# Fluctuations in Regular and Chaotic Many-Body Systems

**Sven Åberg, Lund University, Sweden**

International Workshop on

## Chaos and Collectivity in Many-Body Systems

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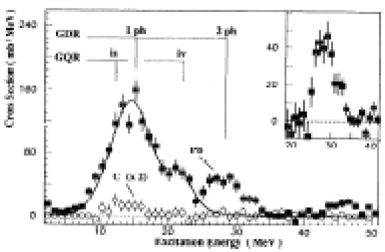
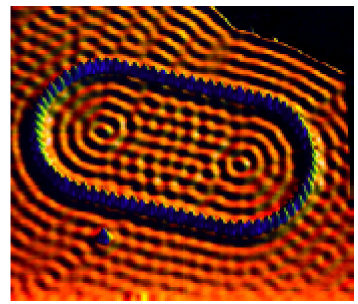



*Scientific coordination:*

**Mahir S. Hussein**
MPI für Physik komplexer Systeme
Dresden, Germany

*Organisation:*

**Marita Schneider**
MPI für Physik komplexer Systeme
Dresden, Germany



# Fluctuations in many-body systems

➢**Fluctuations of energy (or BCS gap) as a parameter is changed:**

**Parameter: shape, number of particles, etc.**

➢**Many-body systems:**

- **Atomic nuclei**
- **Ultracold gases of Fermionic/bosonic atoms**
- **Metallic grains**
- **Quantum dots**

➢**Role of dynamics: order/chaos**

# Fluctuations in Regular and Chaotic Many-Body Systems

## I. Introduction:

(a) Quantum chaos in one-body and many-body systems

## II. Fluctuations of shell energy

(a) Ground-states in nuclei – nuclear masses

(b) Ground-states in ultracold gases of atoms: bosonic/Fermionic

## III. Fluctuations of BCS pairing gap

(a) Atomic nuclei

(b) Nanosized metallic grains

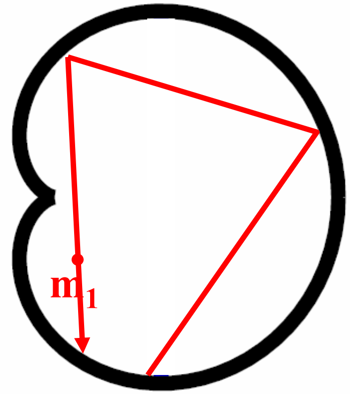
(c) Ultracold Fermionic gases

## 1. One-body system:

$$H_1 = T_1 + V_1$$

**Dynamics determined by potential $V_1$**

**Billiards,**
**H-atom in B-field**



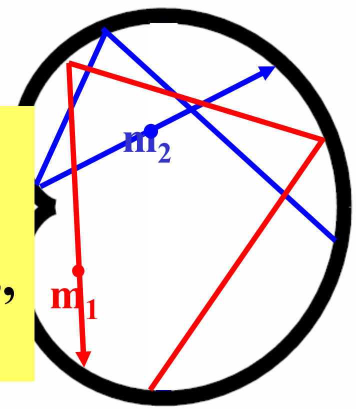

**Classical chaos - quantum chaos established**

## 2. Many-body system:

$$H = \sum_i H_i \,, \quad H_i = T_i + V_i$$

**$V_i$ self-consistent mean field.**
**Determines dynamics.**

***Ground states* of**
**many-body systems:**
**Nuclei, metallic grains,**
**ultracold Fermi gases**



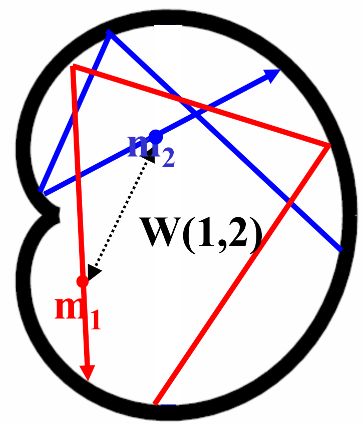

**Classical chaos - quantum chaos established**

## 3. Interacting many-body system:

$$H = \sum_i H_i + \tfrac{1}{2} \sum_{i,j} W(i,j)$$

***W(i,j)* residual interaction**

***Excited states* of**
**many-body systems**




**What determines dynamics, mean field**
**$V_i$ or res. int. *W(i,j)* ?**

**Classical chaos - quantum chaos not established**

## *II. Non-interacting particles in mean field:*
## *- Many-body ground states*

# Periodic orbit theory for non-interacting many-body system [1]

Using the expression for the fluctuating part of **level density**:

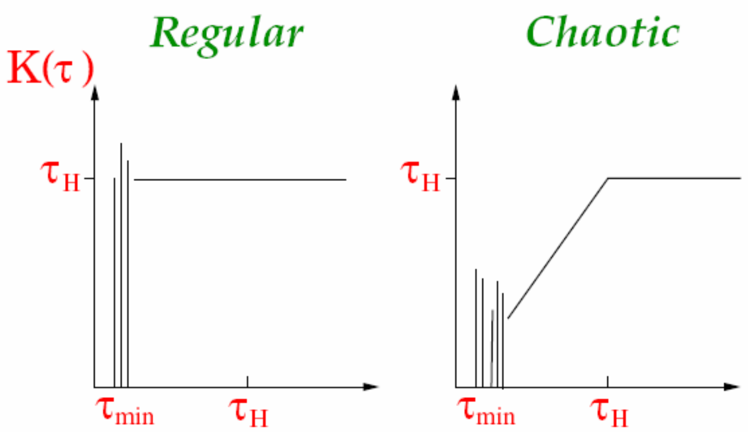
$$\rho_{osc.} = 2 \sum_{\text{periodic orbits}, p} \sum_{r=1}^{\infty} A_{p,r} \cdot \cos(rS_p/\hbar + \nu_{p,r})$$

the fluctuating part of the **total energy** for A particles is obtained:

$$E_{\rm osc}(\mathrm{A}) = \int_0^{e_F} e\rho_{osc}(e)de = 2\hbar^2 \sum_p \sum_{r=1}^{\infty} \frac{A_{p,r}}{r^2\tau_p^2} \cos(rS_p/\hbar + \nu_{p,r})$$

The second moment of $E_{\rm osc.}$ can be evaluated: $\left\langle E_{\rm osc.}^2 \right\rangle = \frac{\hbar^2}{2\pi^2} \int_0^{\infty} \frac{d\tau}{\tau^4} K_D(\tau)$

where K is the spectral form factor (Fourier transform of 2-point corr. function):




giving:

$$E_{regular}^{RMS} = \sqrt{< E_{regular}^2 >} = \sqrt{\frac{\hbar^2}{6\pi^2} \frac{\tau_H}{\tau_{\min}^3}}$$

$$E_{chaos}^{RMS} = \sqrt{< E_{chaos}^2 >} = \sqrt{\frac{\hbar^2}{2\pi^2} \frac{1}{\tau_H^2}}$$

$\tau_{\rm min}$ : shortest periodic orbit

$\tau_{\rm H}$ : Heisenberg time, h/δ


**[1] P. Leboeuf and A.G. Monastra, Ann of Phys 297, 127 (2002)**

# II.a Ground-states of atomic nuclei – nuclear masses

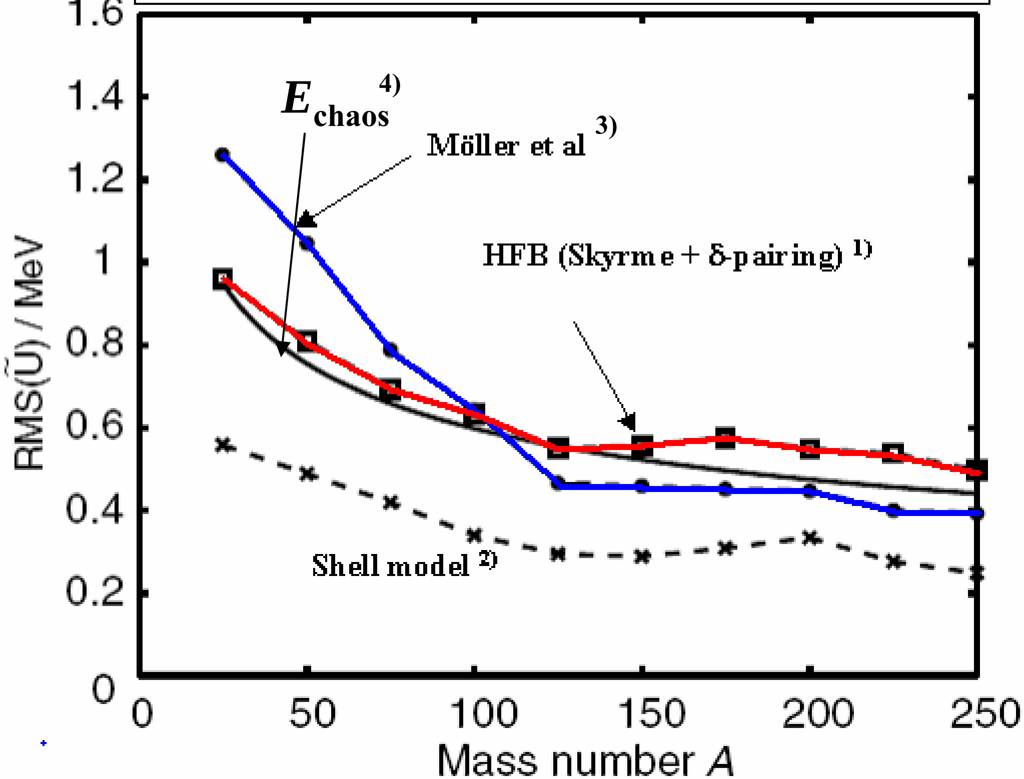
**Nuclear mass:** $m(N,Z)/c^2 = Nm_n + Zm_p - (E_{L.D.} + E_{shell} + E_{error})$

**Inserting proper estimates gives [4]:**

$E^{RMS}_{regular} \approx 2.8\,\text{MeV}$ ⟷ $E^{RMS}_{shell} \approx 3\,\text{MeV}$

$E^{RMS}_{chaos} \approx \frac{2.78}{A^{1/3}}\,\text{MeV}$ ⟷ $E^{RMS}_{error} \approx 0.7\,\text{MeV}$

$E_{L.D.} \approx 8A\,\text{MeV}$



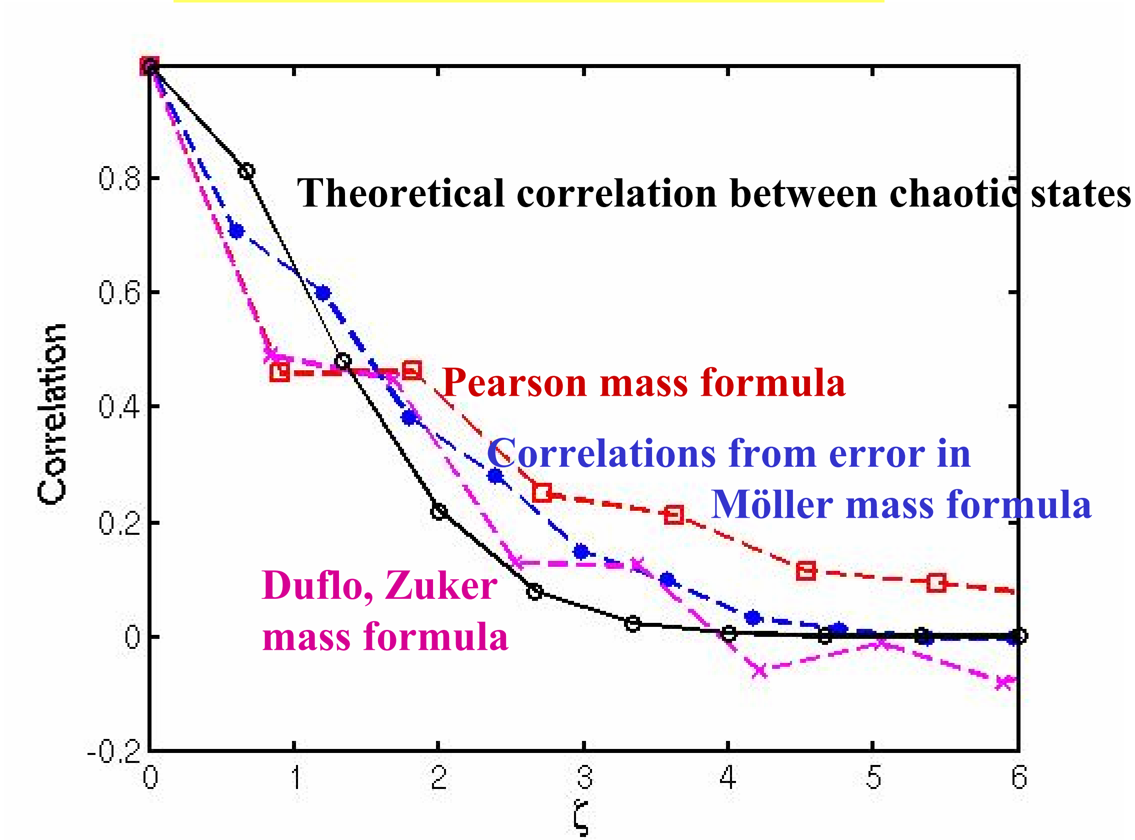

**Agreement suggests a chaotic component in nuclear ground state.**

**This is supported by autocorrelations between error in a sequence of nuclei:**


1) Samyn, Goriely, Bender, Pearson, PRC 70 (2004) 044309
2) Duflo, Zuker, PRC 52 (1995) R23
3) Möller et al, At Data and Nucl. Data Tables 59 (1995) 185.
4) O.Bohigas and P.Leboeuf, PRL 88 (2002) 092502.

# *Autocorrelations in error in mass formuale [1]*

**$C(x)= \langle E(A-x_N/2)E(A+ x_N/2)\rangle$**




**Scaled differens in neutron number**

**Chaotic energy is *not* random but strongly correlated!**

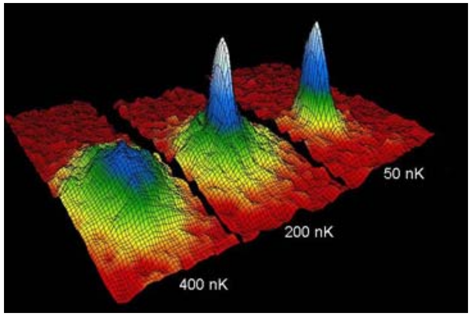
**Supports the idea of a chaotic component in ground state**

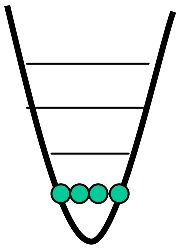

**[1] H. Olofsson. S. Åberg, O. Bohigas and P. Leboeuf, Phys. Rev. Lett. 96 (2006) 042502.**

# II.b Ground states of ultracold gases of atoms

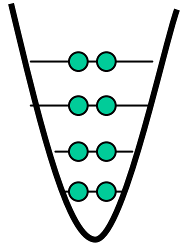
## Trapped quantum gases of bosons or fermions

**T≈0**



**Bose condensate**



**Degenerate fermi gas**

**Dilute gas confined in *V*:**

$$H = \sum_{i=1}^{N} \left( \frac{p_i^2}{2m} + V(r_i) \right) + 4\pi \frac{\hbar^2 a}{m} \sum_{i<j} \delta^{(3)}(r_i - r_j)$$

**S-wave scattering length, *a*, can be experimentally controlled both in size and in sign.**

**Constants:** $g \equiv 2\pi \frac{a}{a_{osc}}$ $\quad \hbar = m = a_{osc} = 1$

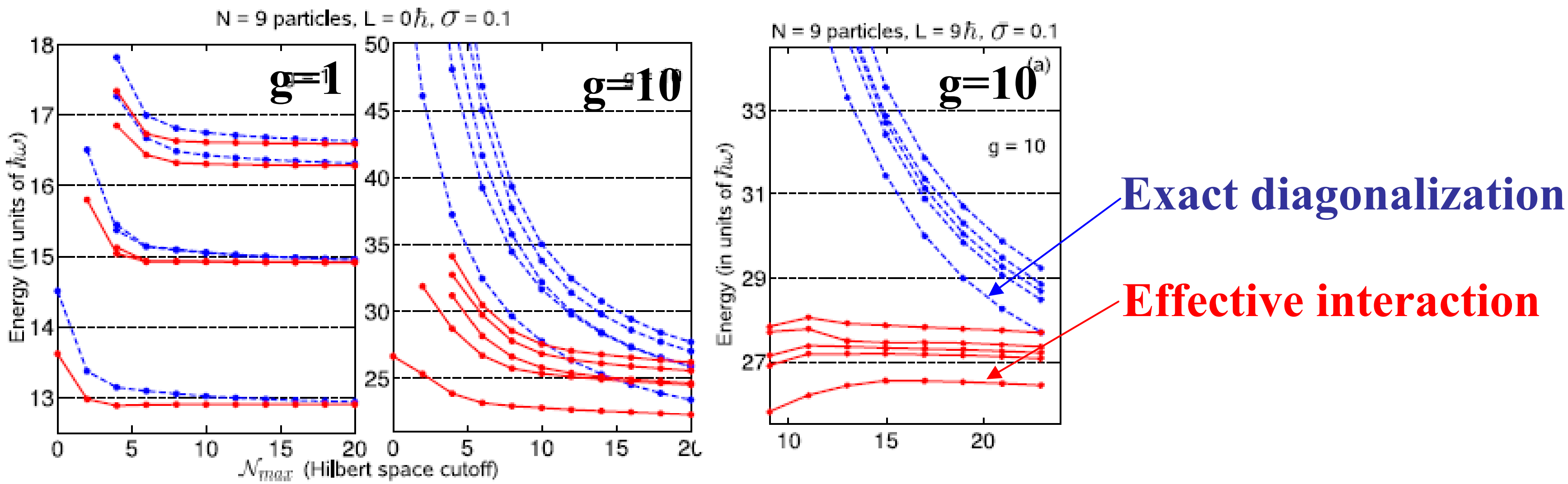
## *Bosonic atoms in harmonic confinement, repulsive interaction (a>0)*

# Effective interaction approach to the many-boson problem

***N*** **spinless bosons in a 2D harmonic confinement interacting via short-range int. with strength g.**
**Effective interaction derived from Lee-Suzuki method.**

***N*=9**

**L=0** **L=9**




**Cutoff energy in Fock space**

- **Method works well for strong correlations**
- **Ground-state AND excited states**
- **All angular momenta**

J. Christensson, Ch. Forssén, S. Åberg and S. Reimann, arXiv:0802.2811

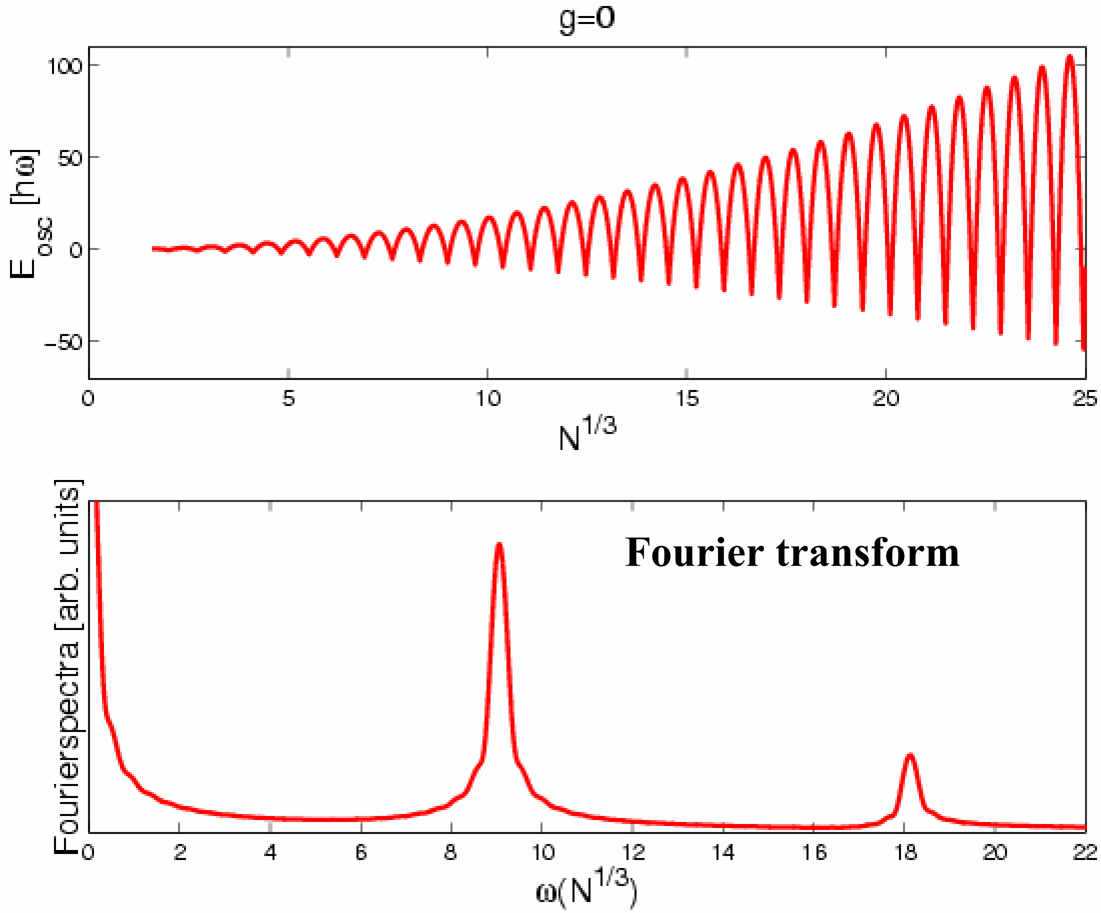
## *Fermionic atoms in harmonic confinenment, repulsive interaction (a > 0)*

# N Fermionic atoms in harmonic trap - Repulsive interaction

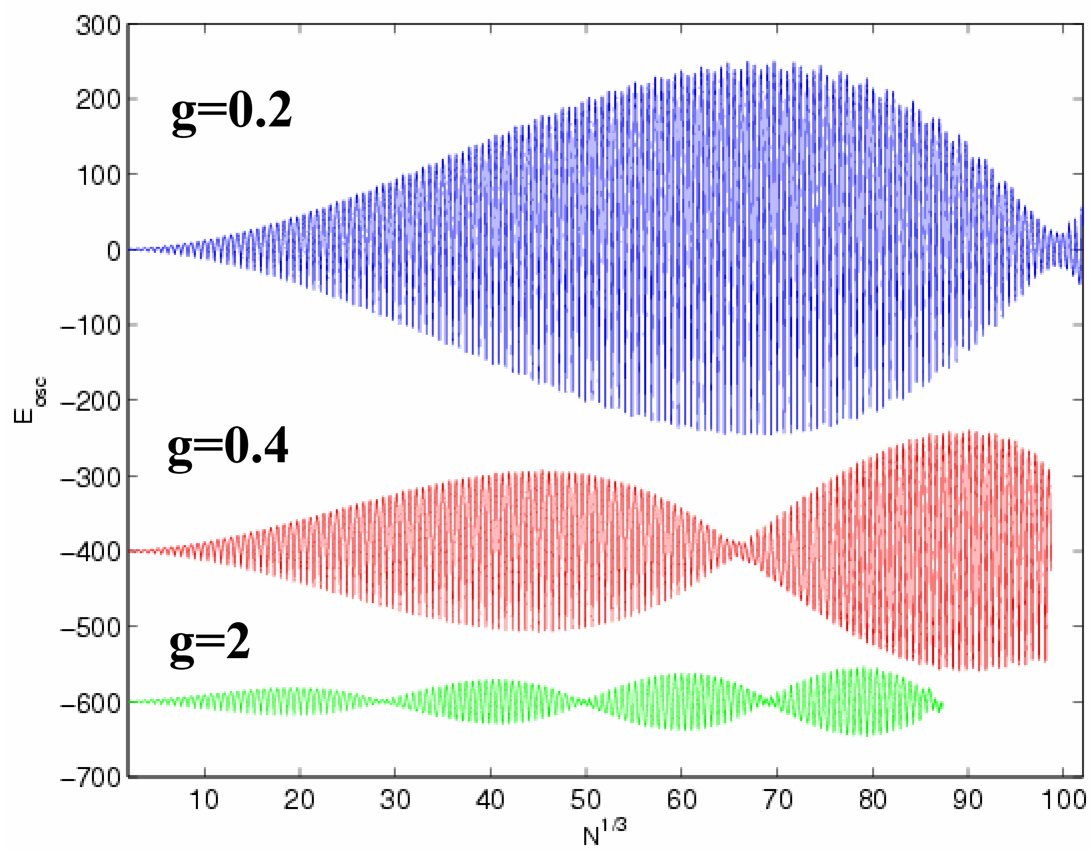
**Hartree-Fock approximation**

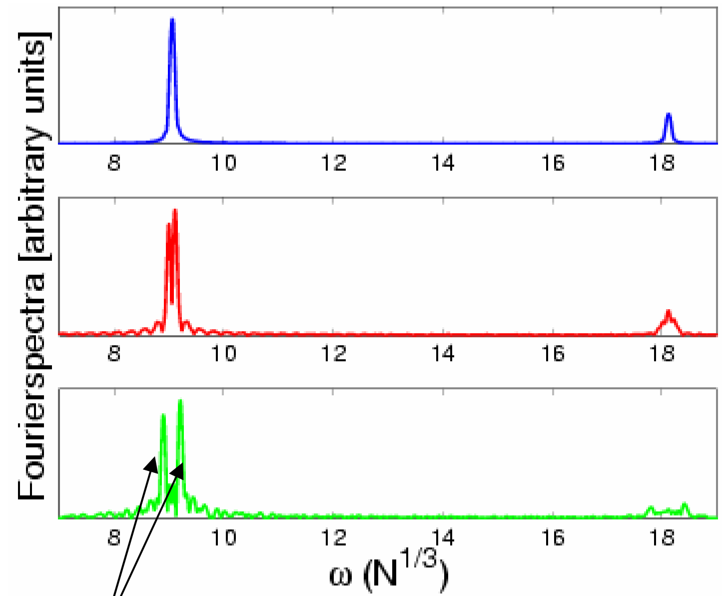
**Shell energy: $E_{osc} = E_{tot} - E_{av}$**

**Shell energy vs particle number for pure H.O.**



# Super shell structure observed for g>0 [1]




**Two close-lying frequencies give rise to the beating pattern: circle and diameter periodic orbits**

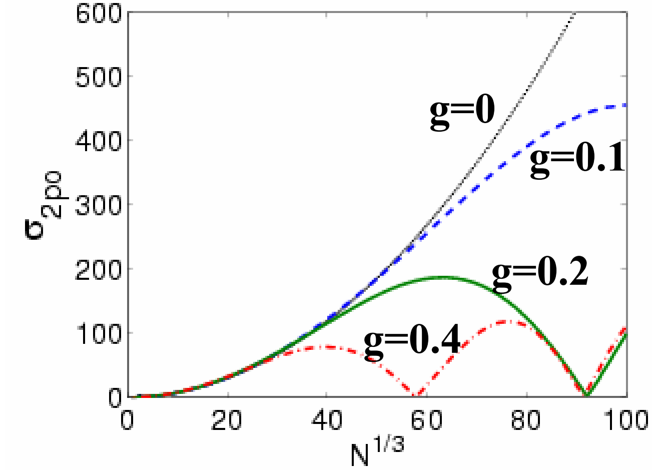
**Effective potential:** $V_{eff} = \frac{1}{2}\,\omega_{eff}\, r^2 + \frac{1}{4}\,\varepsilon r^4$

[1] Y. Yu, M. Ögren, S. Åberg, S.M. Reimann, M. Brack, PRA 72, 051602 (2005)

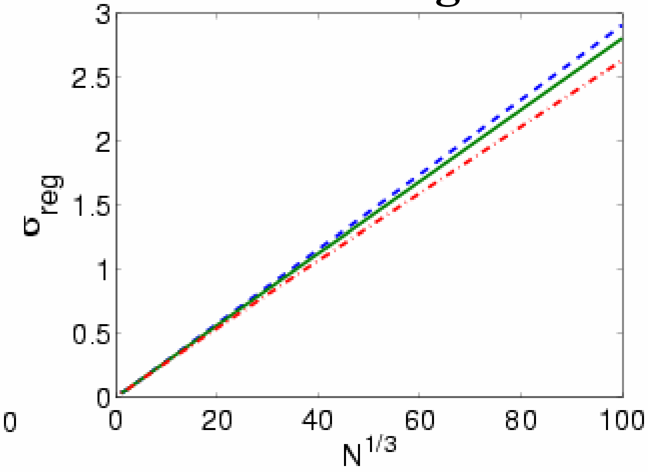
# Fluctuations of shell energy

## Semiclassical calculation of energy fluctuation

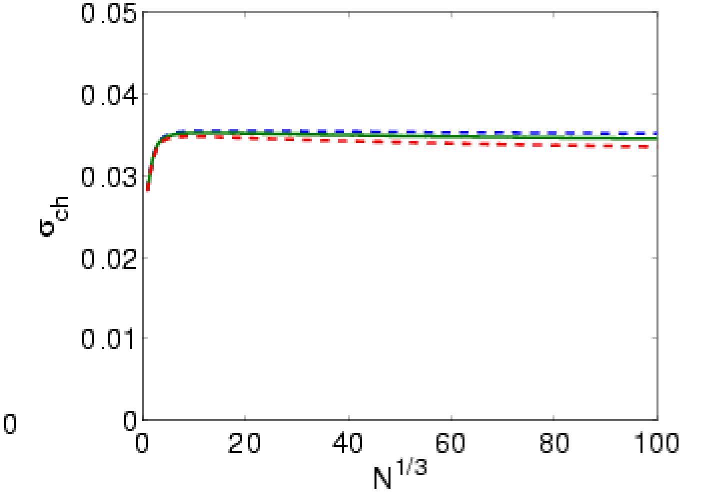
### H.O. confinement




### Generic regular




### Generic chaotic




***Strong*** **suppression of fluctuations from HO, to generic regular, to chaotic**

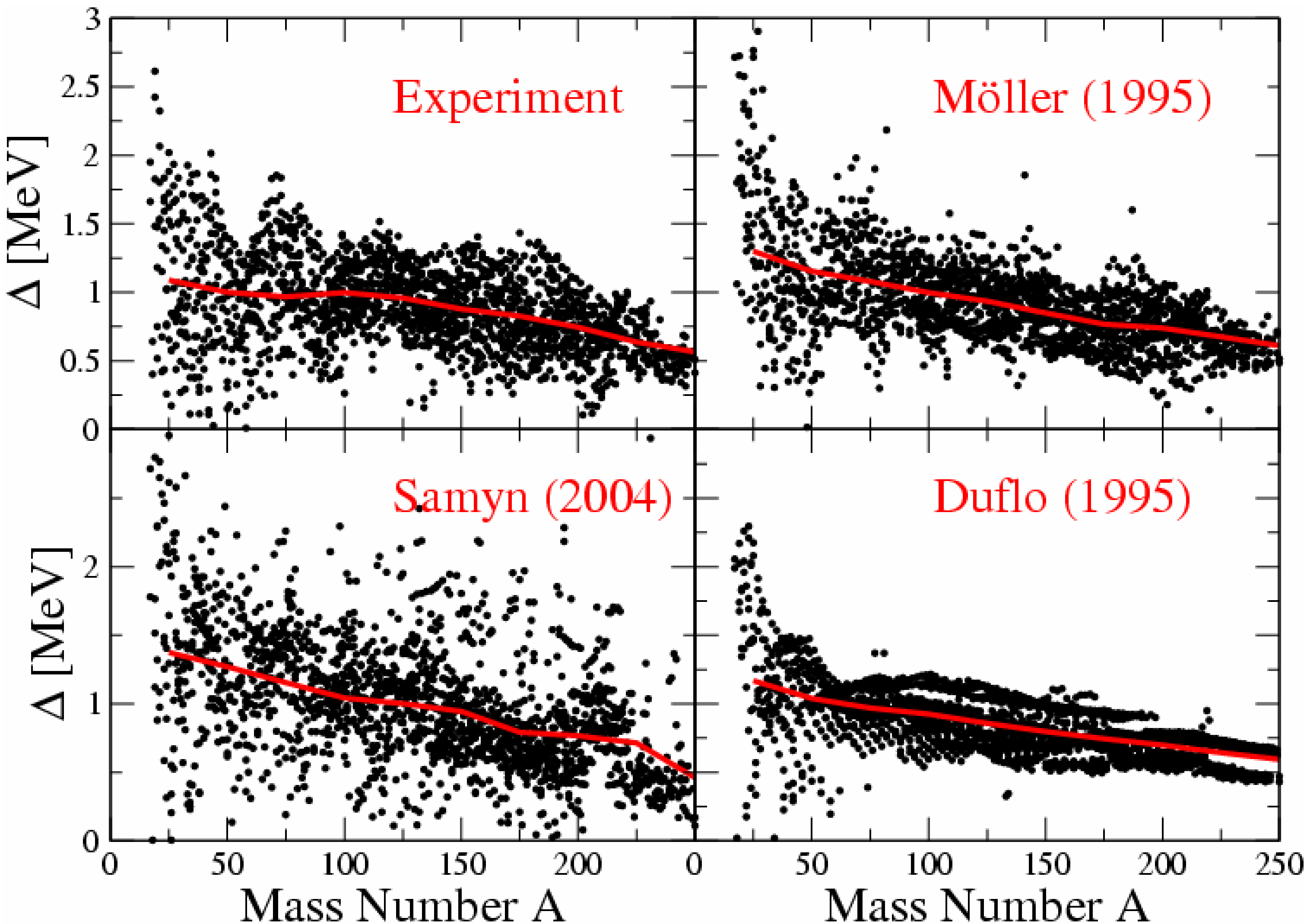
**M. Puig von Friesen, M. Ögren and S. Åberg, PRE 76, 057204 (2007)**

## *III Fluctuations of pairing gap [1] : Role of regular/chaotic dynamics?*

**[1] H. Olofsson, S. Åberg and P. Leboeuf, Phys. Rev. Lett. 100, 037005 (2008)**

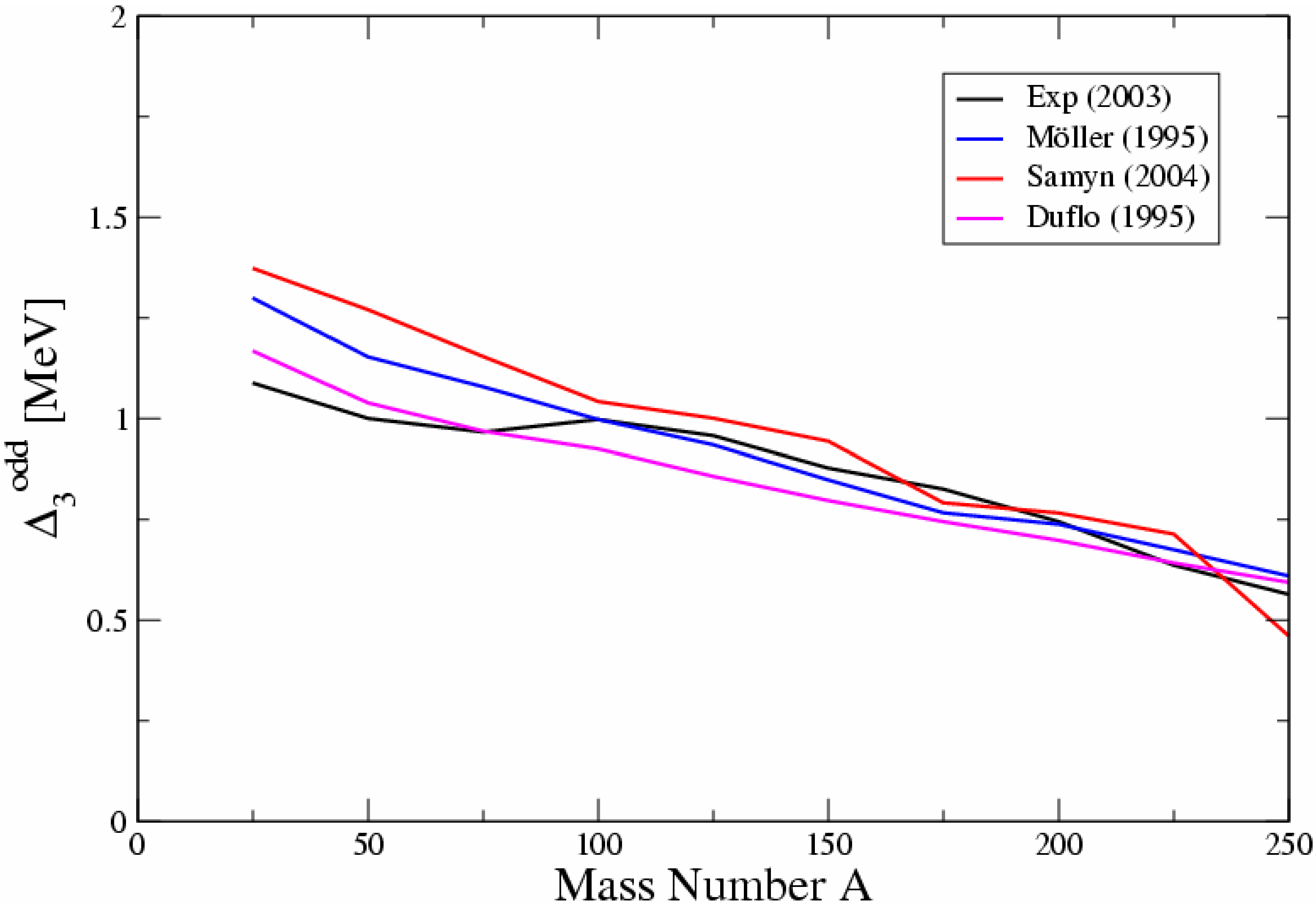
# *III.a BCS pairing gap in nuclei - Odd-even mass difference*

# Pairing gap from different mass models



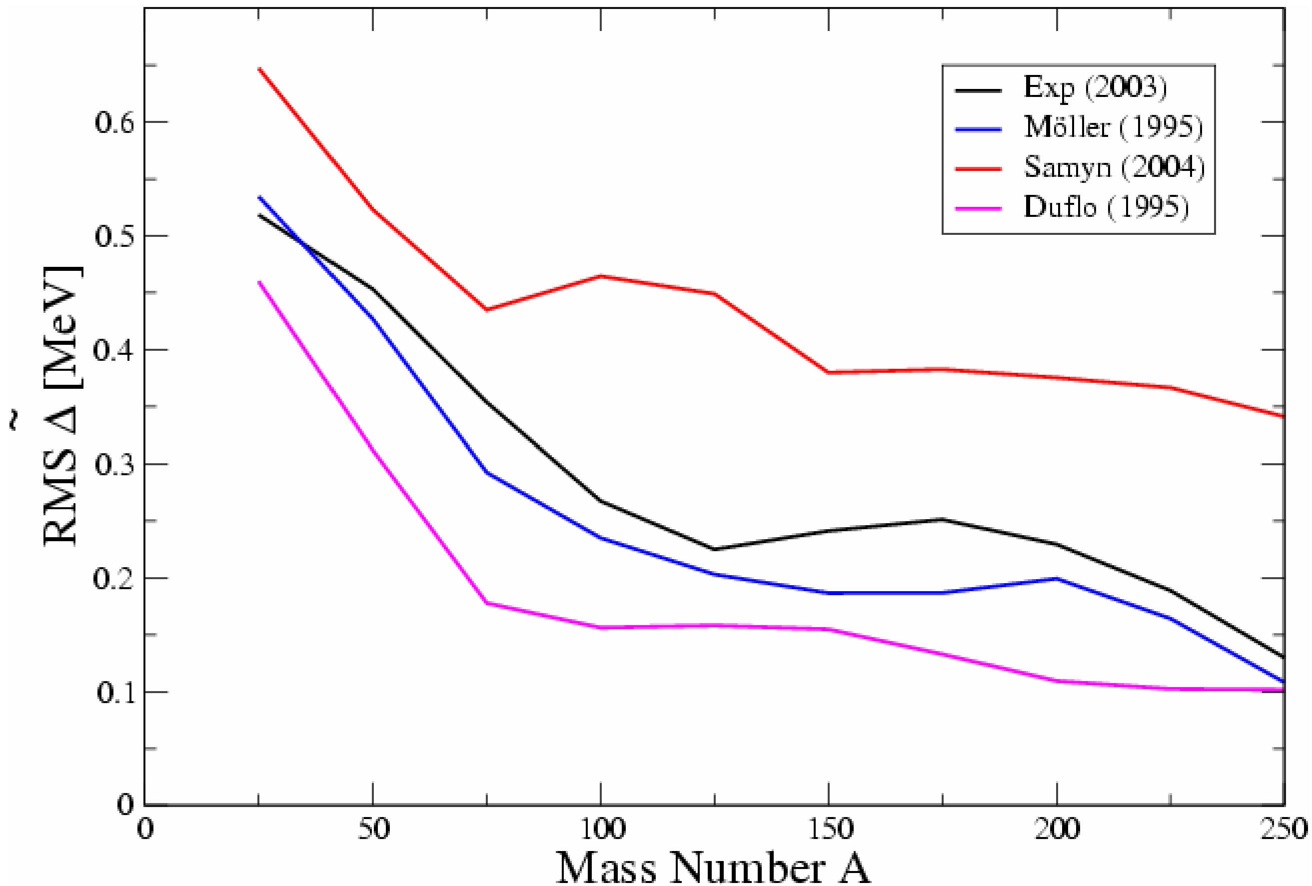

**Average behavior in agreement with exp. but *very* different fluctuations**

# *Pairing gap $\Delta_3^{odd}$ from different mass models*




**Mass models all seem to provide pairing gaps in good agreement with exp.**

# Fluctuations of the pairing gap




**But large deviations in fluctuations of pairing gap**

# BCS theory

**Hamiltonian:**

$$H = \sum_k e_k a_k^+ a_k - G \sum_{kl} a_k^+ a_{\bar{k}}^+ a_{\bar{l}} a_l$$

**Mean field approximation (in pairing space):**

**Pairing gap (”pairing deformation”):**

$$\Delta = \left\langle G \sum_k a_k^+ a_{\bar{k}}^+ \right\rangle$$

**is determined by gap equation:**

$$\frac{2}{G} = \sum_\mu \frac{1}{\sqrt{\left(e_\mu - \lambda\right)^2 + \Delta^2}} \rightarrow \int_{-L}^{L} \frac{\rho(e)de}{\sqrt{e^2 + \Delta^2}}$$

# Periodic orbit description of pairing

**Divide pairing gap in smooth and fluctuating parts:**

$$\Delta = \overline{\Delta} + \tilde{\Delta} \qquad \overline{\Delta} \approx 2L \exp\left(-\frac{1}{\overline{\rho} G}\right)$$

**Insert into gap equation and assuming** $\overline{\Delta} << L$

**Expand to lowest order in fluctuations gives**

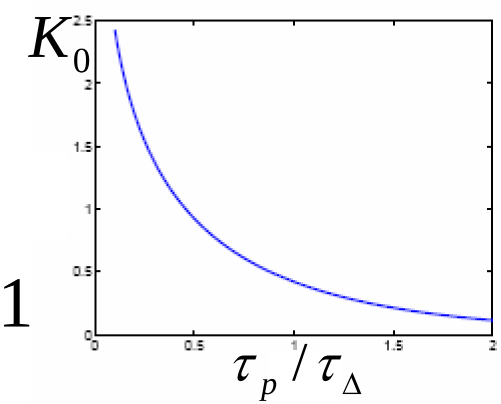
$$\tilde{\Delta} = 2\frac{\overline{\Delta}}{\overline{\rho}} \sum_{p,r} A_{p,r} K_0(r\tau_p / \tau_\Delta) \cos\left(rS_p(e)/\hbar + \nu_{p,r}\right)$$

**where**

$$\tau_\Delta = \frac{h}{2\pi\overline{\Delta}}$$ **is "pairing time"** $\tau_p$ **is period time of periodic orbit p**

$$K_0(x) = \int_0^\infty \frac{\cos(xt)}{\sqrt{1+t^2}} dt$$

$$\rightarrow \exp(-x)/\sqrt{x}, \ \ \mathrm{x} >> 1$$




**i.e. no contribution from orbits with**

$$\tau_p >> \tau_\Delta$$

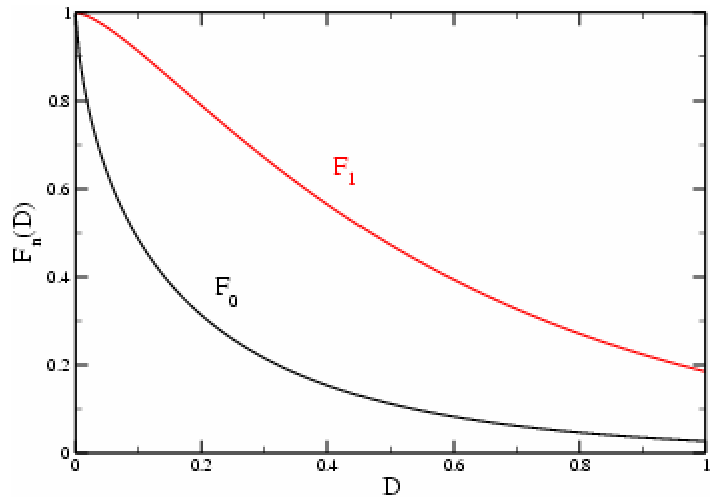
# Fluctuations of pairing – simple expressions

**Fluctuations of pairing, expressed in single-particle mean level spacing, δ:**

$$\sigma = \sqrt{\langle \tilde{\Delta}^2 \rangle} / \delta$$

**If regular:** $$\sigma_{reg}^2 = \frac{\pi}{4} \frac{\overline{\Delta}}{\delta} F_0(D)$$

**If chaotic:** $$\sigma_{ch}^2 = \frac{1}{2\pi^2} F_1(D)$$




**where** $$F_n(D) = 1 - \frac{\int_0^D x^n K_0^2(x)dx}{\int_0^\infty x^n K_0^2(x)dx}$$

**where** $D = \tau_{min}/\tau_\Delta$

| | |
|---|---|
| **Or, dimensionless ratio:** | $D = 2R/\xi_0$ |
| **Size of system:** | **2R** |
| **Correlation length of Cooper pair:** | $\xi_0 = \hbar v_F / 2\Delta$ |

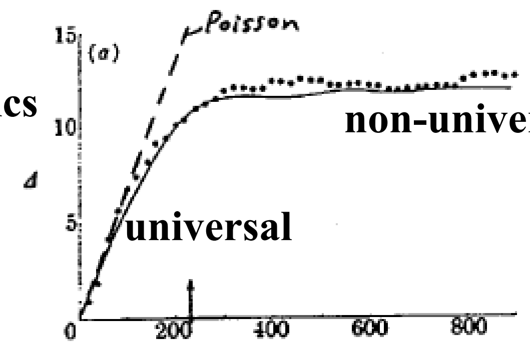
# *Universal/non-universal fluctuations*

$$D = \frac{\tau_{\min}}{\tau_{\Delta}} = \frac{2\pi}{g}\frac{\overline{\Delta}}{\delta}$$

$$g = \frac{\tau_H}{\tau_{\min}}$$ **"dimensionless conductance"**

**Non-universal spectrum fluctuations for energy distances larger than g:**

$\Delta_3$ **statistics**




**g=L$_{max}$**

**Random matrix limit: g → ∞ (i.e. D = 0) corresponding to pure GOE spectrum (chaotic) or pure Poisson spectrum (regular)**

# Fluctuations of pairing in nuclei

**Size of D:**

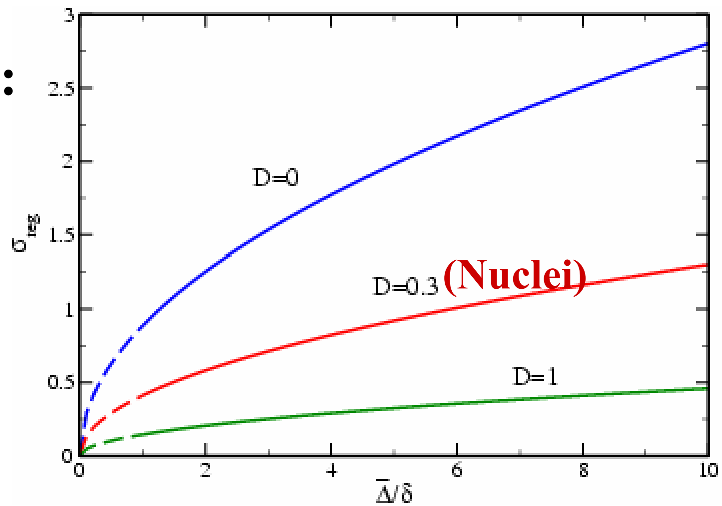
**Size of system:** $2R = 2*1.2A^{1/3}$ fm

**Pairing length:** $\xi_0 = \hbar v_F / 2\Delta = 11.3\ A^{-1/4}$ fm

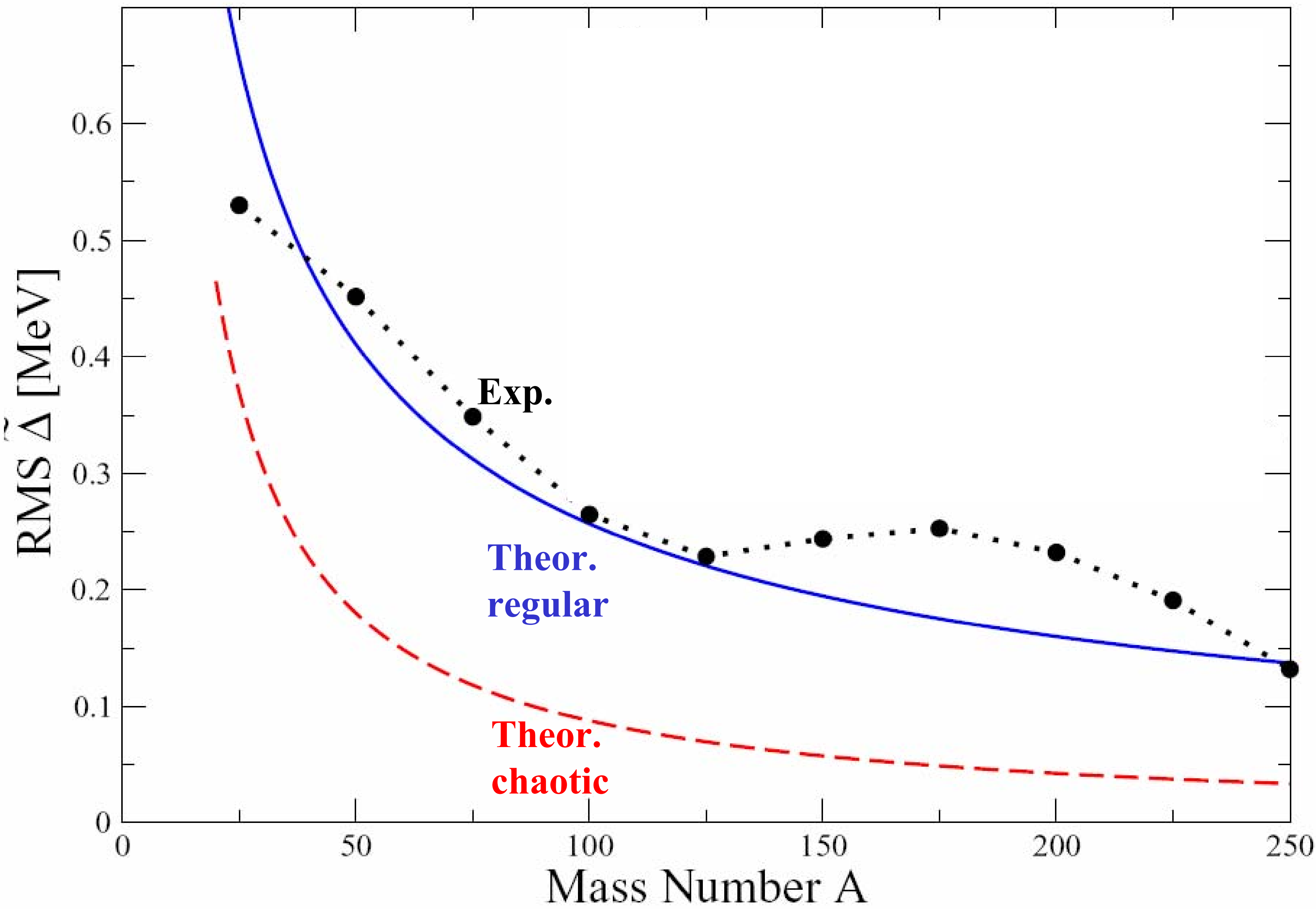
$$\Rightarrow \quad D = 2R/\xi_0 = 0.22A^{1/12} = 0.27 - 0.33 \quad (A=25\text{-}250)$$

**Cooper pairs non-localized in nuclei**

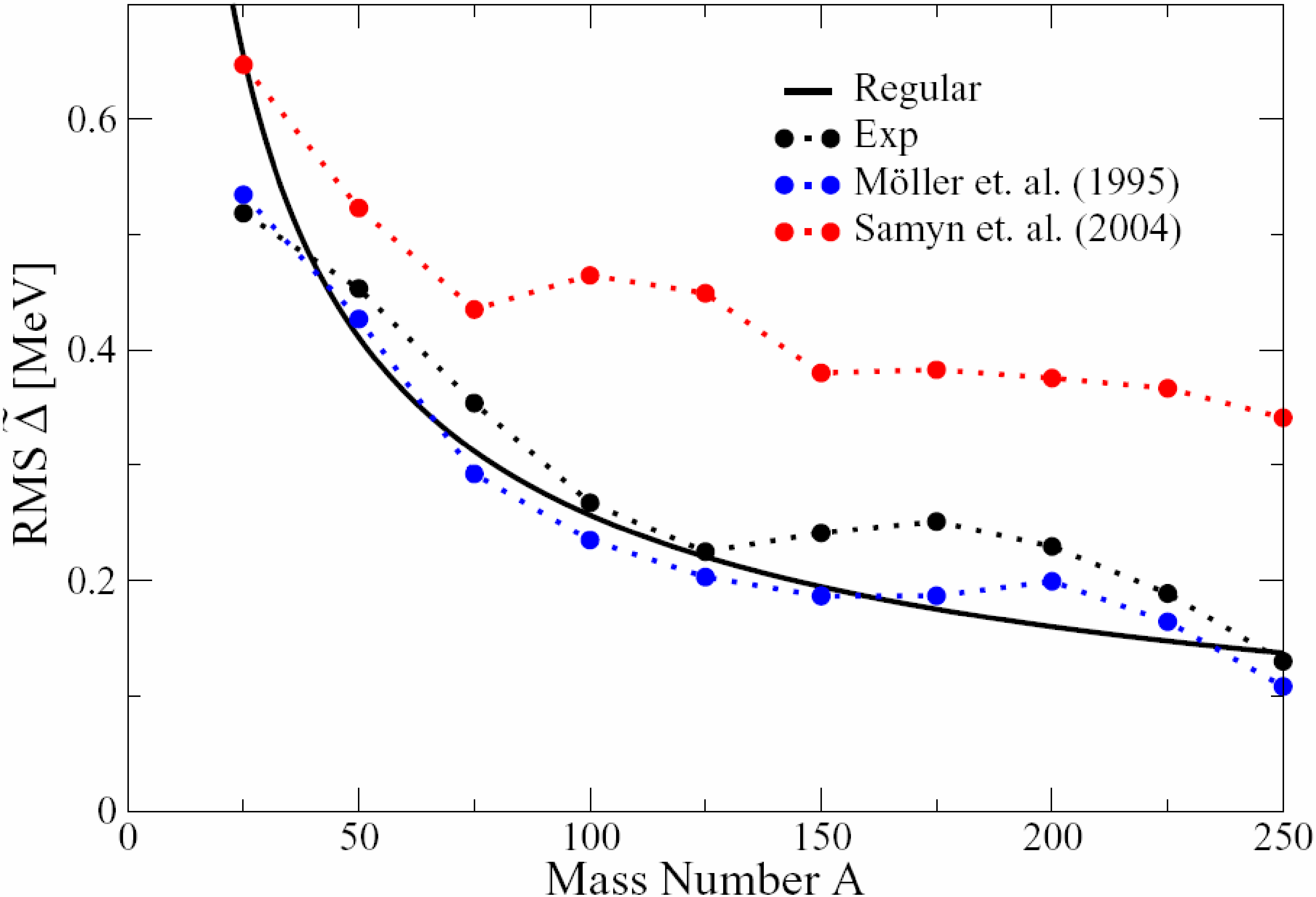
**Pairing fluctuations for regular system:**



# Fluctuations of nuclear pairing gap



# Fluctuations of nuclear pairing gap from mass models



# III.b Pairing fluctuations in nanosized metallic grains

Work by D.C. Ralph, C.T. Black and M. Tinkham, PRL 74, 3241 (1995); PRL 76, 688 (1996)

- **Discrete exc. spectrum**
- **Irregular shape of grain ⇒ chaotic dynamics**
  **- No symmetries – only time-rev. symm.**
  **- Energy level statistics described by GOE**
- **Excitation gap – pairing gap (>>δ) observed for even *N***
- **Applied B-field ⇒ gap disappears**

$$\mathrm{N} \sim 10^3 - 10^5$$

$$\bar{\Delta} \approx 0.38\times 10^{-3}\, eV \qquad \delta = 2.1/N \text{ eV}$$

$$\mathrm{D} = \frac{2R}{\xi_0} = \frac{2\pi}{5.5} N^{1/3}\bar{\Delta} \approx 0.004 - 0.02 \;\Rightarrow\; \mathrm{F}_1 \approx 1$$

**⇒ Universal pairing fluctuations:** $$\sigma_{ch}^2 = \frac{1}{2\pi^2}$$ **(GOE limit)**

**Universal GOE limit derived in: K.A. Matveev and A. Larkin PRL 78, 3749 (1997)**

# III.c Pairing fluctuations in ultracold fermionic gases

**In dilute BCS region: (attractive interaction g < 0)**

$$\overline{\Delta}/\delta = (2/e)^{7/3}\frac{3N}{2}\exp\left(-\frac{\pi}{2k_F|a|}\right)$$

$$D = \frac{2R}{\xi_0} = 2\pi(2/e)^{7/3}(3N)^{1/3}\exp\left(-\frac{\pi}{2k_F|a|}\right)$$

**Recent experiments [1] using $^6$Li reach $k_F|a| = 0.8$ and about 100 000 atoms gives Δ/δ=100 000, D=60 and:**
**negligible fluctuations of the pairing gap**

**However, for example, for $k_F|a| = 0.2$ and 50 000 atoms gives Δ/δ=12, D=0.06 and:**

$\frac{\Delta}{\overline{\Delta}} = 1 \pm 0.24$ **for regular system** $\frac{\Delta}{\overline{\Delta}} = 1 \pm 0.02$ **for chaotic system**

**Making the system chaotic strongly supresses pairing fluctuations!**

$$\tilde{\Delta}_{RMS,regular}/\tilde{\Delta}_{RMS,chaotic} \approx 4\sqrt{\overline{\Delta}/\delta}$$

[1] C.H. Schunck et al, PRL **98** (2007) 050404

# SUMMARY

I. Mixture of chaos in nuclear ground state. Supported by energy statistics, errors in masses and correlation measures.

II. Effective interaction scheme (Lee-Suzuki) works well for many-body boson system

III. Ultracold Fermionic gases show supershell structure in harmonic confinement. Energy fluctuations strongly supressed in generic regular and chaotic systems

IV. Fluctuations of BCS gaps in nuclei well described by periodic orbit theory. Regular part masks possible chaotic mixture.

V. Non-universal corrections to BCS fluctuations important
 - beyond random matrix theory.