TOPOLOGY AND DYNAMICS IN FERROMAGNETIC MEDIA

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Collaborators:

TN Tomaras (1991)
WJ Zakrzewski (1995)
PN Spathis (1999)
S Komineas (1996-2008)

• Landau – Lifshitz equation

Magnetization: $\mathbf{M} = \mathbf{M}(\mathbf{r}, t), \qquad \mathbf{M}^2 = 1$

$$\frac{\partial \mathbf{M}}{\partial t} = \mathbf{M} \text{ x } (\Delta \mathbf{M} + q M_3 e_3 + \mathbf{H})$$

$$\nabla \text{ x } \mathbf{H} = 0 = \nabla \cdot (\mathbf{H} + 4\pi \, \mathbf{M})$$

In the limit of a very thin film, the effect of the magnetostatic field **H** is equivalent to an additive renormalization of the anisotropy constant q.

. Topological vorticity (2D)

local vorticity: $\gamma = (\frac{\partial \mathbf{M}}{\partial x} \times \frac{\partial \mathbf{M}}{\partial y}) \cdot \mathbf{M}$

Pontryagin index
or Skyrmion number: $N = \frac{1}{4\pi} \int \gamma \, dxdy$

. Conservation laws (2D)

Linear momentum (impulse):

$$P_x = -\int y\gamma \, dxdy, \qquad P_y = \int x\gamma \, dxdy$$

Angular momentum (impulse):

$$L = \frac{1}{2} \int \rho^2 \gamma \, dxdy, \qquad \rho^2 = x^2 + y^2.$$

• Magnetic vortices (easy plane)

Numerical solution with asymptotics:

$\rho \to \infty;$ $\quad M_z \to 0, \quad M_x + iM_y = e^{ik\phi}$

$$k = \pm 1 = \textbf{vortex number}$$

$\rho \to 0;$ $\quad M_z \to \lambda = \pm 1 = \textbf{polarity}$

Skyrmion number of a single vortex (k, λ):

$$N = -\frac{1}{2} k\lambda = \pm \frac{1}{2}$$

Skyrmion number of a pair (k_1, λ_1) and (k_2, λ_2):

$$N = -\frac{1}{2}(k_1\lambda_1 + k_2\lambda_2) = 0, \pm 1$$

and so on.

• Dynamics of magnetic vortices

- A single vortex is spontaneously pinned within the medium. It can move only when an external **field gradient** is applied or **other vortices** are present in its vicinity.

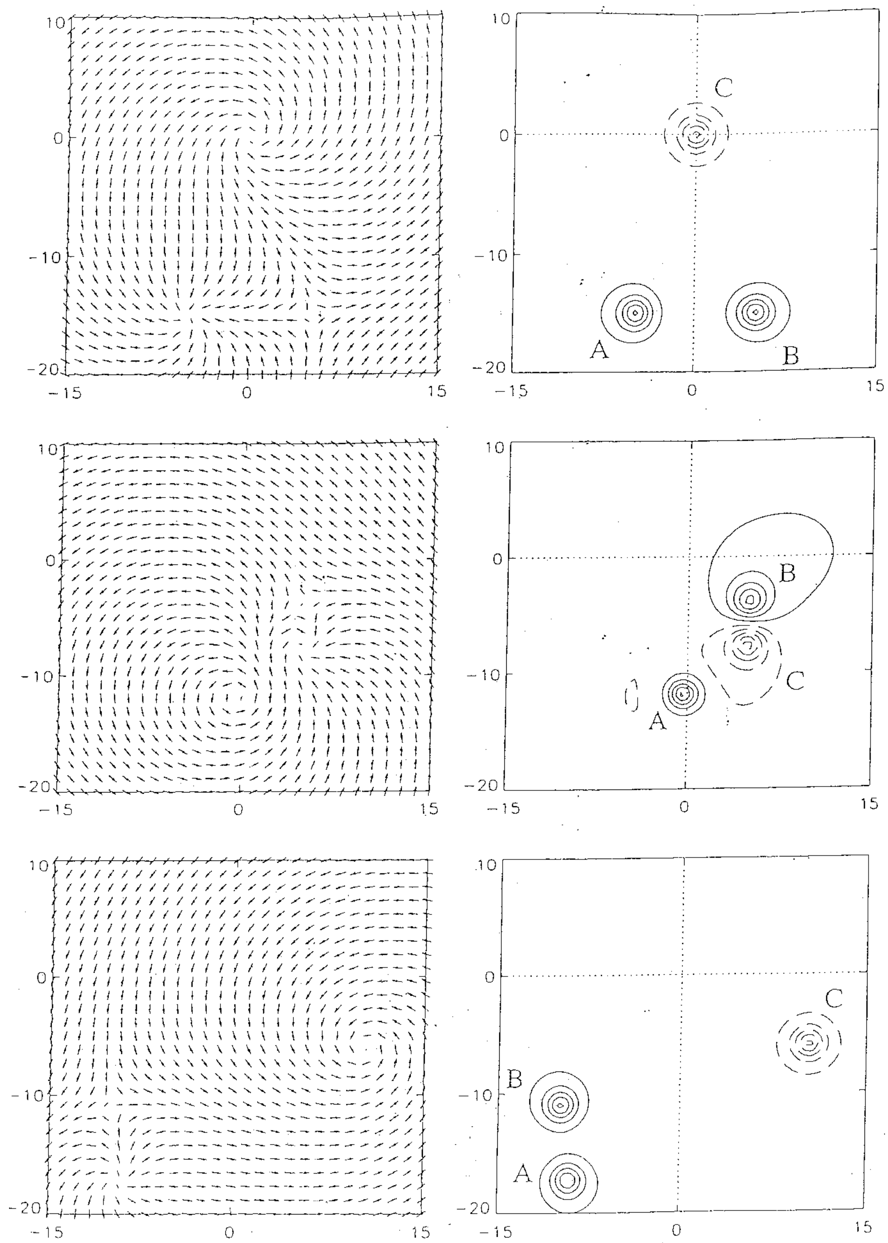
- A vortex tends to move in a direction **perpendicular** to the applied force (as in **Hall** electron motion, skew deflection of magnetic bubbles, etc)

- A vortex pair with zero skyrmion number (N=0) undergoes **Kelvin motion** (Spathis et al 1999)

- A vortex pair with nonzero skyrmion number (N= ±1) undergoes **rotating motion** around a fixed guiding center (Komineas 2007)

. Three-vortex collision

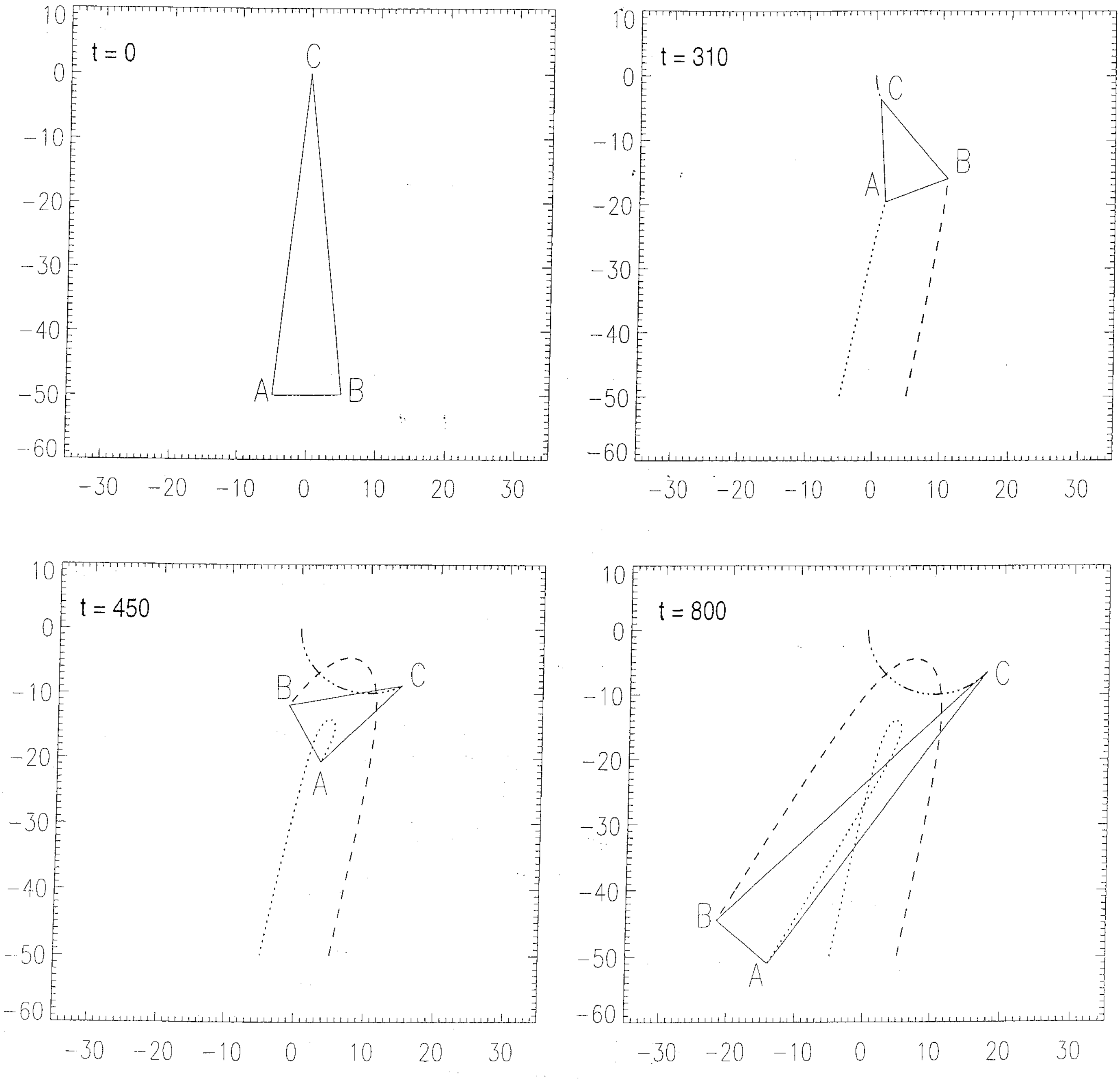
Vortex:	$A=(1,1)$	$B=(-1,1)$	$C=(1,-1)$
Skyrmion N:	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$



Transmutation of momentum into position

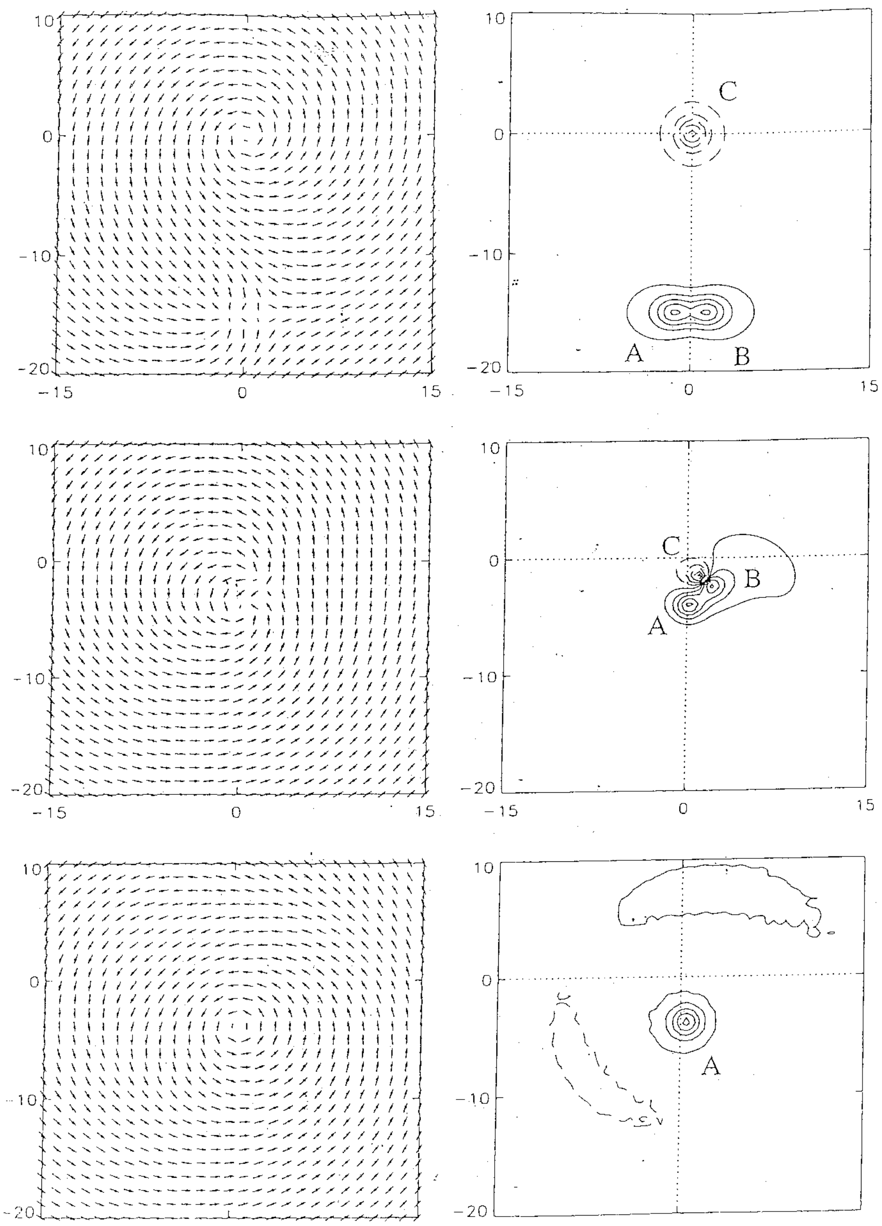
. Collective coordinates

$$2\pi\, k_i \lambda_i \left(\frac{d\mathbf{R}_i}{dt} \wedge \mathbf{z} \right) = 2\pi \sum_{j \neq i} k_i k_j \frac{\mathbf{R}_i - \mathbf{R}_j}{(\mathbf{R}_i - \mathbf{R}_j)^2}\,, \quad i = A, B, C$$



• Switching of vortex polarity

Vortex:	$A=(1,1)$	$B=(-1,1)$	$C=(1,-1)$
Skyrmion N:	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$



Skyrmion number N changes by one unit

. Three dimensions (3D)

topological vorticity: $\gamma_i = \frac{1}{2}\varepsilon_{ijk}(\partial_j \mathbf{M} \times \partial_k \mathbf{M})\cdot\mathbf{M}$

$$\nabla\cdot\gamma = 0$$

- Vortex lines are generically closed.
- **M** remains constant along a vortex line.
- **Linking number** is the same for any pair of vortex lines and is called the **Hopf index** H = 0, ±1, …

There exist 3D solitons with zero or nonzero Hopf index, in the form of vortex rings propagating with constant velocity.

Linear momentum: $\mathbf{P} = \frac{1}{2}\int(\mathbf{r} \times \gamma)dV$

Angular momentum: $\mathbf{L} = \frac{1}{3}\int\mathbf{r} \times (\mathbf{r} \times \gamma)dV$