Emergence of symmetry from random n-body interactions

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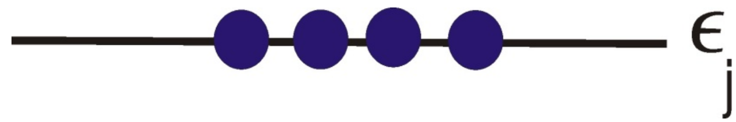
The simple model



- Single-j level
- Ω=2j+1 single-particle orbitals: m=-j, j-1, … j
- Number of nucleons N: $0 \le N \le \Omega$
- Number of many-body states: $\Omega!/((N!(\Omega-N)!)$
- Many-body states classified by rotational symmetry: (J,M)

Dynamics

- Rotational invariance and two-body interactions
  particle-particle pair operator $P_{L\,M}=(a\ a)_{L\,M}$
  particle-hole pair operator $M_{K\,\kappa}=(a\ a^{\dagger})_{K\,\kappa}$
- Hamiltonian $$H = \sum_L V_L \sum_M P^{\dagger}_{LM} P_{LM}$$
- Dynamics is fully determined by j+1/2 parameters V_L

Ground state statistics[1]

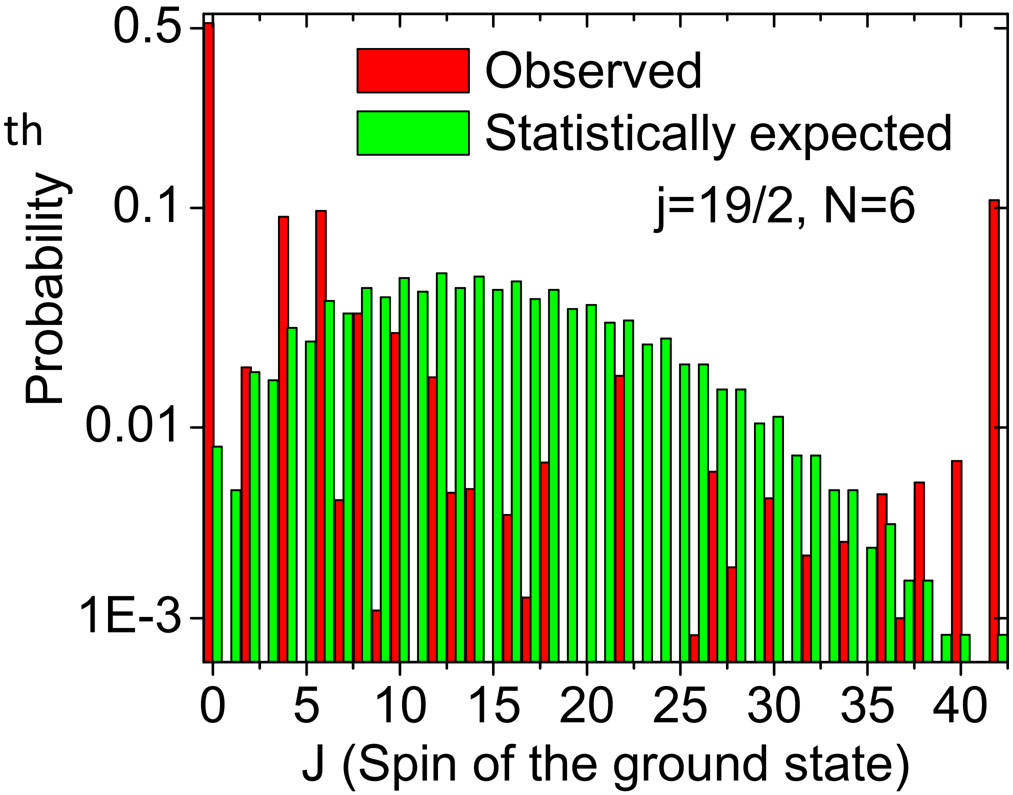
Dynamics versus symmetry

Take V_L at random

(Gaussian distribution centered at 0, width 1)

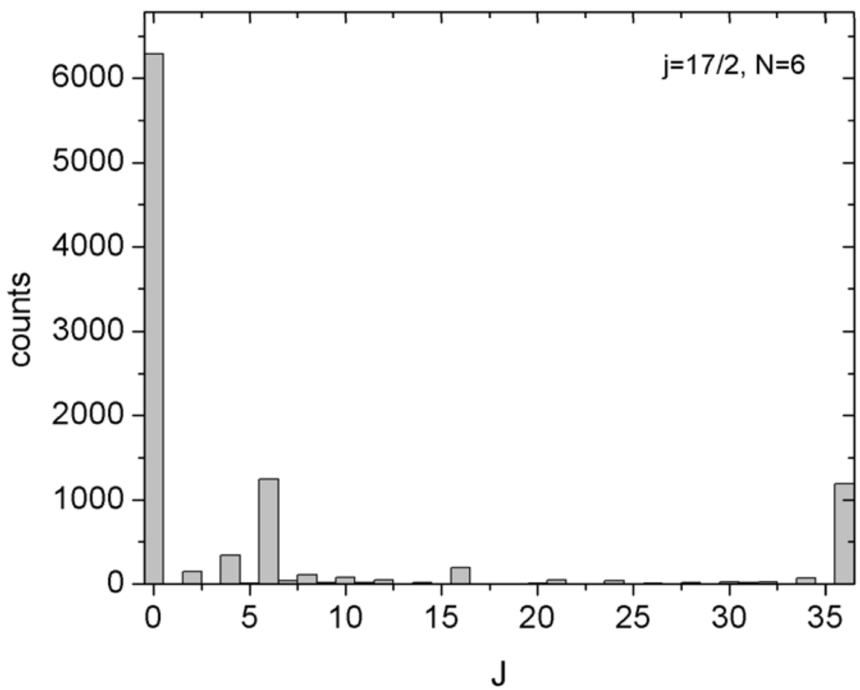
What is the probability for the ground state to have spin J?

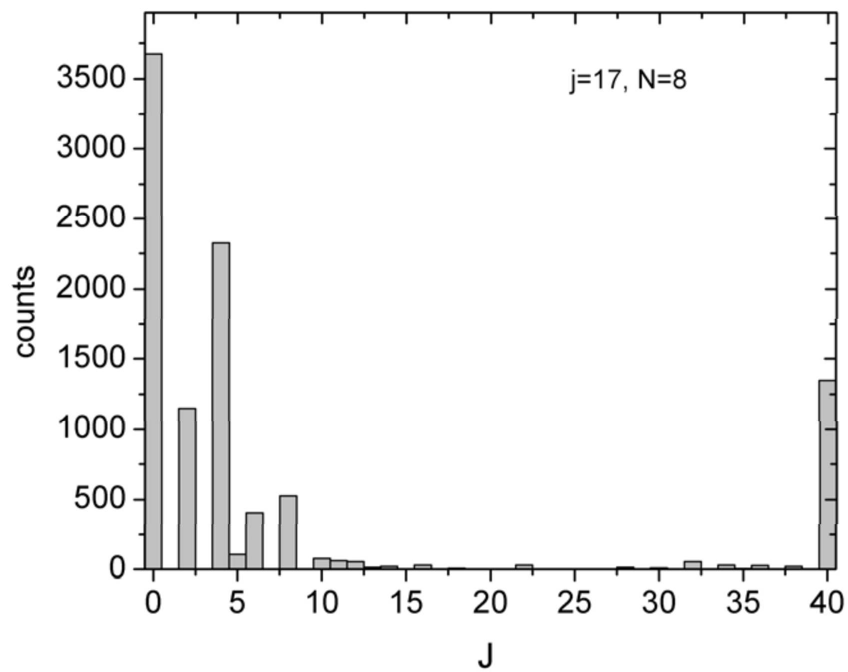
- J=0 is enhanced
- $J=J_{max}$ is enhanced



[1] C. W. Johnson, G. F. Bertsch, and D. J. Dean, Phys. Rev. Lett. **80**, 2749 (1998).

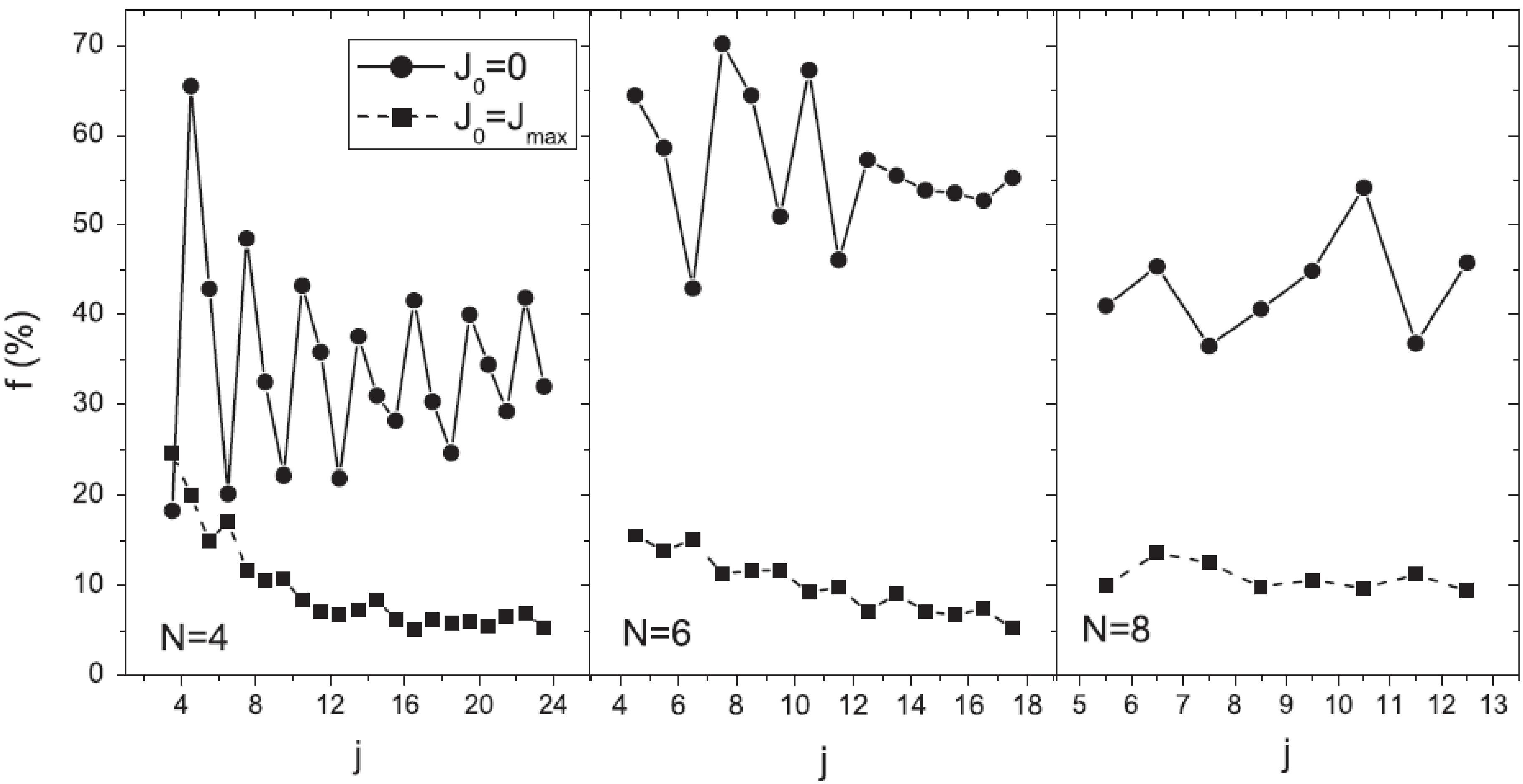
Ground State Spin Statistics





Statistics is based on 10000 random realizations.

Robust preponderance of zero and maximum spin g.s.

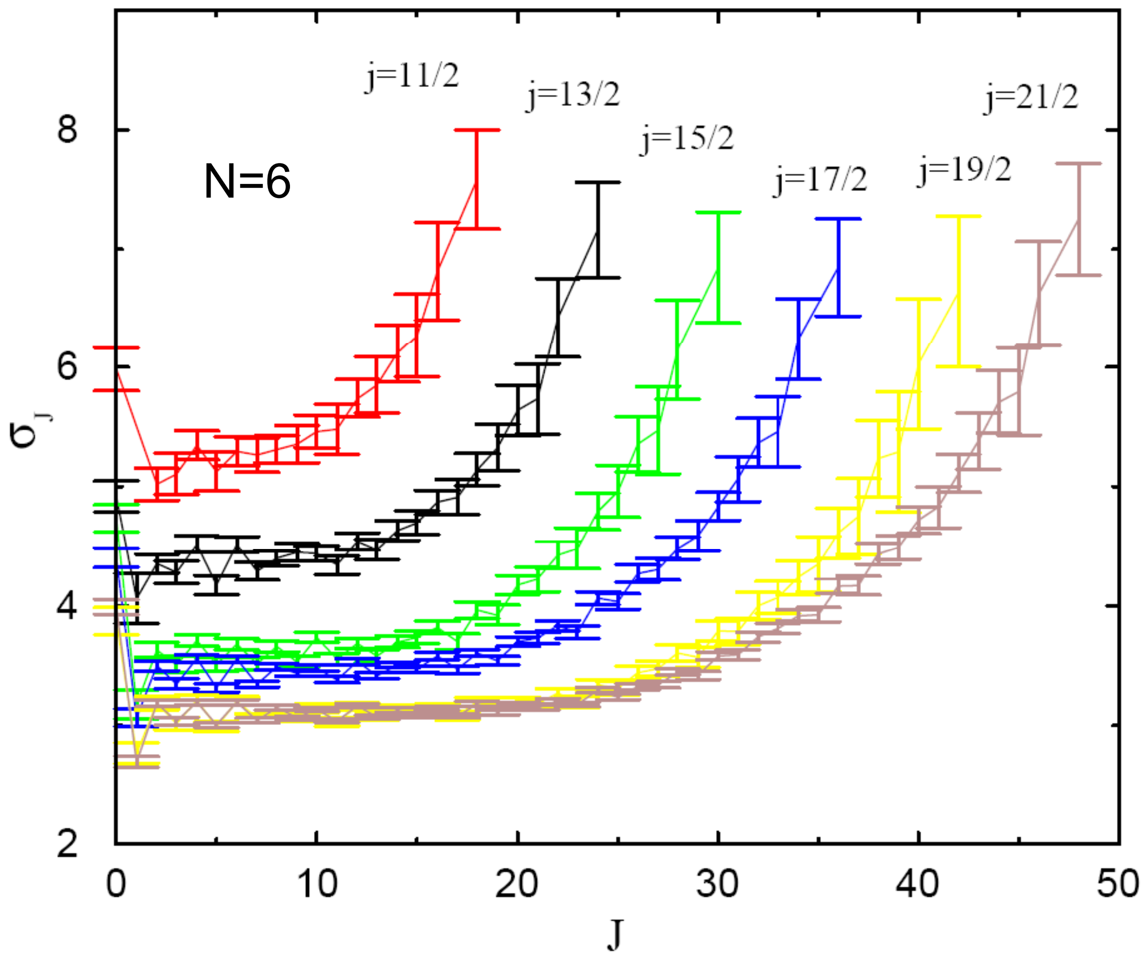


The role of statistical widths

$$\bar{E}_J = \frac{1}{d_J}\,\mathrm{Tr}_J\, H \equiv \langle H \rangle_J$$

$$\sigma_J^2 = \langle (H - \langle H \rangle_J)^2 \rangle_J$$

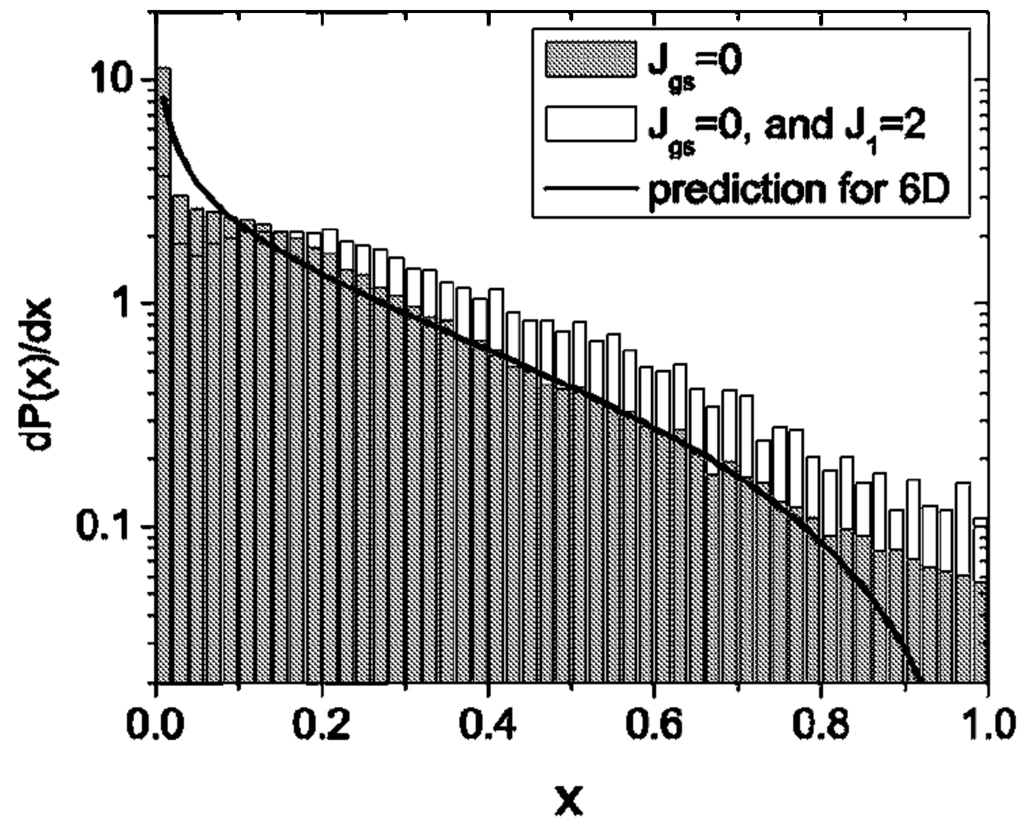
Statistics of widths does not explain the systematics. Correlations may be important



Random Interactions and pairing

Spin	0	48
Fraction of states	0.61	0.05
Gaussian	61.9	11.5
Forced pairing V_0=-1	80.5	9.2
No pairing V_0=0	64.1	11.7
Anti pairing V_0=+1	55.2	12.4

Statistics with pairing and without pairing for j=21/2 N=6



System: N=6 on j=15/2.

$$x = |\langle 0|\mathrm{paired}\rangle|^2$$

Solid line: statistical expectation in 6D

Dashed histogram: J_0=0

Unshaded histogram: J_0=0 and J_1=2

Tackling the TBRE and symmetry puzzle

- C.W. Johnson, G.F. Bertsch, D.J. Dean. Phys Rev Lett. 80 (1998) 2749.
- R. Bijker, A. Frank. Phys Rev Lett. 84 (2000) 420-422.
- D. Mulhall, A. Volya, V. Zelevinsky. Phys Rev Lett. 85 (2000) 4016-4019.
- T. Papenbrock, H.A. Weidenmuller. Phys Rev Lett. 93 (2004) 132503.

- Y.M. Zhao, A. Arima, N. Yoshinaga. Physics Reports. 400 (2004) 1-66.
- T. Papenbrock, H.A. Weidenmuller. Reviews of Modern Physics. 79 (2007) 997-1013.
- V. Zelevinsky, A. Volya. Physics Reports. 391 (2004) 311-352.

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n-body Random Ensemble (n-BRE)

n-body Hamiltonian

$$H^{(n)} = \sum_{\alpha\beta} \sum_{L} V_L^{(n)}(\alpha\beta) \sum_{M=-L}^{L} T_{LM}^{(n)\dagger}(\alpha)\, T_{LM}^{(n)}(\beta)$$

Operator

$$T_{LM}^{(n)\dagger}(\alpha) = \sum_{12\ldots n} C_{12\ldots n}^{LM}(\alpha)\, a_1^\dagger a_2^\dagger \ldots a_n^\dagger$$

n-body operator is eigenstate $T_{LM}^{(n)\dagger}(\alpha)|0\rangle$

of the reference 2-body Hamiltonian $H_0^{(2)}$

Random Gaussian ensemble of interactions

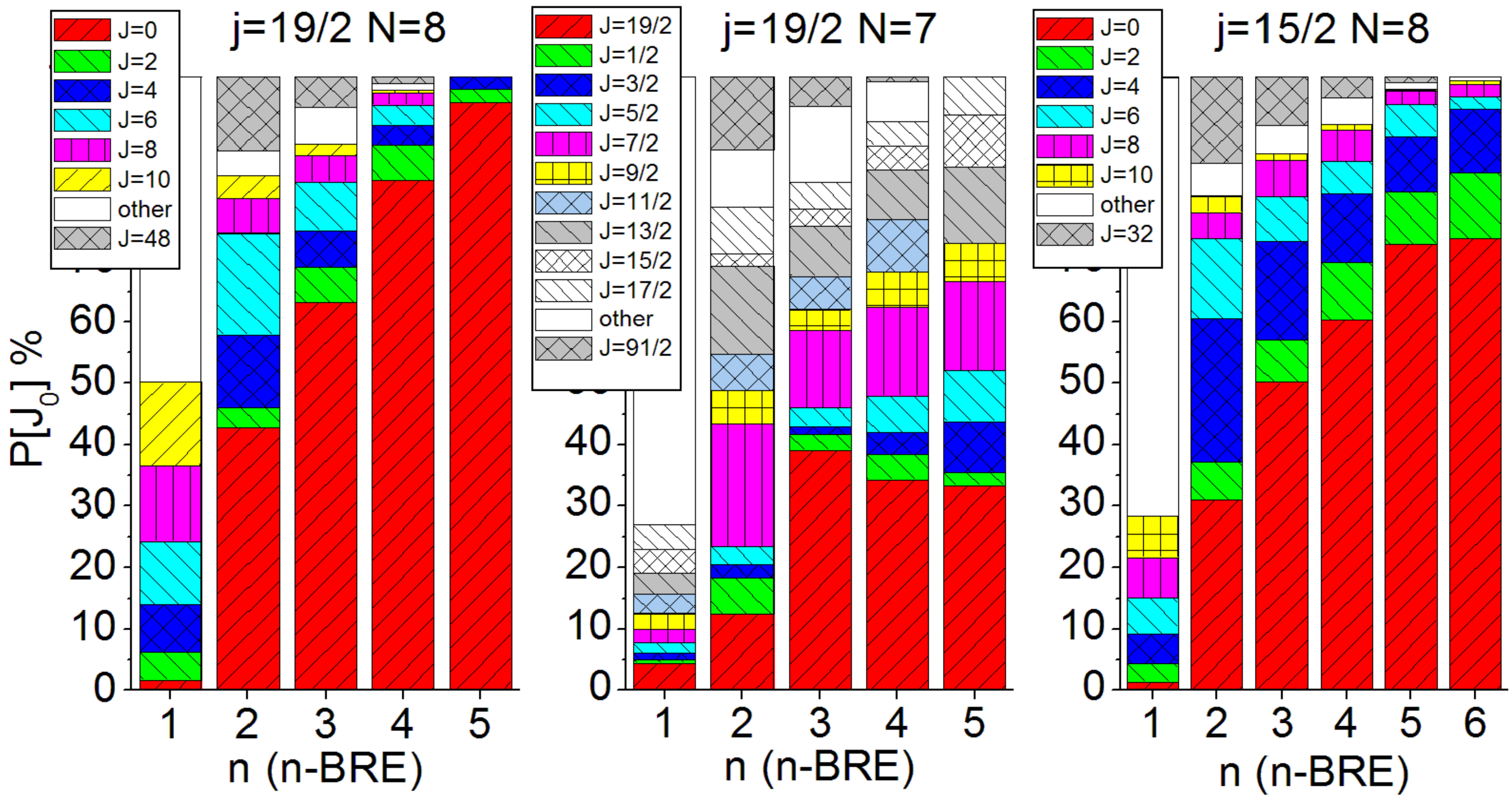
$$\langle V_L^{(n)}(\alpha,\beta)\rangle = 0 \qquad V_L^{(n)}(\alpha,\beta) = V_L^{(n)}(\beta,\alpha)$$

$$\langle V_L^{(n)}(\alpha,\beta)\, V_{L'}^{(n)}(\alpha',\beta')\rangle = \delta_{LL'}\delta_{\alpha\alpha'}\delta_{\beta\beta'}(1+\delta_{\alpha\beta})/2$$

The ensemble does not depend on the choice of reference Hamiltonian

For n = N the ensemble is GOE in each symmetry class

Statistics of g.s. spins



Conserved quantum numbers, coherent property effective interaction

2-BRE $\langle H^{(2)} \rangle_J = \tilde{V}_1 J^2$

In the particle-hole representation:
monopole term (particle number); moment of inertia (angular momentum)

$$\tilde{V}_1 = \sum_L \frac{3(2L+1)}{j(j+1)(2j+1)} \left\{ \begin{array}{ccc} j & j & 1 \\ j & j & L \end{array} \right\} V_L^{(2)}$$

- Predicts equal probability for $P[0_0] = P[(J_{max})_0] = 1/2$
- Particle number independent
- Correlation across different N

Statistical treatment

Minimize energy as a function of density under geometric constraints

$$E(\{n_m\}) = \frac{1}{2} \sum_{mm'} V_{mm'} \langle n_m n_{m'} \rangle$$

$$\sum_m n_m = N, \quad \sum_m m n_m = M$$

$$E(N, M) = \frac{1}{2}[N\mu(N, M) + M\gamma(N, M)]$$

$$\mu = \frac{2N}{(2j+1)^2} \sum_L (2L+1) V_L,$$

$$\gamma = \frac{3M}{(2j+1)^2 j(j+1)} \sum_L (2L+1)[L(L+1) - 2j(j+1)] V_L$$

Statistical treatment [1]

Constants of motion and corresponding terms

•Particle number N   monopole (mass) term

$$\tilde{V}_0 = [\Omega(\Omega - 1)]^{-1} \sum_L (2L + 1) V_L$$

•Angular momentum J ❄ moment of inertia

$$\tilde{V}_1 = (2j^4\Omega^2)^{-1} \sum_L (2L + 1)(L^2 - 2j^2) V_L$$

Average energy

$$\langle H \rangle_{NJ} = \tilde{V}_0 N(N - 1) + \tilde{V}_1 J(J + 1)$$

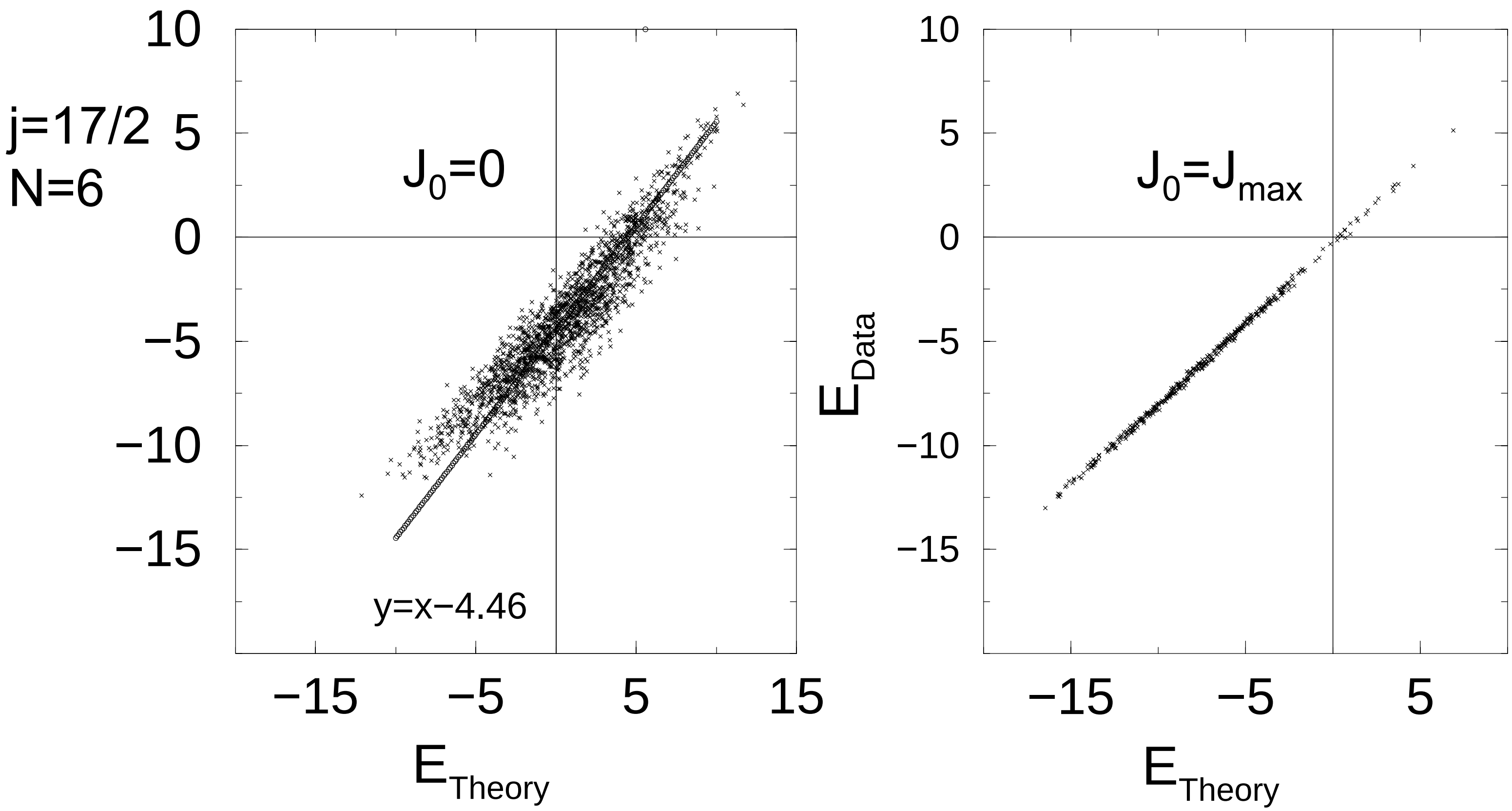
Statistical prediction

$\tilde{V}_1 > 0$ Ground state has J_0=0

$\tilde{V}_1 < 0$ Ground state has J_0=J_{max} (maximum possible J)

[1] D. Mulhall, A. Volya, and V. Zelevinsky, Phys. Rev. Lett. **85**, 4016 (2000);

Exact energy and statistical prediction



Measuring Correlations

Important property of joint probability in independent events

$$P\left[J(N_1)_0, J(N_2)_0, \ldots\right] = \prod_i P\left[J(N_i)_0\right]$$

In the 2-BRE we have correlations

$$j = 19/2 \text{ with } N = 5, 6 \ldots 10 \; P\left[J_{max}(5)_0 \ldots J_{max}(10)_0\right] = 6.6\%$$
$$P\left[J_{max}(5)_0\right] P\left[J_{max}(6)_0\right] \ldots P\left[J_{max}(10)_0\right] = 2.1 \cdot 10^{-4}\%$$

Measure correlation with Total Weighted Correlation (TWC)

$$\mathcal{C}\left[J, J', J'', \ldots\right] = \frac{\log\left(P[J]\,P[J']\,P[J'']\ldots\right)}{\log\left(P\left[J, J', J'', \ldots\right]\right)} - 1$$

- TWC is zero for uncorrelated events
- TWC in X-1 for X totally correlated events
- Measures Informational content: gives the number of redundant sets of data

Correlations across different particle number

	2-BRE		3-BRE		4-BRE	
N	J_{min}	J_{max}	J_{min}	J_{max}	J_{min}	J_{max}
5	16.0	10.1	36.3	2.9	7.7	0.2
6	52.3	10.5	66.4	3.1	83.0	0.0
7	12.4	11.8	39.1	4.8	33.0	0.5
8	42.7	12.1	63.2	5.0	84.3	1.1
9	9.5	12.3	31.1	6.5	33.7	2.3
10	31.2	11.5	48.6	7.1	65.5	2.6
$\mathcal{C}$	1.883	3.806	1.560	3.266	0.435	0.000

Summary of g.s. statistics for minimum and maximum spin, and correlations across different mass numbers N for a single j=19/2 valence space with 2,3, and 4-body random interactions.

Coherent effective interaction components

2-BRE $\langle H^{(2)} \rangle_J = \tilde{V}_1 J^2$

- Predicts equal probability for $P[0_0] = P[(J_{max})_0] = 1/2$
- Particle number independent
- Correlation across different N

3-BRE $\langle H^{(3)} \rangle_J = (\tilde{V}' N + \tilde{V}) J^2$

- Enhancement of $P[(J_{max})_0]$ is lower
- $P[(J_{max})_0]$ increases with N
- Correlation across different N are lower

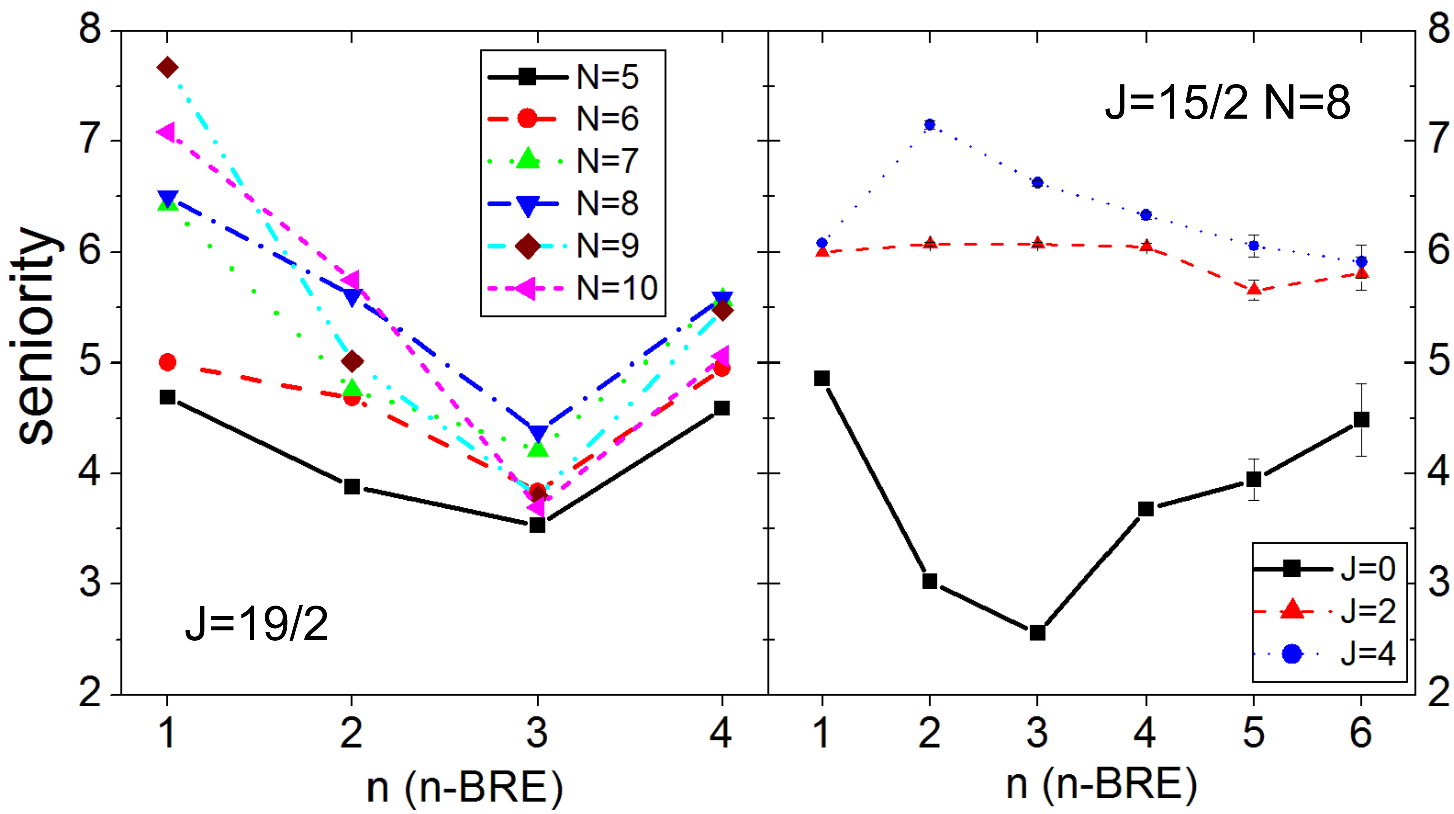
4-BRE $\langle H^{(4)} \rangle_J = (V_1'' \tilde{N}^2 + \tilde{V}_1' N + \tilde{V}_1) J^2 + \tilde{V}_2 J^4$

- The $P[(J_{max})_0]$ is nearly zero.
- $P[0_0]$ increases as it is always a local minimum
- Correlation across different N disappear.

Enhancement of paring

Seniority (average number of unpaired particles)

$$\langle N,\alpha|T^{(2)\dagger}_{00}T^{(2)}_{00}|N,\alpha\rangle = \frac{(N-s)(2j+3-N-s)}{2(2j+1)}$$



Searching for rotational sequences

	2-BRE		3-BRE		4-BRE	
	P	E_1/E_2	P	E_1/E_2	P	E_1/E_2
6	3.7(2)	0.55	4.2(2)*	0.69	4.4(7)*	0.69
8	4.2(2)*	0.59	5.5(2)	0.67	7.4(11)	0.75
10	2.1(1)*	0.72	5.2(2)	0.69	7.3(10)	0.62

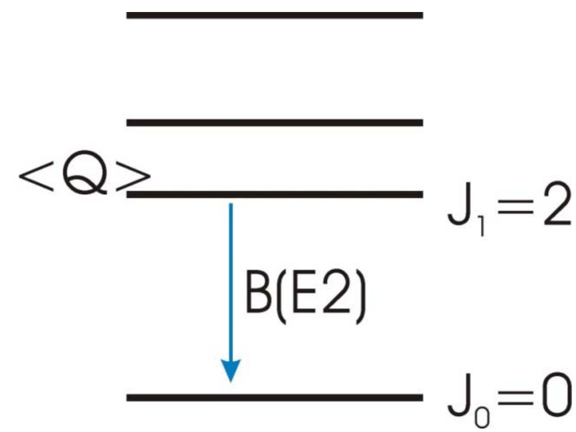
Probability of finding the three lowest states as a sequence 0,2,4, $P[0_0, 2_1, 4_2]$, labeled as P, expressed in percent, and the ratio of excitation energies between 2_1 and 4_2 states. In all cases the sequence 0,2,4 in the most likely g.s. sequence except for those marked with *.

For exact rotor $E_1/E_2 = 0.3$

In all cases $\mathcal{C}\left[0_0, 2_1, 4_2\right] \approx 0.4$

Rotational properties in systems with random interactions

Select realizations where J_0=0 and J_1=2



Quadrupole moment J=2 state

$$Q = \langle J, J \mid M_{20} \mid J, J \rangle$$

Transition strength

$$B(\mathrm{E2}) = \sum_{M_f, \kappa} \left| \langle J, m_f \mid M_{2\kappa} \mid J, m_f \rangle \right|^2$$

Prediction for rotor
(Alaga intensity rules)

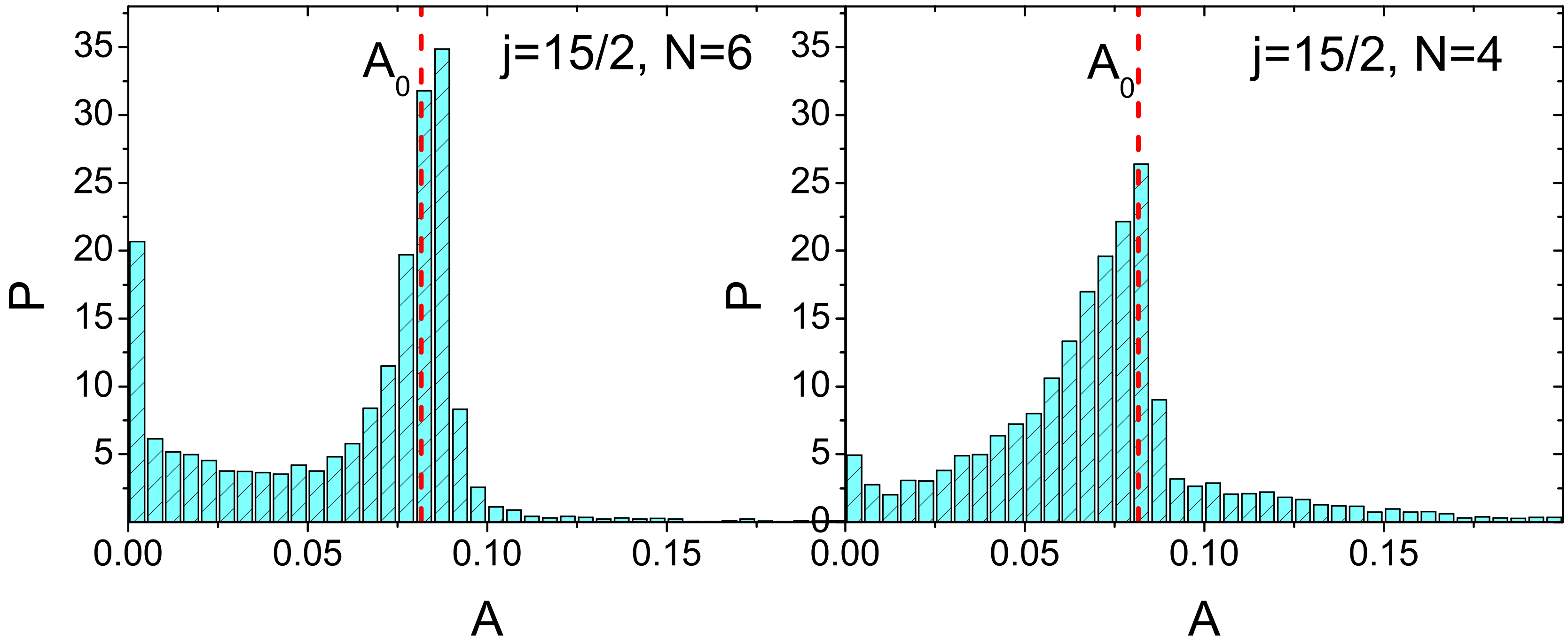
$$A = Q^2 / B(\mathrm{E2}) = 4/49$$

Distribution of Alaga ratio

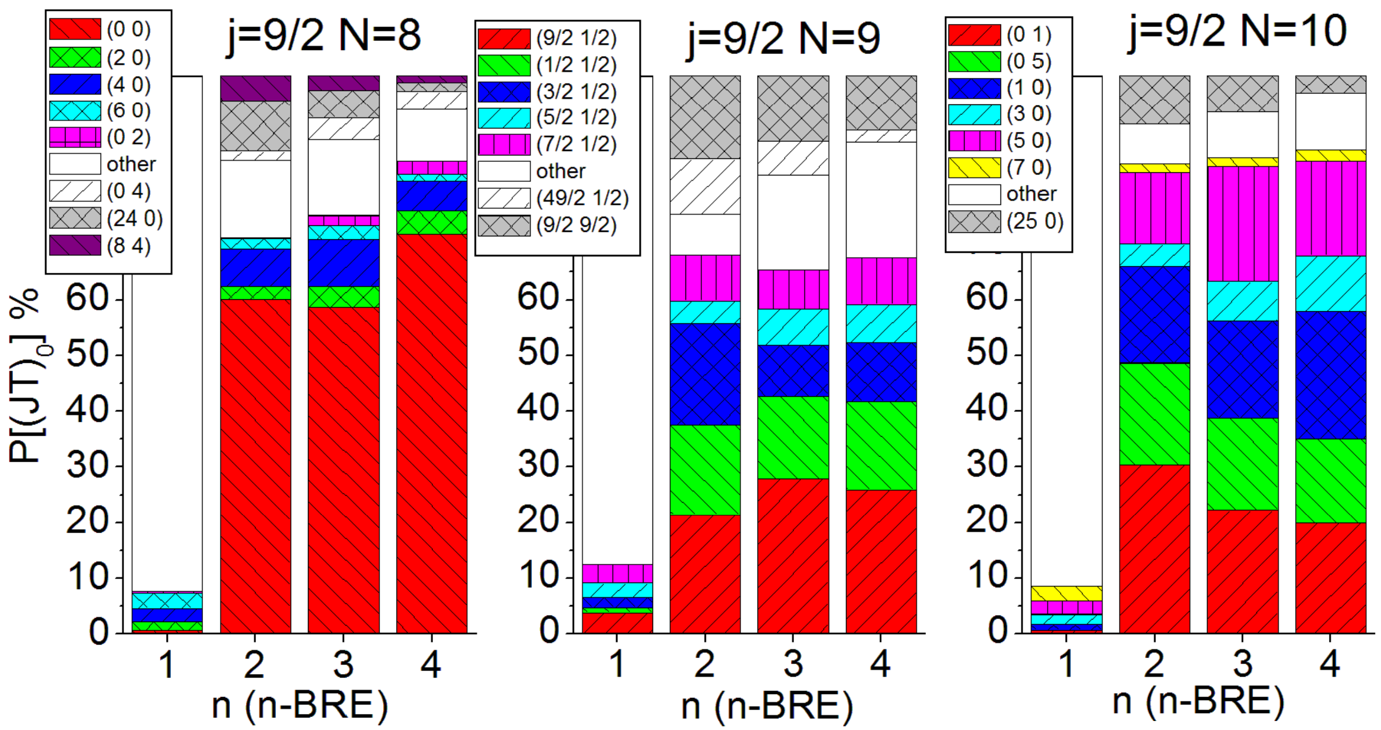
$$A = \frac{\langle Q \rangle_{J=2}^2}{B(\mathrm{E2}; 0 \to 2)}$$

N=6, j=15/2
9.2% cases J_0=0 and J_1=2

N=4, j=15/2
In ~6.7% cases J_0=0 and J_1=2



One-level system with isospin



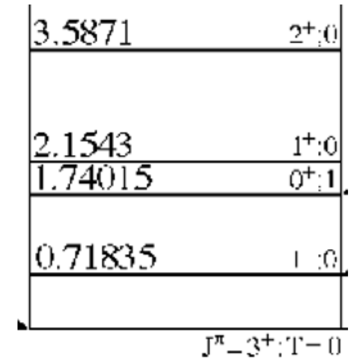
- Even-Even system (J T)=(0,0) is enhanced, grows with n, J+T even
- Odd-Odd (0,0) does not appear as g.s. J+T odd enhanced
- J_{max} and T_{max} appear in 2-BRE and less expected for higher n.

Is order of states a statistical evidence for 3-body forces?

p-shell ^{10}B

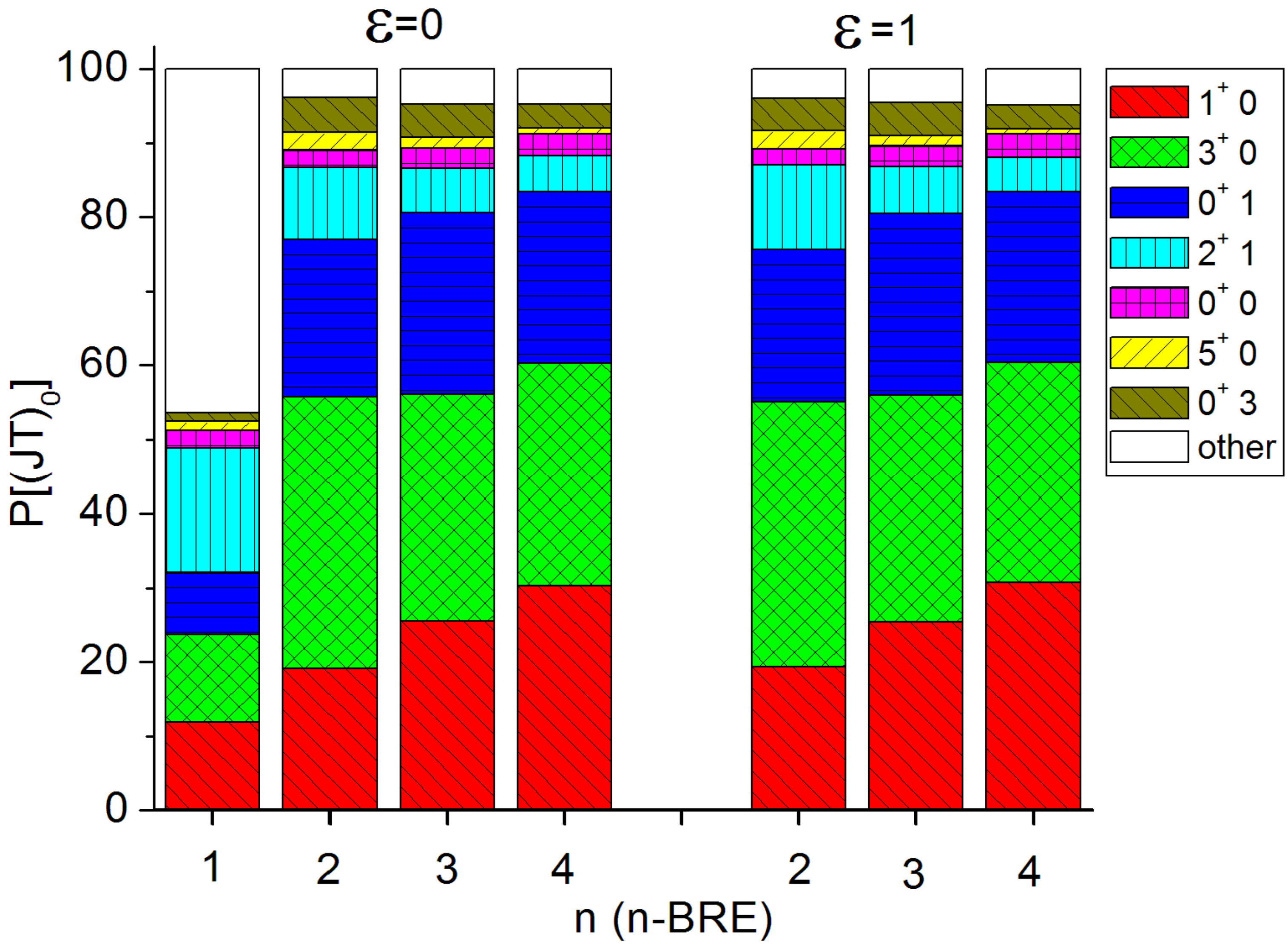
J	T	1-body	2-body	3-body	4-body	1-body	2-body	3-body	4-body
		degenerate $\epsilon = 0$				non degenerate $\epsilon = 1$			
1	0	11.9	19.1(4)	25.6(5)	30.3(6)	25	19.3(4)	25.4(5)	30.8(6)
3	0	11.9	36.6(6)	30.5(6)	29.9(6)	25	35.8(6)	30.5(6)	29.6(5)
0	1	8.3	21.3(5)	24.5(5)	23.2(5)	25	20.5(5)	24.6(5)	23.0(5)
2	1	16.7	9.7(3)	5.9(2)	4.8(2)	25	11.4(3)	6.3(3)	4.6(2)
0	0	2.4	2.3(2)	2.8(2)	3.0(2)		2.2(2)	2.8(2)	3.2(2)
5	0	1.2	2.4(2)	1.5(1)	0.8(1)		2.4(2)	1.4(1)	0.7(1)
0	3	1.2	4.8(2)	4.5(2)	3.3(2)		4.5(2)	4.5(2)	3.2(2)

Experiment



^{10}B

Many-body forces and structure of ^{10}B



Conclusions

- n-Body ensemble and preponderance of symmetry
- Conserved quantities in effective interactions
- Coherent components: pairing, rotations, vibrations

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Thanks: V. Abramkina, D. Mulahll, and V. Zelevinsky.
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Further reading:

Alexander Volya, *Emergence of symmetry from random n-body interactions*,
http://lanl.arxiv.org/abs/0712.3754