International Workshop "Chaos and Collectivity in Many-Body Systems", 5-8 March 2008, MPIPKS Dresden, Germany

# The Efficiency of Topological Defect Formation by the Strongly-Nonequilibrium Phase Transformations in BECs

Yurii V. Dumin

Theoretical Department,
Institute of Terrestrial Magnetism, Ionosphere,
and Radio Wave Propagation (IZMIRAN),
Russian Academy of Sciences,
142190 Troitsk, Moscow region, Russia

Department of Finite Systems,
Max-Planck-Institut fuer Physik komplexer Systeme (MPIPKS),
Noethnitzer Strasse 38,
01187 Dresden, Germany

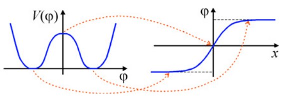
## ABSTRACT

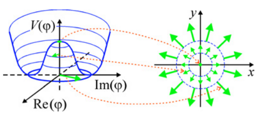
The experiments on measuring the rates of spontaneous formation of the topological defects (such as the vortices and domain walls) in a variety of superconducting and superfluid systems have shown that the defects are formed most efficiently in the samples of small size or low dimensionality, whereas in the macroscopic two- and three-dimensional systems the concentration of defects is substantially suppressed. A reason for universality of such behavior can be explained by the specific quantum entanglement between the phases of BECs in the remote spatial subregions, which was well established recently by the experiments with multi-Josephson-junction loops as well as with condensation of ultracold atoms in periodic optical potentials. Considering a strongly-nonequilibrium symmetry-breaking phase transition in the simplest $\varphi^4$ real field model and taking into account the experimentally revealed corrections for the residual entanglement, we show that the resulting distribution of the defects (domain walls) is formally described by the Ising model. As is known, its critical behavior changes dramatically in passing from finite to infinite size of the system and from the low (D = 1) to higher (D > 1) dimensionality. Just this fact might be a clue to the interpretation of the above-mentioned universality in the efficiency of the spontaneous defect formation.

## Introduction: The Concept of Topological Defects and the Mechanism of Their Formation

A topological defect is the spot in space where the "old" (symmetric) vacuum is preserved after the phase transition because of the global (topological) constraints on the "new" vacua around it. The most typical defects are domain walls, strings (vortices), and monopoles.

The domain wall is formed when a discrete symmetry is broken, for example, the initial symmetric high-temperature phase is transformed to one of the two degenerate low-temperature phases.

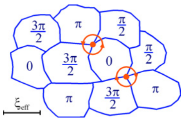



The string is formed when axial symmetry of the degenerate vacuum becomes broken so that circulation of the phase around some spot remains nonzero. The string represents the place where the original symmetric vacuum is preserved to prevent ambiguity in the phase of the field $\varphi$.

A well-known process of the defect formation by a strongly-nonequilibrium symmetry-breaking phase transition is the so-called Kibble–Zurek mechanism.

The phases of the order parameter are assumed to be established independently in the subregions of size $\xi_{\rm eff} = c_{\rm ph}\tau$, where $\tau$ is the characteristic time of the phase transformation, and $c_{\rm ph}$ is the speed of transfer of the information about the order parameter (*e.g.* the speed of the second sound in superfluids). The topological defects are formed during the subsequent relaxation of the order parameter in the vertexes where a few of the initial domains contact each other.




The resulting concentration of the topological defects formed by the Kibble–Zurek mechanism, for example, in a 3D system will be

$$n \approx 1/\,\xi_{\rm eff}^{d}\,,$$

where $d = 3$, 2, and 1 for the monopoles, strings, and domain walls, respectively.

## Summary of Experiments on Topological Defect Formation in the Condensed-Matter Systems

| Experimental object | | Dimensionality | Detection technique | Institution | First publications: Authors | First publications: Journal |
|---|---|---|---|---|---|---|
| Superfluid helium | He-4 | 3D macro (–) | Second sound absorption | Lancaster University, England | P.C.Hendry, N.S.Lawson, R.A.M.Lee, P.V.E.McClintock, C.D.H.Williams | Nature **368**, 315 (1994) |
| | | | | | M.E.Dodd, P.C.Hendry, N.S.Lawson, P.V.E.McClintock, C.D.H.Williams | PRL **81**, 3703 (1998) |
| | He-3 | 3D micro (+) | Calorimetry | CNRS, Grenoble, France | C.Bäuerle, Yu.M.Bunkov, S.N.Fisher, H.Godfrin, G.R.Pickett | Nature **382**, 332 (1996) |
| | | | NMR | Helsinki Univ. of Technology, Finland | V.M.H.Ruutu, V.B.Eltsov, A.J.Gill, T.W.B.Kibble, M.Krusius, Yu.G.Makhlin, B.Plaçais, G.E.Volovik, Wen Xu | Nature **382**, 334 (1996) |
| Superconductors | Thin films | 2D macro (–) | SQUID | IBM Research Center, New York, USA | J.R.Kirtley, C.C.Tsuei, F.Tafuri | PRL **90**, 257001 (2003) |
| | | | | Technion – Israel Inst. of Technology, Haifa | R.Carmi, E.Polturak | PR **B60**, 7595 (1999) |
| | | | | | A.Maniv, E.Polturak, G.Koren | PRL **91**, 197001 (2003) |
| | Multi-Josephson junctions | 1D macro (+) | | | R.Carmi, E.Polturak, G.Koren | PRL **84**, 4966 (2000) |
| | Annular Josephson tunnel junctions | | Voltage measurement | Istituto di Cibernetica del CNR, Italy | R.Monaco, J.Mygind, R.J.Rivers | PRL **89**, 080603 (2002) |

As is seen in the table, the topological defects are formed most efficiently (in agreement with Kibble–Zurek estimates) in the quasi-one-dimensional superconductor structures and microscopic hot bubbles of $^3$He produced by neutron irradiation. On the other hand, the topological defect concentration was found to be substantially suppressed in the macroscopic two- and three-dimensional samples of superfluids and superconductors.

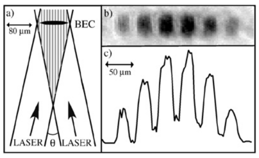
As will be shown in the subsequent sections of our report, the above-mentioned behavior can be reasonably explained by the residual entanglement in the spatially separated parts of BECs after the phase transitions.

## Entanglement of BECs Formed in the Disconnected Spatial Regions

By now, there are two groups of experiments pointing to the entanglement of BECs formed in the disconnected spatial subregions. First evidence came from the interference between a few condensates of ultracold atoms formed in the periodic optical potentials. Unfortunately, the interpretation of such interference is somewhat ambiguous and does not enable one to reveal the degree of entanglement / disentanglement as function of the parameters of the system. On the other hand, this can be done by studying the fast phase transitions in the superconducting multi-Josephson-junction loops (MJJL). Just these measurements give information about dependence of the residual entanglement on the crucial parameters of the system.

### *Bose–Einstein Condensation of Ultracold Atoms in the Periodic Optical Potentials*

The experiments on the formation of condensates of ultracold atoms in periodic optical potentials were started in 1990s and now the number of the simultaneously formed condensates can be as large as 30 [Z. Hadzibabic, *et al.*, Phys. Rev. Lett., **93**, 180403 (2004)].

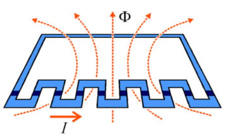



When the potential is switched off and the gas clouds are allowed to expand freely, a high-contract interference is observed. This takes place even if the initial potential walls were high enough to prevent any tunneling between the condensates, and therefore they evolved for a long time independently of each other. Moreover, the interference still exists when the lattice was ramped up on a thermal cloud with a temperature much higher than the condensation temperature, so that the subsequently produced condensates "have never seen one another".

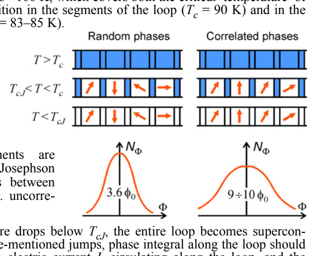
Just the last fact may serve as indication to the residual entanglement of BECs formed in the disconnected spatial subregions. Nevertheless, it needs to be proved more carefully by studying stability of the interference pattern over a lot of realizations.

### *Multi-Josephson-Junction-Loop Experiment*

As was already mentioned, the most clear evidence for the residual entanglement follows from the MJJL experiment [R. Carmi, *et al.*, Phys. Rev. Lett., **84**, 4966 (2000)], which is briefly outlined here. A thin quasi-one-dimensional winding strip was engraved at the boundary between two crystalline grains of a high-temperature superconductor film, thereby forming a loop of 214 superconductor segments separated by the grain-boundary Josephson junctions. The system experienced multiple heating-cooling cycles in the temperature range 77 K to ~100 K, which covers both the critical temperature of superconducting phase transition in the segments of the loop ($T_c$ = 90 K) and in the junctions between them ($T_{cJ}$ = 83–85 K).




There is evidently no order parameter in the entire loop above $T_c$. Next, when the temperature drops below $T_c$ but remains above $T_{cJ}$, some value of the order parameter should be established in each segment. Since these segments are separated by nonconducting Josephson junctions, the phase jumps between them should be random (*i.e.* uncorrelated to each other).




Finally, when the temperature drops below $T_{cJ}$, the entire loop becomes superconducting and, due to the above-mentioned jumps, phase integral along the loop should be nonzero. As a result, the electric current $I$, circulating along the loop, and the corresponding magnetic flux $\Phi$, penetrating the loop, will be spontaneously generated.

Under assumption of uncorrelated phases, the distribution of the magnetic flux in the particular experimental setup would be given by the normal (Gaussian) law with characteristic width ~3.6 $\phi_0$ (where $\phi_0$ is the magnetic flux quantum). But the actual experimental distribution was found to be over two times wider; and this anomaly was well interpreted by the authors of the experiment assuming that the phase jumps in the intermediate state were not random but correlated to each other so that the probability $P(\delta_i)$ of the phase difference $\delta_i$ in the $i$'th junction was

$$P(\delta_i) \sim \exp[-E_J(\delta_i)\,/\,T_c]\,,$$

where $E_J$ is the energy concentrated in the junction, and $T_c$ is the phase transition temperature. Just the last formula can be used to estimate the effects of the BEC's entanglement in a variety of other systems.

## Theoretical Model of the Defect Formation

To estimate the effect of residual entanglement on the systems of various size and dimensionality, let us consider the simplest $\varphi^4$-model of the real scalar field (order parameter), whose Lagrangian

$$L(\mathbf{r},t) = \frac{1}{2}\left[(\partial_t\varphi)^2 - (\nabla\varphi)^2\right] - \frac{\lambda}{4}\left[\varphi^2 - \left(\mu^2/\lambda\right)\right]^2$$

admits the discrete $\mathbf{Z}_2$ symmetry breaking. Two stable vacuum states of this field (which will be marked by the oppositely directed arrows) are

$$\varphi_{\uparrow\downarrow} = \pm\varphi_0 = \pm\mu/\sqrt{\lambda}\,;$$

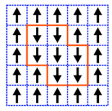
the structure of a domain wall between them, located at $x_0$, is described as

$$\varphi(x) = \pm\varphi_0 \tanh\left[\frac{\mu}{\sqrt{2}}\,(x - x_0)\right];$$

and the specific energy concentrated in this wall equals

$$E = \frac{2\sqrt{2}}{3}\,\frac{\mu^3}{\lambda}\,.$$

Let a domain structure formed after a strongly-nonequilibrium phase transition be approximated by a regular rectangular grid with a cell size about the effective correlation length $\xi_{\rm eff}$. As can be shown by integration of the energy corresponding to the above Lagrangian over the symmetry-broken field configurations, the elementary domains are distributed exactly as spins in the Ising model; the energy of the wall playing the role of the spin–spin interaction constant.



If the final symmetry-broken states of the field $\varphi$ in two neighboring cells are independent of each other, then the probability of the domain wall formation between them is given by the ratio of the number of statistical configurations involving the wall to the total number of configurations: $P_{\rm uncor}$ = 2/4 = 1/2; and the resulting concentration of the defects (domain walls) in the entire system will be

$$n_{\rm uncor} = \frac{1}{2}\,\frac{D}{\xi_{\rm eff}^{D}}\,,$$

where $D$ is the dimensionality of the system.

In fact, this estimate is not sufficiently accurate because the order parameter formed by the phase transition represents an entangled state of BEC, in which the specific quantum correlations may occur even at the distances exceeding the effective correlation length $\xi_{\rm eff}$ derived from the "classical" arguments.

As follows from the results of MJJL experiment, refined distribution of the field configurations should be calculated taking into account the energy concentrated in the defects. As a result, the pattern of domains formed by the strongly-nonequilibrium phase transition in the $\varphi^4$ lattice model will look like a distribution of spins in the Ising model at a nonzero temperature $T$, formally equivalent to the critical temperature $T_c$ of the initial $\varphi^4$-model. Then, probability of the domain wall formation should be calculated as

$$P = \frac{T_c^2}{E\,\mathrm{D}N^{\mathrm{D}}}\,\frac{\partial}{\partial T_c}\ln Z^{(\mathrm{D})},$$

where $E$ is the energy of the elementary domain wall, $N$ is the number of cells along each side of the lattice, and

$$Z^{(\mathrm{D})} = \sum_i \exp(-\varepsilon_i\,/\,T_c)$$

is the statistical sum over all possible spin configurations of the Ising model, where $\varepsilon_i$ is the total energy of $i$'th configuration.

Particularly, for the one-dimensional system we have

$$Z^{(1)} = \sum_{i=1}^{N}\sum_{s_i=\pm1} \exp\left\{-\frac{E}{T_c}\sum_{k=1}^{N}\frac{1}{2}\,(1 - s_k s_{k+1})\right\};$$

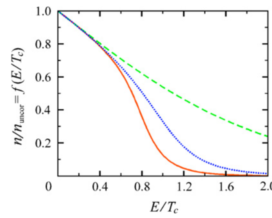
for two-dimensional system,

$$Z^{(2)} = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s_{ij}=\pm1} \exp\left\{-\frac{E}{T_c}\sum_{k=1}^{N}\sum_{l=1}^{N}\frac{1}{2}\,(2 - s_{kl}s_{k+1,l} - s_{kl}s_{k,l+1})\right\};$$

and so on. Here, $s_k$ and $s_{kl}$ are the spin-like variables describing the broken-symmetry states in the $k$'th and $(kl)$'th cells, respectively.

As is known, the Ising model for one-dimensional as well as the finite-size higher-dimensional systems does not experience a phase transition to the ordered state at any value of the ratio $E/T$. This means that concentration of the defects will not differ considerably from the standard Kibble–Zurek estimate, because the probability of defect formation $P$ at the scale of the effective correlation length $\xi_{\rm eff}$ will not deviate substantially from $P_{\rm uncor}$ = 1/2 ~ 1. On the other hand, the Ising model for the sufficiently large (infinite-size) two- and three-dimensional systems does experience a phase transition to the ordered state at some value of $E/T \sim 1$. As a result, the concentration of domain walls in the corresponding field model should be suppressed dramatically, due to formation of macroscopic regions with the same value of order parameter, covering a large number of cells of the effective correlation length $\xi_{\rm eff}$.

These general conclusions can be illustrated by the particular example in figure, which represents the concentration of domain walls $n$ normalized to its uncorrelated value $n_{\rm uncor}$ as function of $E/T_c$ for the infinite 1D system (*green dashed curve*), small-size 2D system (*blue dotted curve*), and infinite 2D system (*solid red curve*).




As is seen, at $E/T_c$~1 the concentration of defects in one-dimensional system differs from the uncorrelated case by less than two times; in microscopic two-dimensional system, by three times; while in the macroscopic two-dimensional system it is suppressed by an order of magnitude. Furthermore, the suppression becomes much stronger when the ratio $E/T_c$ increases: for example, at $E/T_c$~2 the difference between each of the curves is over an order of magnitude.

## Discussion and Conclusions

As follows from the above consideration, residual entanglement between the phases of BECs in separated spatial regions, revealed most clearly in the MJJL experiment, can show a promising way for explanation of the entire diversity of experimental data on the efficiency of topological defect formation.

Unfortunately, the simplified model with the real field $\varphi$ cannot be confronted quantitatively with the experimental data cited in the table, since superfluids and superconductors possess more complex order parameters and Lagrangians than the one written above. Consideration of the strongly-nonequilibrium symmetry-breaking phase transition in the two-dimensional system with a complex scalar field, possessing $U(1)$ symmetry, can be approximately reduced to treatment of the so-called $xy$ lattice model (with spins in the plane of the lattice). A presence of the well-known Kosterlitz–Thouless phase transition in this model implies that the above-stated general conclusions remain valid.