Dualities in 2+1 quantum field theories and the link between composite fermions and the exciton condensate

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Plan of talk

- The boson - boson vortex duality.
- The fermion - fermion vortex duality.
- The link between composite fermions and exciton condensate.

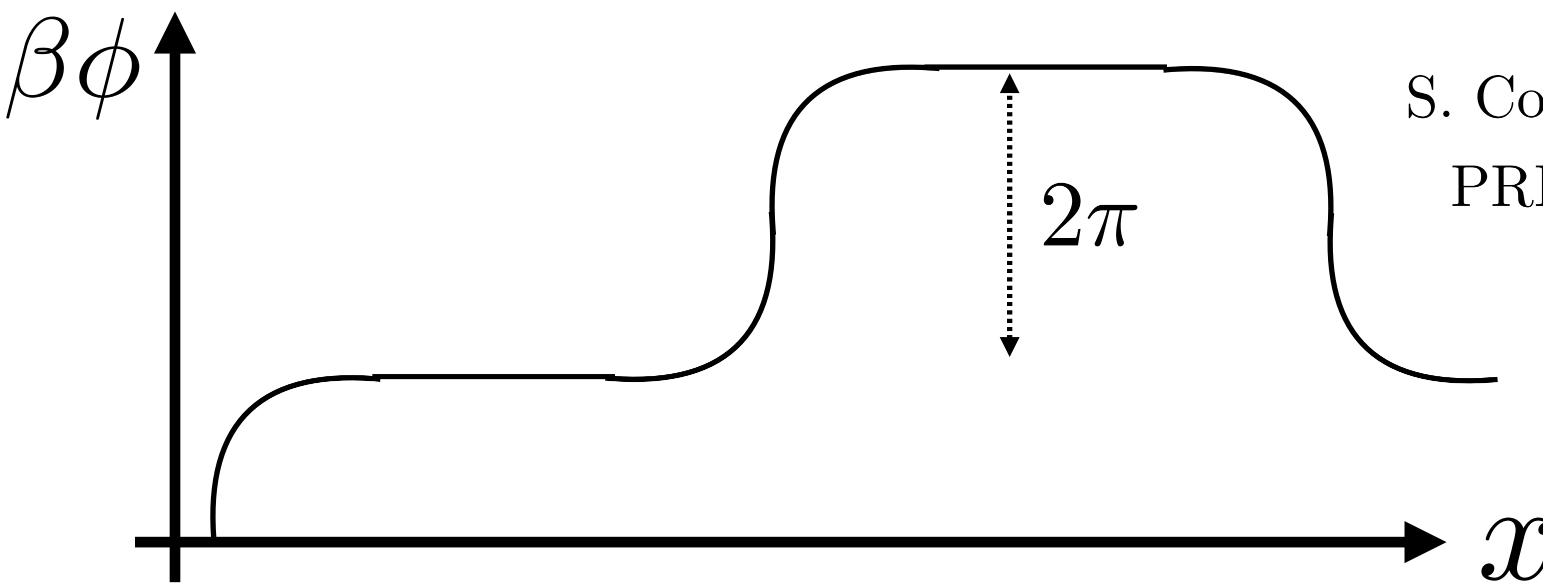
"UV" duality of 1D QFTs

Coleman Luttinger Haldane

$$1+1 \ \mathrm{Sine-Gordon} \qquad \equiv \qquad 1+1 \ \mathrm{Massive-Thirring}$$

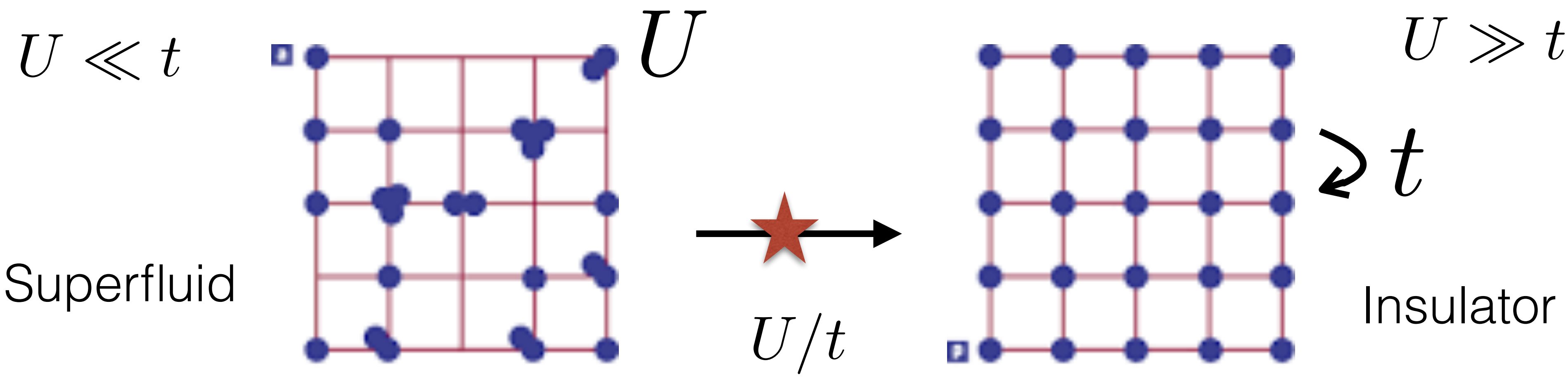
$$\frac{1}{2}(\partial\phi)^2 + (m/\beta)^2\cos(\beta\phi) \qquad \equiv \qquad \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{g}{2}(\bar{\psi}\gamma_\mu\psi)^2$$

$$\frac{4\pi}{\beta^2} = 1 + \frac{g}{\pi} \qquad \psi^\dagger\psi = \frac{\beta}{2\pi}\partial_x\phi$$



S. Coleman, PRD 1975

Superfluid to insulator transition of bosons



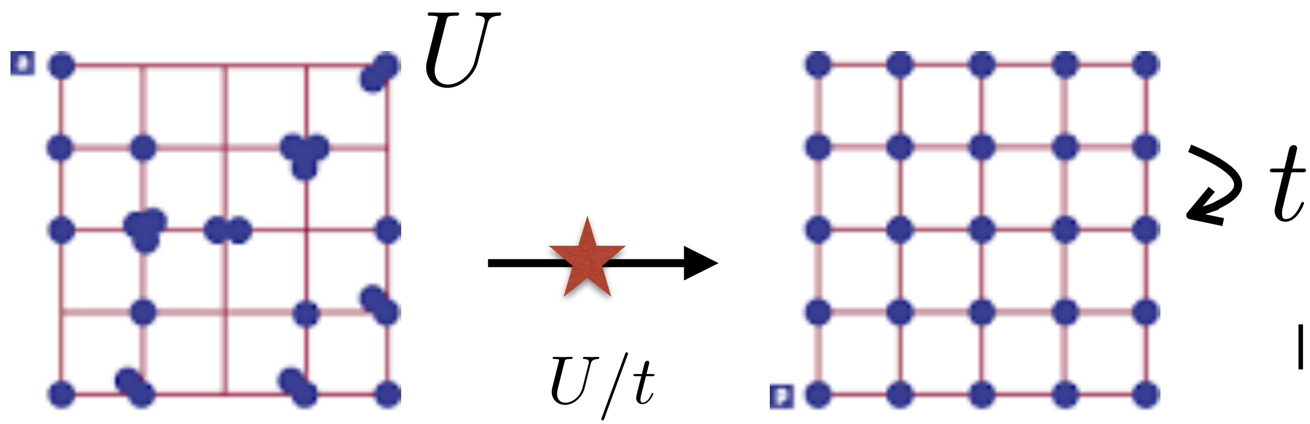
Key ingredients:

- Local Hamiltonian of microscopic lattice bosonic operators.
- Microscopic U(1) symmetry of total particle number conservation.
- Integer filling of the lattice.

At critical point a relativistic field theory emerges.

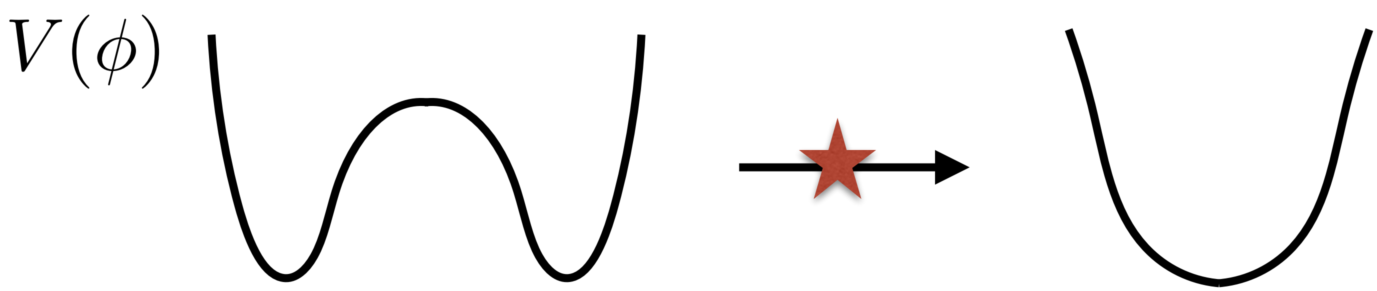
Field theory of superfluid-insulator transition

Superfluid



Insulator

$\langle \phi \rangle \neq 0$ $\quad$ $\langle \phi \rangle = 0$



ϕ boson creation operator

$$\phi = |\phi_0| e^{i\varphi}$$

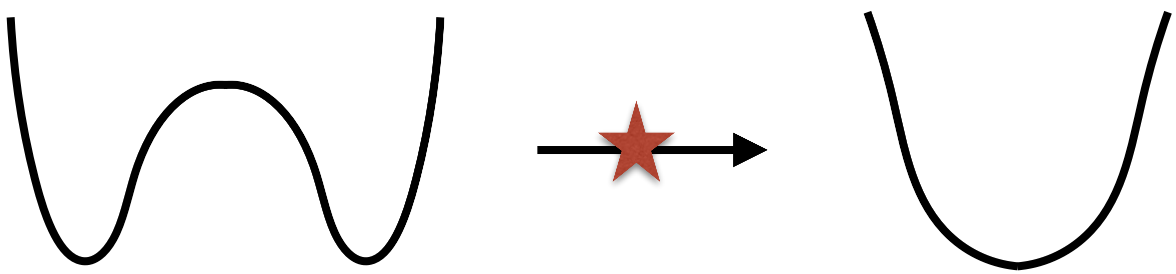
Phase fluctuations are gapless and describe sound

Superfluid vortices

Superfluid $\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$ Insulator

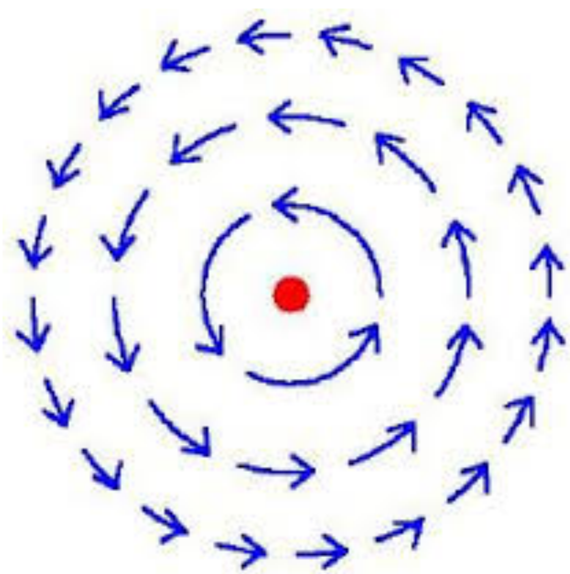
$V(\phi)$



boson creation operator ϕ

$$\phi = |\phi_0| e^{i\varphi}$$

Vortex



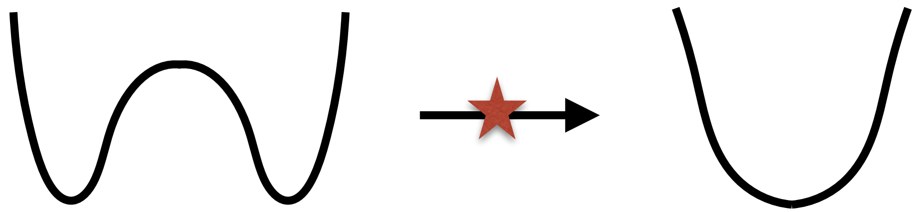
$$j = \rho_s \nabla \varphi$$

$$j \propto \frac{1}{r}$$

vortex is non-local:
carries a power law current

Boson-vortex duality in 2+1 D

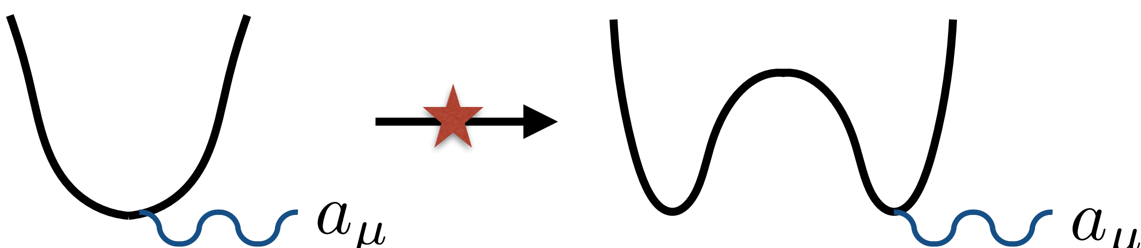
Superfluid $\langle \phi \rangle \neq 0$ $\langle \phi \rangle = 0$ Insulator



boson creation operator ϕ

$$|(i\partial_\mu - A_\mu)\phi|^2 + r|\phi|^2 + u|\phi|^4$$

Superfluid $\langle \psi \rangle = 0$ $\langle \psi \rangle \neq 0$ Insulator



vortex creation operator ψ

$$|(i\partial_\mu - a_\mu)\psi|^2 + s|\psi|^2 + v|\psi|^4 + \frac{1}{2\pi}\epsilon_{\mu\nu\sigma}A_\mu\partial_\nu a_\sigma$$

Peskin, Ann, Phys. 113, 122 (1978). Dasgupta, Halperin, PRL 47, 1556 (1981).

Boson-vortex duality dictionary

$$|(i\partial_\mu - A_\mu)\phi|^2 + r|\phi|^2 + u|\phi|^4$$ ϕ boson

$$|(i\partial_\mu - a_\mu)\psi|^2 + s|\psi|^2 + v|\psi|^4 + \frac{1}{2\pi}\epsilon_{\mu\nu\sigma}A_\mu\partial_\nu a_\sigma$$ ψ vortex

$$\delta n(r) = \phi^\dagger \phi = \frac{\nabla \times \vec{a}}{2\pi}$$

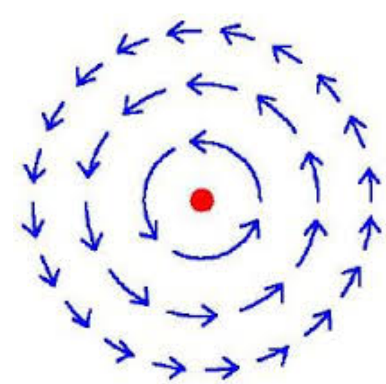
Incompressibility of insulator =
"Meissner effect"

$$\hat{z} \times \vec{j}(r) = \vec{e}(r) = -\frac{\nabla a_0 + \partial_t \vec{a}}{2\pi}$$

Gauss law =
non-local current around vortex

Faraday law =
continuity equation

ψ

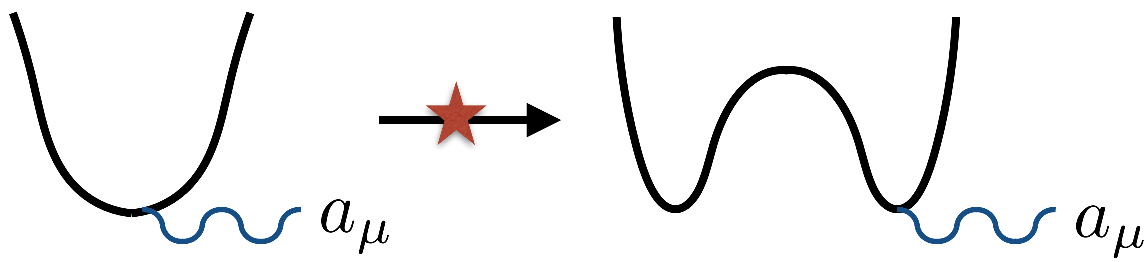


$$j \propto \frac{1}{r}$$

Boson-vortex duality in 2+1 D

Superfluid $\langle \psi \rangle = 0$ $\langle \psi \rangle \neq 0$ Insulator

vortex creation operator ψ



$$|(i\partial_\mu - a_\mu)\psi|^2 + s|\psi|^2 + v|\psi|^4 + \frac{1}{2\pi}\epsilon_{\mu\nu\sigma}A_\mu\partial_\nu a_\sigma$$

$$\delta n(r) = \phi^\dagger \phi = \frac{\nabla \times \vec{a}}{2\pi} \qquad \hat{z} \times \vec{j}(r) = \vec{e}(r) = -\frac{\nabla a_0 + \partial_t \vec{a}}{2\pi}$$

Photon = sound

Abrikosov vortex = boson $\phi^\dagger \leftrightarrow M_{2\pi}$

Higgs mechanism = Absence of low energy excitations in the insulator

quantization of vorticity = boson number quantization

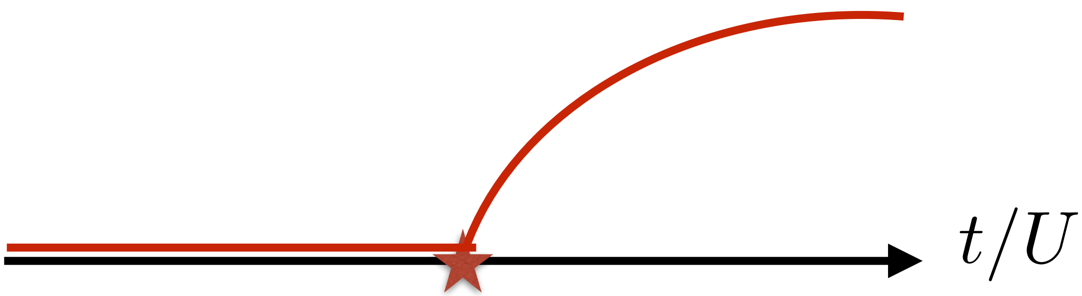
Polyakov confinement of U(1) gauge theory

Insulator

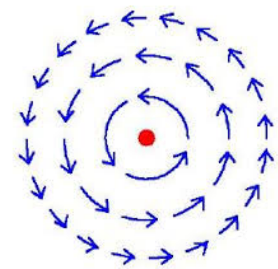
$\langle \phi \rangle = 0$

Superfluid

$\langle \phi \rangle \neq 0$



Gapless sound = massless photon



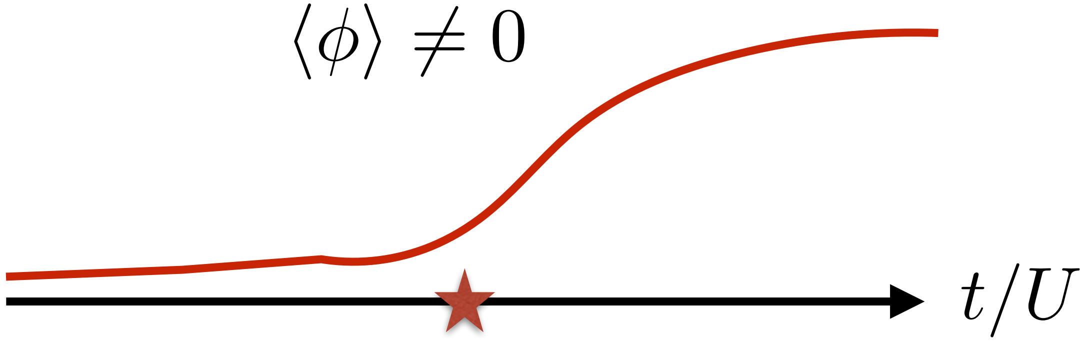
Vortices have 1/r force

Break explicitly U(1) boson conservation

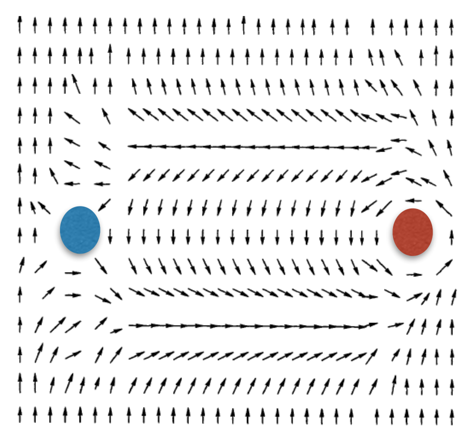
$$\phi^\dagger \leftrightarrow M_{2\pi}$$

Insulator

$\langle \phi \rangle \neq 0$



“Pinned” phase = massive photon



Linearly confined vortices

Boson and vortex fractionalization

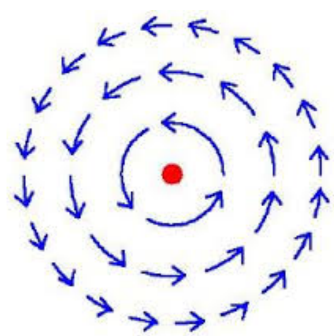
“Trivial” Insulator | “Trivial” superfluid

$\langle \phi \rangle = 0$ | $\langle \phi \rangle \neq 0$

“Trivial” Insulator | “paired” superfluid

$\langle \phi \rangle = 0$ | $\langle \phi\phi \rangle \neq 0$, $\langle \phi \rangle = 0$

$$\phi^2 = |\phi_0|^2 e^{2i\varphi}$$



“half-vortices” allowed

$$\Delta(2\varphi) = 2\pi$$

$$\Delta(\varphi) = \pi$$

Boson and vortex fractionalization

"trivial" superfluid | "trivial" Insulator

$$\langle\psi\rangle = 0 \qquad\qquad \langle\psi\rangle \neq 0$$

"trivial" superfluid | "paired" insulator

$$\langle\psi\rangle = 0 \qquad\qquad \begin{matrix}\langle\psi\rangle = 0 \\ \langle\psi\psi\rangle \neq 0\end{matrix}$$

$\mathbb{Z}_2$
spin liquid

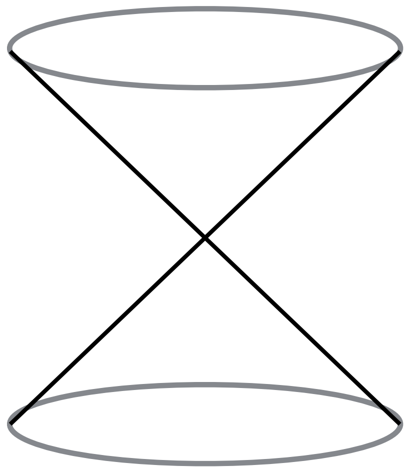
Abrikosov vortices will trap half flux quantum!

Excitations carry fractional charge!

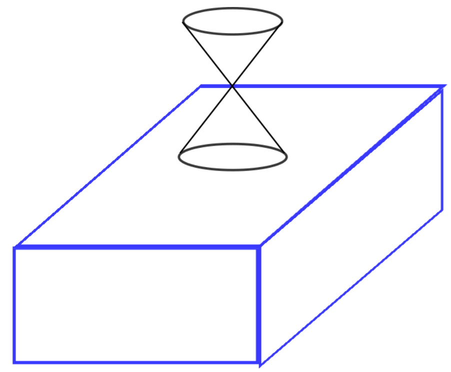
$$\delta n(r) = \phi^\dagger \phi = \frac{\nabla \times \vec{a}}{2\pi} \quad \Rightarrow \quad \Delta N = \int d^2r \; \delta n(r) = \pm\frac{1}{2}$$

Fermion vortex duality

Dirac fermion

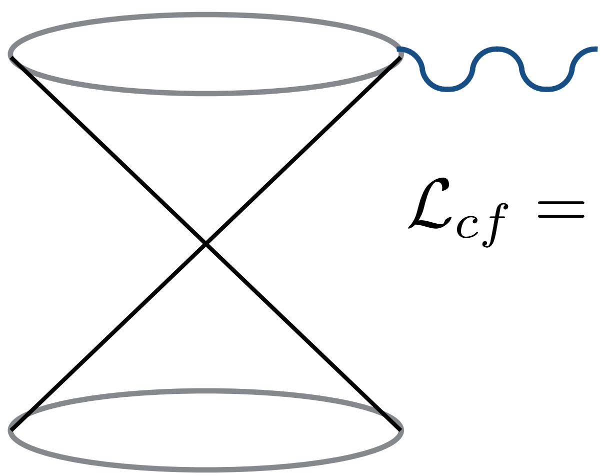


$$\mathcal{L}_e = \bar{\psi}_e(i\partial\!\!\!/ - A\!\!\!/)\psi_e + \mathcal{L}_{\text{int}}$$



Topological insulator

Dirac fermion vortex



a_μ

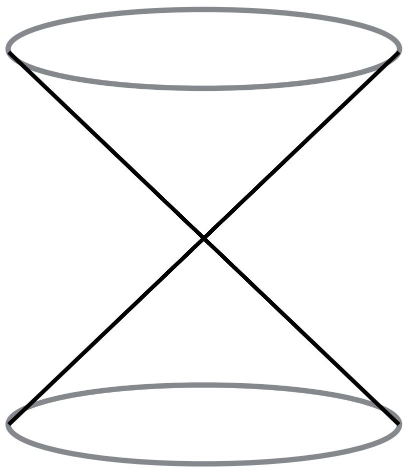
$$\mathcal{L}_{cf} = \bar{\psi}_{cf}(i\partial\!\!\!/ - a\!\!\!/)\psi_{cf} + \frac{adA}{4\pi} + \mathcal{L}_{\text{int}}$$

cf = composite fermion

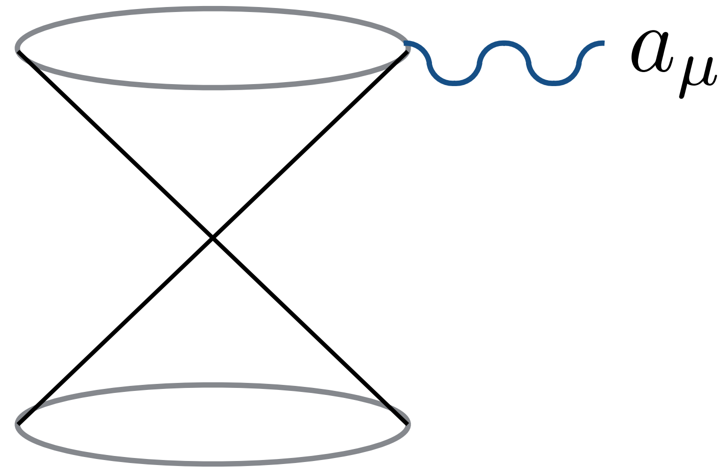
Son PRX (2015). Wang, Senthil PRB (2016). Metlitski, Vishwanath, PRB (2015). Mross, Alicea, Motrunich PRL (2016). Seiberg, Wang, Senthil, Witten Ann. Phys. (2016).

Fermion vortex duality

Dirac fermion



Dirac composite fermion vortex



$$\mathcal{L}_e = \bar{\psi}_e(i\not\partial - \not A)\psi_e + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{cf} = \bar{\psi}_{cf}(i\not\partial - \not a)\psi_{cf} + \frac{adA}{4\pi} + \mathcal{L}_{\text{int}}$$

$$\delta n_{elec}(r) = \frac{\nabla \times \vec{a}}{4\pi}$$

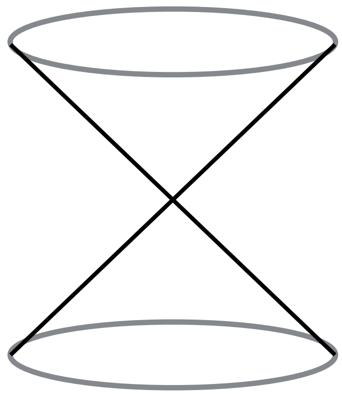
$$\hat{z} \times \vec{j}_{elec}(r) = \frac{\nabla a_0 + \partial_t \vec{a}}{4\pi}$$

$$\psi_e^\dagger \leftrightarrow M_{4\pi}$$

Electron creation is flux insertion operator

IR "stronger version" of fermion vortex duality

"free" Dirac fermion



$$\mathcal{L}_e = \bar{\psi}_e(i\partial\!\!\!/ - A\!\!\!/)\psi_e + \mathcal{L}_{\text{int}}$$

Fixed point has no relevant perturbations

$\mathcal{T}$

$$\mathcal{T}\psi_e\mathcal{T}^{-1} = i\sigma^y\psi_e$$

$$\mathcal{T}\vec{A}\mathcal{T}^{-1} = -\vec{A}$$

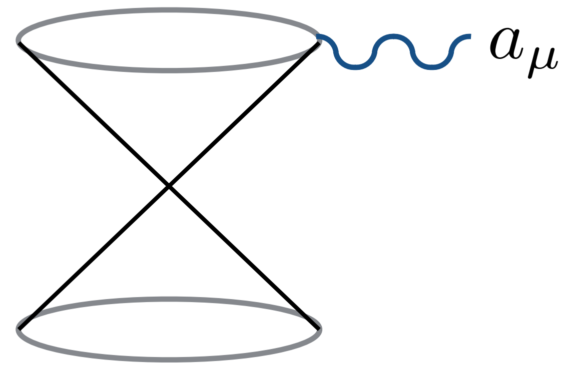
$\mathcal{CT}$

$$\mathcal{CT}\psi_e\mathcal{CT}^{-1} = \sigma^x\psi_e^\dagger$$

$$\mathcal{CT}\vec{A}\mathcal{CT}^{-1} = \vec{A}$$

Roscher, Torres & Strack, JHEP 11 (2016) 017

(2+1) QED



$$\mathcal{L}_{cf} = \bar{\psi}_{cf}(i\partial\!\!\!/ - a\!\!\!/)\psi_{cf} + \frac{adA}{4\pi} + \mathcal{L}_{\text{int}}$$

What's the fixed point here???

$\mathcal{T}$

$$\mathcal{T}\psi_{cf}\mathcal{T}^{-1} = \sigma^x\psi_{cf}^\dagger$$

$$\mathcal{T}\vec{a}\mathcal{T}^{-1} = \vec{a}$$

$\mathcal{CT}$

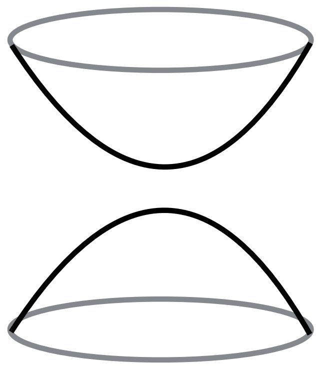
$$\mathcal{CT}\psi_{cf}\mathcal{CT}^{-1} = i\sigma^y\psi_{cf}$$

$$\mathcal{CT}\vec{a}\mathcal{CT}^{-1} = -\vec{a}$$

Perturbative RG seems to find different fixed points?

T-Pfaffian symmetry respecting surface topological order

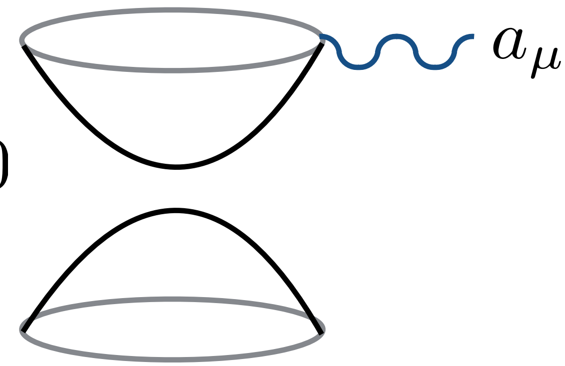
T-Pfaffian



$$\mathcal{L}_e = \bar{\psi}_e(i\partial\!\!\!/ - A\!\!\!/)\psi_e + \mathcal{L}_{\text{int}}$$

Fu-Kane paired (2+1) QED

$$\langle \psi^\dagger_{cf}\psi^\dagger_{cf}\rangle \neq 0$$



$$\mathcal{L}_{cf} = \bar{\psi}_{cf}(i\partial\!\!\!/ - a\!\!\!/)\psi_{cf} + \frac{adA}{4\pi} + \mathcal{L}_{\text{int}}$$

	0	1	2	3	4	5	6	7
I	1		i		1		i	
σ		1		-1		-1		1
ψ	-1		$-i$		-1		$-i$	
Charge	0	$e/4$	$e/2$	$3e/4$	e	$5e/4$	$3e/2$	$7e/4$

Respects $\mathcal{T}$ $\mathcal{CT}$

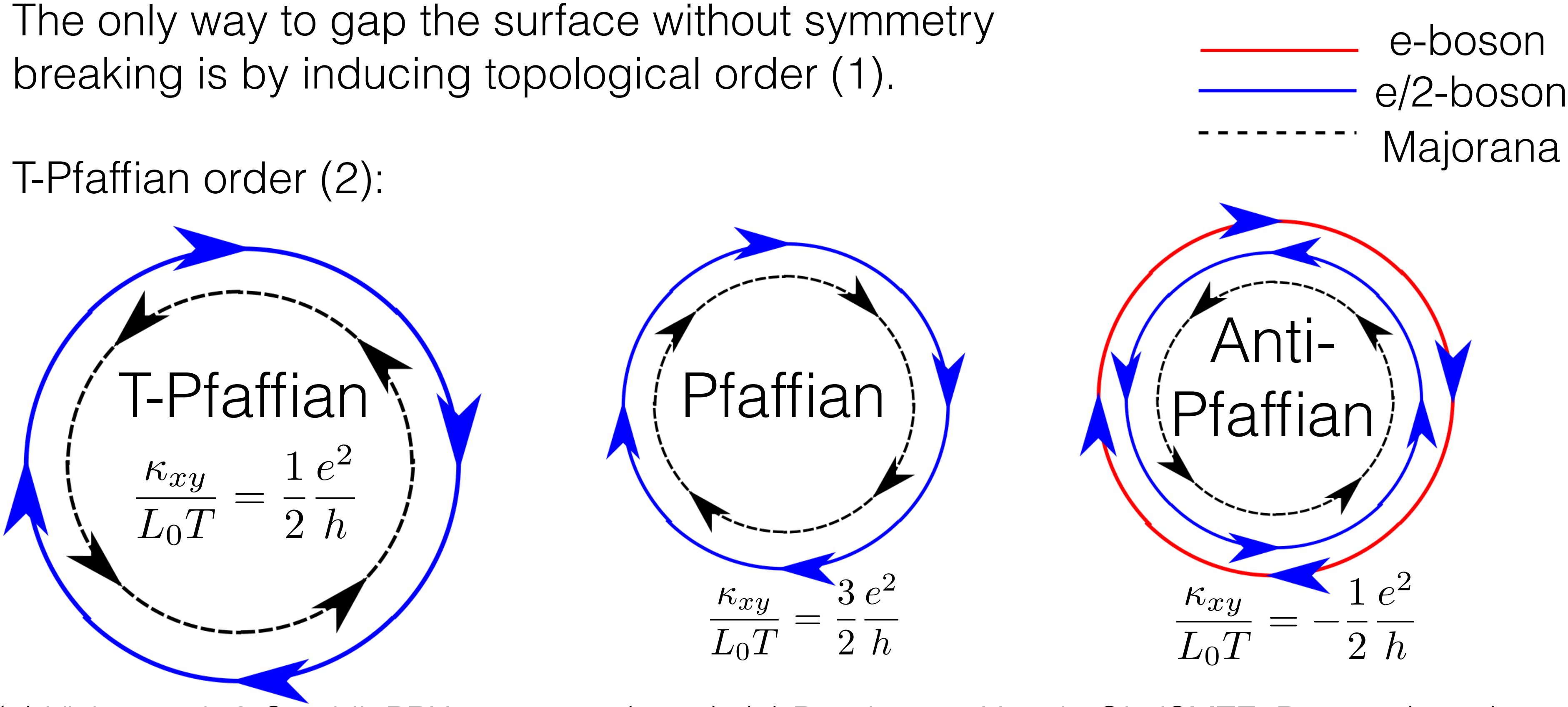
Abrikosov vortex $\delta N_{elec} = \frac{1}{4}$

$\mathrm{T-Pfaffian}$ Subset of $\mathrm{Ising} \times U(1)_{-8}$

(1) Vishwanath & Senthil, PRX 3, 011016 (2013). (2) Bonderson, Nayak, Qi, JSMTE, P09016 (2013). Wang, Potter, Senthil PRB 88, 115137 (2013). Chen, Fidkowski, Vishwanath, PRB 89, 165132 (2014). Metlitski, Kane, Fisher, PRB 92, 125111 (2015).

T-Pfaffian symmetry respecting surface topological order

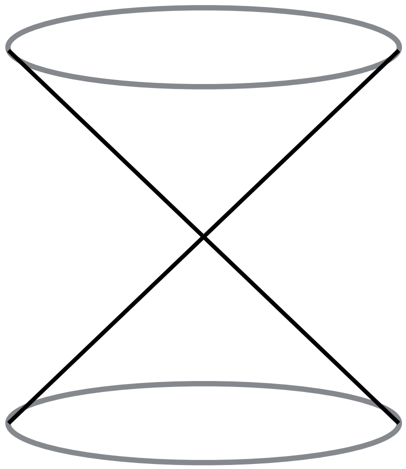
- Topological order = quasiparticle fractionalization.
- The only way to gap the surface without symmetry breaking is by inducing topological order (1).
- T-Pfaffian order (2):

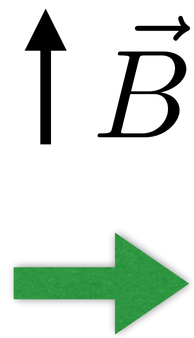


- (1) Vishwanath & Senthil, PRX 3, 011016 (2013). (2) Bonderson, Nayak, Qi, JSMTE, P09016 (2013). Wang, Potter, Senthil PRB 88, 115137 (2013). Chen, Fidkowski, Vishwanath, PRB 89, 165132 (2014). Metlitski, Kane, Fisher, PRB 92, 125111 (2015).

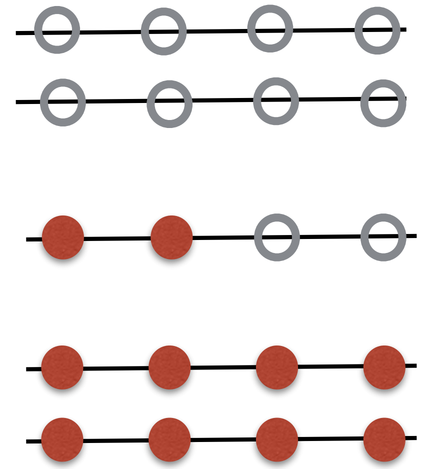
Fermion vortex duality

Dirac fermion





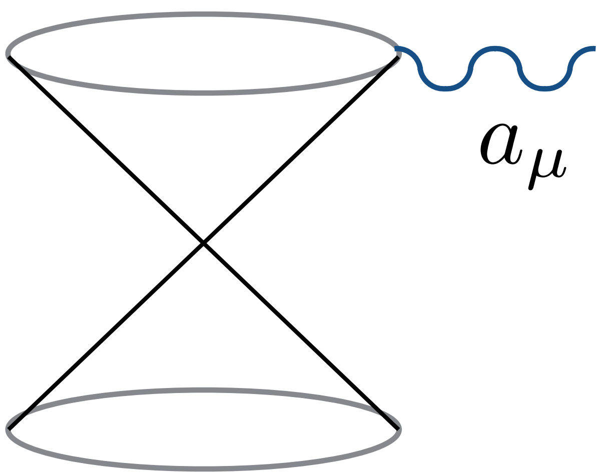
Particle-hole symmetry

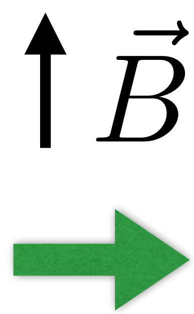


Zeroth Landau level

Half filled

composite fermion vortex



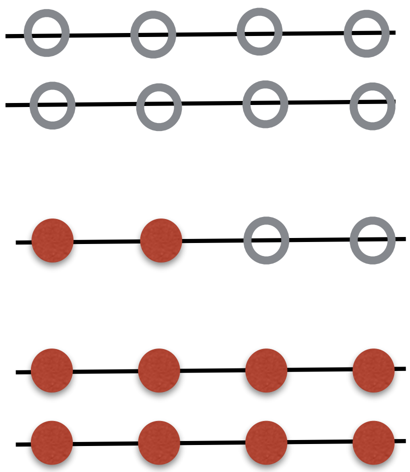


Fermi surface (metal) of vortices

$$\int d^2r \ \psi^\dagger_{cf}\psi_{cf} = \frac{BA}{4\pi} = \frac{N_\phi}{2}$$

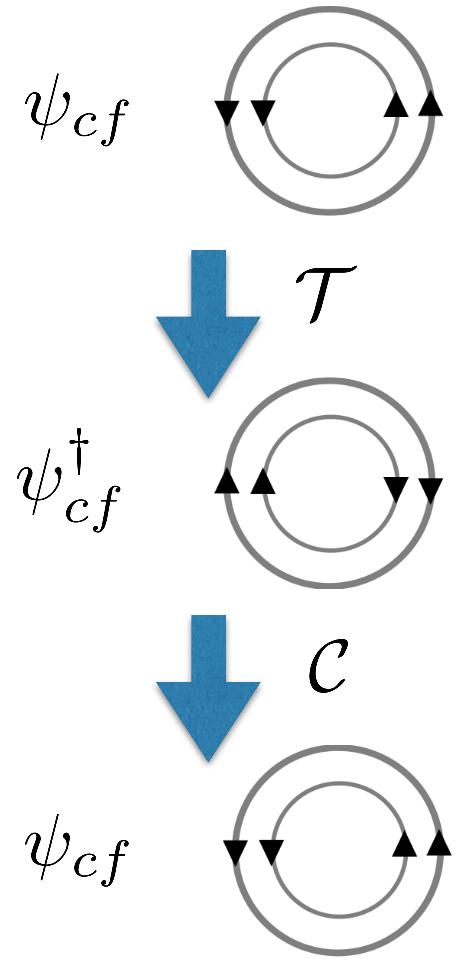
Dirac composite fermi liquid

Particle-hole symmetry



Zeroth Landau level

Half filled



$$(\mathcal{C}\mathcal{T})\psi_{cf}(\mathcal{C}\mathcal{T})^{-1} = i\sigma_y\psi_{cf},$$
$$(\mathcal{C}\mathcal{T})a^0(\mathcal{C}\mathcal{T})^{-1} = a^0,$$
$$(\mathcal{C}\mathcal{T})\vec{a}(\mathcal{C}\mathcal{T})^{-1} = -\vec{a}.$$

- Son, PRX 5, 031027 (2015).
- Wang, Senthil, Phys. Rev. B 93, 085110 (2016); arXiv:1604.06807 (2016).

I. Kimchi

C. Wang

T. Senthil

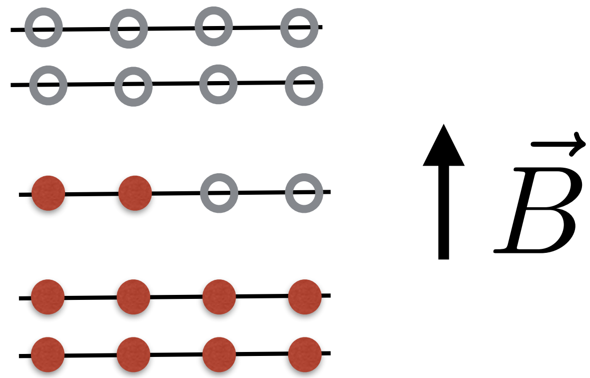
Main message

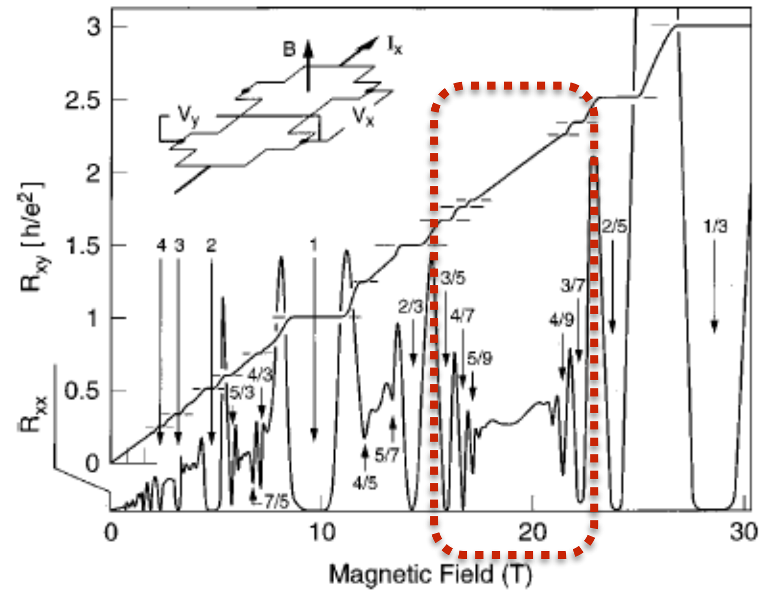
- The celebrated exciton condensate in quantum Hall bilayers is identical to a BCS-type inter-layer paired state of composite fermions.

I. Sodemann, I. Kimchi (MIT), C. Wang (Harvard), T. Senthil (MIT), Phys. Rev. B **95**, 085135 (2017).

Composite fermion metal

Half filled zero Landau level



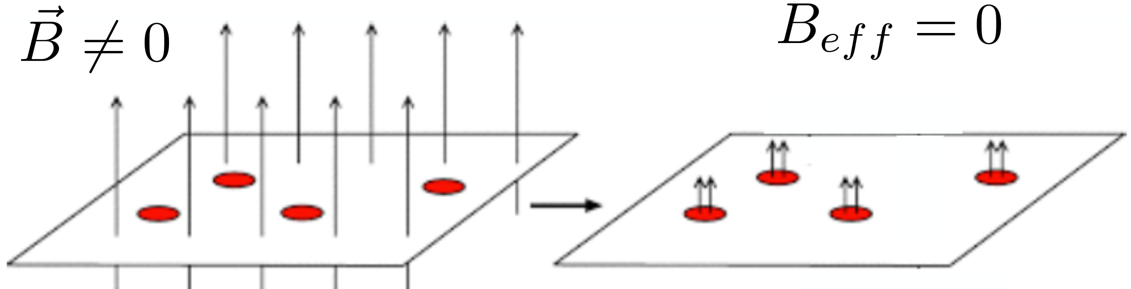


- Fractionalized metal for half filled landau level:

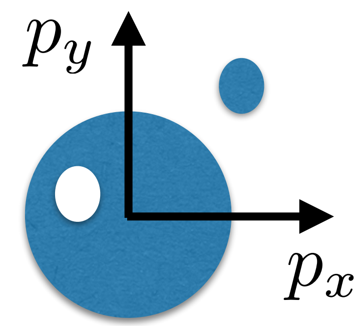
$$N_e = \frac{1}{2}N_\phi$$

- Composite fermion: electron bound to two vortices

$\vec{B} \neq 0$ $\vec{B}_{eff} = 0$



electrons **composite fermions**

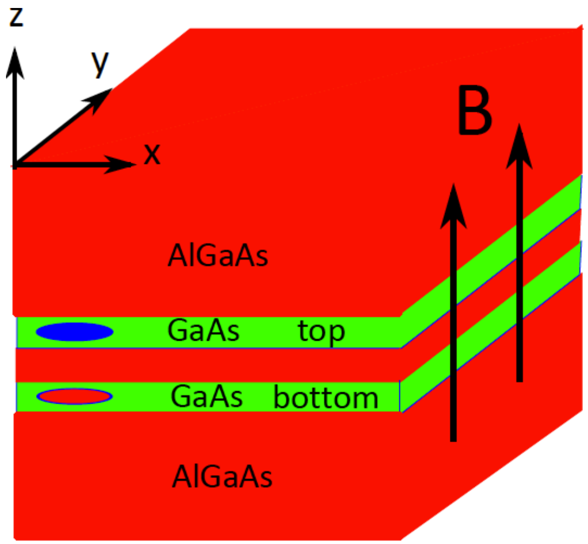


composite fermion fermi surface

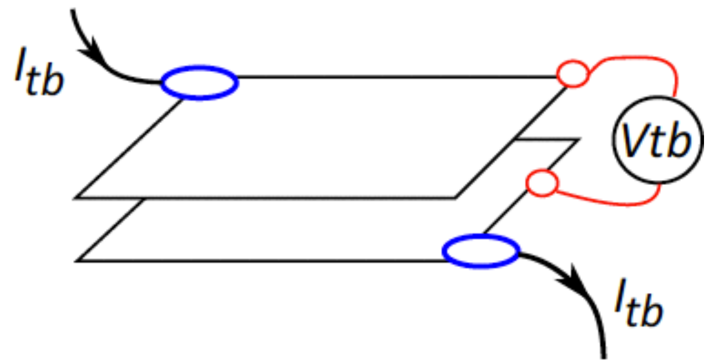
- Jain, PRL 63, 199 (1989).
- Halperin, Lee, Read, PRB 47, 7312 (1993).

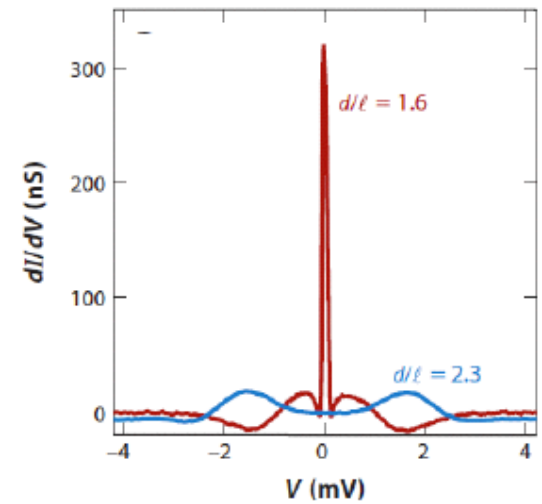
Exciton condensate

- No tunneling but strong interactions



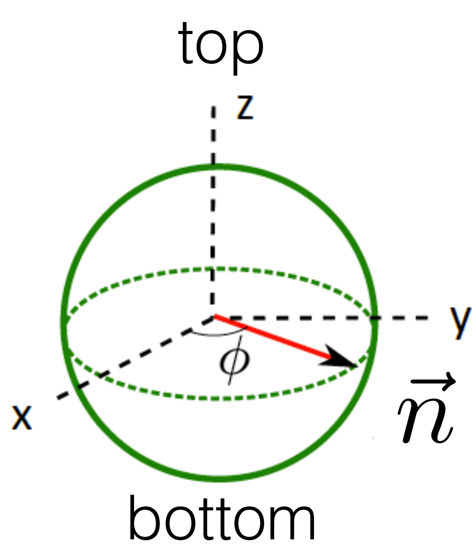
$$\nu = \nu_{top} + \nu_{bottom} = 1/2 + 1/2$$





Spielman *et al.*, PRL (2000)

- Exciton condensate:



$$|top\rangle + e^{i\phi}|bottom\rangle$$

$$\langle c^{\dagger}_{\text{bottom}} c_{\text{top}} \rangle \propto e^{i\phi}$$

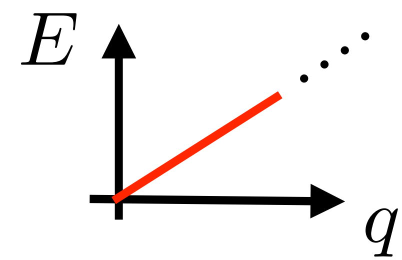
Properties of exciton condensate

- Superfluidity for charge imbalance:

$$Q_- = Q_{top} - Q_{bottom} \qquad [Q_-, \phi] = i$$

- Linearly dispersing Goldstone mode of ϕ (pseudo-spin wave).

Spin-wave

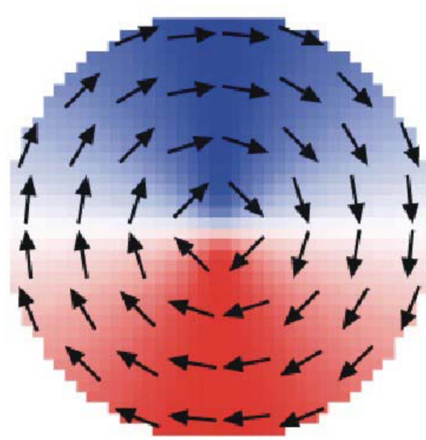


- Half-charged vortices (merons):

2π winding

$$v = 1$$

$$Q_+ = e/2$$



$$Q_+ = (v n_z)\,\frac{e}{2}$$

$$v \in \mathbb{Z}$$

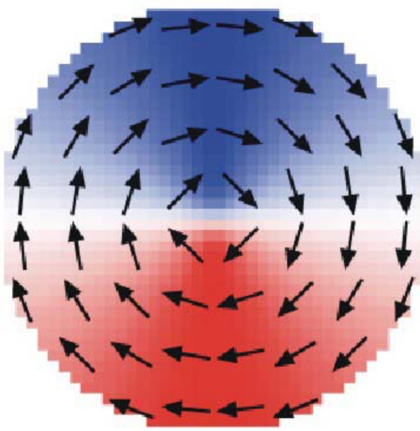
$$n_z = \pm 1$$

- Wen, Zee, PRL 69, 1811 (1992).
- Moon, Mori, Yang, Girvin, MacDonald, Zheng, Yoshioka, Zhang, PRB 51, 5138 (1995).

Properties of exciton condensate

- Half-charged vortices (merons):

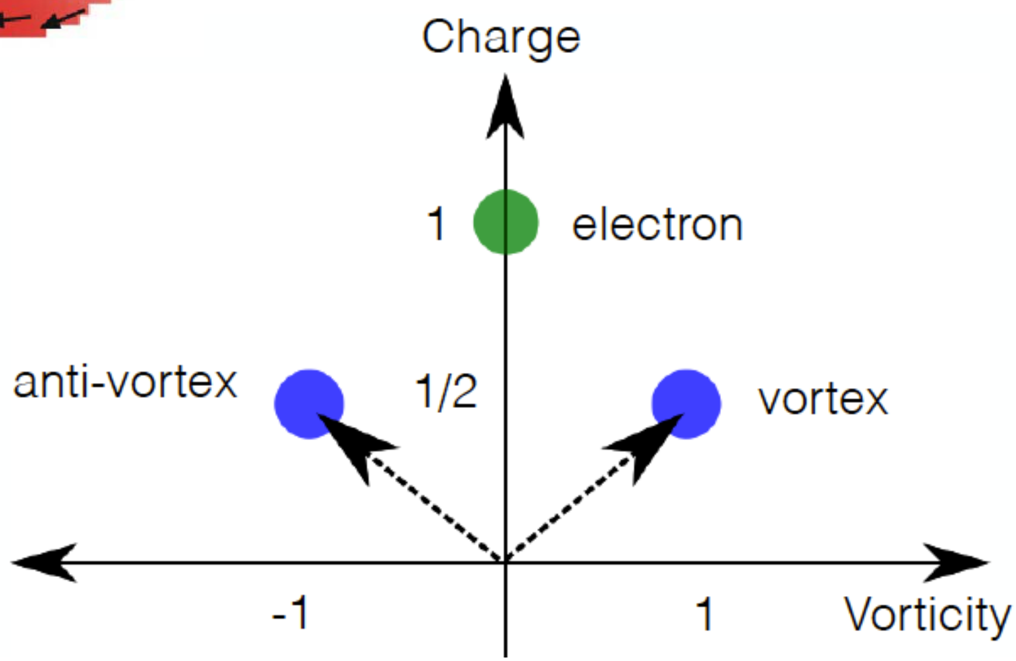
2π winding



$$Q_+ = (vn_z)\ \frac{e}{2}$$

$v = 1$
$Q_+ = e/2$

$v = \pm 1$
$n_z = \pm 1$



Bilayer exciton condensate and Composite fermion metal

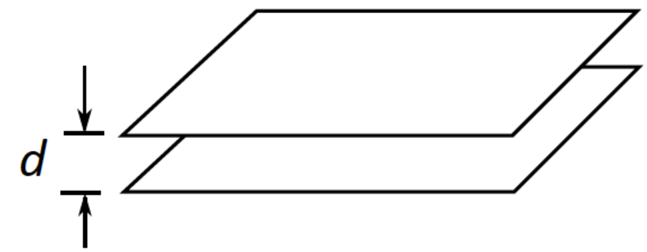
- Are zero and infinity connected?

$$\nu = \nu_{top} + \nu_{bottom} = 1/2 + 1/2$$

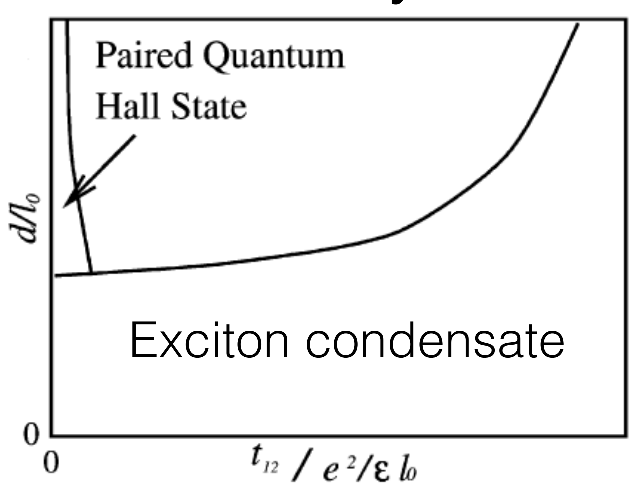
$d = 0$

$d = \infty$

Exciton condensate ? Two fermi liquids

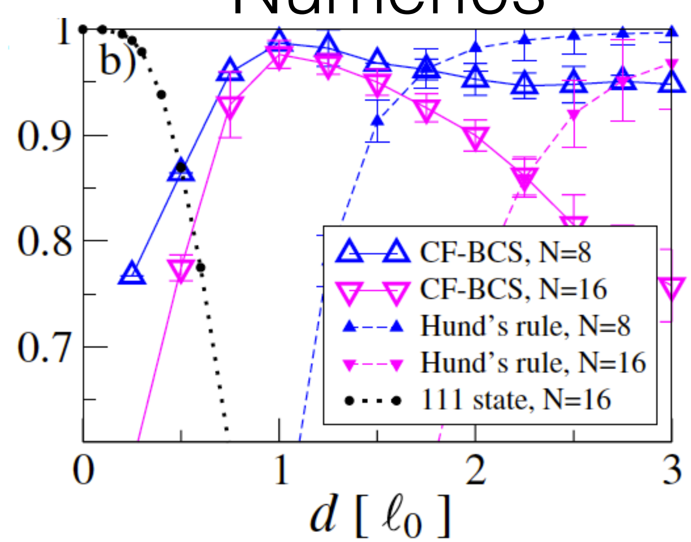


Theory



Bonesteel et al. PRL(1996)

Numerics

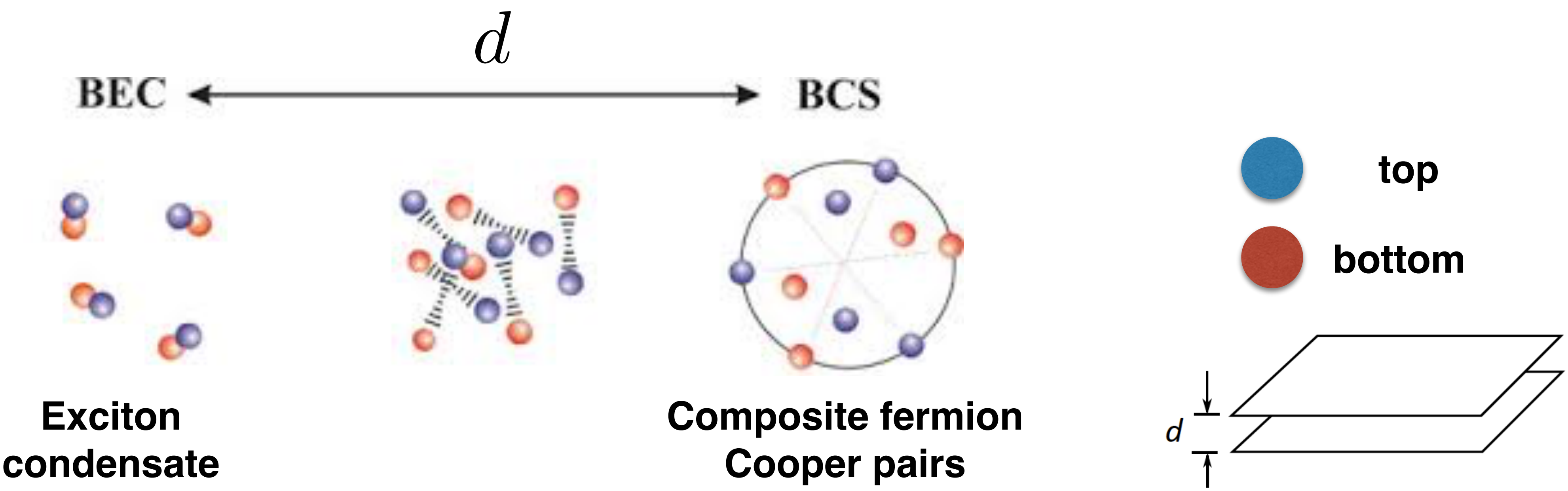


Möller et al. PRL (2008)

Bilayer exciton condensate and Composite fermion metal

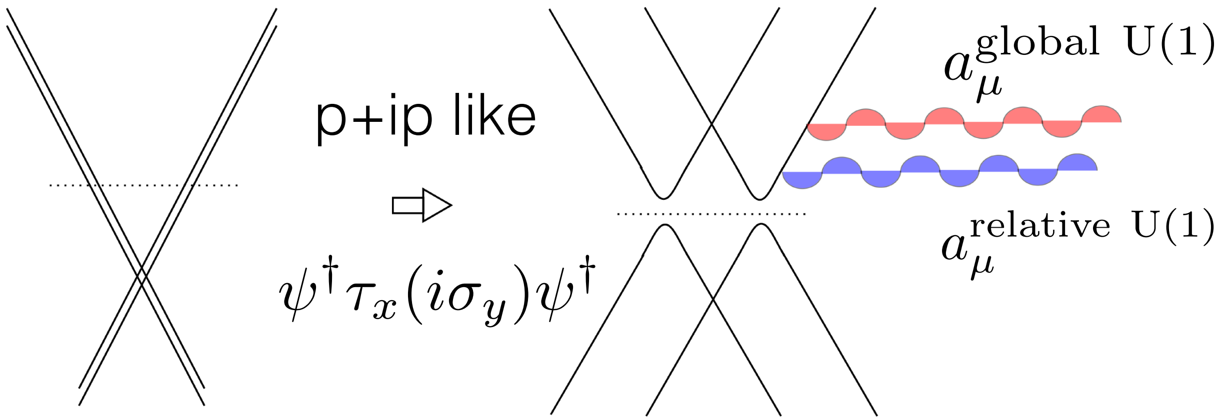
- A special particle-hole invariant "cooper pairing" of composite fermions is equivalent to exciton condensate:

$$\hat{\Delta} = i\psi^\dagger \sigma_y \tau_x \psi^\dagger \sim i\psi^\dagger_{\text{top}} \sigma_y \psi^\dagger_{\text{bottom}}$$



I. Sodemann, I. Kimchi, C. Wang, T. Senthil, arxiv.1609.08616 (2016).

Goldstone mode and photons



$$a_+ = \frac{a_1 + a_2}{2}$$

$$a_- = \frac{a_1 - a_2}{2}$$

- Global gauge field is gapped via Higgs.

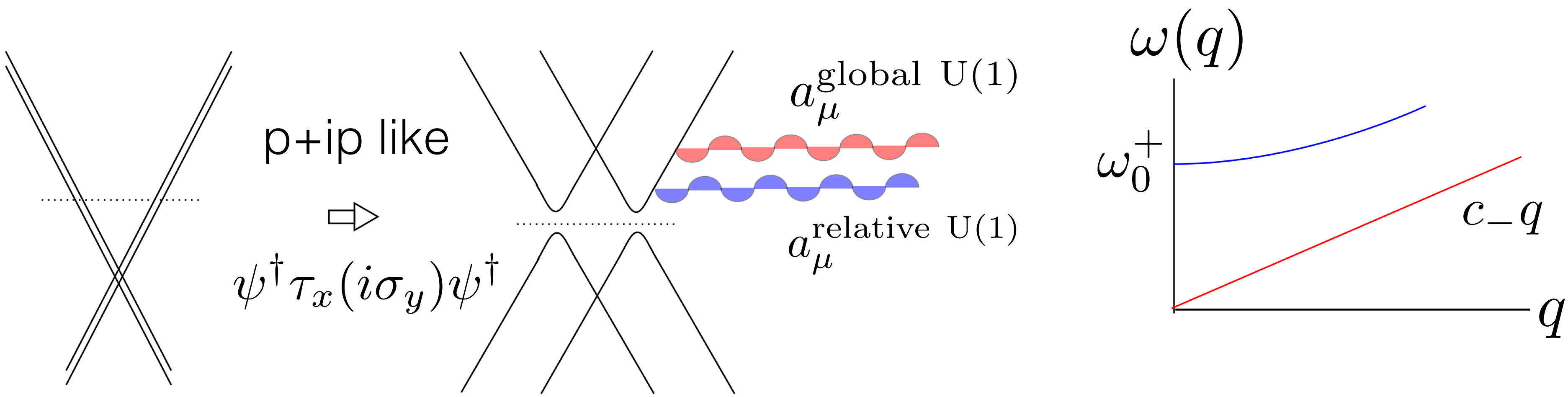
- Relative gauge field remains gapless. 2+1 Maxwell theory has a spontaneously broken symmetry:

$$\langle \mathcal{M}_-(r) \mathcal{M}_-^\dagger(0) \rangle \xrightarrow{|r| \to \infty} \text{const} \qquad n^e_{\text{top}} - n^e_{\text{bottom}} = \frac{\nabla \times \vec{a}_-}{2\pi}$$

$$\langle c^\dagger_{\text{bottom}} c_{\text{top}} \rangle \propto e^{i\phi}$$

The state is an exciton condensate!

Relative u(1) photon = Goldstone mode



- Photon is exciton condensate “spin-wave”.
- Electric charges under field a_- are vortices of condensate order parameter:

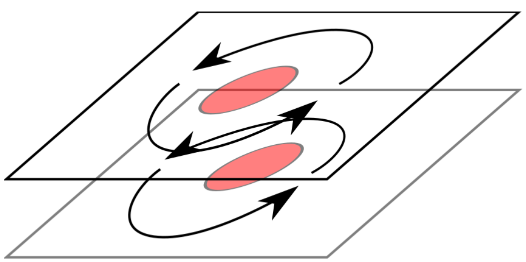
$$4\pi q_- \leftrightarrow vorticity$$

$$\hat{z} \times (\vec{j}_{\text{top}}(r) - \vec{j}_{\text{bottom}}(r)) = \frac{\nabla a_0 + \partial_t \vec{a}}{4\pi}$$

Abrikosov vortices and merons

- Abrikosov vortices carry half charge:

$\pi - \text{vortex}$



$Q = 1/2$

$$n^e_{\text{top}} + n^e_{\text{bottom}} = \frac{\nabla \times \vec{a}_+}{2\pi} \Rightarrow Q_\pi = \pm\frac{1}{2}$$

Abrikosov vortices = merons

- Abrikosov vortices carry half charge:

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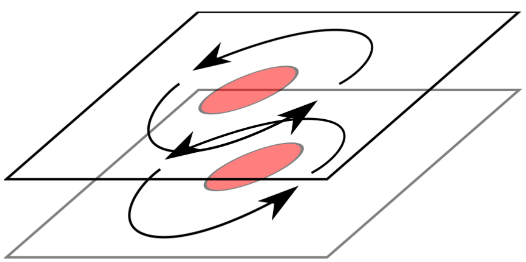


$Q = 1/2$

$$n^e_{\mathrm{top}} + n^e_{\mathrm{bottom}} = \frac{\nabla \times \vec{a}_+}{2\pi} \quad \longrightarrow \quad Q_\pi = \pm\frac{1}{2}$$

- Abrikosov vortices have a complex fermion zero mode:

Layer X-change

$$|0\rangle \longrightarrow |1\rangle$$

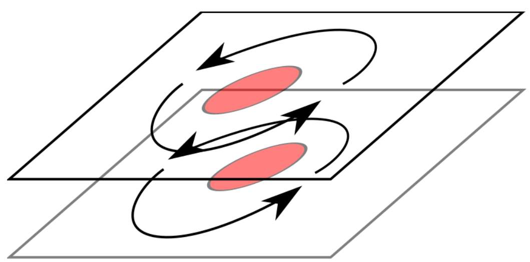
$$|1\rangle \equiv \psi_0^\dagger |0\rangle \longrightarrow |0\rangle$$

	q_-	(vorticity)
$\vert 0\rangle$	$1/2$	2π
$\vert 1\rangle$	$-1/2$	-2π

Abrikosov vortices = merons

- Two π Abrikosov vortices of opposite vorticity are mutual semions

$\pi - \text{vortex}$



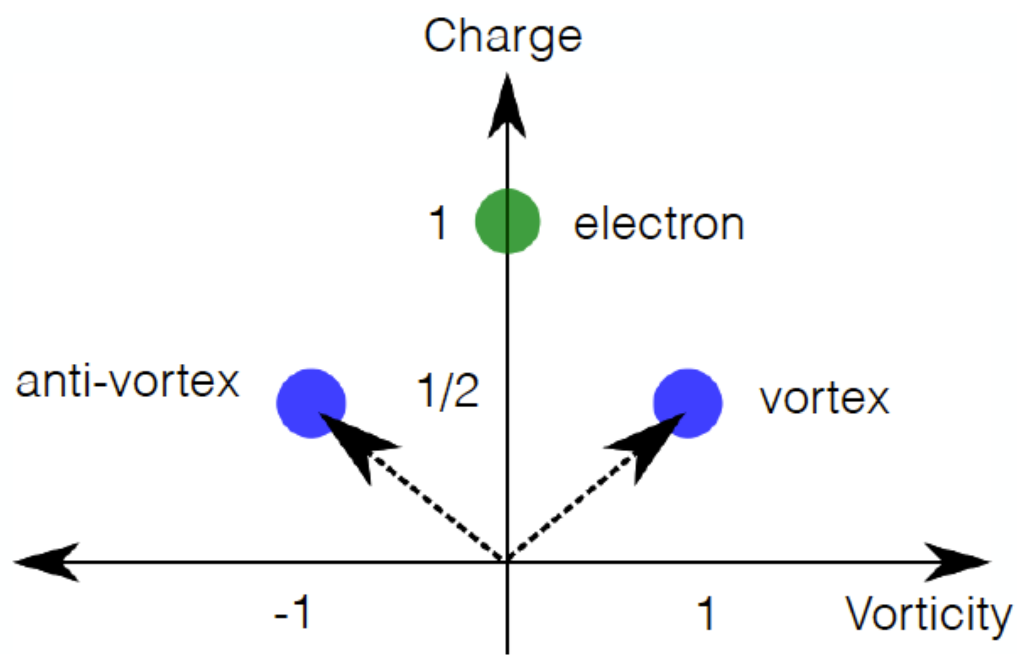
$Q = 1/2$

$$|0\rangle$$

$$|1\rangle \equiv \psi_0^\dagger |0\rangle$$

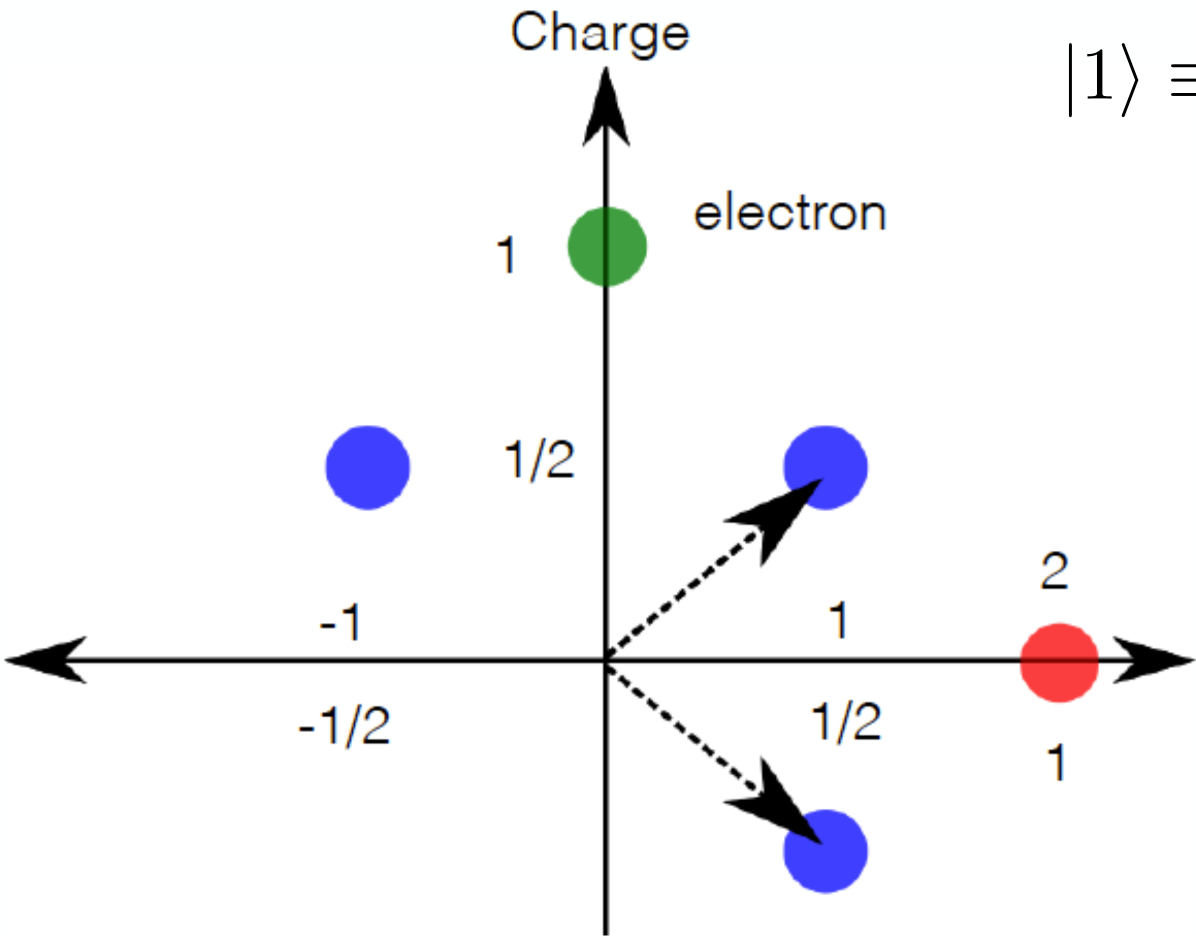
- Their fusion is a fermion:

The electron (with layer charge imbalance neutralized by condensate).

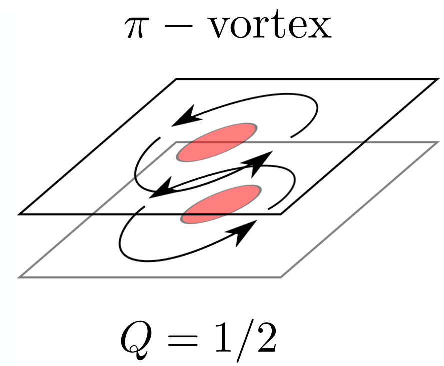


Bogoliubov fermion

- Consider fusing two Abrikosov vortices of opposite flux but same a_- charge (order parameter vorticity):



$$|1\rangle \equiv \psi_0^\dagger|0\rangle$$



Bogoliubov fermion

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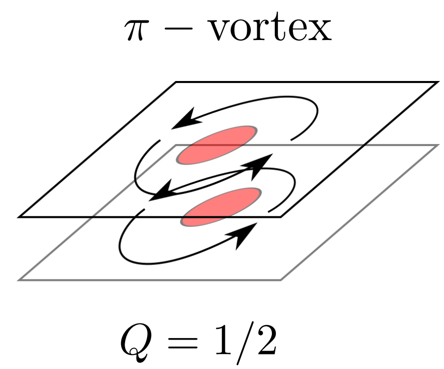
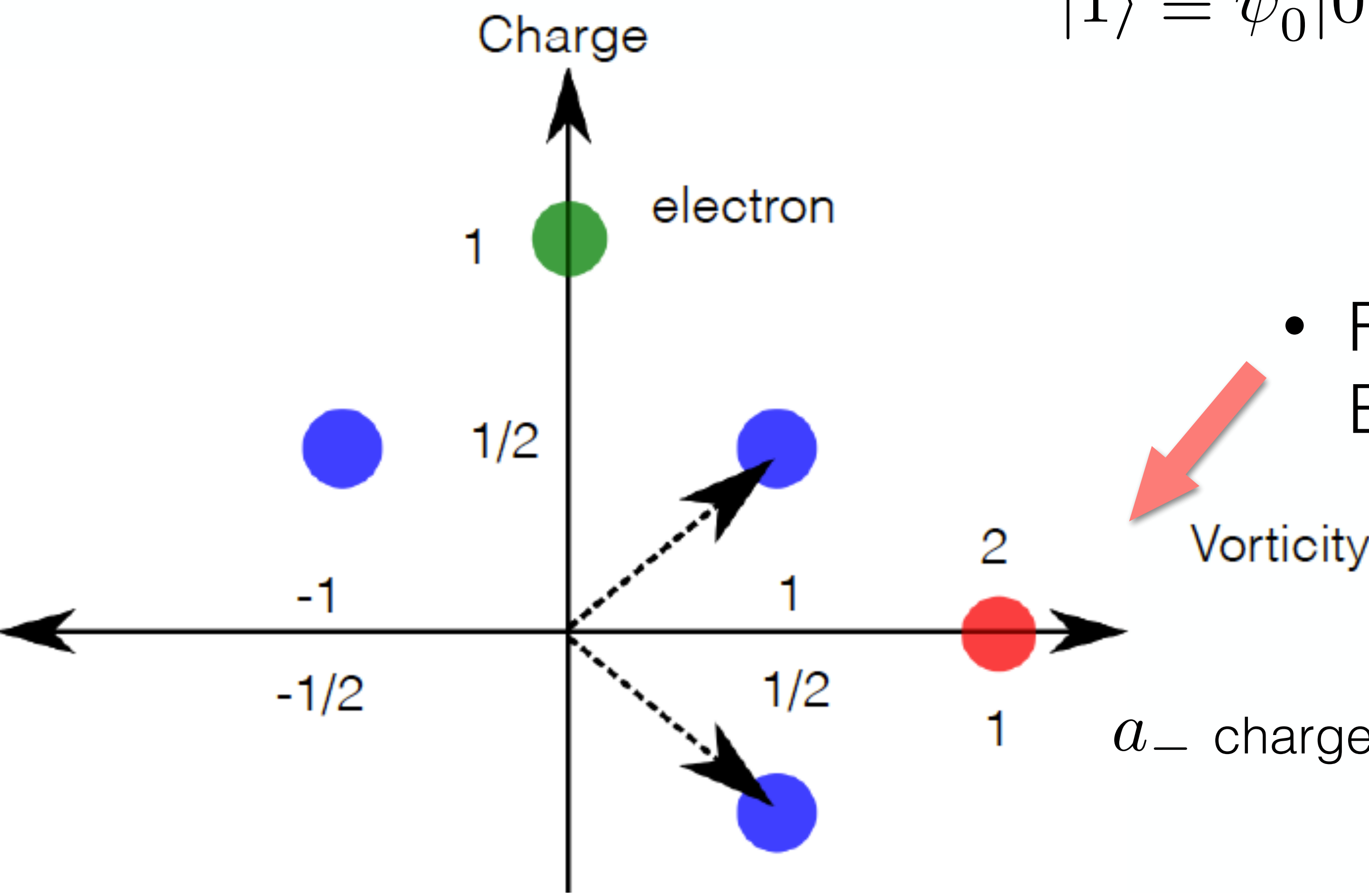




- Fusion is Bogoliubov fermion

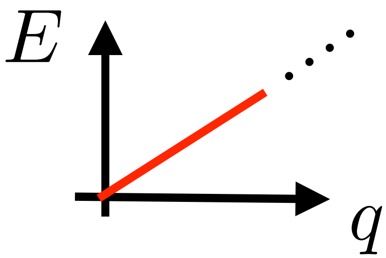
4π charge neutral Kramers vortex

Dictionary

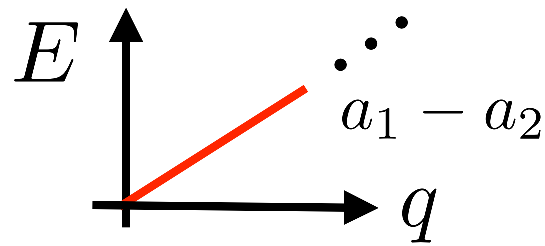
Phys. Rev. B **95**, 085135 (2017).

Exciton condensate | Composite fermion superconductor

Spin-wave



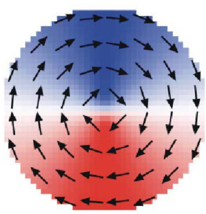
Photon



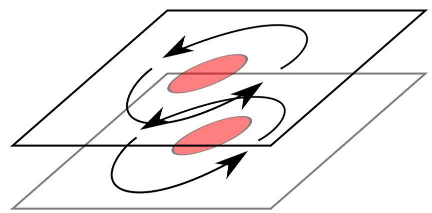
XY vortex

$Q = 1/2$

2π winding



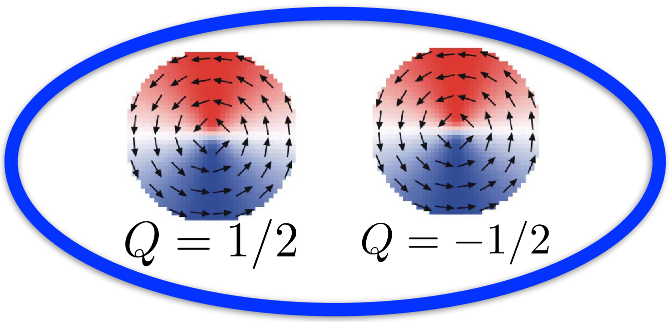
Abrikosov vortex



π flux

$Q = 1/2$

4π **neutral vortex**



Composite fermion

Charge neutral

Dipole carrying

Claim: Interlayer CT-invariant (p_x+ip_y) paired state equals exciton condensate

- Interlayer pairing is:

$$\hat{\Delta} = i\psi^\dagger \sigma_y \tau_x \psi^\dagger \sim i\psi^\dagger_{\rm top} \sigma_y \psi^\dagger_{\rm bottom}$$

- In HLR picture this channel corresponds to p_x+ip_y interlayer paring:

$$\hat{\Delta} \sim i\psi^\dagger \tau_x (p_x + ip_y) \psi^\dagger$$

$$\Psi = \Phi_{BCS}(\{z_i, w_j\}) \prod_{i<j} (z_i - z_j)^2 \prod_{i<j} (w_i - w_j)^2$$

$$\Phi_{BCS} \sim \frac{|top\rangle_i |bottom\rangle_j + |bottom\rangle_i |top\rangle_j}{\bar{z}_i - \bar{w}_j}$$

Wave-function argument

- Exciton condensate wave-function:

$$\Psi_{111} = \prod_{i<j}(z_i - z_j)\prod_{i<j}(w_i - w_j)\prod_{i,j}(z_i - w_j)$$

z :top

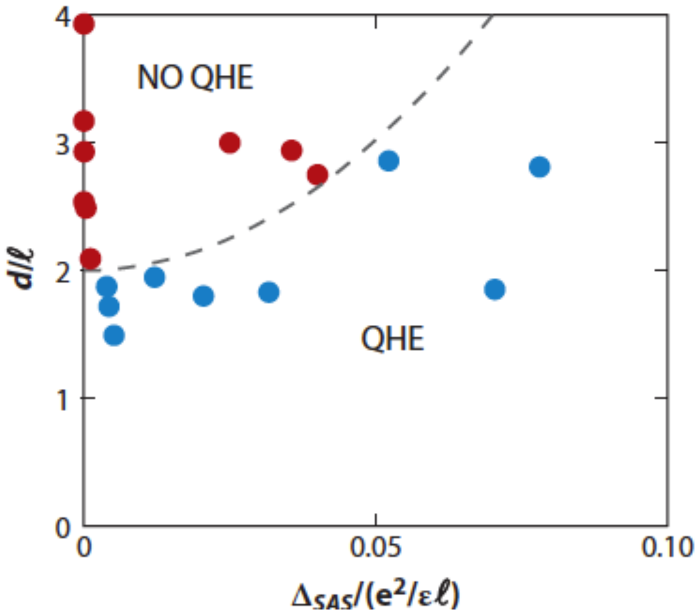
w :bottom

- p+ip paired CF wave-function:

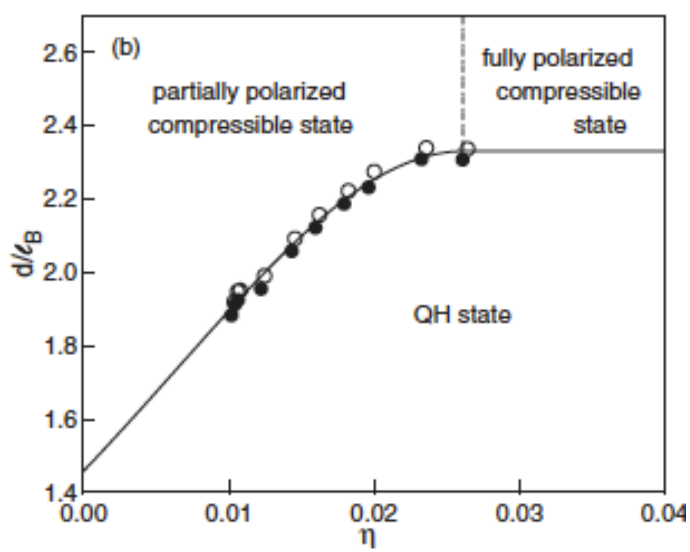
$$\Psi_{\text{pair}} = \frac{\prod_{i,j}|z_i - w_j|^m}{\prod_{i<j}|z_i - z_j|^n|w_i - w_j|^n} \times \det\left[\frac{1}{\bar{z}_i - \bar{w}_j}\right]\prod_{i<j}(z_i - z_j)^2\prod_{i<j}(w_i - w_j)^2$$

$$\boxed{\Psi_{\text{pair}} = \frac{\prod_{i<j}|z_i - z_j|^{2-n}|w_i - w_j|^{2-n}}{\prod_{i,j}|z_i - w_j|^{2-m}}\Psi_{111}}$$

Experiments

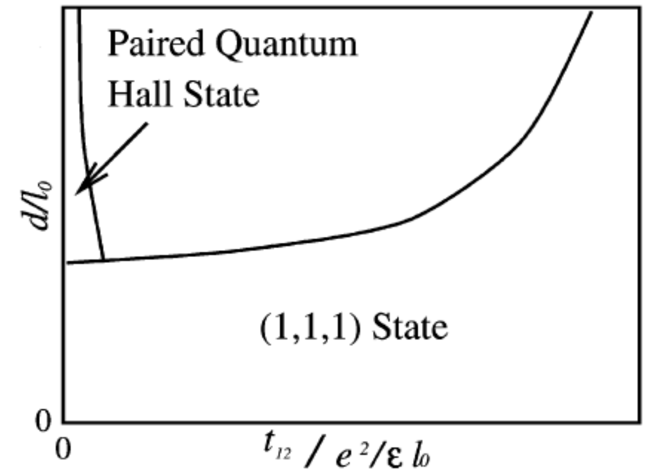


Eisenstein, **ARCMP** (2014)

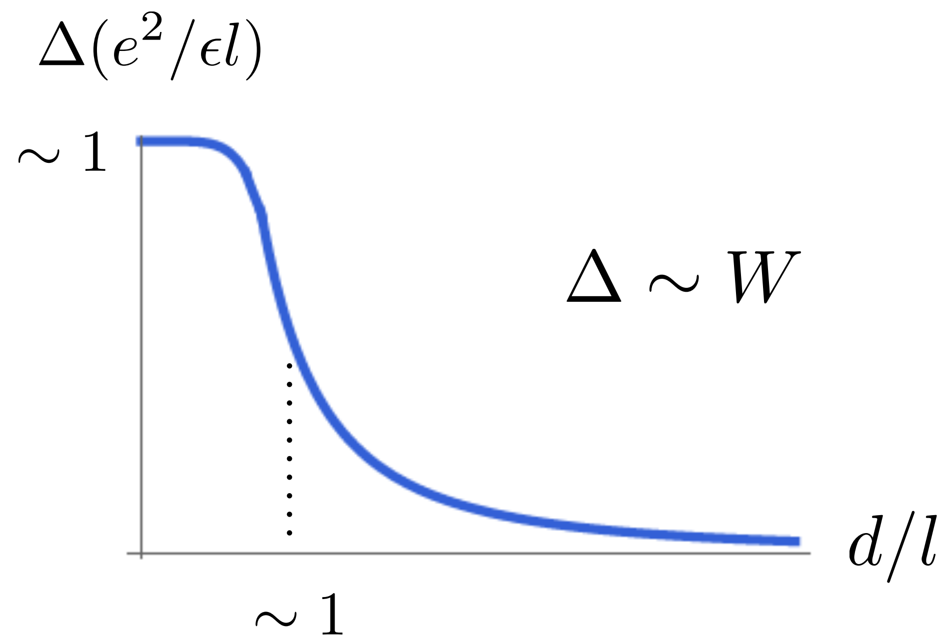


P. Giudici, et al. **PRL** (2008)

No support for this:
Bonesteel et al. **PRL** (1996)



Experimental phase transition could be disorder driven:

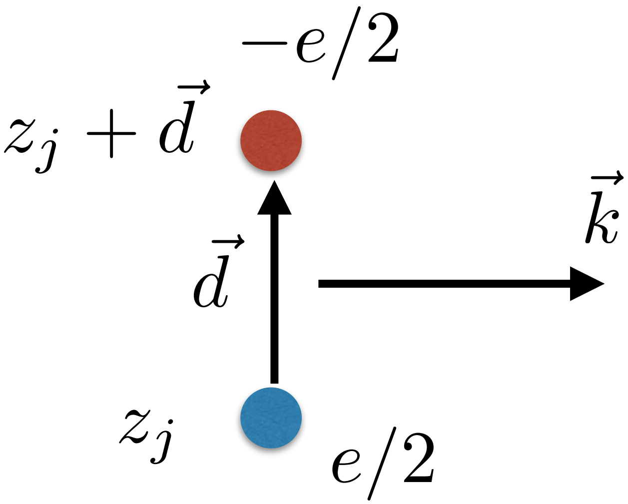


Anatomy of Composite fermion metal

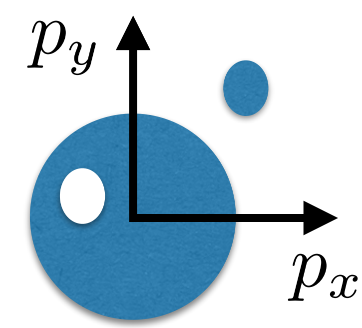
- Composite fermion is a dipolar object electron bound to two vortices:

Bosons: $$\Psi^{1/2}_{Laughlin} \sim \prod_{i<j}(z_i - z_j)^2$$

Fermions: $$\Psi^{1/2}_{CFL} \sim \prod_{i<j}(z_i - z_j)(z_i - d_i - z_j - d_j)$$



$$k_i = l^2 d_i \times \hat{z}$$



composite fermion fermi surface

Read, Sem. Sci. & Tech. (1994).

Wang & Senthil, PRB (2016).