Probing Localization in Scattering Systems via Fidelity

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Abstract

Using scattering measurements from a microwave cavity filled with randomly distributed scatterers, we evaluate the scattering fidelity. We show that depending on the degree of localization inside the sample, the fidelity decays in a novel way that reflects the degree of localization (or randomness) inside the cavity. The outcome of the experimental measurements are explained on the basis of a parametric Banded Random Matrix modeling which incorporates localization phenomena. The theoretical results are in good agreement with those of the experiment.

Experimental Apparatus

Scattering Fidelity\(^{[2]}\)

\[ f_w(t) = \frac{S_w(t) \cdot S_0(t)}{\sqrt{|S_w(t)|^2 \cdot |S_0(t)|^2}} \]

where:

\[ S_w(t) = \int S_w(E) e^{-iE t} \, dE \quad \text{and} \quad S_0(t) = \int S_0(E) e^{-iE t} \, dE \]

Fidelity in the Localized Regime

Theoretical Model\(^{[3]}\)

\[ H_w = H_0 + x V \quad \text{such that} \quad H_0, V \text{ are either:} \]

Extended

\[ l_\infty \approx b^2 \]

Localized

Quantum Fidelity

\[ F(t) = \langle \psi | \exp (i H_0 t) \exp ( - i H_w t) | \psi \rangle^2 = | \langle f(t) \rangle |^2 \]

In the Extended Regime\(^{[4]}\)

\[ F(t) \approx \exp ( - 4 \pi^2 x^2 C(t) ) \]

\[ C(t) = t^2 + \frac{t}{2} - \int_0^t \, d \tau \int_0^\tau b_2(\tau') \, d \tau' \]

What about the Localized Regime?

Experiment vs. Theory

References