi} \right)^2 \frac{1}{\text{Re}[\sigma_{ce}(\omega)]} \]

Upturn might indicate other physics at lower temperature

The end of the beginning!

Conceptual frontier:
• Topological matter beyond free fermions.
• Fractionalization and topology in 3D.
• Gapless fractionalized phases in 2D and 3D.
• Novel non-perturbative approaches to interacting systems.

Real world frontier:
• New probes for fractionalized matter.
• Fractionalization beyond quantum Hall and frustrated magnets.
• More cross talk between materials and models.

Non-equilibrium and transport frontier:
• Transport in fractionalized and topological matter.
• Collective behavior and broken symmetries in topological and fractionalized matter.
• Dynamics of nearly conserved quantities (hydrodynamics).