A VORTEX DIPOLE IN A TRAPPED 2D BOSE-EINSTEIN CONDENSATE

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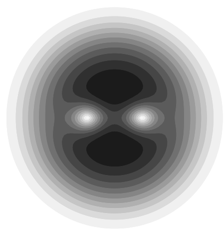
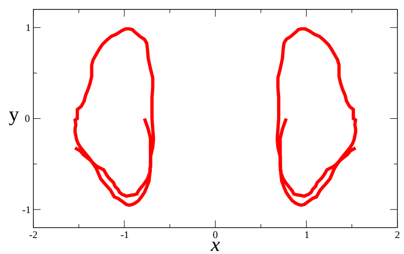
Stationary states.



Conservative dynamics – defect trajectories.



Many open questions

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Reference:

arxiv:0712.4217

Why vortex dipoles?

- Vortex-antivortex pairs appear in many 2D situations.

  Normal fluids and superfluids; during turbulent flow; flow over a sharp barrier, ...

- Vital for physics of Kosterlitz-Thouless transitions in 2D.

  Recently probed experimentally in ENS (Paris) experiments.

- Fascinating analogies to vortex rings in 3D, solitons in 1D.

  Some of our results have analogs in 3D vortex ring physics.

- Created recently experimentally in BEC's by sudden perturbation.

- Can in principle be created & studied by phase imprinting.

Overview

- Setup, approximations.
  Time-dependent Gross-Pitaevskii equation.
- Insights from a variational (approximate) calculation.
  Analytically tractable calculations; relatively simple results.
- Stationary vortex dipole solutions.
  Similar stationary soliton-like & vortex ring solutions.
- Dynamics: trajectories of the two defects.
  Characteristic trajectory shapes. Large and small g. Deviations.
- Open questions.
  (many...)

TIME-DEPENDENT GROSS-PITAEVSKII EQUATION.

$$i\frac{\partial\psi(t)}{\partial t} = -\frac{1}{2}\nabla^2\psi + V_{\text{tr}}(\mathbf{r})\psi + g|\psi|^2\psi \qquad \text{(trap units)}$$

- Isotropic (CIRCULAR) 2D trap →

$$V_{\text{tr}}(x,y) = \tfrac{1}{2}(x^2+y^2)$$

- $|\psi|^2 =$ areal density.
- g is an EFFECTIVE 2D interaction parameter.

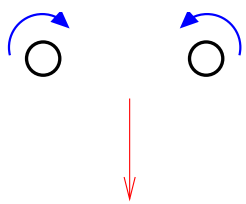
$$g \;\propto\; g_{3\text{D}} \times N \times \sqrt{\omega_z}$$

Neglected

- Dissipation effects.
- Temperature, quantum depletion & fluctuations.
- Vortex pair creation/annihilation.
- Axial dynamics.

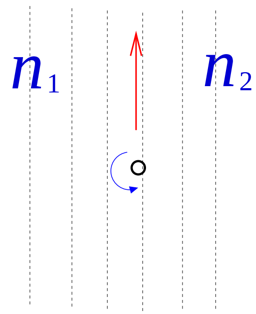
Two competing effects

Vortex dipole in uniform condensate is self-propelled.

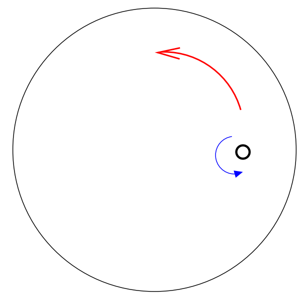


Fetter, Phys. Rev. 1965

Single vortex in non-uniform condensate is driven by inhomogeneity.



inhomogeneous BEC



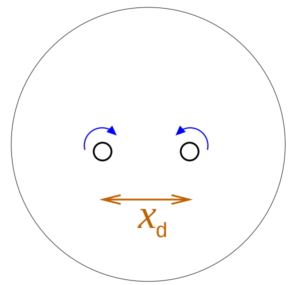
precession in trap

Rokhsar, PRL 1997

Vortex dipole in trap $\longrightarrow$

Small distance: mutually driven motion dominates.

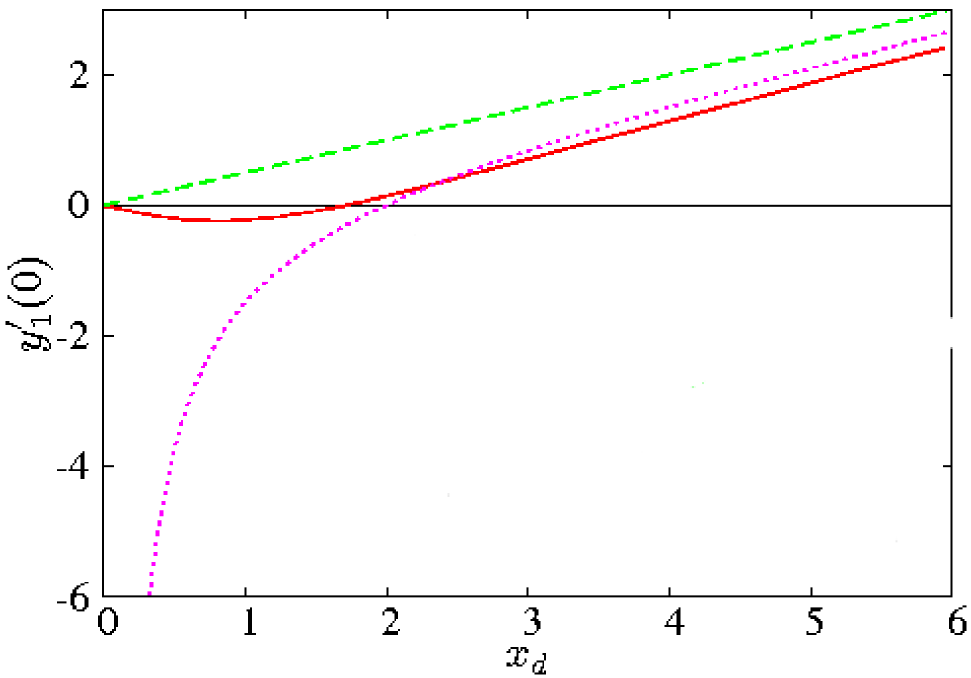
Large distance: inhomogeneity-driven motion dominates.



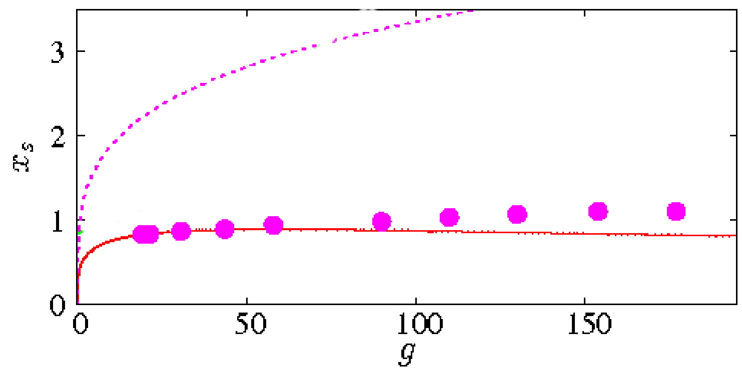
(Simple) Variational Calculation: Results (1)

$$\psi = [z - z_1(t)] \; [z^* - z_2^*(t)] \; f_{\mathrm{C}}(|z|^2) \qquad \begin{cases} z_1 = x_1 + iy_1 & \text{vortex} \\ z_2 = x_2 + iy_2 & \text{antivortex} \end{cases}$$

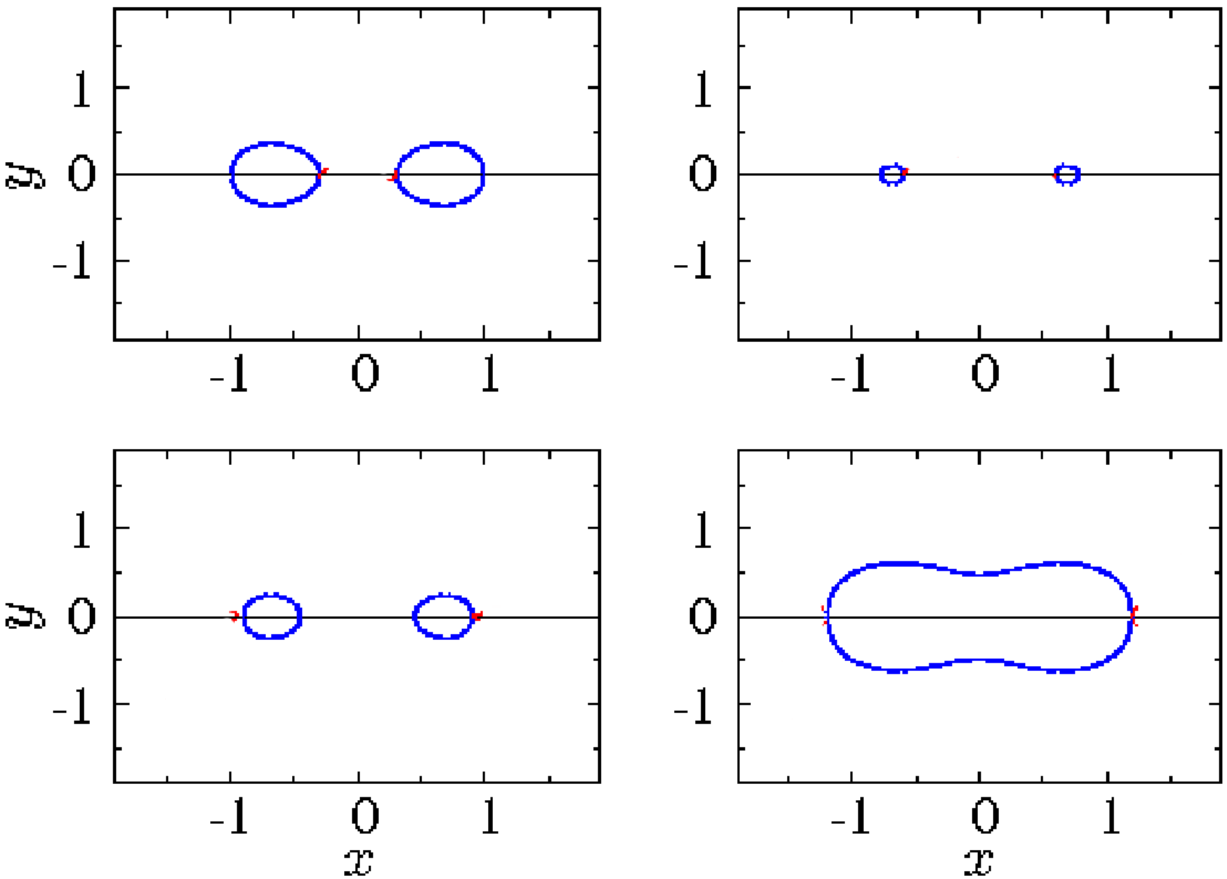
Good phase structure, unreliable (rigid) vortex size.



Stationary solution for
$x_1(0) = x_{\mathrm{s}}$, $x_{\mathrm{d}} = 2x_{\mathrm{s}}$



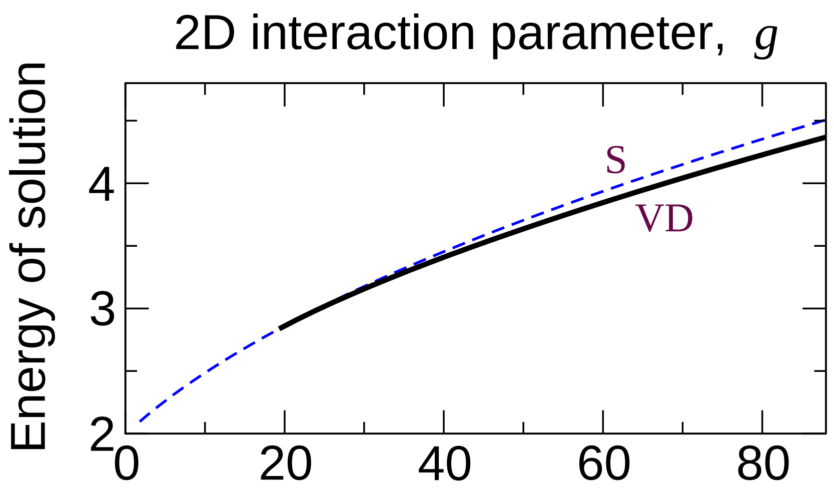
Insights from variational calculation (2) - dynamics

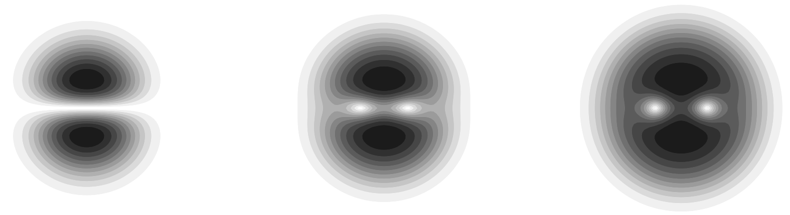


- Increasing initial distance $x_d = 2x_1(0)$.
- Each defect revolves around a stationary point.
- This trajectory type occurs in full GPE solutions.
  Not periodic, additional effects...
- Results qualitative only.
- Last trajectory is an artifact.

STATIONARY SOLUTIONS (1)

- Stationary 'soliton'-like solution at small g.
- Bifurcation at $g \approx 18$.
- For $g \gtrsim 18$, one 'soliton' and one vortex-dipole branch.





$g = 11$ $g = 25$ $g = 60$

Stationary solutions (2)

Non-interacting case ($g = 0$)

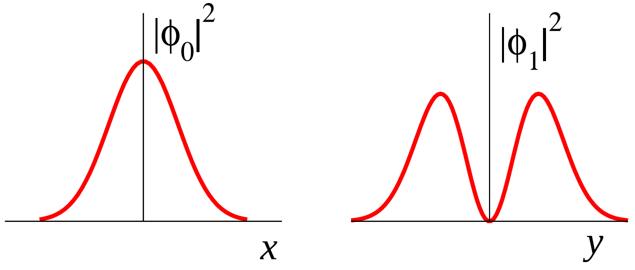
'Soliton'-like stationary solution $\rightarrow$

First excited state,

$$\psi(x,y) \;\propto\; \phi_0(x)\,\phi_1(y)$$

$$\phi_0(x) \sim e^{-x^2/2} \qquad \phi_1(y) \sim y e^{-y^2/2}$$

Stationary because energy eigenstate.



Similar bifurcations

In 3D condensate

Vortex RING instead of vortex dipole

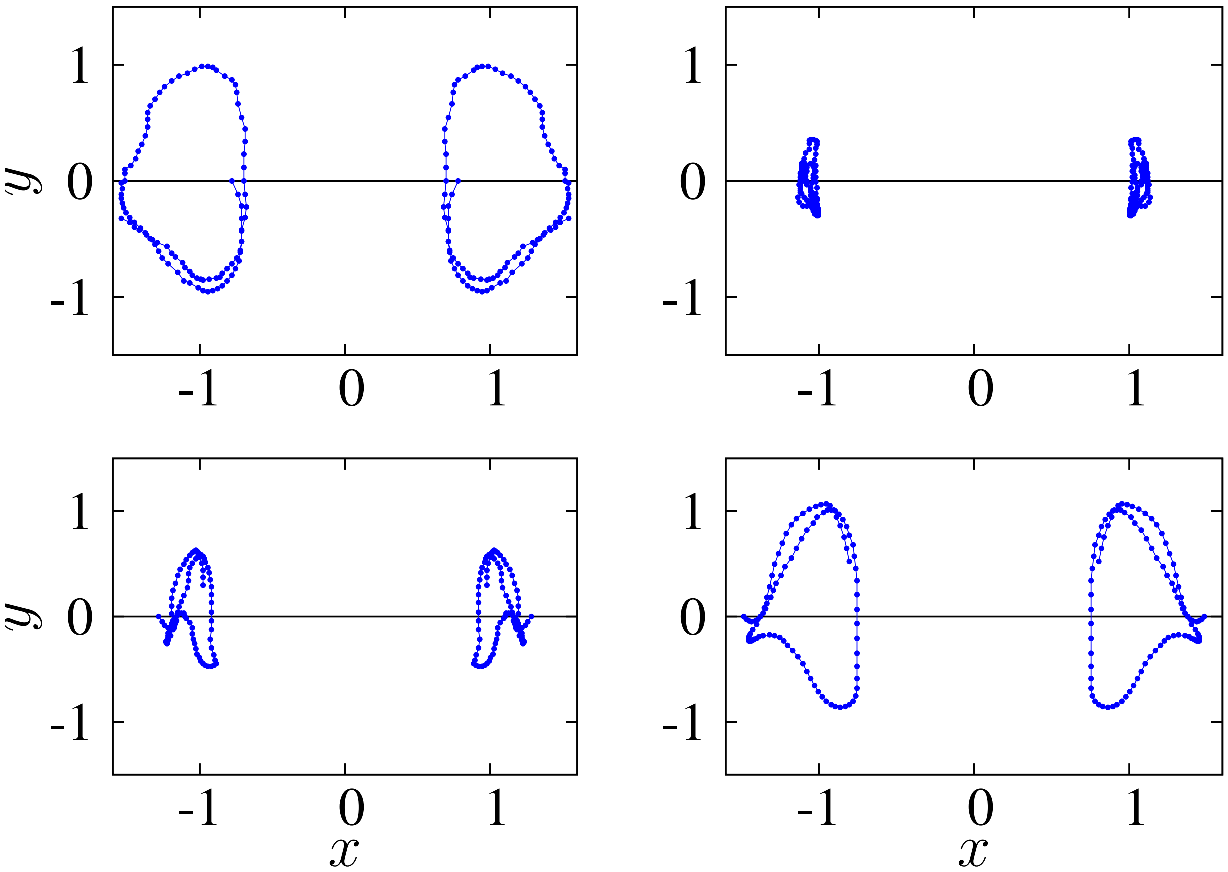
In elongated condensate

Term 'soliton' more appropriate.

Open issues

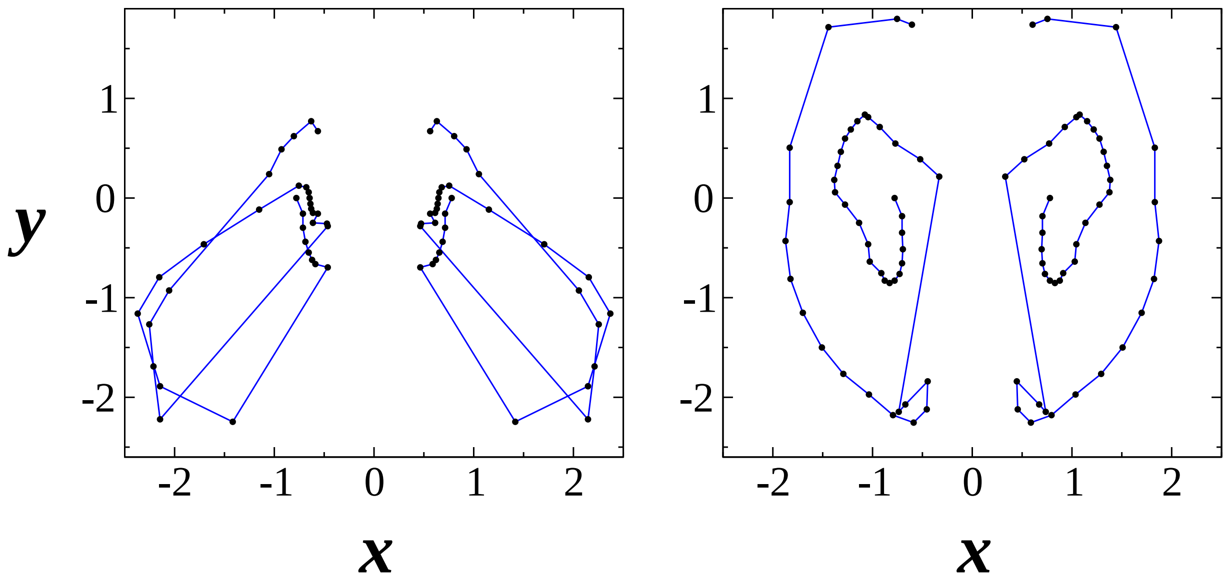
crossover of bifurcation between circular and elongated trap; between 2D and 3D

Dynamics (vortex trajectories); large g



- Simpler at large g.
- $g = 150$ shown here.
- Not periodic ('almost').
- Trajectories elongated in y direction.
  'Reflection' at edges clearer.
- Extra features.
  Curvatures at outer parts.
  Pointy feature at outer edge.
  Direction reversal for large initial x_{d}).

Dynamics (vortex trajectories); small g



$g = 10$ and $g = 50$. Many more unexplained features.

Significant intervals where it is difficult to identify original defect pair.

Creation of extra defect pairs, even without dissipation.

Additional physics affecting trajectories

Large g (tiny vortices) better understood.

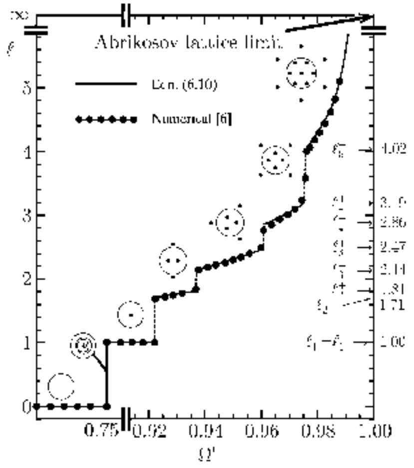
Smaller g $\rightarrow$ VORTEX POSITIONS alone are not sufficient description.
Additional dynamics possibilities!

- Vortex shape distortion dynamics.
- Extra pairs created easily.
- Tendency to morph into soliton-like objects.
  SOLITON $\equiv$ LINE OF VORTEX DIPOLES
- Annihilation into collective excitations better favored.
- Effects of condensate boundary.

Details are open issues...

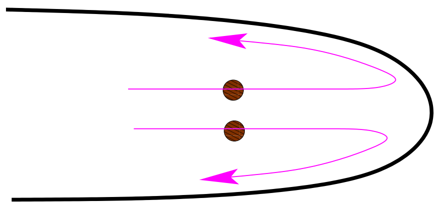
Analog in many-vortex dynamics (rotating trapped BEC) $\rightarrow$

VORTEX POSITIONS provide sufficient description of dynamics in fast-rotation limit, where vortices are 'tiny'.



Questions & open issues

- Don't understand all features of defect trajectories.
  Even at large g!!
- Do trajectories become periodic in $g \to \infty$ limit?
  Above some critical g value?
- Do trajectories lose features in $g \to \infty$ limit?
  i.e., become smoother?
- Details of reflection, *e.g.*, in elongated condensate.



- How does vortex shape dynamics couple to vortex position dynamics?
- Pair annihilation & creation without dissipation.
- How does existence of 'nearby' solitonic solution affect vortex dipole dynamics?
- Vortex dipoles not placed initially symmetrically in trap.
  Chaotic motion?
- Dissipation.

Variational formulation; wavefunctions.

Lagrangian :
$$L = \int dr \left[\frac{i}{2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right) + \tfrac{1}{2}\psi^* \nabla^2 \psi - V_{\mathrm{tr}}(\mathbf{r})\ |\psi|^2 - \tfrac{1}{2} g\ |\psi|^4 \right]$$

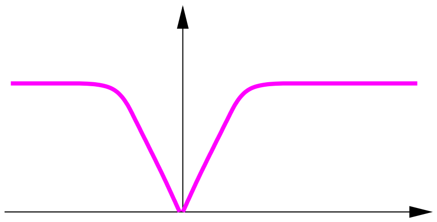
Trial w.f. :
$$\psi = A(t)\ g_{\mathrm{v}}(u_1)e^{i\phi_1}\ g_{\mathrm{v}}(u_2)e^{-i\phi_2}\ f_{\mathrm{c}}(|z|^2) \qquad \begin{cases} u_i = |z - z_i|/\xi \\ \phi_i = \tan^{-1}\left(\frac{y-y_i}{x-x_i}\right) \end{cases}$$

2D coordinates bundled into complex $z = x + iy$. Vortex at z_1, antivortex at z_2.

Euler-Lagrange equations for $z_i = x_i + iy_i$:
$$\frac{\partial L}{\partial x_1} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right)$$

$\longrightarrow$ equations of motion for x_1, y_1, x_2, y_2.

Vortex shape function $g_{\mathrm{v}}(u)$: ideally



We used $g_{\mathrm{v}}(u) = u \longrightarrow$

$$g_{\mathrm{v}}(u_1)\ e^{i\phi_1} = |z - z_1|e^{i\phi_1} = z - z_1$$

$$\psi = [z - z_1(t)]\ \left[z^* - z_2^*(t)\right]\ f_{\mathrm{c}}(|z|^2)$$