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Generation of Subharmonics in Ultracold Plasma Clusters

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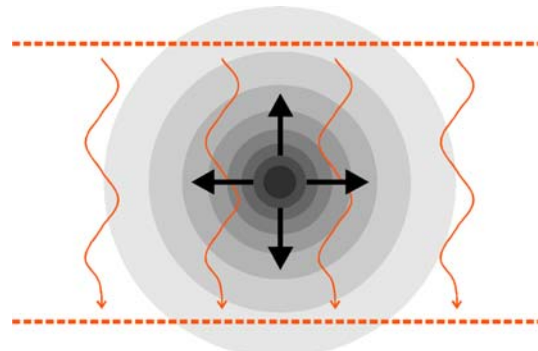
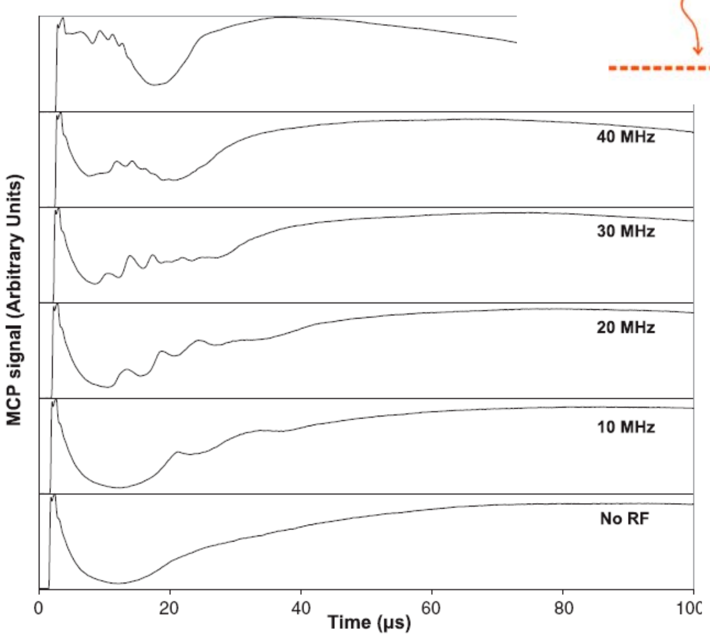
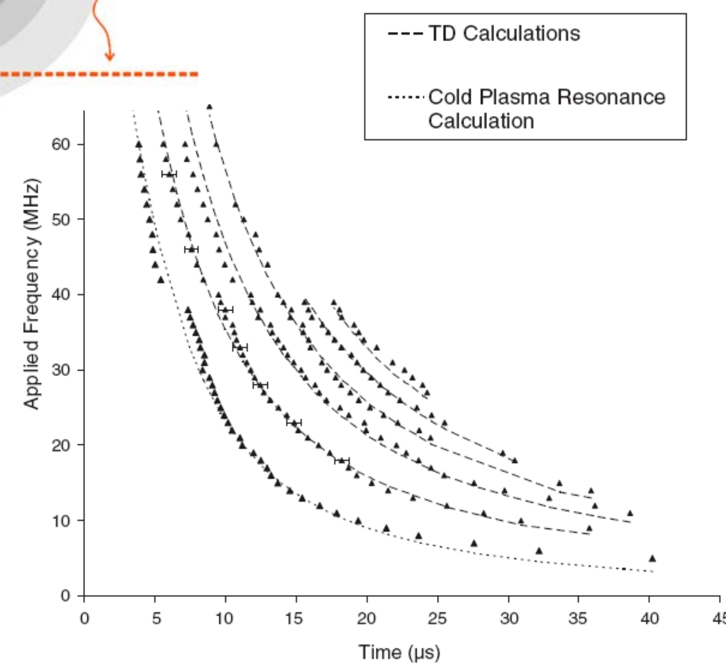
Observation of Collective Modes of Ultracold Plasmas

R. S. Fletcher, X. L. Zhang, and S. L. Rolston

Applying a radio-frequency electric field to an expanding ultracold neutral plasma leads to the observation of as many as six peaks in the emission of electrons from the plasma. These are identified as collective modes of the plasma and are in qualitative agreement with a model of Tonks-Dattner resonances, electron sound waves propagating in a finite-sized, inhomogeneous plasma.

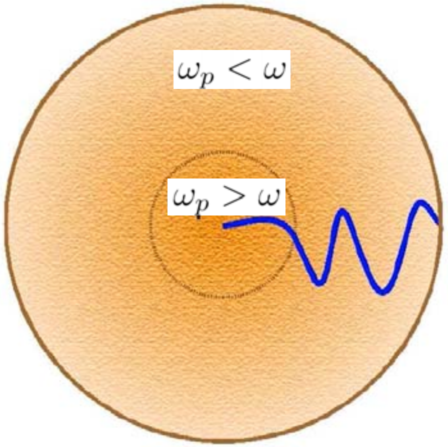






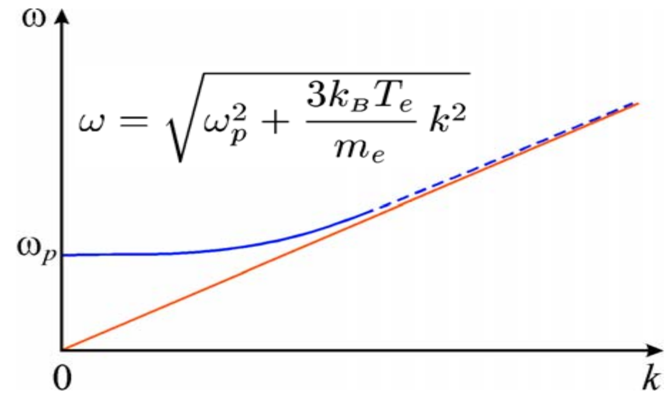
Collective (Tonks-Dattner) plasma resonances in a finite-size cloud

[Figure by A. Lyubonko]



The main problem is to explain formation of the plasma waves with very small wavelengths (as compared to the external irradiation):

λ_{ext}~ 10 m,
R_{cloud}~ 1 mm.



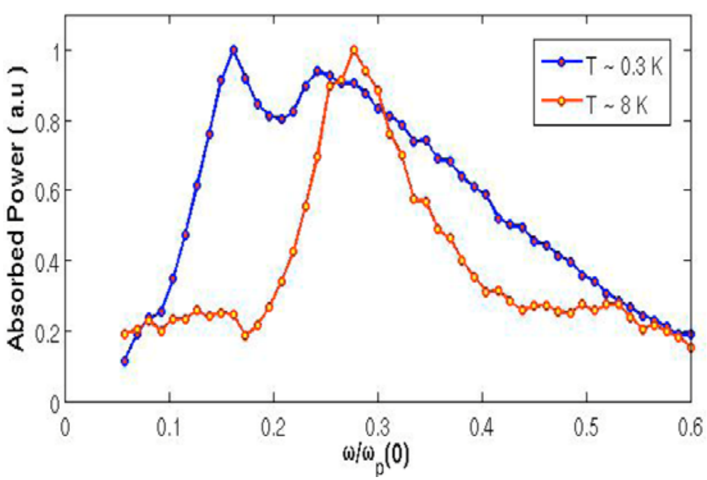
Obstacles in the interpretation of resonances as the collective modes (analytical approach):

(1) The second term in the Bohm-Gross dispersion relation is valid only as a small correction.

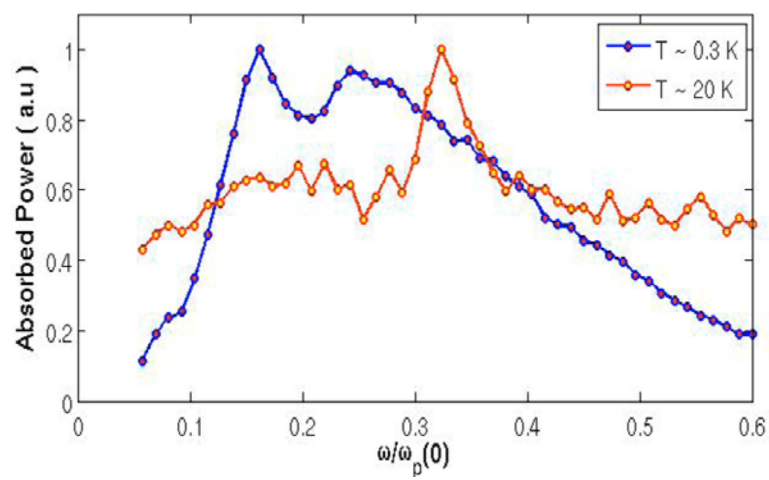
(2) The electron temperature required for this interpretation (20 K) is a few times greater than the one measured by another method [R.S. Fletcher, *et al*., Phys. Rev. Lett., **99**, 145001 (2007)].

(3) There are no walls or sharp boundaries, from which the waves could be well reflected.

Numerical simulation of collective modes in a small plasma cloud (by A. Lyubonko)**:**



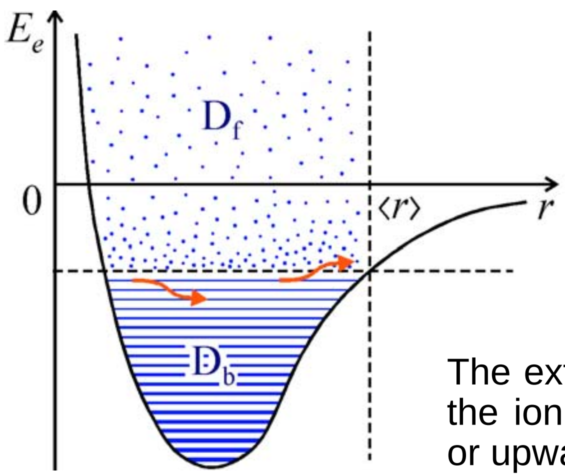
[Figure by A. Lyubonko]



[Figure by A. Lyubonko]

Resonant ionization of Rydberg atoms in ultracold plasmas / 1

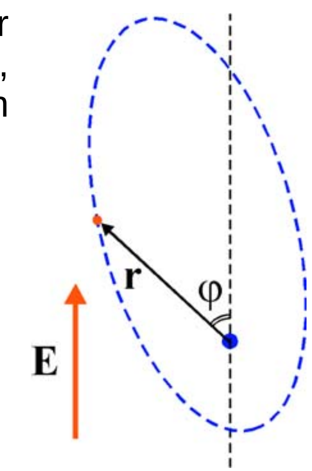
Interpretation of the subharmonics as a result of excitation of the collective plasma modes is hardly compatible with the experimental finding [R.S. Fletcher, *et al*., Phys. Rev. Lett., **96**, 105003 (2006)] that the number of peaks increases when concentration of the free electrons (responsible for the collective processes) decreases. This experimental finding forced us to seek for an alternative explanation, associated with the strongly-coupled electrons near the ionization threshold of the Rydberg atoms.



Resonant escape of electrons near the ionization threshold under the action of the external electric field:

As is seen in figure, electron in the region of the phase space $\mathbf{D_b}$ cannot go away from the nearest ion by the distance <*r*>; and therefore it is captured by this ion, forming the Rydberg atom. On the other hand, the electron in the region $\mathbf{D_b}$ can reach the average interparticle distance <*r*> and thereby leave the ion.

The external oscillating electric field can move an electron near the ionization threshold either downwards, to the deeper level, or upwards, to the free state (depending on the relation between the phases of the electron motion and the external field).



The perturbed motion of the outer electrons can be described in the framework of classical mechanics:

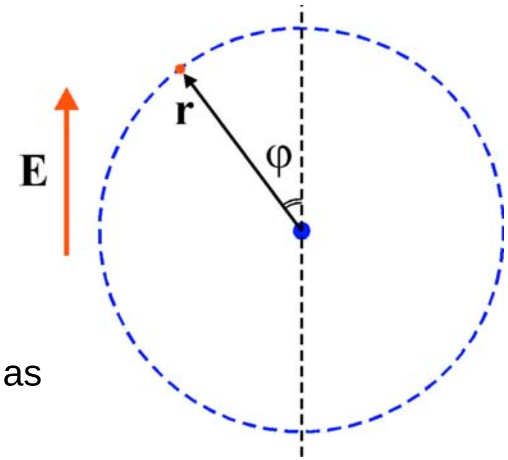
$$\begin{cases} \delta\ddot{r} - \left(2\dfrac{e^2}{m_e}\dfrac{1}{r_0^3} + \dot{\varphi}_0^2\right)\delta r - 2r_0\dot{\varphi}_0\delta\dot{\varphi} = -\varepsilon_0 \sin(\omega t + \psi^*)\cos\varphi_0\,, \\ r_0\delta\ddot{\varphi} + 2\dot{r}_0\delta\dot{\varphi} + 2\dot{\varphi}_0\delta\dot{r} + \ddot{\varphi}_0\delta r = \varepsilon_0 \sin(\omega t + \psi^*)\sin\varphi_0\,. \end{cases}$$

Resonant ionization of Rydberg atoms in ultracold plasmas / 2

$$m_e \frac{d^2\mathbf{r}}{dt^2} = -e^2 \frac{\mathbf{r}}{r^3} - e\mathbf{E}(t)$$

$$\begin{cases} m_e(\ddot{r} - r\dot{\varphi}^2) = -\dfrac{e^2}{r^2} - eE(t)\cos\varphi\,, \\ m_e(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}) = eE(t)\sin\varphi\,, \end{cases}$$

where $E(t) = E_0 \sin(\omega t + \psi^*)\,.$



The unperturbed circular motion takes the form:

$$\begin{cases} r_0 = a\,, \\ \varphi_0 = \Omega\, t + \varphi^*, \end{cases}$$

where

$$\Omega = \left(\frac{e^2}{m_e\, a^3} \right)^{1/2}$$

is the Keplerian frequency.

The perturbed circular motion can be written as

$$\begin{cases} r(t) = r_0 + \delta r(t)\,, \\ \varphi(t) = \varphi_0(t) + \delta\varphi(t)\,, \end{cases}$$

where the perturbed orbital elements are determined by the equations:

$$\begin{cases} \delta\ddot{r} - 3\Omega^2 \delta r - 2a\Omega\, \delta\dot{\varphi} = -\varepsilon_0 \sin(\omega t + \psi^*) \cos(\Omega t + \varphi^*)\,, \\ a\, \delta\ddot{\varphi} + 2\Omega\, \delta\dot{r} = \varepsilon_0 \sin(\omega t + \psi^*) \sin(\Omega t + \varphi^*)\,, \end{cases}$$ where $\varepsilon_0 = eE_0/m_e\,.$

The equation for radial perturbation can be also rewritten as

$$\delta\ddot{r} - 3\Omega^2\delta r - 2a\Omega\,\delta\dot{\varphi} = -\frac{\varepsilon_0}{2}\left\{ \sin\left[(\omega+\Omega)t + (\psi^* + \varphi^*)\right] + \sin\left[(\omega-\Omega)t + (\psi^* - \varphi^*)\right] \right\}.$$

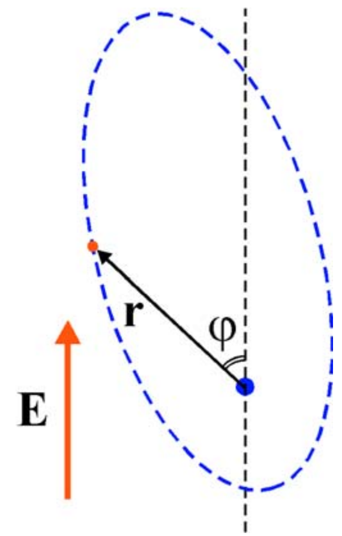
Therefore, at $\boxed{\omega = \Omega}$ we get a permanent (time independent) force, which should lead to a secular (monotonic) change in the radius. This may be either increase or decrease, depending on the relation between the phases of the electric field and the electron motion. It is interesting to mention also that, for a classical electron orbit near the ionization threshold, this frequency coincides up to some numerical factor with the plasma frequency ω_p.

Resonant ionization of Rydberg atoms in ultracold plasmas / 3

By now, only one resonant frequency was found. Is it possible to obtain the additional frequencies by considering a more general case of the elliptic electron motion?

The equations for unperturbed elliptical motion have the form:

$$\begin{cases} \ddot{r}_0 + \dfrac{e^2}{m_e}\dfrac{1}{r_0^2} - r_0\dot{\varphi}_0^2 = 0\,, \\ r_0\ddot{\varphi}_0 + 2\dot{r}_0\dot{\varphi}_0 = 0\,; \end{cases}$$



and the equations for the orbital perturbations are

$$\begin{cases} \delta\ddot{r} - \left(2\dfrac{e^2}{m_e}\dfrac{1}{r_0^3} + \dot{\varphi}_0^2\right)\delta r - 2r_0\dot{\varphi}_0\delta\dot{\varphi} = -\varepsilon_0 \sin(\omega t + \psi^*)\cos\varphi_0\,, \\ r_0\delta\ddot{\varphi} + 2\dot{r}_0\delta\dot{\varphi} + 2\dot{\varphi}_0\delta\dot{r} + \ddot{\varphi}_0\delta r = \varepsilon_0 \sin(\omega t + \psi^*)\sin\varphi_0\,. \end{cases}$$

Since the solutions of unperturbed equations satisfy the conditions

$$r_0\left(t + \frac{2\pi}{\Omega}\right) = r_0(t)\,, \qquad \varphi_0\left(t + \frac{2\pi}{\Omega}\right) = \varphi_0(t) \pm 2\pi\,,$$

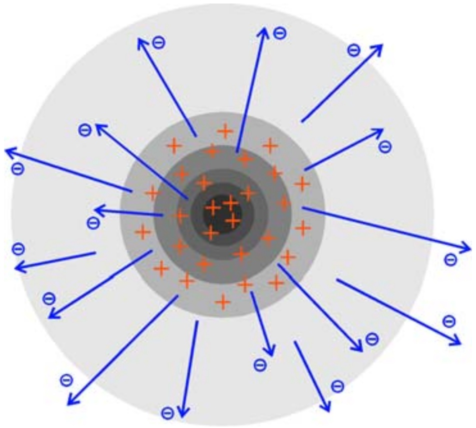
the right-hand sides of the equations for perturbations can be expanded into Fourier series. Particularly, the equation for radial perturbations will take the form:

$$\delta\ddot{r} - \left(2\frac{e^2}{m_e}\frac{1}{r_0^3} + \dot{\varphi}_0^2\right)\delta r - 2r_0\dot{\varphi}_0\delta\dot{\varphi} = -\frac{\varepsilon_0}{2}\sum_{n=0}^{\infty} a_n \Big\{ \sin\Big[(\omega + n\Omega)t + (\psi^* + \varphi_n^*)\Big] + \sin\Big[(\omega - n\Omega)t + (\psi^* - \varphi_n^*)\Big] \Big\}.$$

Consequently, the resonances occur at the frequencies $\boxed{\omega = n\Omega.}$

It is important to emphasize that atoms with various orientation with respect to the external electric field will produce the peaks of electron emission with various amplitudes but at the same frequencies. Therefore, the total pattern of the peaks will not be smoothed out after summation over all the atoms.

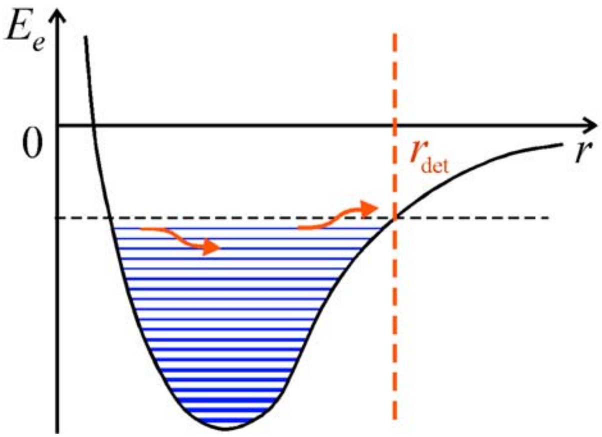
Multiple resonances of electromagnetic-wave absorption in the electronic halo of a non-neutral plasma cloud / 1



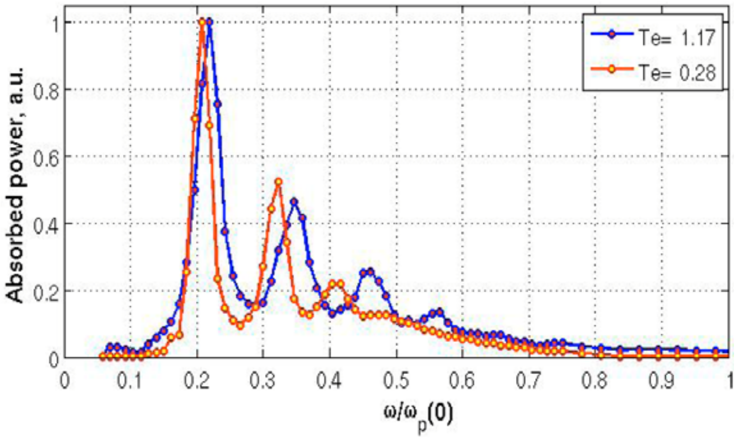
Yet another possibility for the interpretation of subharmonics follows from the experimental finding [R.S. Fletcher, *et al*., Phys. Rev. Lett., **96**, 105003 (2006)] that some number of the most energetic electrons escape from the cloud just after its photoionization, resulting in the overall attractive potential for the remaining electrons.

The entire non-neutral plasma cluster looks like a huge Rydberg atom.

Electrons in the halo of such a cluster should experience resonant perturbations by the external electromagnetic irradiation exactly by the same way as outer electrons in the Rydberg atoms.

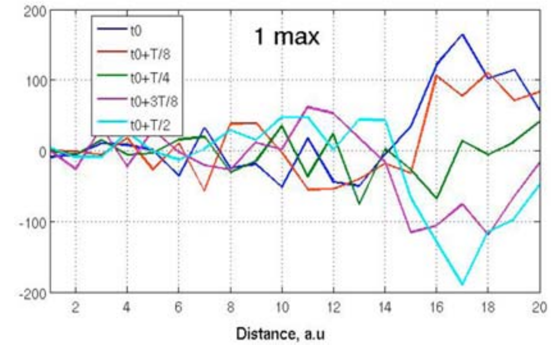


Multiple resonances of electromagnetic-wave absorption in the electronic halo of a non-neutral plasma cloud / 2

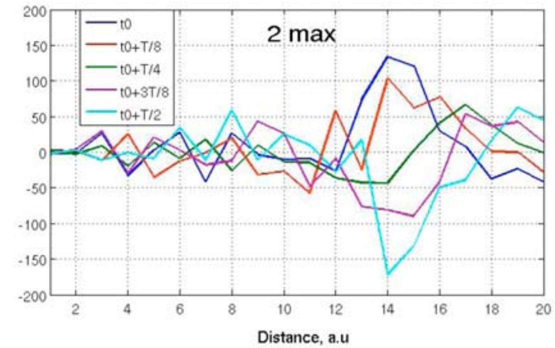


[Figure by A. Lyubonko]

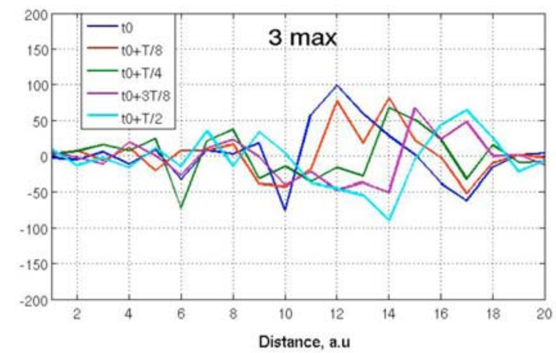
$$Y_{01} = \sum_i r_i \cdot \cos(\theta_i)$$



[Figure by A. Lyubonko]



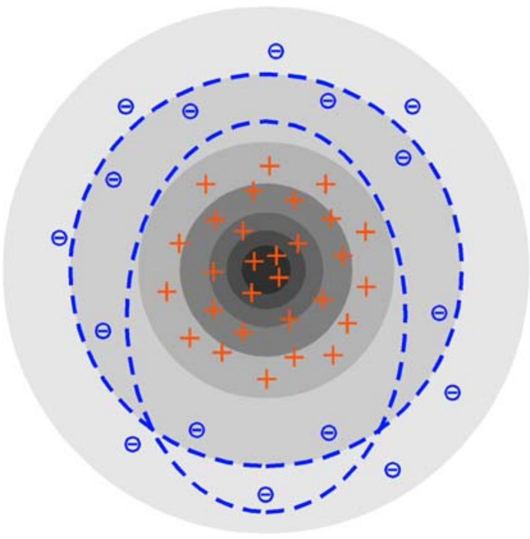
[Figure by A. Lyubonko]



[Figure by A. Lyubonko]

Circular orbit:

$$\delta\ddot{r} - 3\Omega^2\delta r - 2a\Omega\,\delta\dot{\varphi} = -\frac{\varepsilon_0}{2}\left\{\sin\left[(\omega+\Omega)t + (\psi^*+\varphi^*)\right] + \sin\left[(\omega-\Omega)t + (\psi^*-\varphi^*)\right]\right\}$$



Elliptical orbit:

$$\delta\ddot{r} - \left(2\frac{e^2}{m_e}\frac{1}{r_0^3} + \dot{\varphi}_0^2\right)\delta r - 2r_0\dot{\varphi}_0\delta\dot{\varphi} = -\frac{\varepsilon_0}{2}\sum_{n=0}^{\infty} a_n\left\{\sin\left[(\omega+n\Omega)t + (\psi^*+\varphi_n^*)\right] + \sin\left[(\omega-n\Omega)t + (\psi^*-\varphi_n^*)\right]\right\}$$

Conclusions:

1. A few mechanisms for generation of subharmonics of the electron emission from ultracold plasma clouds were proposed by now:

   (a) excitation of collective plasma oscillations,

   (b) resonant ionization of Rydberg atoms,

   (c) resonant escape of electrons from the electronic halo of a non-neutral plasma cloud.

2. The last two mechanisms, based on the resonant perturbation of individual electronic orbits, do not suffer from the drawbacks inherent for the collective plasma processes, and they show a reasonable agreement between the analytical estimates and the results of numerical simulation.