

Advanced Study Group 2011: Towards a Semiclassical Theory of Dynamical Tunneling

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The Advanced Study Group (ASG) *Towards a Semiclassical Theory of Dynamical Tunneling* took place over four and a half months beginning April, 2011. The main goals and activities were applied to two central topics:

1. Combination and unification of different semiclassical approaches to tunneling

To a very rough extent, the existing semiclassical approaches to describe quantum tunneling processes can be classified into WKB-type methods, employed to describe direct and resonance-assisted tunneling, which are essentially based on the complexification of classical invariant manifolds, and path-integral-type methods which are based on the representation of the quantum propagator in terms of complex classical trajectories. Both types of methods have their strong points and limitations, and a central task of the ASG was to establish a common unifying framework for both of them. This task has been achieved to a limited extent. Indeed, a genuinely semiclassical theory of direct tunneling could be obtained through the inclusion of complex trajectories (Mertig *et al.*, Ref. [1]) The role of nonlinear resonances in complex trajectory-based semiclassical approaches, on the other hand, still represents an open issue under debate (Shudo and Ikeda, Ref. [2]).

2. Exploration of dynamical tunneling in systems with more than two degrees of freedom

Tunneling processes in molecular systems, to mention a relevant example, most generally involve more than two degrees of freedom, and a semiclassical determination of the corresponding time scales characterizing tunneling-induced decay or transfer processes is therefore concerned with a high-dimensional phase space. A central aim of the ASG was therefore to extend the existing semiclassical descriptions of tunneling to systems with more than two degrees of freedom. As far as resonance-assisted tunneling is concerned, the theoretical foundations of this task were laid by an internal working note written by P. Schlagheck (Ref. [7]) and dis-

tributed among the members of the ASG. Two main questions are identified in this context:

- (a) How does direct and resonance-assisted tunneling proceed in systems with more than two degrees of freedom, and how can the associated coupling rates be evaluated in practice?
- (b) How does the possibility of classical diffusion within the Arnol'd web manifest in the eigenstates or transport observables of the quantum system?

These questions are at present addressed in a joint effort focusing on the near-integrable (S. Keshavamurthy/J. Madronero/P. Schlagheck/S. Tomsovic) and mixed regular-chaotic regime (A. Bäcker/R. Ketzmerick/P. Schlagheck).

In order to update each other on the progress that we have made on these two main topics since the end of the ASG, we organized an informal one-week post ASG meeting at the MPIPES from 15th to 19th October 2012. The invited key participants of this meeting were Srihari Keshavamurthy (Kanpur), Akira Shudo (Tokyo), Amaury Mouchet (Tours), and Javier Madronero (Duisburg), in addition to Peter Schlagheck (Liège), Arnd Bäcker and Roland Ketzmerick (Dresden) who were the main co-organizers.

Seminars:

We organized weekly Thursday seminars every week except for during the focus event. They were intended to be very informal, with introductory material presented, and questions asked by the listeners. This was very successful and rarely did a talk last less than 2 hours due to all the extensive questions being asked. The talks are listed below. In addition, every Friday we held a meeting to discuss open physics research questions of interest to the ASG, except one or two Fridays dedicated to administrative organization of the focus event. Usually, one person was responsible for initiating a discussion in which they had some particular interest. These discussions also mostly lasted a couple of hours.

Thursday talks:

Jérémy Le Deunff: *Resonant tunneling and normal forms*

Akira Shudo: *Dynamical tunneling in the anti-integrable limit*

Akira Shudo: *Dynamical tunneling in the anti-integrable limit II*

Steven Tomsovic: *What is being measured in the scanning gate microscopy of a quantum point contact?*

Stephen Creagh: *Looking for tunnelling in boundary integral methods*
 Steven Tomsovic: *Extreme value statistics for random waves*
 Alfredo Miguel Ozorio de Almeida: *Initial value representations for the Loschmidt echo*
 Peter Schlagheck and Martin Richter: *Secular perturbation theory in systems with two and three degrees of freedom*
 Srihari Keshavamurthy: *Classical and Quantum transport on the Arnold Web: an introduction*
 Clemens Löbner: *Integrable Approximation for Regular Islands in Billiards*
 Arseni Goussev: *A Huygens-Fresnel-type approximation for quantum propagators: Wave packet diffraction and interference*
 Andreas Buchleitner: *Nonlinear resonances - from atoms to ultracold atoms*
 Srihari Keshavamurthy: *Transport on the Arnold web: Chirikov's calculation*
 Javier Madronero: *Complexity in the 3-body Coulomb problem: fluctuations in the spectrum of highly doubly excited states*
 Italo Guarneri: *Projection approach to resonance assisted tunneling*
 Normann Mertig: *Power-law level statistics due to dynamical tunneling*
 Steven Tomsovic: *Random matrix theory for long-range ocean acoustics*

Focus event:

We held a four day focus event organized to facilitate discussions with a larger group of people. There were 25 participants, but only twelve seminars during the event, three per morning. Each afternoon, discussion sessions were organized to explore further the morning talks in greater detail. There were half a dozen posters up the entire event.

Stephen Creagh: *Resonant tunnelling from cavities*
 Shmuel Fishman: *Quantum chaos of a mixed, open system of kicked cold atoms*
 Kensuke Ikeda: *Complex semiclassical description of chaotic tunneling in quasi-stationary regime*
 Srihari Keshavamurthy: *Dynamical Tunneling in three degrees of freedom and beyond: The final frontier?*
 Steffen Löck: *Regular-to-chaotic tunneling rates using the fictitious integrable system approach*
 Amaury Mouchet: *The role of resonances in tunnelling*
 Evgenii Narimanov: *Quantum Chaos in Hyperbolic Metamaterials*
 Louis Pecora: *Changes in Distributions of Tunneling Rates Through a Barrier between Regular and Chaotic Symmetric Well Systems*
 Susumu Shinohara: *Chaos-assisted emission from microcavity lasers*

Peter Schlagheck: *Resonance-assisted tunneling in mixed regular-chaotic systems*

Akira Shudo: *What does theory of complex dynamics tell us?*

Kin'ya Takahashi: *Stable-unstable manifold guided tunneling for continuous time systems*

Publications and unpublished documents:

- [1] N. Mertig, S. Löck, A. Bäcker, R. Ketzmerick, and A. Shudo *Complex paths for regular-to-chaotic tunneling rates*, arXiv:1207.0985 (2012).
- [2] A. Shudo and K. S. Ikeda, *Tunneling Effect and the Natural Boundary of Invariant Tori*, Phys. Rev. Lett. 109, 154102 (2012).
- [3] A. Ishikawa, A. Tanaka, K. S. Ikeda, and A. Shudo, *Diffraction and tunneling in systems with mixed phase space*, Phys. Rev. E 86, 036208 (2012).
- [4] S. Löck, A. Bäcker, and R. Ketzmerick, *Coupling of bouncing-ball modes to the chaotic sea and their counting function*, Phys. Rev. E 85, 016210 (2012).
- [5] J. Le Deunff, O. Brodier, and A. Mouchet, *A primer for resonant tunnelling*, Eur. J. Phys. 33, 1771 (2012).
- [6] A. Sethi and S. Keshavamurthy, *Driven coupled Morse oscillators: visualizing the phase space and characterizing the transport*, Molecular Physics 110, 717 (2012).
- [7] P. Schlagheck, *Resonance-assisted tunneling in systems with many degrees of freedom*, unpublished working note (2011).
- [8] J. Le Deunff, A. Mouchet, and P. Schlagheck, *Semiclassical description of resonance-assisted tunneling in one-dimensional integrable systems*, in preparation.