FACHBEREICH PHYSIK

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Superscars in Billiards —
A Model for the Doorway Mechanism

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Chaos and Collectivity in Many–Body Systems,
MPIPKS, Dresden, 2008

Collaborators

experiment:

this work: M. Miski–Oglu, A. Richter (Darmstadt)

previous results: J. Enders, A. Heine, N. Huxel,
P. von Neumann–Cosel (Darmstadt),
C. Rangacharyulu (Saskatoon)

theory:

this work: S. Åberg (Lund), H. Kohler (Duisburg–Essen)

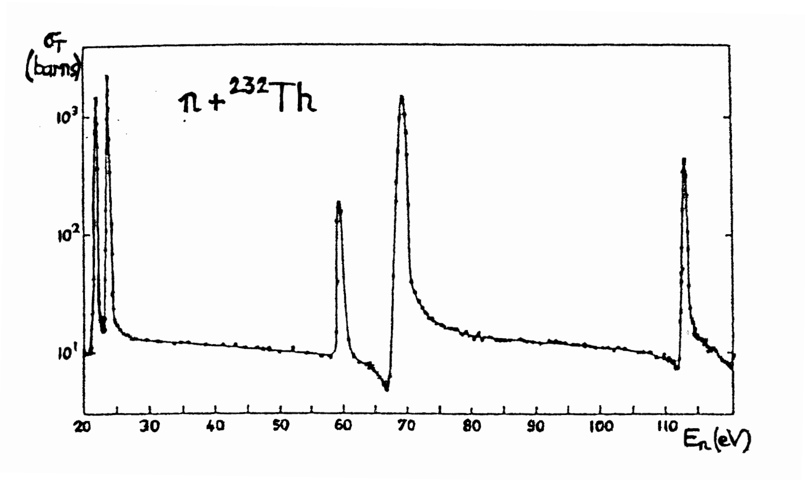
previous results: V.Y. Ponomarev, J. Wambach (Darmstadt)

Outline

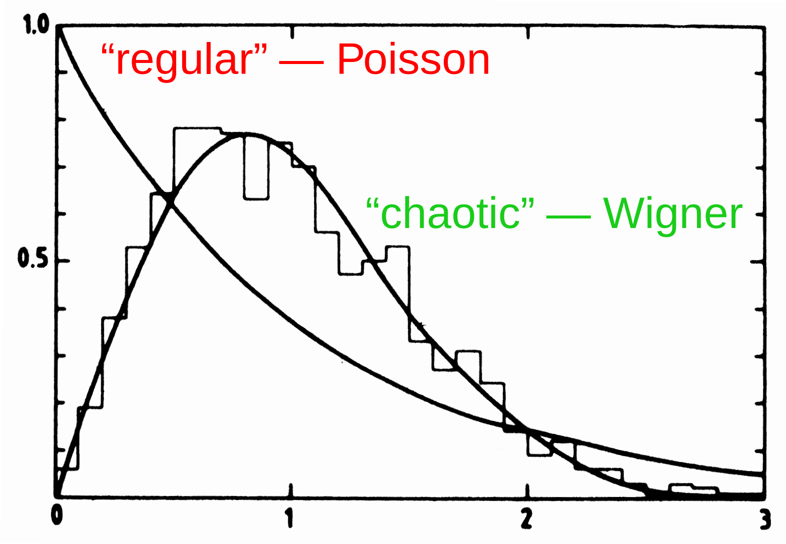
- spectral statistics and the doorway mechanism
- three examples in nuclear physics:
  giant dipole, scissors, pygmy dipole mode
- discuss surprises, limits and implications
- barrier billiard and superscars
- first hints at doorway mechanism
- quantitative study with new observables
  - distribution of maximum coupling coefficient
  - spatial correlator in extended Berry model
- summary and conclusions

Quantum Chaos in Nuclei

nuclear data ensemble (far from groundstate)



resonances



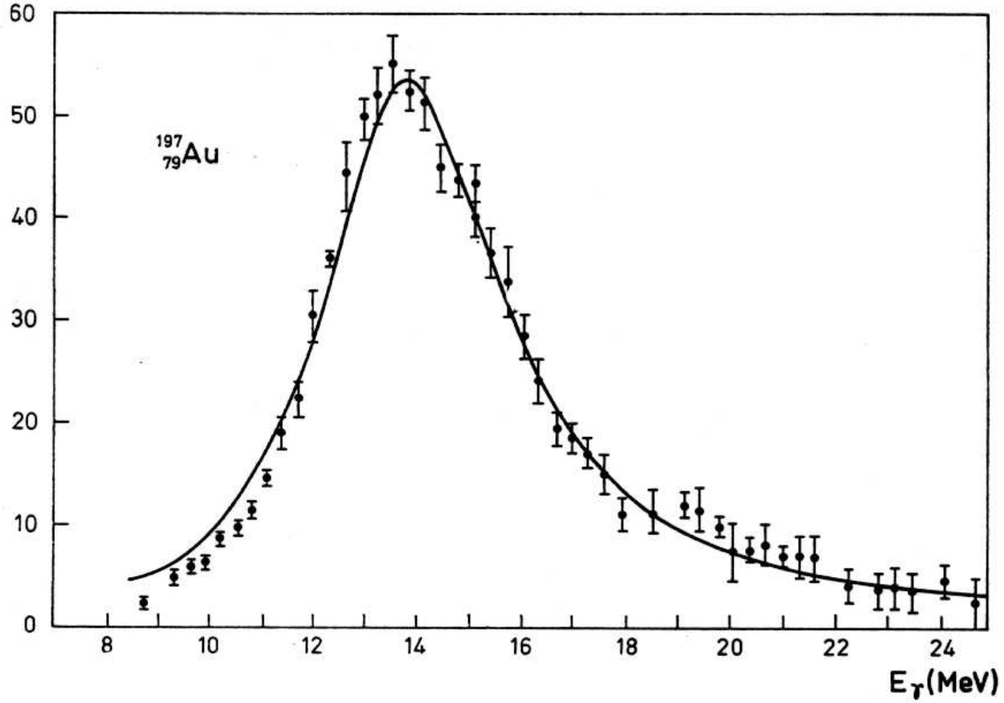
spacing distribution

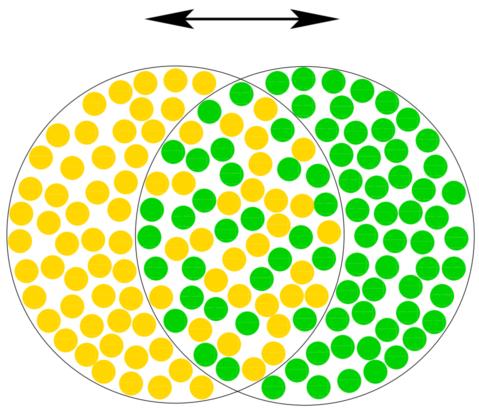
⟶ quantum chaos ⟶ random matrix theory

Bohigas, Haq, Pandey (1983)

Electric Giant Dipole Resonance

seen in many nuclei, here for Gold ($A = 197$)





high energies

Fultz, Bramblett, Caldwell, Kerr, PR 127 (1962) 1273

Strength Function

cross section contains huge number of individual states (fragmentation) which cannot be resolved

strength function is related to this: consider state $|\mathrm{GDR}\rangle$ with resonance energy E_{GDR} and couple many states to it
$\longrightarrow$ density around $|\mathrm{GDR}\rangle$

$$\varrho_{\mathrm{GDR}}(E) = \frac{1}{\pi} \frac{\Gamma/2}{(E - E_{\mathrm{GDR}})^2 + \Gamma^2/4}$$

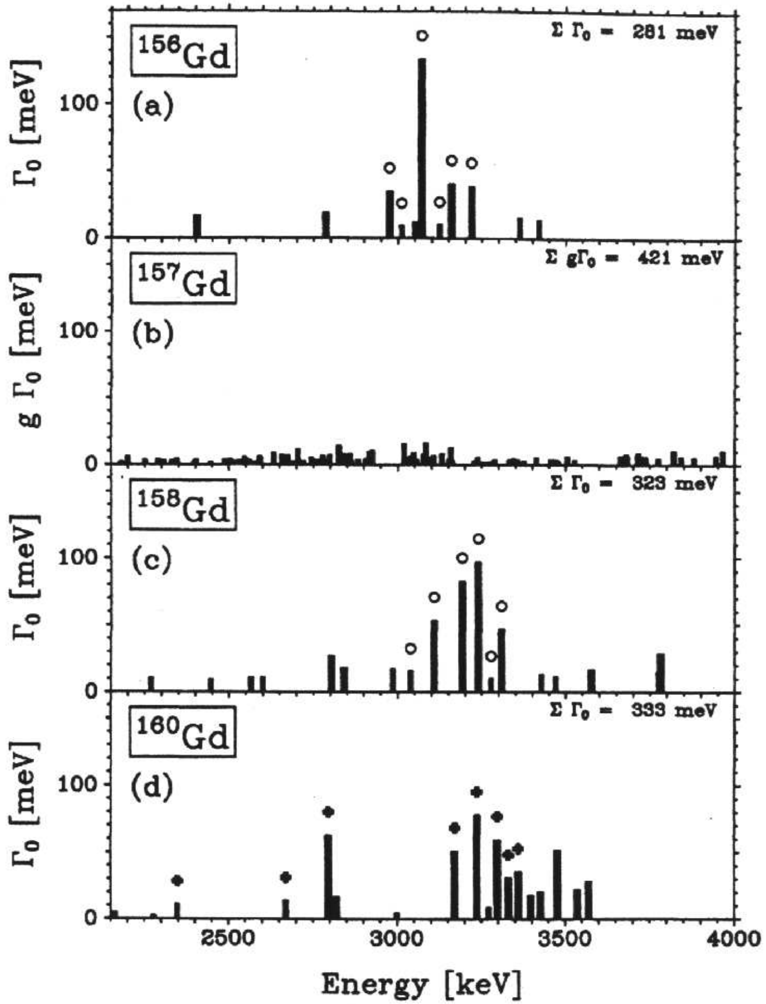
(Breit–Wigner)

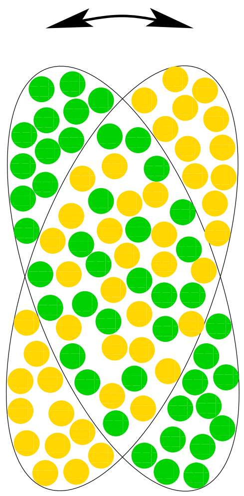
under rather general conditions!

$\longrightarrow$ strictly, one cannot conclude chaotic fluctuations, but at these GDR energies one certainly expects them

Bohr, Mottelson, Nuclear Structure (1969); Zelevinsky, Annu Rev Nucl Part Sci 46 (1996) 237

Magnetic Scissors Mode in Rare Earth Nuclei





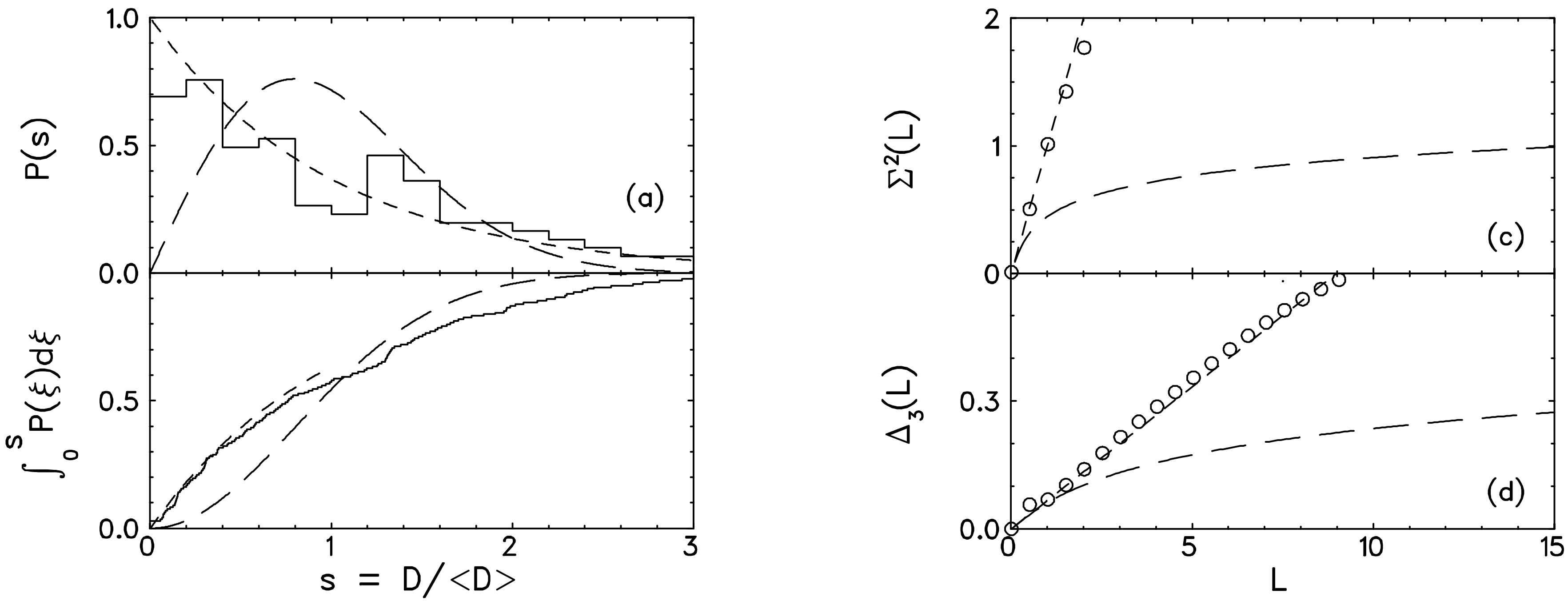
low energies

fragmentation resolved
in experiment

Margraf et al, PRC 52 (1995) 2429

Spectral Statistics of Scissors Mode Resonances

152 states in 13 heavy deformed nuclei (at least 8 per nucleus)

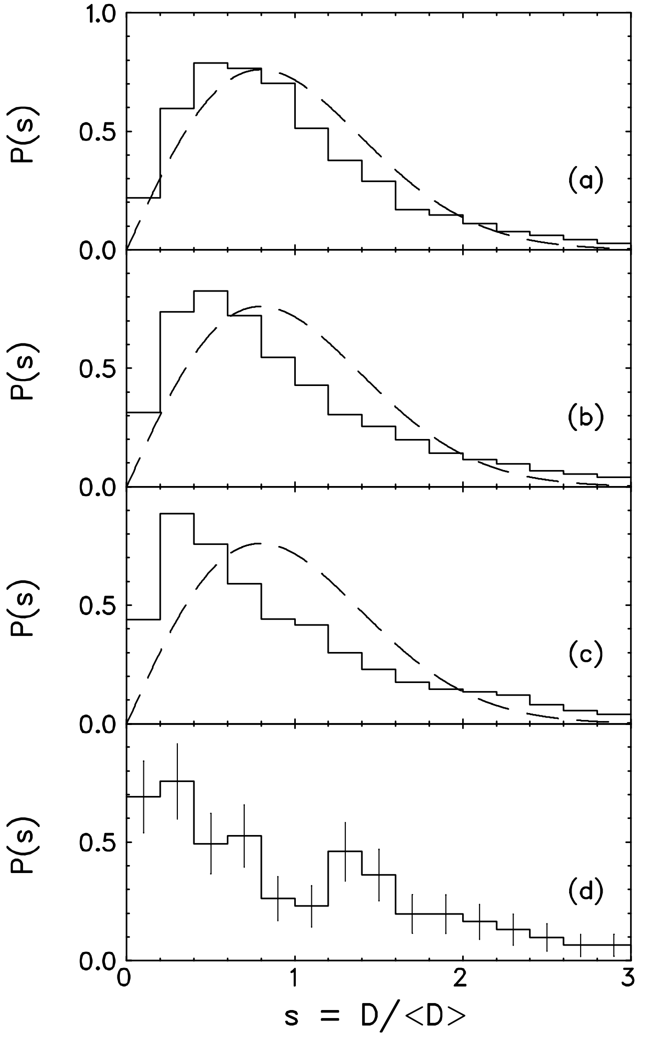


$\longrightarrow$ Poissonian (regular) behavior!

Enders, Guhr, Huxel, von Neumann–Cosel, Rangacharyulu, Richter, PLB 486 (2000) 273

Influence of Detection Threshold

numerical simulation: assume chaos (Wigner for levels and Porter–Thomas for intensities) $\rightarrow$ worst case scenario



71% observed, $I > 0.2\langle I \rangle$

50% observed, $I > 0.5\langle I \rangle$

33% observed, $I > 1.0\langle I \rangle$

experimental data
$>$50% observed (sum rule)

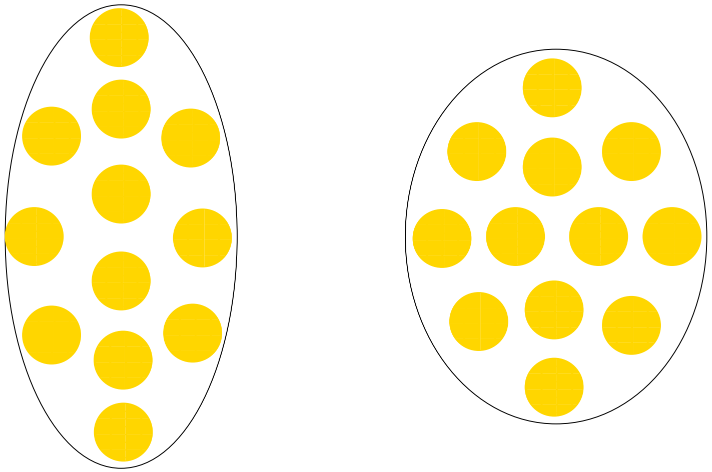
How to interpret this?

- no state is distinguished as a doorway
- level density low, no states available to mix in
- all states are collective in their own right
- fragmentation does not imply correlations
- these are conclusions based on an experiment

is there a semiclassical interpretation?

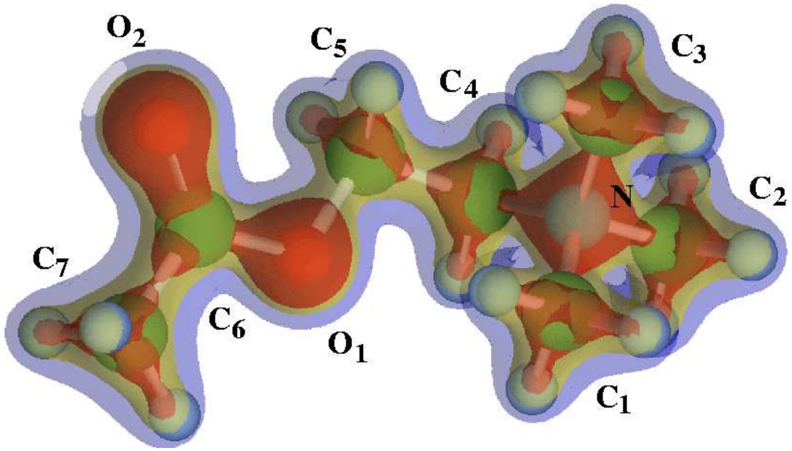
... here comes an attempt

Low Energies and Rearrangement of Particles



12 nucleons

different inertia

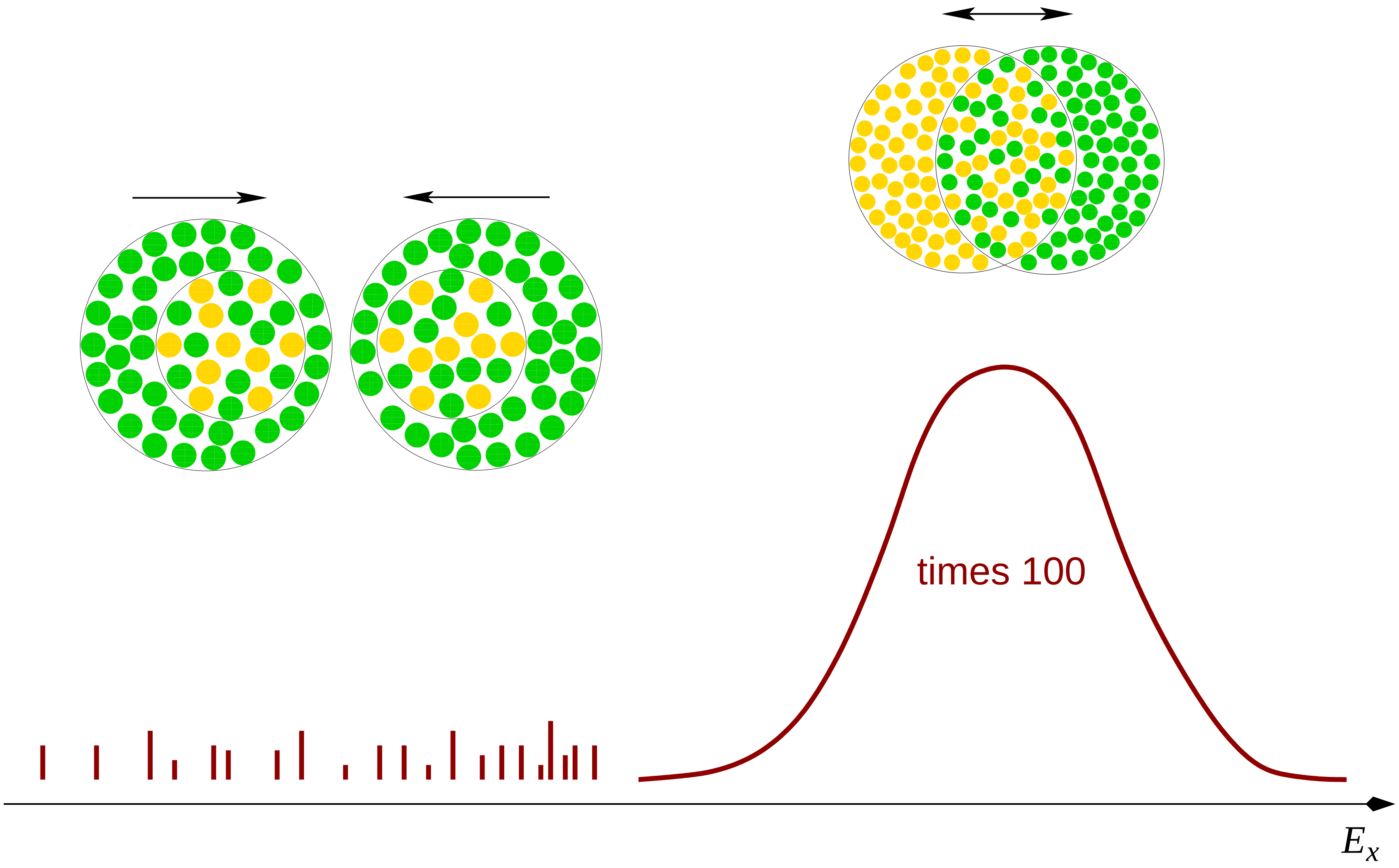


large molecule

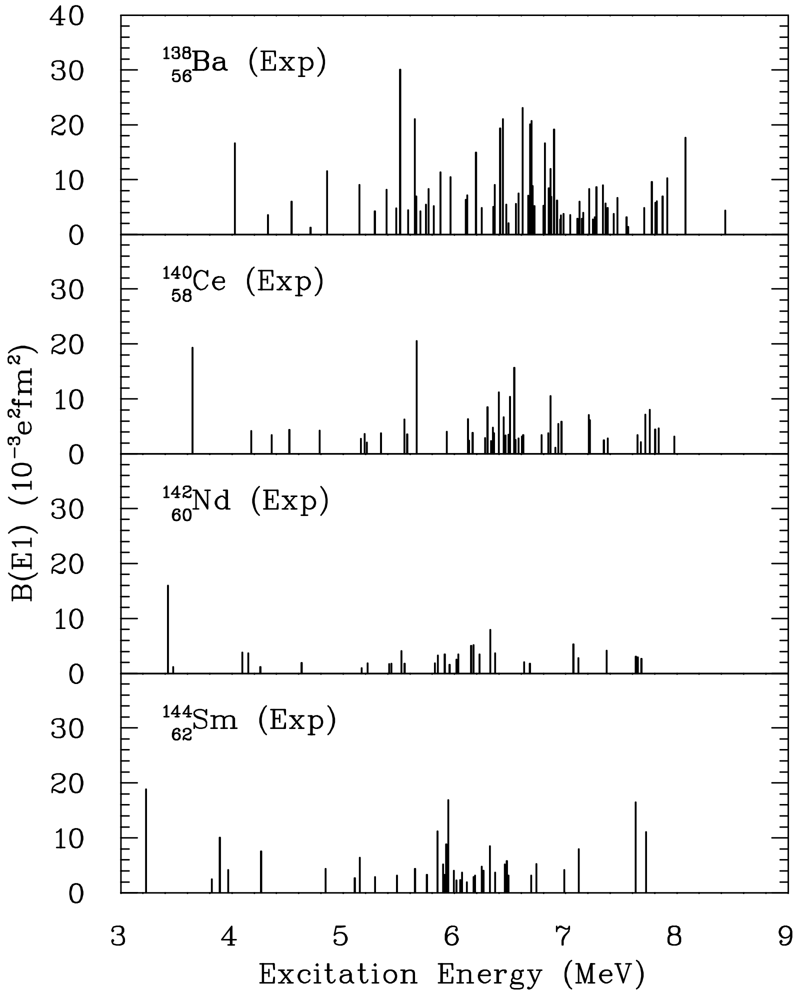
slightly different arrangement can yield slightly different energies

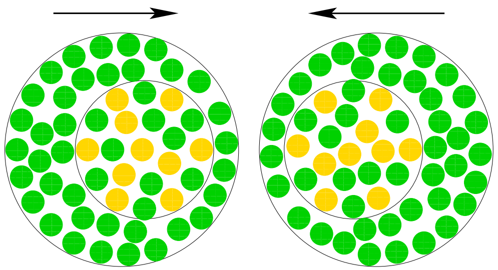
more general feature in such nuclei ?

Electric Pygmy Dipole Resonances



Pygmy Resonances in $N = 82$ Nuclei





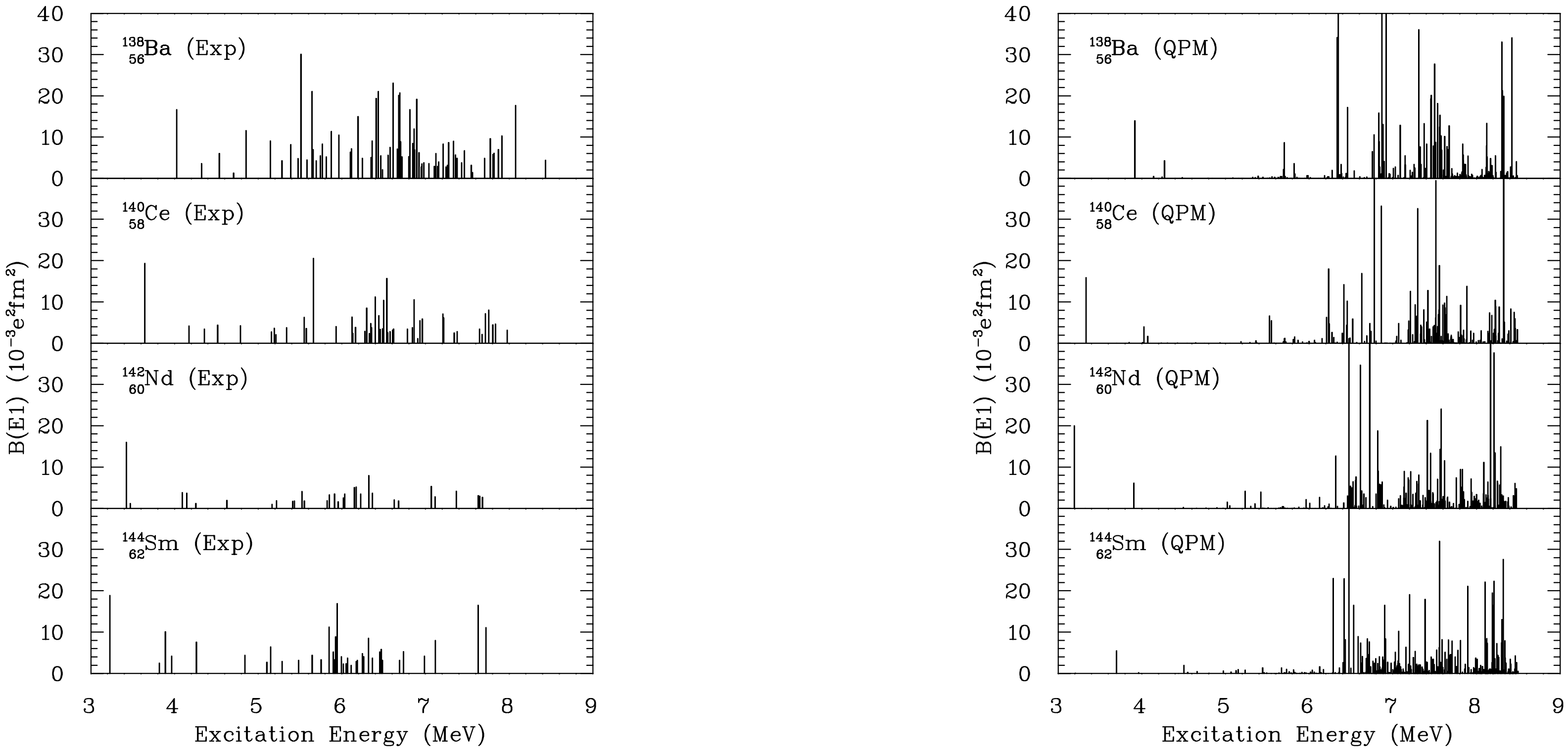
intermediate energies

fragmentation resolved
in experiment

Zilges et al, PLB 542 (2002) 43

Comparison Experiment ↔ Theory

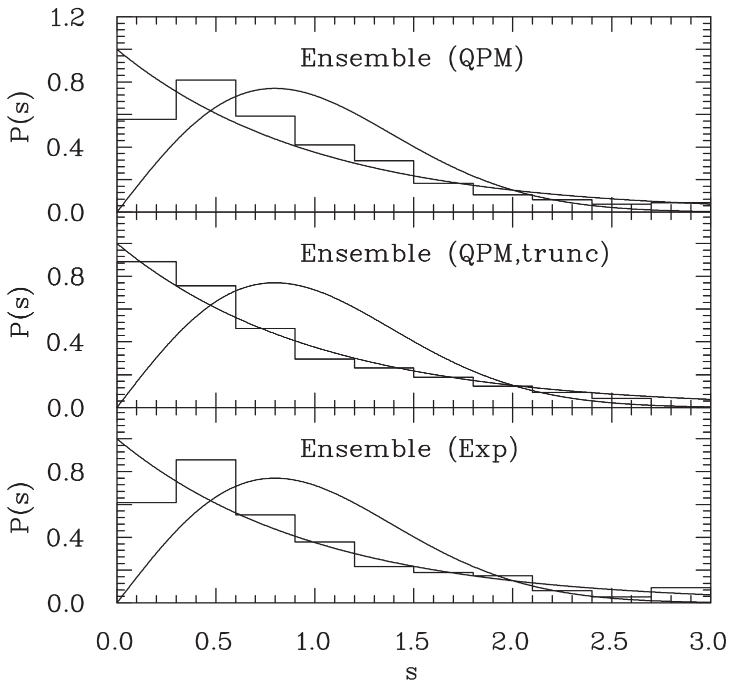
microscopic calculation within quasiparticle phonon model (QPM)

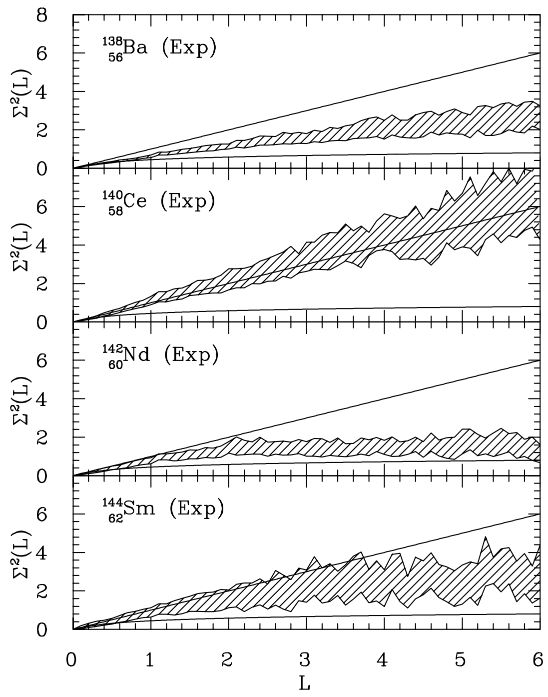


grouping of levels around several doorway states seems possible in experimental data

such a hierachy is a built–in feature of QPM

Spacing Distribution and Level Number Variance





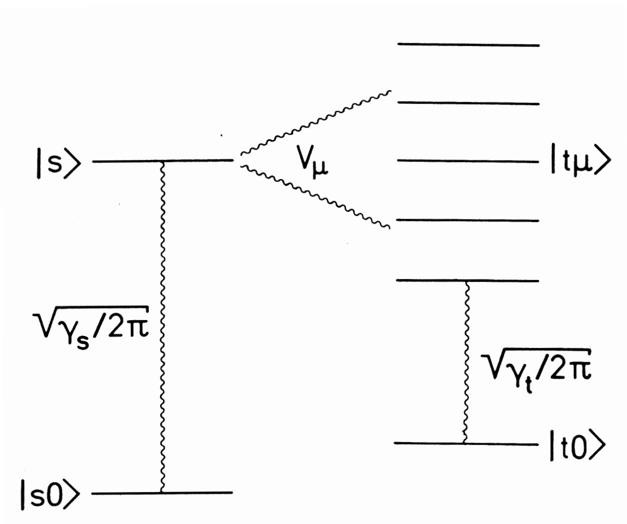
strong missing level effect — expect largely chaotic statistics

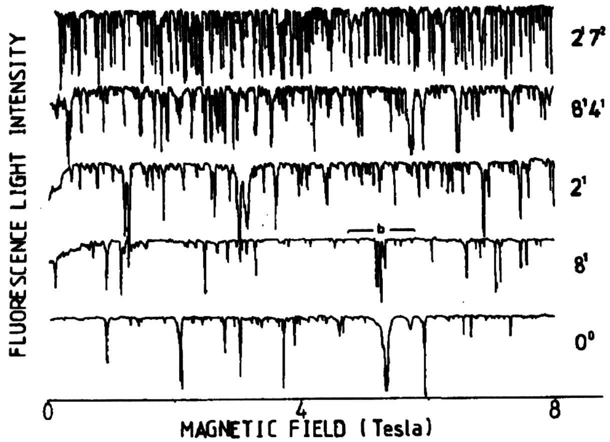
we are at the limits of such an analysis for nuclear data

Enders, Guhr, Heine, von Neumann–Cosel, Ponomarev, Richter, Wambach, NPA 741 (2004) 3

Anticrossing Spectroscopy in Molecular Physics

doorway mechanism: singlet state $|s\rangle$ couples with matrix elements V_μ to background of triplet states $|t\mu\rangle$



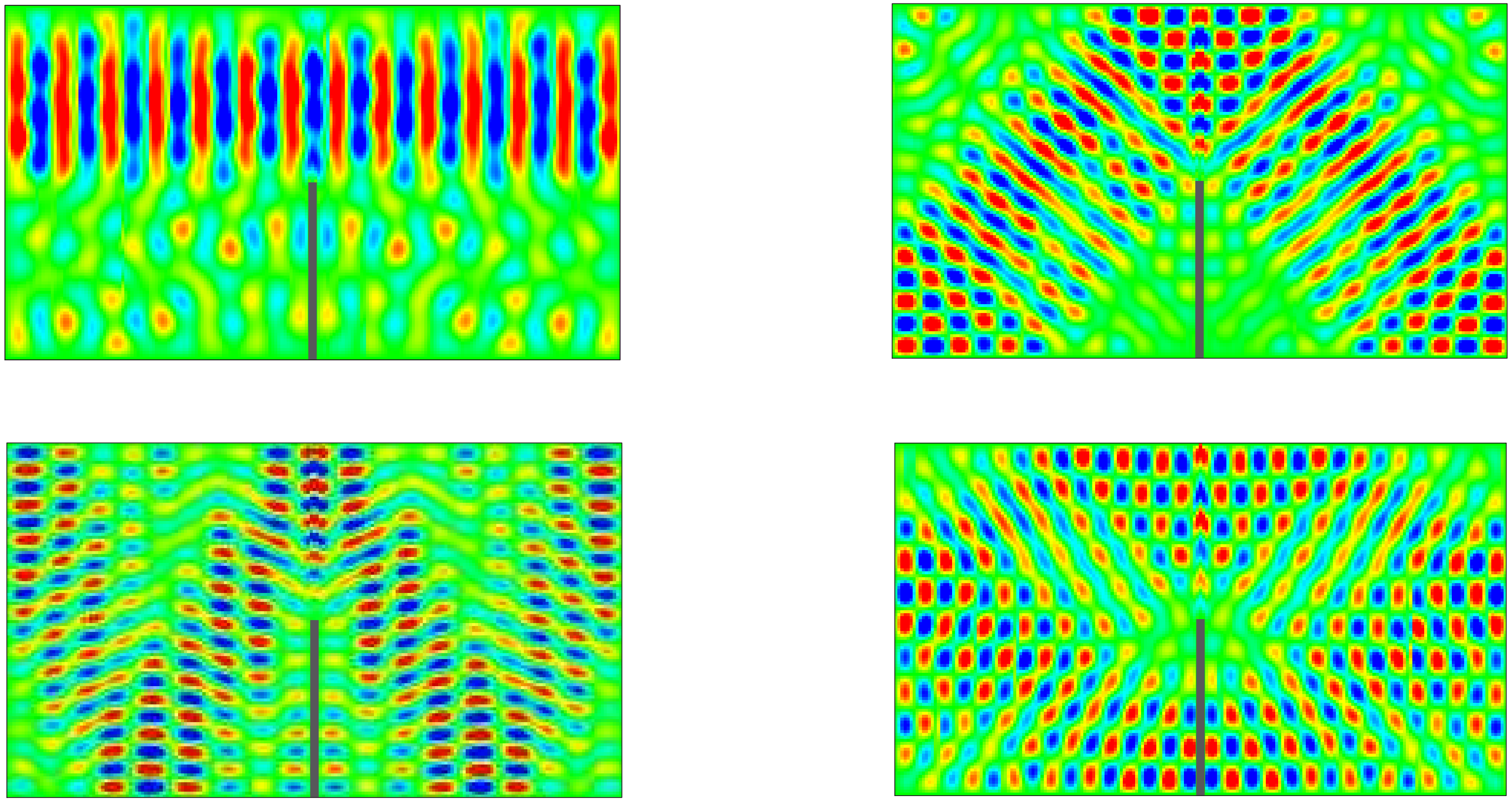


shift triplets with magnetic field $\rightarrow$ resonance fluorescence yield

root mean square coupling V_{st} enters statistical observables

Pebay Peyroula, Jost, Lombardi, Dupré, CP 102 (1986) 417

Superscars in a Pseudointegrable Barrier Billiard



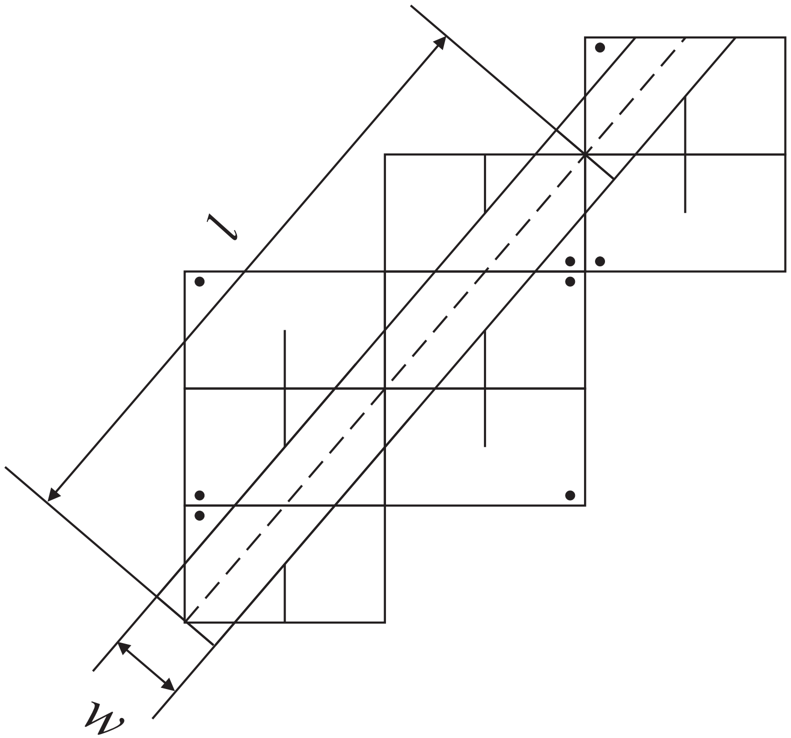
ordinary scars “vanish” at high energies, superscars do not !

Bogomolny, Schmit, PRL 92 (2004) 244102

Bogomolny, Dietz, Friedrich, Miski–Oglu, Richter, Schäfer, Schmit, PRL 97 (2006) 254102

Constructed Superscars

integrable approximation in Periodic Orbit Channels



inside POC:

$$\Psi^{(\mathsf{F})}_{m,n}(\vec{r}) \sim \sin\left(\frac{\pi m \xi}{l} + \delta\right) \sin\left(\frac{\pi n \eta}{w}\right)$$

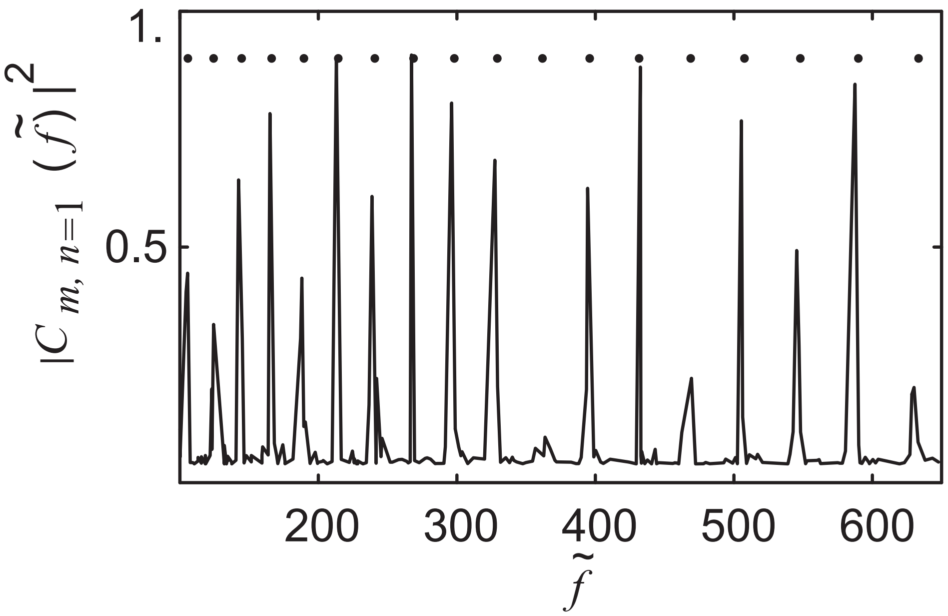
outside POC:

$$\Psi^{(\mathsf{F})}_{m,n}(\vec{r}) = 0$$

families $\mathsf{F} \in \{\mathsf{BB}, \mathsf{V}, \mathsf{D}, \mathsf{W}\}$

Overlap with Measured States

rescaled frequency $\tilde{f}$ according to Weyl's law



$$c_{m,n}(\tilde{f}_\lambda) = \langle \Psi^{(\mathsf{F})}_{m,n} | \Psi_{\tilde{f}_\lambda} \rangle$$

quantum number perpendicular to POC chosen as $n = 1$

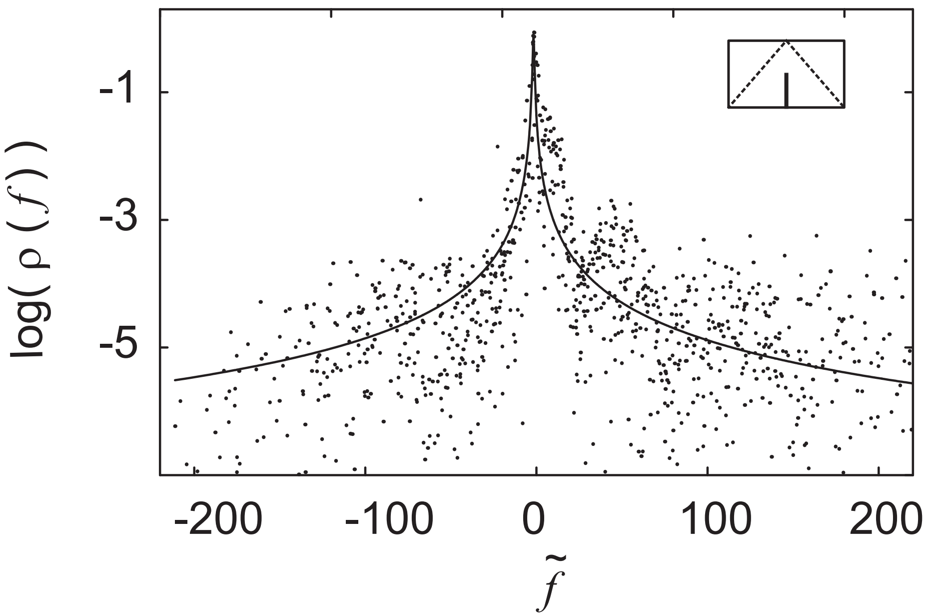
points indicate predicted superscar positions

Bogomolny, Dietz, Friedrich, Miski–Oglu, Richter, Schäfer, Schmit, PRL 97 (2006) 254102

Local Density of States

sum over measured states λ and over m quantum number

$$\rho_n(\tilde{f}) = \left\langle \sum_{\lambda} |c_{m,n}(\tilde{f})|^2 \delta\left(\tilde{f} - \tilde{f}_\lambda + \tilde{f}_{m,n}\right) \right\rangle_m$$



Breit–Wigner shape

doorway mechanism

Modeling the Doorway Mechanism

total Hamilonian $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{V}$

one doorway $\mathcal{H}_S|s\rangle = E_s|s\rangle$, background $\mathcal{H}_B|b\rangle = E_b|b\rangle$

orthogonality $\langle s|b\rangle = 0$

coupling $\langle s|\mathcal{V}|s\rangle = 0 = \langle b|\mathcal{V}|b'\rangle$ and $\langle s|\mathcal{V}|b\rangle = V_{bs}$

solve $\mathcal{H}|\nu\rangle = \mathcal{E}_\nu|\nu\rangle$ get $\mathcal{E}_\nu = E_s - \sum_b \frac{V_{bs}^2}{E_b - \mathcal{E}_\nu}$

coupling coefficients $c_{s\nu} = \langle \nu|s\rangle = \left(1 + \sum_b \frac{V_{bs}^2}{(E_b - \mathcal{E}_\nu)^2}\right)^{-1/2}$

Matrix Representation for Statistical Model

N background states, matrix H_B, vector V

$$H = \begin{bmatrix} E_s & V^T \\ V & H_B \end{bmatrix}$$

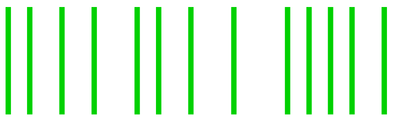
total Hamiltonian has $N+1$ states, eventually $N \to \infty$

average over random Hamiltonian H_B

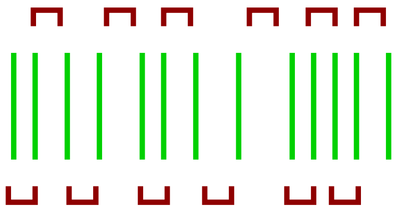
$\longrightarrow$ observables only depend on $\langle V^2 \rangle = \frac{1}{N} V^T V$

Local Density is Breit–Wigner with width $\Gamma^{\downarrow} = 2\pi \frac{\langle V^2 \rangle}{D}$

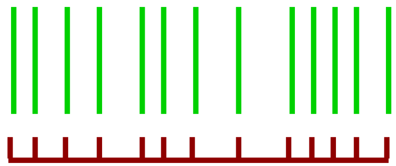
Statistical Properties of Background States



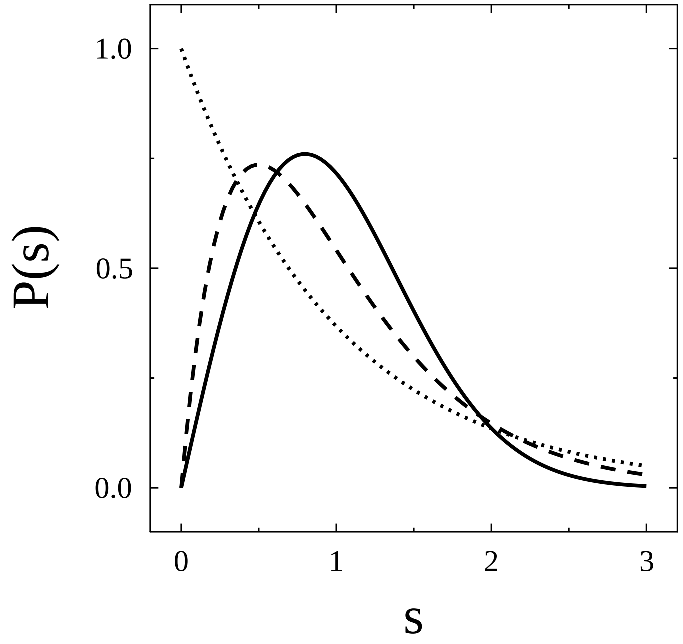
Poisson



semi − Poisson



Wigner − Dyson

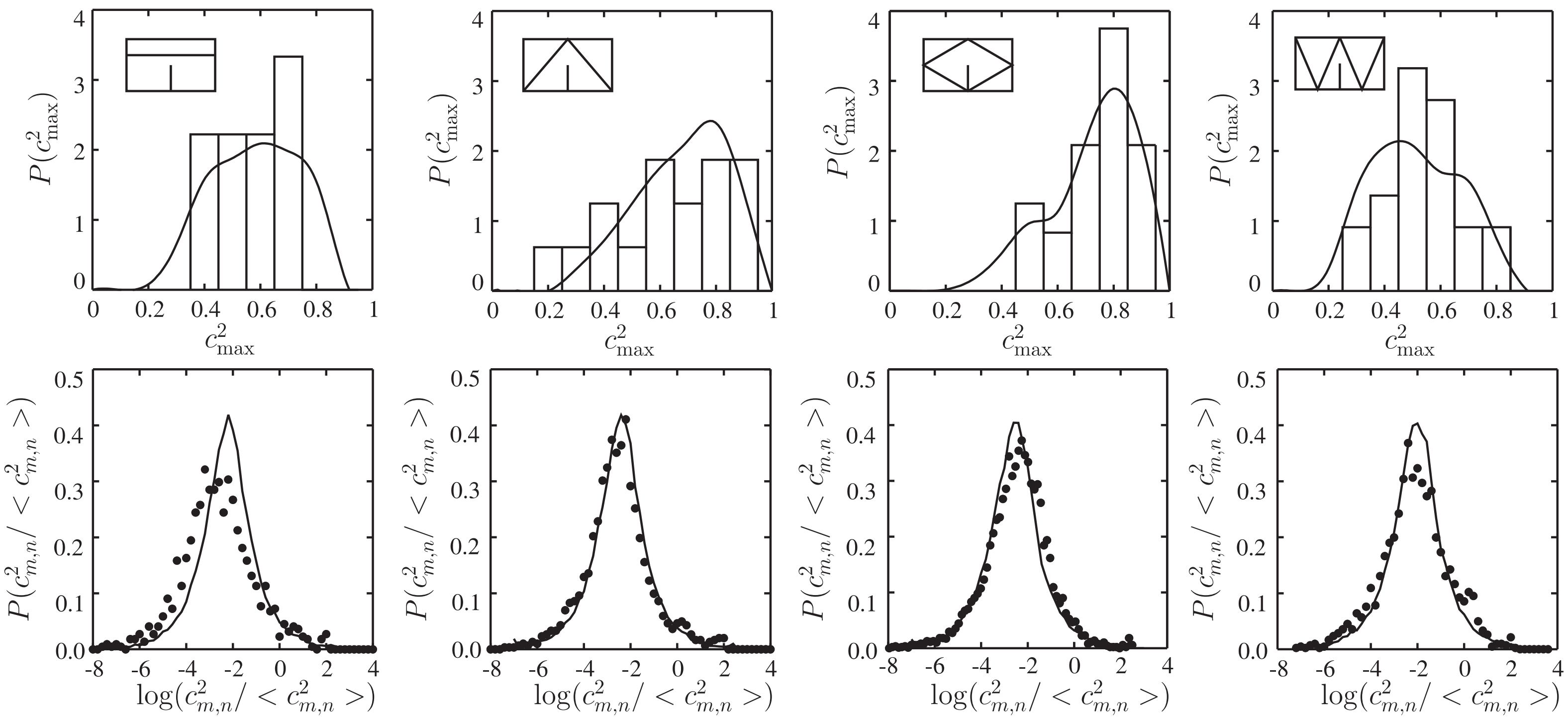


barrier billiard is a Veech billiard $\longrightarrow$ semi–Poisson

Coefficient Statistics versus Numerical Simulation

distribution of squared maximum coefficient $c^2_{\max} = \max\left(c^2_{m,n}\right)$

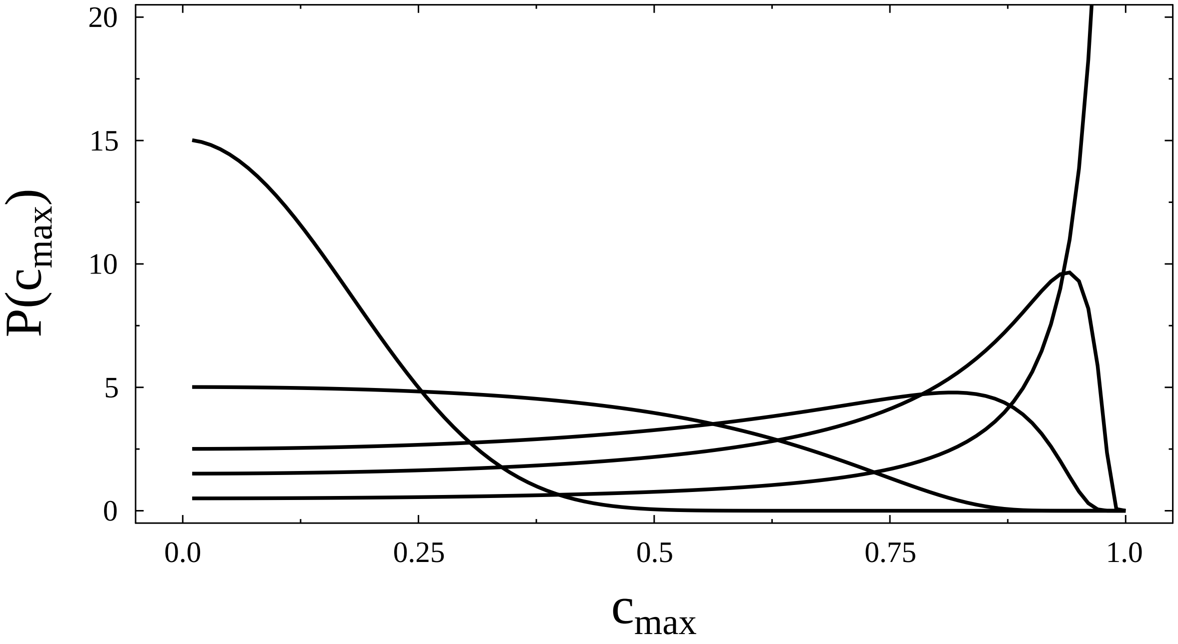
distribution of all coefficients $c^2_{m,n}$



extract the only parameter $\sqrt{\langle V^2 \rangle}/D$

Analytical Treatment — Poisson Case

distribution $P(c_{\max})$ of maximum coupling coefficient
in case of background states with Poisson statistics



gross features as in numerical simulation, details quite different

Analytical Treatment — Wigner–Dyson Case

distribution $P(c_{\mathrm{max}})$ of maximum coupling coefficient
in case of Wigner–Dyson statistics (unitary)

integral over two ordinary 2×2 matrices for finite N

$$\int d[\sigma] \exp\left(-\mathrm{tr}\,\sigma^2\right) \int d[\tau] \exp\left(-\mathrm{tr}\,\tau^2\right)$$

$$\det{}^{N-1}\left(\sigma + g\left(\langle V^2\rangle\right)\tau\right)\,\det\tau$$

$\longrightarrow$ two–matrix model $\longrightarrow$ surprising generality

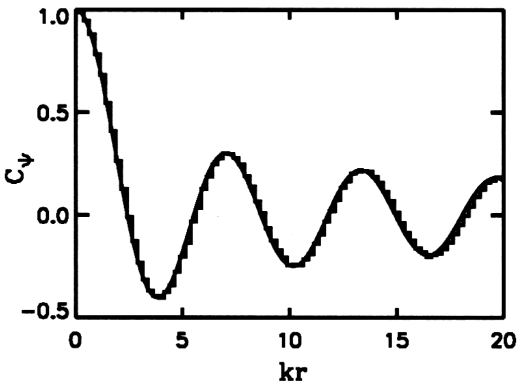
... work in progress ...

Berry's Random Wave Model

spatial wavefunction correlator: isotropic average, fixed $k = |\vec{k}|$

$$C(kr) = \frac{\langle \psi_k(\vec{R} - \vec{r}/2)\psi_k^*(\vec{R} + \vec{r}/2)\rangle}{\langle |\psi_k(\vec{R})|^2 \rangle}$$

in ergodic regime, wave function equivalent to superposition of random plane waves $\rightarrow$ in two dimensions $C(kr) = J_0(kr)$



microwave experiment

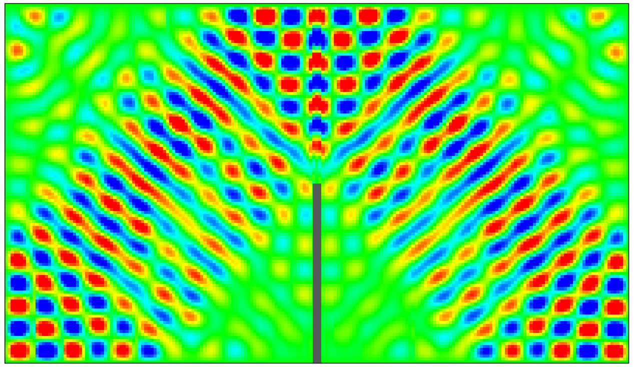
Kim, Barth, Kuhl, Stöckmann, PTPS 150 (2003) 105

Introducing Directed Spatial Correlators

correlator parallel to Periodic Orbit Channel, fixed $k = |\vec{k}|$

$$C^{||}(kr) = \frac{\langle \psi_k(\vec{R} - \vec{r}/2)\psi_k^*(\vec{R} + \vec{r}/2)\rangle}{\langle |\psi_k(\vec{R})|^2 \rangle} \quad \text{with} \quad \vec{r} \,||\, \mathrm{POC}$$

similarly, correlator $C^{\perp}(kr)$ with $\vec{r} \perp \mathrm{POC}$ perpendicular to Periodic Orbit Channel



Extended Random Wave Model

measured superscar states modeled as superpositions of constructed superscar states and ergodic states

$$\Psi^{(\mathsf{F})}_{\tilde{f}}(\vec{r}) = c_{\max}\Psi^{(\mathsf{F})}_{m,n}(\vec{r}) + \sqrt{1-c^2_{\max}}\,\widetilde{\chi}_k(\vec{r})$$

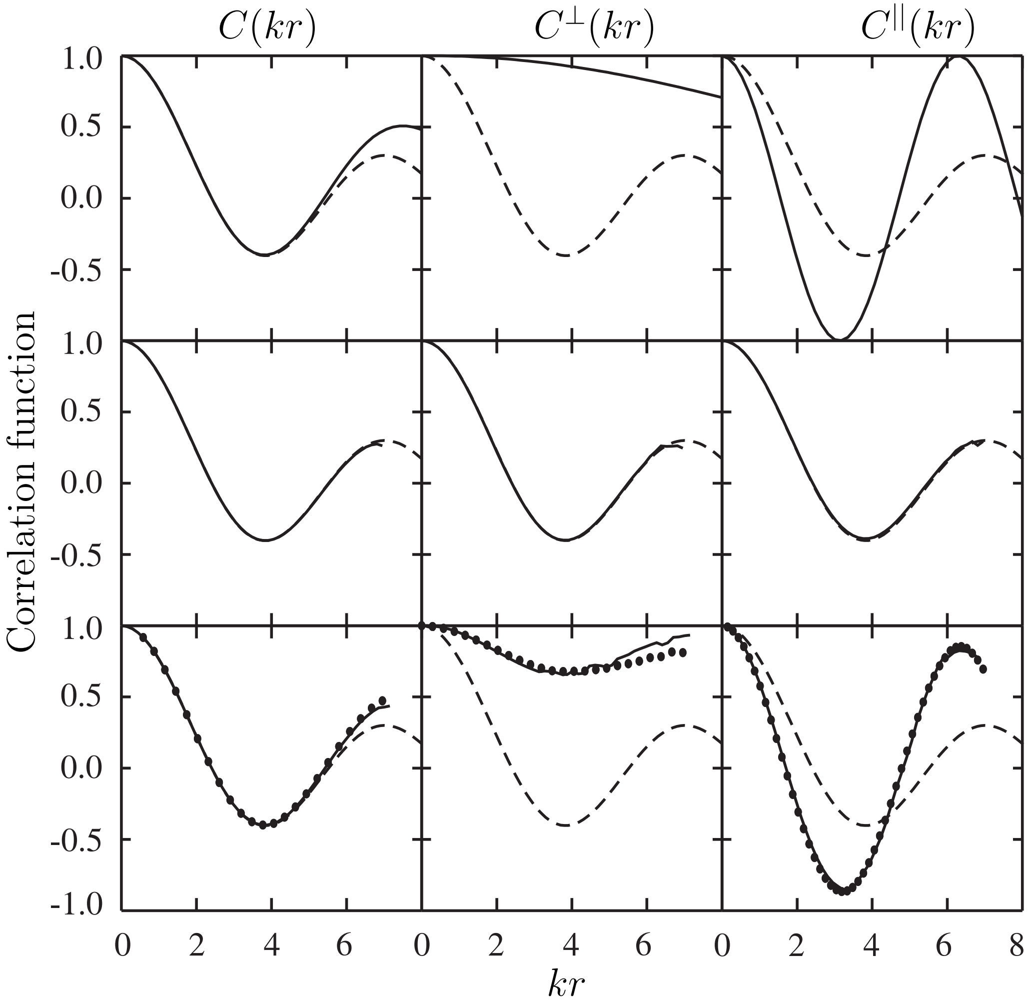
with maximum coupling coefficient $c_{\max}$

ergodic states modeled as “scarless” plane waves

$$\widetilde{\chi}_k(\vec{r}) = \frac{\chi_k(\vec{r}) - \langle\Psi^{(\mathsf{F})}_{m,n}|\chi_k\rangle\Psi^{(\mathsf{F})}_{m,n}(\vec{r})}{\sqrt{1-\langle\Psi^{(\mathsf{F})}_{m,n}|\chi_k\rangle^2}}$$

orthogonality with constructed superscars ensured

Analysis of all Correlators



constructed
V superscars

all measured
wave functions

measured
V superscars and fit

Results for Maximum Coefficients and Widths

F	$\langle c^2_{\max} \rangle$			$\Gamma^{\downarrow}$	
	Exp	RMT	Corr	Exp	RMT
BB	0.58 ± 0.05	0.58	0.81	0.9 ± 0.1	1.3
V	0.63 ± 0.05	0.68	0.69	0.8 ± 0.1	0.8
D	0.74 ± 0.03	0.72	0.69	0.9 ± 0.1	0.6
W	0.54 ± 0.03	0.51	0.49	1.0 ± 0.1	1.9

Summary and Conclusions

- doorway mechanism and spectral statistics
- scissors mode regular, pygmy dipole largely chaotic
- exploring limits of such an analysis in nuclei
- superscars provide a beautiful model for doorway mechanism
- used two new observables: distribution of maximum coefficient, spatial correlators in extended Berry model
- they yield consistent picture
- brief comment on preliminary analytical results